2 Exponents

2.1 Mon., Mar. 5: Expansion of b^x

We often want to consider powers of a number where:

$$b^x = \underbrace{b \cdot b \cdot b}_{x \text{ times}} = y$$

From this idea we can derive "rules" for exponents.

$$x^0 = 1 \tag{11}$$

$$x^a \cdot x^b = x^{a+b} \tag{12}$$

$$(x^a)x^b = x^{a \cdot b} \tag{13}$$

$$\frac{x^a}{x^b} = x^{a-b} \tag{14}$$

$$\frac{x^a}{x^a} = (\frac{x}{x})^a \tag{15}$$

$$x^{a/b} = \sqrt[b]{x^a} \tag{16}$$

The logarithm language rewrites this in the format:

$$\log_b y = x \tag{17}$$

Where:

$$b^x = y \tag{18}$$

I've used the variables x and y in a consistent manner here, so you can copy between the formulas. But usually logs will be written such that the dependent variable, y, is the exponent... making them the inverse of b^x. The change of base format is very useful and helpful to consider as we understand logs:

$$\log_a x = \frac{\log_b x}{\log_b a} \tag{19}$$

2.2 Wednes., Mar. 7: Logarithms

The logarithm language rewrites this in the format:

$$\log_b y = x \tag{20}$$

Where:

$$b^x = y \tag{21}$$

I've used the variables x and y in a consistent manner here, so you can copy between the formulas. But usually logs will be written such that the dependent variable, y, is the exponent... making them the inverse of b^x. The change of base format is very useful and helpful to consider as we understand logs:

$$\log_a x = \frac{\log_b x}{\log_b a} \tag{22}$$

Which we can prove by observing that:

$$y \log_b x = \log_b x^y \tag{23}$$

This is the power property of logs, and there are also properties for products

$$\log_a uv = \log_a u + \log_a v \tag{24}$$

...and quotients:

$$\log_a \frac{u}{v} = \log_a u - \log_a v \tag{25}$$

Another important identity that we should think about is:

$$b^{\log_b x} = x \tag{26}$$