

4 Systems of equations

4.1 Tues., Mar. 19:

We've worked with many types of algebraic expressions. Now we can start putting them together, or thinking about where several expressions are simultaneously true. Let's work a few to warm up:

$$\begin{cases} 8 = 2x + 3y \\ -2 = x \end{cases}$$

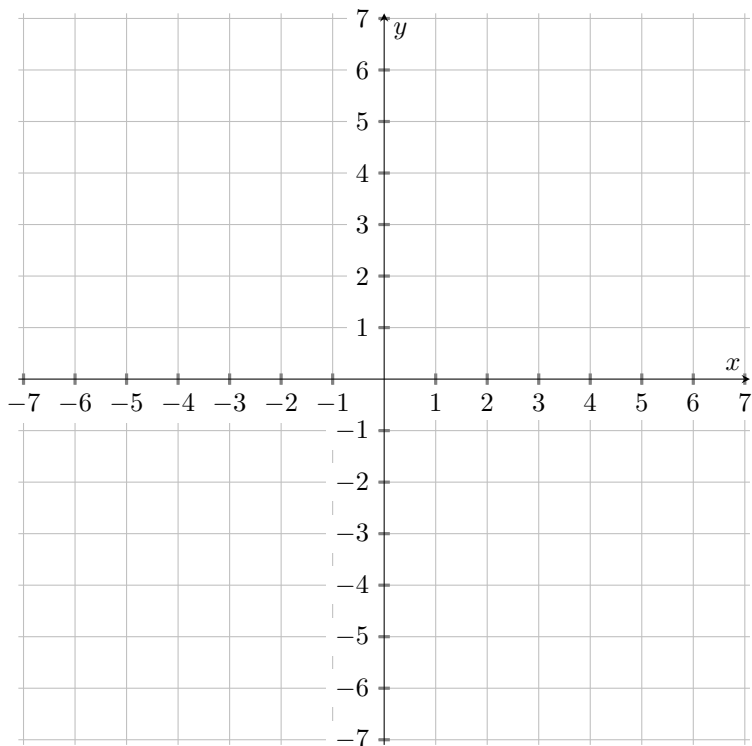
$$\begin{cases} -6x + \frac{1}{2}y = 4 \\ y = 4 \end{cases}$$

$$\begin{cases} 5x - y = 17 \\ x = y + 1 \end{cases}$$

We can solve these with substitution. But will that continue to work as the equations get more complicated? Let's consider another system of equations and solve it graphically.

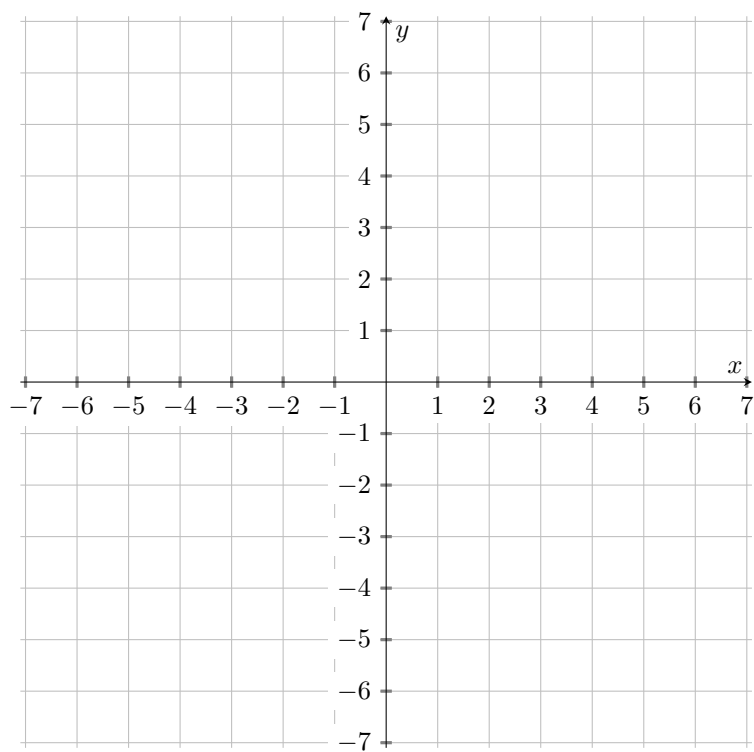
Exercise 9.

