

5 Trigonometry and Euclid

5.1 Tues., Mar. 19: Representative triangles

In calc we turn from degree measure to radian measure... as we learned from deriving π , there are 6.28 radius lengths in a circle and so we have the formula:

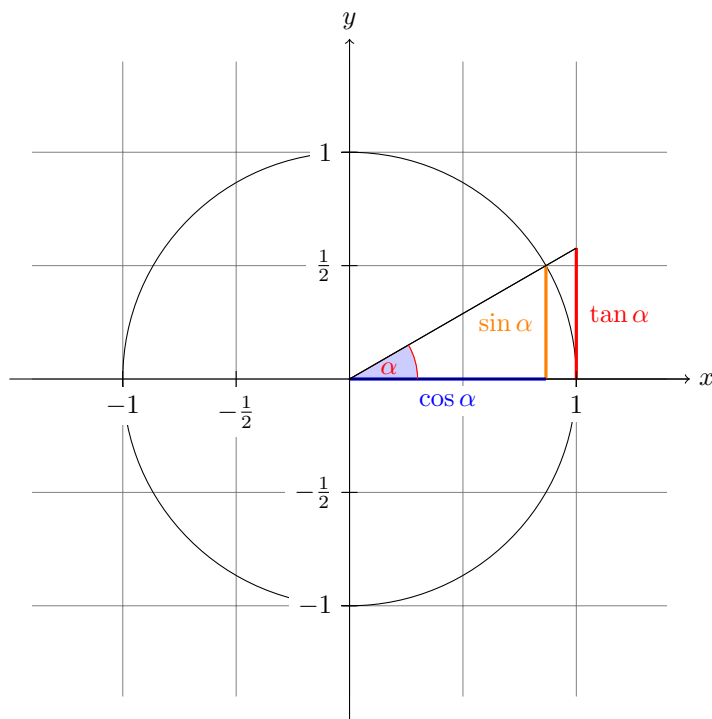
$$s = 2\pi r \quad (41)$$

So $\frac{2\pi}{2}$ is a half revolution, or 180 degrees. It's also useful to know that $\frac{\pi}{6} = 30^\circ$, $\frac{\pi}{4} = 45^\circ$, $\frac{\pi}{2} = 90^\circ$.

We can convert from degrees to radians easily by observing:

$$360^\circ = 2\pi \text{ rad} \quad (42)$$

$\frac{\pi}{180^\circ}$ is the conversion you most often want to multiply by.



The trig functions and functions for their reciprocals are:

$$\begin{aligned} \sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc(\theta) &= \frac{h}{o} \\ \sin(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec(\theta) &= \frac{h}{a} \\ \sin(\theta) &= \frac{\text{opposite}}{\text{adjacent}} & \cot(\theta) &= \frac{a}{o} \end{aligned}$$

Exercise 114. Find the sides and all six trig ratios for the representative triangles 45 – 45 – 90 and 30 – 60 – 90 in each quadrant, i.e. when $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{6}$, etc. (See the last page of this section for all the trig ratios... all you'll have to do is figure out how the signs change.)

Exercise 115. Compute $\sin(\theta)$ for three θ you choose... then compute $\cos(\theta - 90^\circ)$ for the same three θ values. What is their relationship? Why?

Exercise 116. Find $\sin(30^\circ)$.

Exercise 117. Find $\sin(45^\circ)$.

Exercise 118. Find $\cos(270^\circ)$.

Exercise 119. Find $\tan(13^\circ)$.

Exercise 120. Find $\sin(180^\circ)$.

Exercise 121. Find $\sin(330^\circ)$.

Exercise 122. Find $\cos(30^\circ)$.

Exercise 123. Find $\cos(\pi)$.

Exercise 124. Find $\sin(\frac{\pi}{6})$.

Exercise 125. Find $\cos(\frac{\pi}{2})$.

Exercise 126. Find $\tan(\frac{5\pi}{6})$.

Exercise 127. Find $\cos(\frac{\pi}{3})$.

