3 Quadratics and factoring, division

3.1 Tues, Feb. 27: Practice with distributive property of multiplication

We note that multiplication is distributive. This means multiplying each part of a sum is the same as taking the sum and multiplying that... so:

$$A(B + C) = A \cdot B + A \cdot C \tag{31}$$

Verify that this is true with real numbers! The same is true with unknowns.

Exercise 41. *Multiply* $x(2 + x)$

Exercise 42. *Multiply* $(x - 1)(x + 1)$

Exercise 43. *Find a solution for* $7(2 + x) = 63$

Exercise 44. *Multiply* $(x + y + z)(a + b + c)$

What is the geometric nature of multiplication and distribution. Let's explore this with an example?

Exercise 45. *Solve* $3 \cdot (3 + 7)$ *by finding the area of the rectangle with sides* 3 *and* 3*... and the rectangle with sides* 3 *and* 7*. And then find the area of the rectangle with sides* 3 *and* 10*.*

x

3.2 Thurs., Feb. 29: General form of quadratics

Polynomials are important in mathematics and life. It's worth thinking about why polynomials appear so often... I don't expect that you fully understand, but I do want us to spend some time thinking about how they relate important ideas about space and distance. Does this connection seem familiar? It is related to the two interpretations (graphic, algebraic) of linear equations in the form $y = mx + b$.

> Why are quadratic polynomials useful? There seem to be two different but interacting reasons. The first is that quadratic functions of a real variable are always either convex or concave and therefore have a unique maximum or minimum. The second is that quadratic functions are intimately related to bilinear forms and therefore can be accessed using linear algebra. —Paul Siegel, *Mathoverflow*

Polynomials have the form:

$$a_n x^n + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \tag{32}$$

Where the "poly" means many and the "nomial" refers to terms... or things added to each other. A polynomial with only one term is called a mononomial, one with two is called a binomial... and the highest order (or power) term designates whether it is quadratic, cubic, etc. The behavior of a polynomial is largely determined by its first term. Why is this? Think about properties of exponents. Is there a way we could say that x^5 is more "powerful" than x? This gives us the "leading coefficient test". And tells us how to graph polynomials. (Weird behavior often happens between the zeros, not outside of them). We begin investigating polynomials by thinking about quadratics. The standard form of a quadratic is:

$$f(x) = a(x-h)^2 + k \tag{33}$$

How could we derive this form from (12)? In this form, (h, k) will be the vertex. What does f look like if $a > 0$? If $a < 0$? Will the vertex be a min or max? For $ax^2 + bx + c = 0$, where a,b,c $\in \mathbb{R}$ and $a \neq 0$, $h = \frac{-b}{2a}$ and $k = f(h) = f(\frac{-b}{2a})$.

Exercise 46. *Write $f(x) = 2x^2 + 8x + 7$ in standard form by factoring around x, then working on the $x^2 + x$ term and inventing a constant value to make the factorization work... then adding that value to k.*

x

3.3 Fri, Mar. 1: x

x