5 Trigonometry and Euclid

5.1 Representative triangles

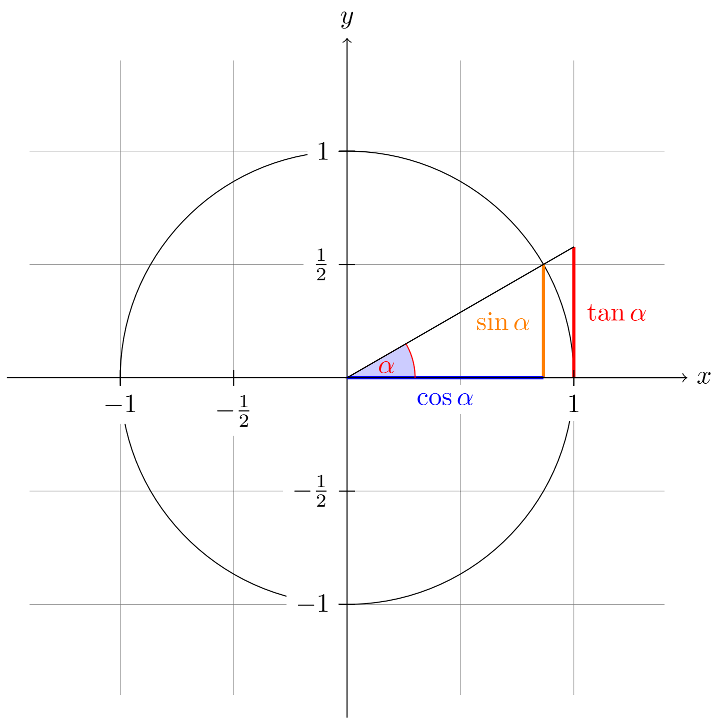
In calc we turn from degree measure to radian measure... as we learned from deriving π, there are 6.28 radius lengths in a circle and so we have the formula:

$$s = 2\pi r \tag{41}$$

So $\frac{2\pi}{2}$ is a half revolution, or 180 degrees. It's also useful to know that $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, $\frac{\pi}{2} = 90°$.
We can convert from degrees to radians easily by observing:

$$360° = 2\pi\,\mathrm{rad} \tag{42}$$

$\frac{\pi}{180°}$ is the conversion you most often want to multiply by.



The trig functions and functions for their reciprocals are:

$$\begin{array}{ll}
\sin(\theta) = \frac{opposite}{hypotenuse} & \csc(\theta) = \frac{h}{o} \\
\sin(\theta) = \frac{adjacent}{hypotenuse} & \sec(\theta) = \frac{h}{a} \\
\sin(\theta) = \frac{opposite}{adjacent} & \cot(\theta) = \frac{a}{o}
\end{array}$$

Exercise 114. *Find the sides and all six trig ratios for the representative triangles* $45-45-90$ *and* $30-60-90$ *in each quadrant,* i.e. *when* $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{6}$, *etc. (See the last page of this section for all the trig ratios... all you'll have to do is figure out how the signs change.)*

Exercise 115. *Compute* $\sin(\theta)$ *for three* θ *you choose... then compute* $\cos(\theta - 90°)$ *for the same three* θ *values. What is their relationship? Why?*

Exercise 116. *Find* $\sin(30°)$.

Exercise 117. *Find* $\sin(45°)$.

Exercise 118. *Find* $\cos(270°)$.

Exercise 119. *Find* $\tan(13°)$.

Exercise 120. *Find* $\sin(180°)$.

Exercise 121. *Find* $\sin(330°)$.

Exercise 122. *Find* $\cos(30°)$.

Exercise 123. *Find* $\cos(\pi)$.

Exercise 124. *Find* $\sin(\frac{\pi}{6})$.

Exercise 125. *Find* $\cos(\frac{\pi}{2})$.

Exercise 126. *Find* $\tan(\frac{5\pi}{6})$.

Exercise 127. *Find* $\cos(\frac{\pi}{3})$.