There are functions that invert the regular trig functions. Instead of taking in an angle and returning a ratio of sides, they take in a ratio of sides and return an angle:

$$\begin{aligned}\arcsin\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) &= \theta \\ \arccos\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) &= \theta \\ \arctan\left(\frac{\text{opposite}}{\text{adjacent}}\right) &= \theta\end{aligned}$$

Exercise 128. Find $\arccos(\frac{1}{2})$

Exercise 129. Find $\arccos(\frac{2\sqrt{2}}{2})$

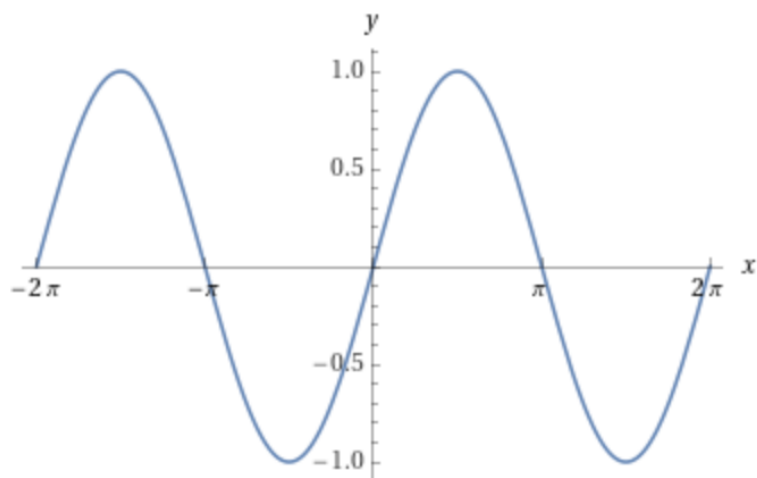
Exercise 130. Find $\arcsin(\frac{\sqrt{3}}{2})$

Exercise 131. Find $\arctan(1)$

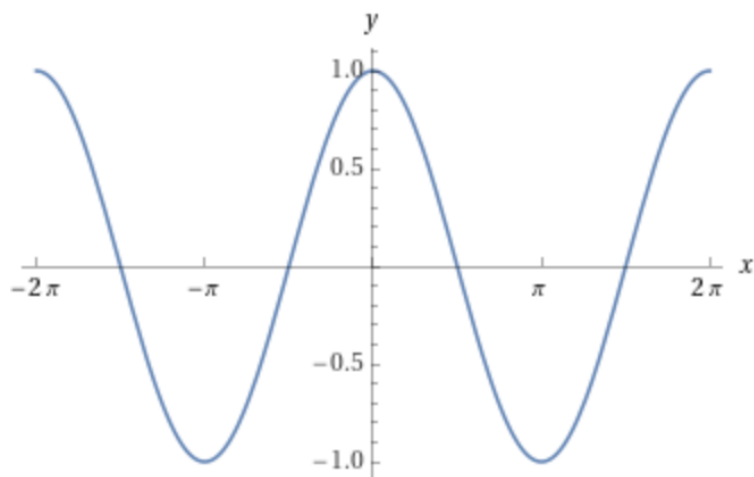
Exercise 132. Find $\arcsin(-1)$

5.2 Thurs., Mar. 21: Plots of trig functions

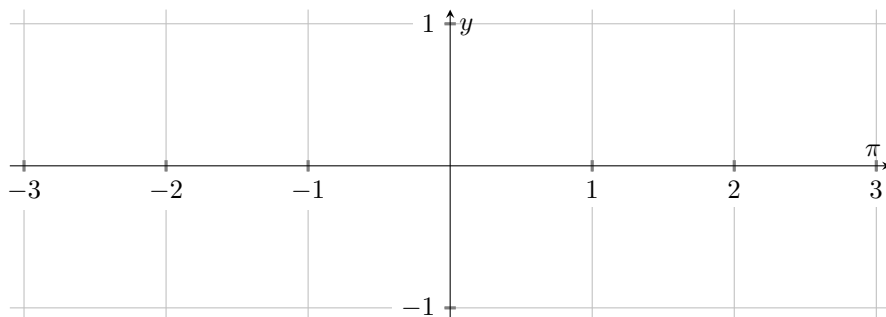
The plot of $y = \sin(x)$ is:



