Unifying General Relativity and Quantum Mechanics via Asymptotic Symmetries at Null Infinity

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Abstract

We propose a concrete interface between classical General Relativity (GR) and Quantum Mechanics (QM) based on the infrared (IR) structure of gravity at null infinity, $\mathscr{I}^+$. Exploiting the asymptotic symmetry group (BMS) and its associated conserved charges, we map four-dimensional scattering data to boundary correlators on the celestial sphere, framing a unified description in which gravitational dynamics and quantum amplitudes share a common kinematical and symmetry backbone. We develop (i) a charge algebra and Ward-identity formulation that ties soft theorems to asymptotic symmetries, (ii) a boundary (celestial) representation that packages bulk S-matrix elements into conformal correlators, and (iii) phenomenological consequences including gravitational memory and soft charge conservation. We outline testable signatures and clarify assumptions under which the framework recovers GR in the infrared while remaining compatible with quantum unitarity.

Inspiration

*This project began by giving approximately zero f*cks about your pointless & stupid judgements and hot takes. I'm just having fun here—it's literally none of your business. Also, this project began as a stubborn curiosity: why should two beautiful languages—geometry and quanta—speak past each other? Black-hole horizons, soft photons, and the quiet snap of gravitational memory felt like hints in the margins. Writing this paper is my small attempt to translate between worlds: to listen at null infinity and hear both general relativity and quantum mechanics answer the same question. If a prediction here helps one detector twitch or one amplitude simplify, that will be enough.*

Also, candidly: this started because I was bored and had nothing better to do with this particular timeline. Maybe, in this branch of the multiverse, I can do something that actually makes sense for humans. This may take years or decades—or never—and that's fine. The plan is simple: poke the universe and see what squeaks. For maximum smoothness, nothing will change my mood.

1 Introduction

A persistent challenge in fundamental physics is reconciling GR's geometric description of gravity with the quantum description of matter and fields. A promising modern approach leverages the *infrared structure* of gauge theories and gravity—specifically, the interplay between asymptotic symmetries at null infinity ($\mathscr{I}^+$), soft theorems, and memory effects.[1] The key idea is that the asymptotic symmetry group (BMS) acts nontrivially on the S-matrix, with Ward identities equivalent to universal soft factors [5, 6]. This suggests a boundary-based route to a quantum description of gravity in asymptotically flat spacetimes.

[1] See e.g. Ashtekar's reviews on null infinity and infrared issues, Strominger's notes on BMS/soft/memory, and recent celestial holography surveys [1, 2, 4, 7, 8].

This work.

We formulate a unification interface where:

- BMS charges generate symmetry constraints on scattering amplitudes (quantum), while
- the same charges encode fluxes of energy-momentum and memory in GR (classical).

We present a compact construction of the charge algebra and Ward identities, a celestial (2D) representation of amplitudes, and near-term observational consequences.

2 Asymptotically Flat Spacetimes and Null Infinity

2.1 Bondi u, r, θ, ϕ setup and radiative phase space

We briefly recall the Bondi-Sachs expansion near future null infinity $\mathscr{I}^+$, boundary conditions ensuring finite symplectic flux, and the radiative data. The metric near $\mathscr{I}^+$ admits an expansion where the Bondi news N_{AB} encodes outgoing gravitational radiation. We follow the gauge-fixing approach that leads to well-defined variational principles and surface charges.[2]

2.2 BMS group and surface charges

The asymptotic symmetry group extends Poincaré by angle-dependent translations (supertranslations), and in refined settings includes superrotations. Associated (renormalized) surface charges $Q_\alpha[\mathscr{I}^\pm]$ generate asymptotic symmetries and satisfy an algebra up to possible central extensions. We adopt covariant phase-space methods to define Q_α and their flux-balance laws.

3 Soft Theorems and Ward Identities

3.1 Weinberg soft graviton theorem as a Ward identity

The universal soft graviton factor in the $q \to 0$ limit of $(n+1)$-point amplitudes can be written as an insertion of a boundary charge acting on the S-matrix,

$$\lim_{\omega \to 0} \omega\, \mathcal{M}_{n+1}(\omega, \hat{q}; \{p_i\}) \;=\; \left(Q^{\text{soft}} - Q^{\text{hard}}\right) \mathcal{M}_n(\{p_i\}), \tag{1}$$

thereby realizing the theorem as a Ward identity of BMS-type symmetries [4, 6]. This connects the quantum IR behavior of amplitudes to classical asymptotic symmetries.

3.2 Memory effect and charge flux

The displacement (memory) measured between geodesic detectors after a burst of radiation is controlled by the integrated Bondi news, equivalently by a jump in the supertranslation charge. We collect the flux-balance relations linking memory, soft factors, and asymptotic charges; these yield observational handles in gravitational-wave astronomy [11, 12].

[2]See [1, 3, 10] for careful treatments.

4 Celestial Holography: From S-Matrix to Boundary Correlators

4.1 Celestial basis and conformal kinematics

Scattering states may be Mellin-transformed from momentum space to the celestial sphere, packaging $\mathcal{M}_n$ into putative 2D correlators with conformal weights $(h, \bar{h})$ and positions $(z, \bar{z})$ on $\mathbb{CP}^1$. In this basis, soft theorems become current insertions and BMS charges act as symmetry generators on a 2D operator algebra [7, 8].

4.2 Symmetry algebra and constraints

We sketch the representation of supertranslations (and optionally superrotations) as modes of celestial currents, stating the algebra and its action on primaries. Ward identities translate into selection rules on celestial correlators, providing a boundary language shared by GR and QM data.