There are functions that invert the regular trig functions. Instead of taking in an angle and returning a ratio of sides, they take in a ratio of sides and return an angle:

$$\arcsin(\frac{opposite}{hypotenuse}) = \theta$$
$$\arccos(\frac{adjacent}{hypotenuse}) = \theta$$
$$\arctan(\frac{opposite}{adjacent}) = \theta$$

Exercise 128. *Find* $\arccos(\frac{1}{2})$

Exercise 129. *Find* $\arccos(\frac{2\sqrt{2}}{2})$

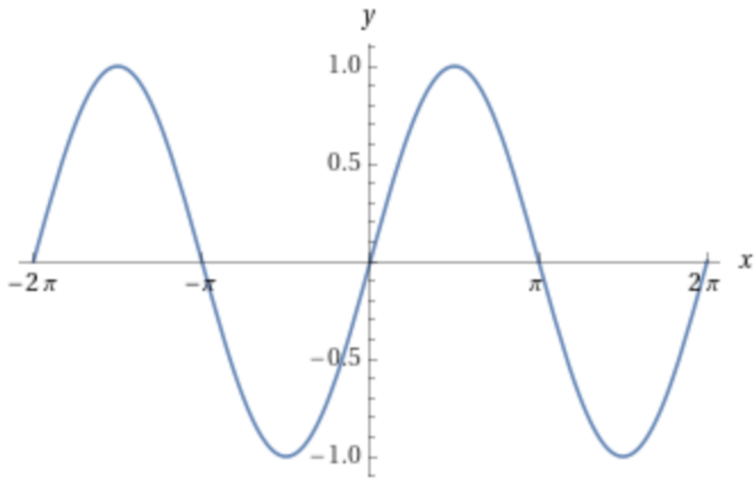
Exercise 130. *Find* $\arcsin(\frac{\sqrt{3}}{2})$

Exercise 131. *Find* $\arctan(1)$

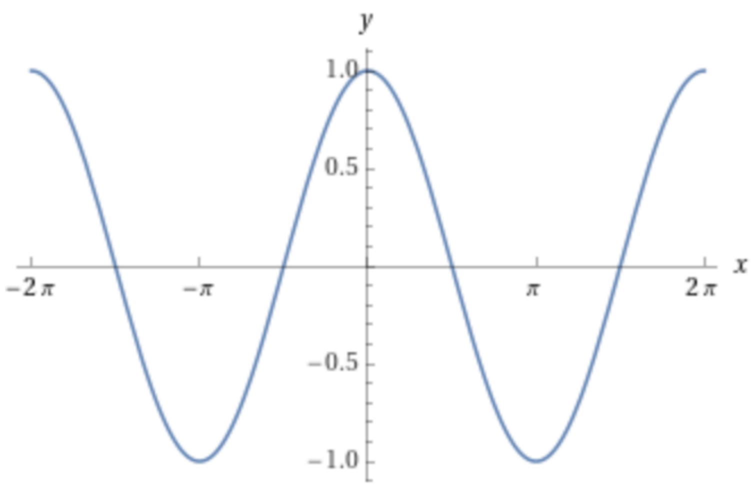
Exercise 132. *Find* $\arcsin(-1)$

5.2 Plots of trig functions

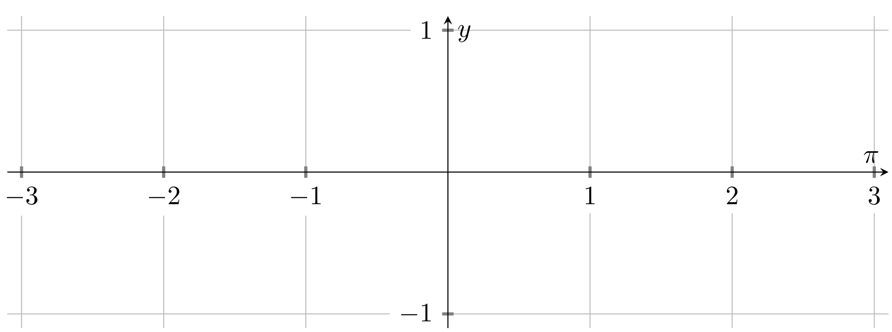
The plot of $y = sin(x)$ is:



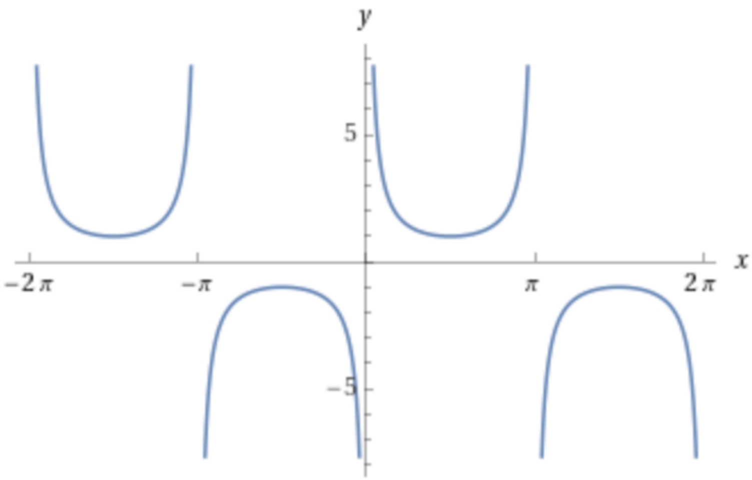
The plot of $y = cos(x)$ is:



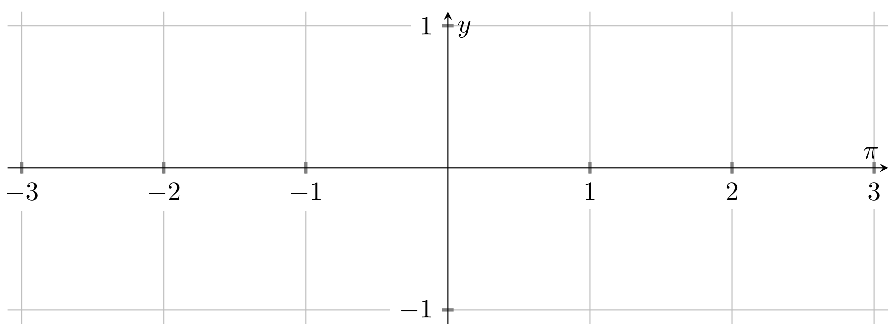
Exercise 133. *What is the plot of $y = tan(x)$? (Hint: use simple 'choose x, find y' but choose points strategically. You can use a calculator to evaluate the trig functions but don't use a graphing calculator.)*



Exercise 134. *I've given the plot of $y = csc(x)$ below. Based on what we know about these two functions, without doing any computation, what is the plot of $y = sec(x)$?*

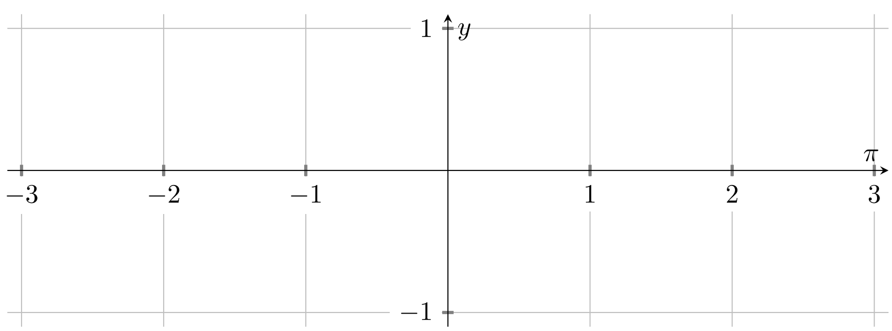


Now your turn!