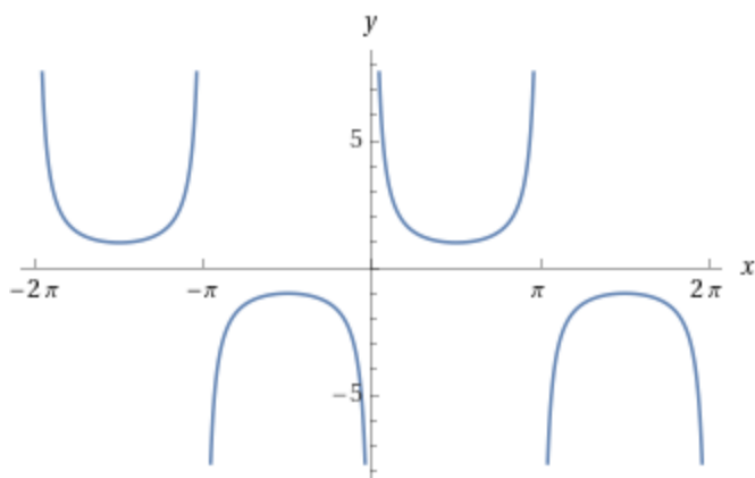
The plot of $y = \cos(x)$ is:



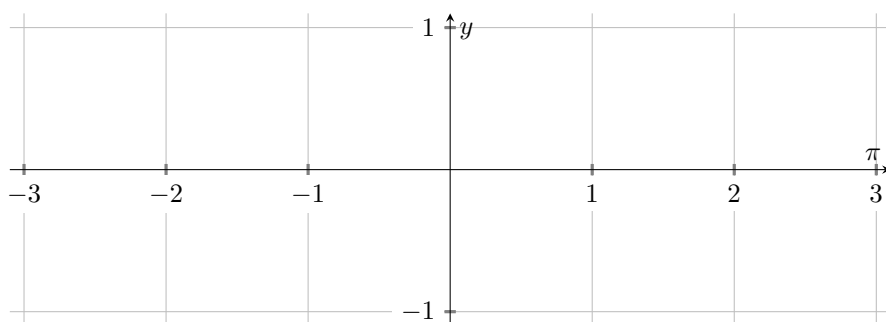
Exercise 133. What is the plot of $y = \tan(x)$? (Hint: use simple 'choose x , find y ' but choose points strategically. You can use a calculator to evaluate the trig functions but don't use a graphing calculator.)



Exercise 134. I've given the plot of $y = \csc(x)$ below. Based on what we know about these two functions, without doing any computation, what is the plot of $y = \sec(x)$?

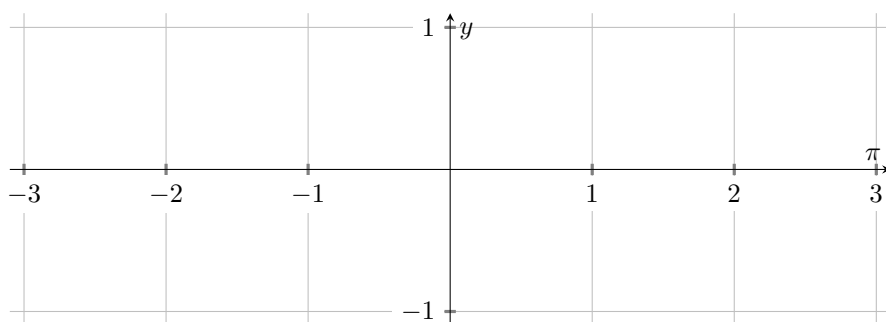


Now your turn!