$$\begin{cases} y = -x^2 + 2x + 8 \\ y = 3x + 2 \end{cases}$$



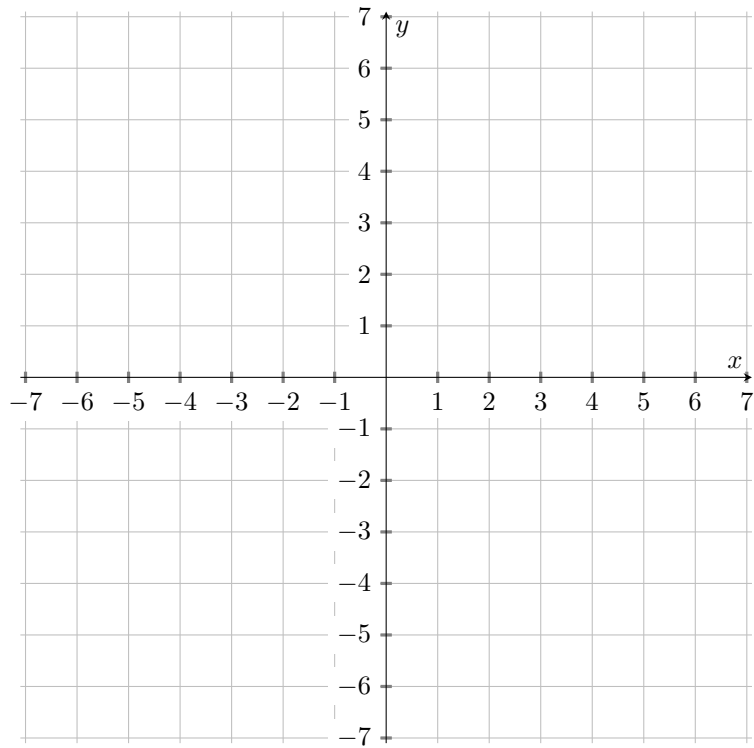
Exercise 10.

$$\begin{cases} y = x^2 - 3x - 4 \\ y = x - 8 \end{cases}$$



Exercise 11.

$$\begin{cases} y = 2x^2 + 4x + 3 \\ y = 4x - 1 \end{cases}$$



4.2 Thurs., Mar. 21:

We now consider a more challenging case, where there may be three equations in three unknowns!

Exercise 12. *Let's try to solve the equation below with substitution:*

$$\begin{cases} x + y + z = -2 \\ 2x - 3y + 2z = -14 \\ 4x + 3y - z = 5 \end{cases}$$

Exercise 13. *Let's try to solve the equation below with substitution:*

$$\begin{cases} 2x - y - 2z = 3 \\ 3x + y - 2z = 11 \\ -2x - y + z = -8 \end{cases}$$

Exercise 14. *Let's try to solve the equation below with elimination:*

$$\begin{cases} x + y + z = -2 \\ 2x - 3y + 2z = -14 \\ 4x + 3y - z = 5 \end{cases}$$

Exercise 15. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 2x - 3y = 7 \\ y + z = -5 \\ x + 2y + 4z = -17 \end{cases}$$

Exercise 16. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 5x + y + 3z = 9 \\ -x - 2y - z = -16 \\ 2x + 4y + 2z = -30 \end{cases}$$

Exercise 17. *Let's try to solve the equation below with elimination:*

$$\begin{cases} x - 4y + 3z = -7 \\ 2x + 3y - 5z = 19 \\ 4x + y - z = 17 \end{cases}$$

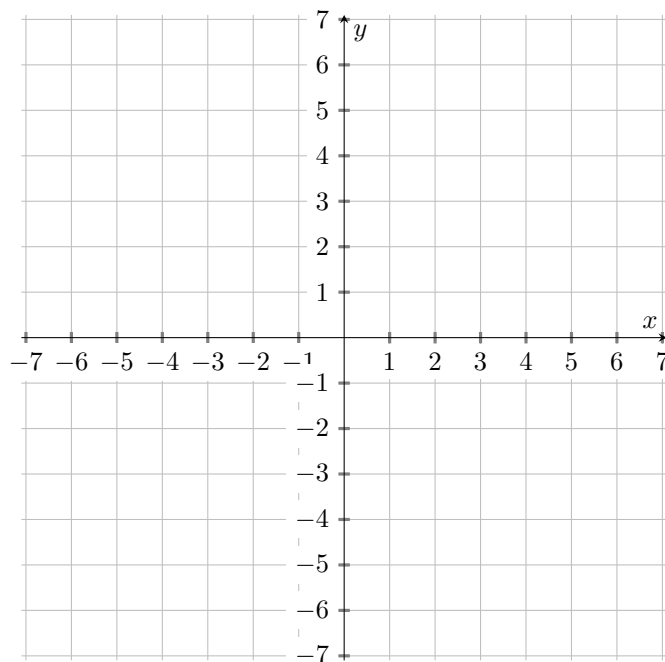
Exercise 18. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 2x - 3z = 4 \\ 2x + y - 5z = -1 \\ 3y - 4z = 2 \end{cases}$$

4.2.1 Applying this to inequalities

Exercise 19. We can also think about graphing systems of inequalities. Graph the lines and shade the region defined in this system of inequalities:

$$\begin{cases} y > x^2 - 3x + 4 \\ y < x + 1 \end{cases}$$



Exercise 20. We can also think about graphing systems of inequalities. Graph the lines and shade the region defined in this system of inequalities:

$$\begin{cases} y < -2x^2 - x - \frac{1}{2} \\ y > \frac{1}{4}x - 1 \end{cases}$$