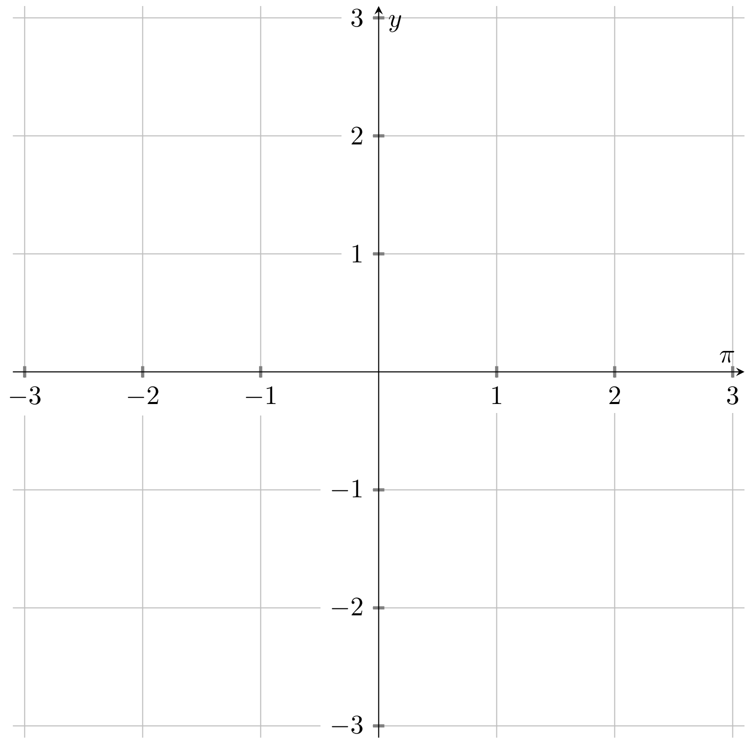


5.3 Dilation and shifting of the trig functions!

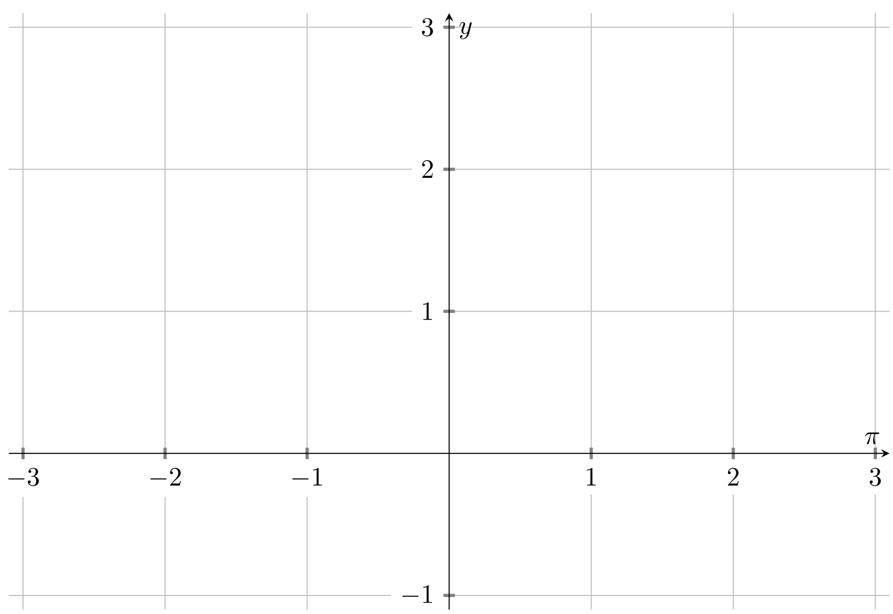
Exercise 135. *What is the plot of $y = sin(\frac{x}{3})$?*



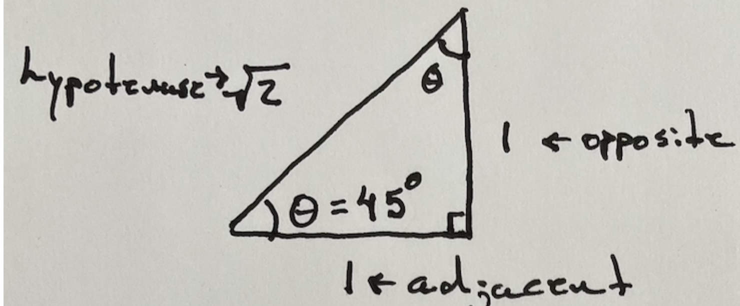
Exercise 136. *What is the plot of* $y = 3sin(x)$*?*

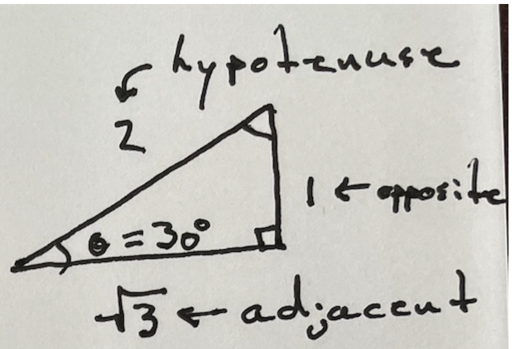


Exercise 137. *What is the plot of* $y = sin(x) + 2$*?*



Here are the values for several common trig functions as a reference:





So:

$$\sin(45^\circ) = \frac{o}{h} = \frac{1}{\sqrt{2}} = \blacksquare\frac{\sqrt{2}}{2} = .71$$

$$\cos(45^\circ) = \frac{a}{h} = \frac{1}{\sqrt{2}} = \blacksquare\frac{\sqrt{2}}{2} = .71$$

$$\tan(45^\circ) = \frac{o}{a} = \frac{1}{1} = 1$$

$$\sin(30^\circ) = \frac{o}{h} = \frac{1}{2} = .5$$

$$\cos(30^\circ) = \frac{a}{h} = \frac{\sqrt{3}}{2} = .87$$

$$\tan(30^\circ) = \frac{o}{a} = \frac{1}{\sqrt{3}} = \blacksquare\frac{\sqrt{3}}{3} = .58$$

■ Note: this is a simplification to eliminate $\sqrt{\ }$ in denominator:

$$\text{b/c} \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$$