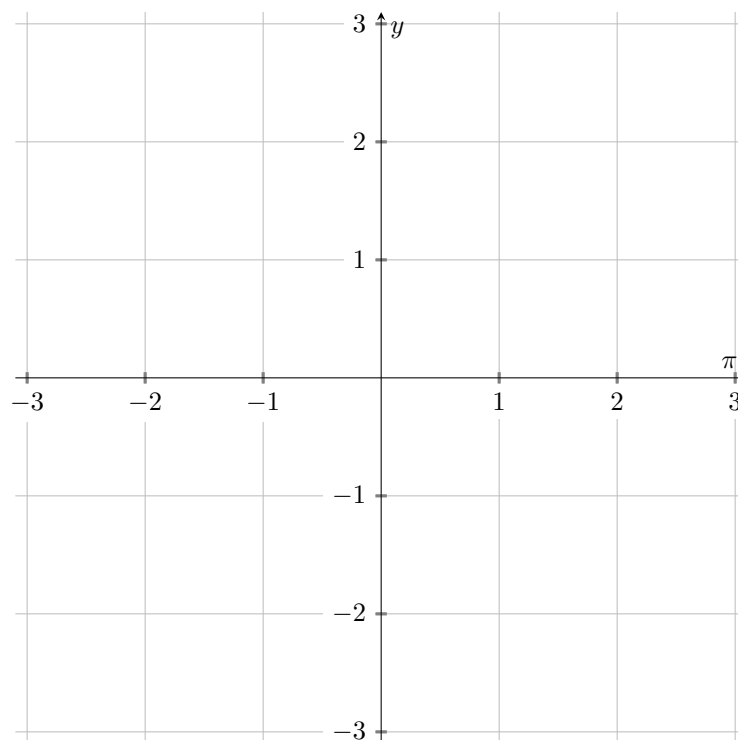


5.3 Fri., Mar. 22: Dilation and shifting of the trig functions!

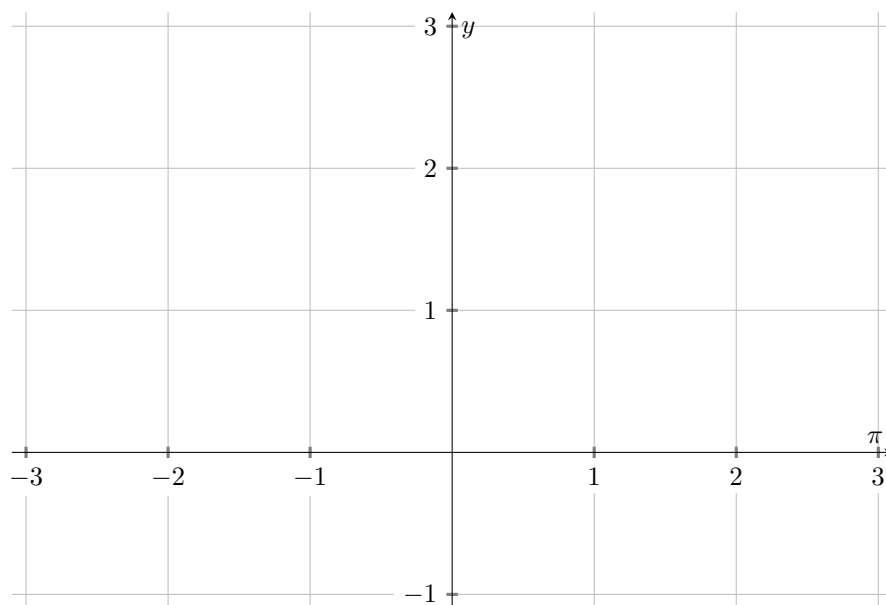
Exercise 135. What is the plot of $y = \sin(\frac{x}{3})$?



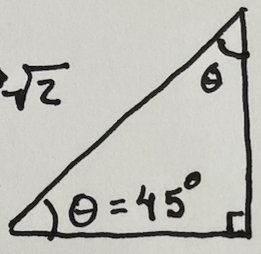
Exercise 136. What is the plot of $y = 3\sin(x)$?



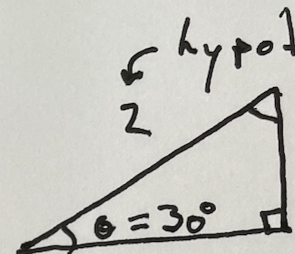
Exercise 137. What is the plot of $y = \sin(x) + 2$?



Here are the values for several common trig functions as a reference:



hypotenuse $\rightarrow \sqrt{2}$
 $\theta = 45^\circ$
 $1 \leftarrow \text{opposite}$
 $1 \leftarrow \text{adjacent}$



hypotenuse
 2
 $\theta = 30^\circ$
 $1 \leftarrow \text{opposite}$
 $\sqrt{3} \leftarrow \text{adjacent}$

so:

$$\sin(45^\circ) = \frac{o}{h} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = .71$$

$$\cos(45^\circ) = \frac{a}{h} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = .71$$

$$\tan(45^\circ) = \frac{o}{a} = \frac{1}{1} = 1$$

$$\sin(30^\circ) = \frac{o}{h} = \frac{1}{2} = .5$$

$$\cos(30^\circ) = \frac{a}{h} = \frac{\sqrt{3}}{2} = .87$$

$$\tan(30^\circ) = \frac{o}{a} = \frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} = .58$$

■ Note: this is a simplification to eliminate $\sqrt{\quad}$ in denominator:

$$\text{b/c } \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$