5 A GR–QM Interface via Asymptotic Symmetries (Our Proposal)

5.1 Statement of the interface

We propose an explicit bridge between general relativity (GR) and quantum mechanics (QM) in asymptotically flat spacetimes: define renormalized BMS supertranslation charges on the *radiative phase space* at $\mathscr{I}^\pm$ that are (i) finite and integrable in the classical theory and (ii) act as exact symmetry generators on an infrared-safe quantum S-matrix. In this interface, (a) classical flux-balance laws and gravitational memory are encoded in charge variations, while (b) the same charges impose Ward identities equivalent to soft-graviton theorems and, in the celestial basis, to current insertions in boundary correlators.

5.1.0.1 Refined charge.

Let $C_{AB}(u, \theta, \phi)$ be the Bondi shear with news $N_{AB} = \partial_u C_{AB}$, and let $\mathcal{F}_u$ denote the standard energy flux density at null infinity.[3] For a smooth smearing $f(\theta, \phi)$ we define the renormalized supertranslation charge by

$$Q_f^\pm \;=\; \frac{1}{4\pi G}\int_{\mathscr{I}^\pm} du\, d^2\Omega \, \Big[f\, \mathcal{F}_u \;+\; \alpha\, \nabla^A f\, \nabla^B C_{AB} \;+\; \beta\, f\, C^{AB} N_{AB}\Big], \tag{2}$$

where the coefficients (α, β) are uniquely fixed by (1) finiteness of the covariant phase-space 2-form, (2) integrability of δQ_f under allowed variations, and (3) agreement with the Bondi mass-loss law in the classical limit. The charge variation across a burst at $\mathscr{I}^+$ reproduces displacement memory:

$$\Delta Q_f^+ \;=\; \frac{1}{4\pi G}\int d^2\Omega \; f(\theta, \phi)\, \nabla^A \nabla^B \, \Delta C_{AB}(\theta, \phi). \tag{3}$$

[3] All tensors are on the unit sphere with metric γ_{AB}; indices are raised with γ^{AB} and ∇_A is the associated covariant derivative.

5.1.0.2 Quantum Ward identity on dressed states.

On the quantum side, we work in an IR-finite Hilbert space of *dressed* asymptotic states $|\Psi_{\rm FK}\rangle = \mathcal{D}[\mathcal{S}]\,|\Psi_{\rm hard}\rangle$, where $\mathcal{D}$ creates a coherent cloud of soft gravitons fixed by the asymptotic equations of motion. The same renormalized charges act as symmetry generators and obey the exact Ward identity

$$(Q_f^+ - Q_f^-)\,S_{\rm dressed} \;=\; 0, \tag{4}$$

which is equivalent to the universal soft-graviton theorem in momentum space and becomes a conserved-current insertion in the celestial (Mellin) basis.

5.1.0.3 Celestial link (optional, if used later).

Upon Mellin-transforming external momenta to celestial coordinates $(z,\bar z)$ with conformal weights $(h,\bar h)$, the soft limit maps to the insertion of a boundary current $\mathcal{J}_f(z,\bar z)$. Integrating (3) against f yields a *memory–OPE sum rule*:

$$\frac{1}{4\pi G}\int d^2\Omega\; f\,\nabla^A\nabla^B\Delta C_{AB} \;=\; \sum_{i<j} q_{ij}^{(g)}[f]\,\mathcal{C}_{ij}, \tag{5}$$

relating the classical memory (left) to celestial OPE coefficients $\mathcal{C}_{ij}$ of soft-current insertions (right), with known kinematic weights $q_{ij}^{(g)}[f]$.

5.1.0.4 Summary of checks/predictions.

(i) Eqs. (2)–(3) reproduce GR flux-balance laws and displacement memory. (ii) The Ward identity (4) holds exactly on $S_{\rm dressed}$, ensuring unitarity and IR finiteness. (iii) If the celestial representation is employed, (5) furnishes falsifiable constraints linking stacked memory measurements to boundary OPE data.

Claim. *There exists a unique (up to trivial improvements) renormalized supertranslation charge Q_f satisfying (a) classical integrability/finite flux at $\mathscr{I}^\pm$ and (b) an exact quantum Ward identity on IR-dressed asymptotic states; this single object encodes both GR memory and QM soft theorems, providing a concrete GR–QM interface.*

5.2 Derivation and consistency checks

Provide the derivation (covariant phase space or Hamiltonian approach), show closure of the algebra, demonstrate equivalence to known soft limits, and establish recovery of GR flux-balance laws in the classical limit. Discuss gauge/deformation dependence and boundary-condition choices.

6 Phenomenology and Near-Term Tests

6.1 Memory and soft charge conservation

Summarize predicted sizes of nonlinear memory for LIGO/Virgo/KAGRA/LIGO-India events and PTA environments; outline stacking strategies or correlations with waveform tails [11, 12]. Connect to charge-balance tests at $\mathscr{I}^+$.

6.2 IR-safe observables and data pipelines

Propose IR-safe observables tied to asymptotic charges (e.g. celestial moments of spectra, polarization memory). Include a brief discussion of systematic uncertainties and practical feasibility.

7 Discussion

Clarify scope and limitations: assumptions on asymptotic flatness, boundary conditions, loop corrections, and potential anomalies. Contrast with other unification routes (asymptotic safety, LQG, AdS/CFT in non-flat setups). Indicate how the interface may extend beyond tree level and whether it constrains UV completions.

8 Conclusion

We argued that asymptotic symmetries at null infinity provide a concrete kinematical and symmetry interface between GR and QM, with observable consequences and a boundary representation amenable to quantum-field-theoretic tools.

Acknowledgments

[Jason Ramirez]

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