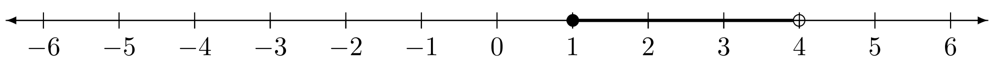
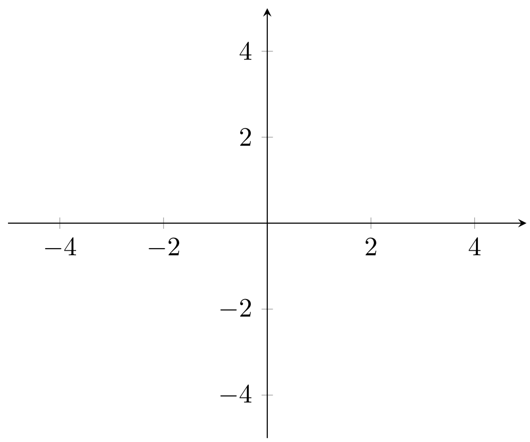
1 Graphing and solving linear equations through rearrangement, substitution and subtraction

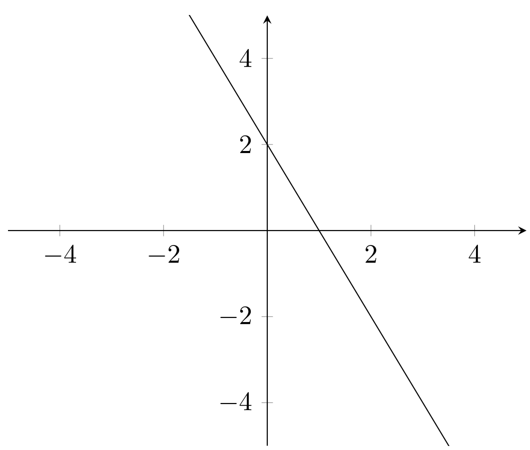
We begin our study of algebra by remembering the number line. If you haven't seen one in a while, it looks like this:



Here I used the number line to show adding $1 + 3$. But often we want to draw on two number lines at the same time, to see where our graphs intersect:



This is good because we can now plot not just one number, but two at the same time... and these relations can be expressed in line equations where each point on the line is a co-value for x and y that is true in the line equation:



These lines are expressed with equations like $y = mx + b$ where:

$$y = \underbrace{m}_{\text{the ratio of x and y}} x + \underbrace{b}_{\text{a constant value}}$$

It can be helpful to think about where m comes from:

$$\frac{y}{x} = \frac{y}{x} \rightarrow y = (\underbrace{\frac{y}{x}}_{\text{m}})x \tag{1}$$

We know slope is a ratio of $\frac{\Delta x}{\Delta y}$ and for any x and y this can be expressed:

$$\frac{y_2 - y_1}{x_2 - x_1} \tag{2}$$

1.1 Mon., Feb. 19: Graphing a line and finding the equation of a line with any two points that lie on it

We can draw the graph for a line equation $y = mx + b$ by:

- identifying the b-value on the y-axis... (hint: this is found by imagining x to be 0)
- drawing from that point the line slope given by m

We can also find a line equation from any two points on the graph $(x_2, y_2)$and (x_1, y_1) by reversing the process:

- finding m, or the slope with $\frac{y_2-y_2}{x_2-x_1}$
- substituting one of our points for x and y and the value we just found for m to solve for b

1.2 Wednes., Feb. 21: Shapes of common functions

Consider the functions:

$$f(x) = c \tag{3}$$

$$f(x) = x^2 \tag{4}$$

$$f(x) = x^3 \tag{5}$$

$$x^2 + y^2 = 1 \tag{6}$$

(Note: You may have heard of the vertical line test. Does this pass the vertical line test? If not a function, what is it? Are there other ways we could describe a circle that would pass the test?)

$$f(x) = |x| \tag{7}$$

$$f(x) = \sqrt{x} \tag{8}$$

$$f(x) = \frac{1}{x} \tag{9}$$

$$f(x) = \frac{1}{x^2} \tag{10}$$

What is the basic shape of each of these functions? How do their compositions with other functions affect their shapes?

Exercise 1. *Let's invent some points and functions and work on translating between the two. Use the method of* 2^n *steps to find points on the graph with the brute force method.*

1.3 Fri, Feb. 23: Shapes... derivation of π

Exercise 2. *What is the perimeter of a circle with radius 4?*

Exercise 3. *What is the volume of a sphere with diameter 6?*

Exercise 4. *What is the volume of a regular pyramid with side length 3?*

Exercise 5. *Imagine a sphere inside a cube. The sphere touches the cube once on all six sides. What is the volume between the shapes?*

Exercise 6. *Why is* π 3.14*? Can you come up with a proof for the value for* π*? Can you come up with a proof for the area of a circle?*

Exercise 7. *Write down all the shape equations for surface and volume for future reference... (You can look these up on the internet. Try to have about 10.)*