4 Systems of equations

4.1 Tues., Mar. 19: Considering what we mean by algebraic substitution

We've worked with many types of algebraic expressions. Now we can start putting them together, or thinking about where several expressions are simultaneously true. Let's work a few to warm up:

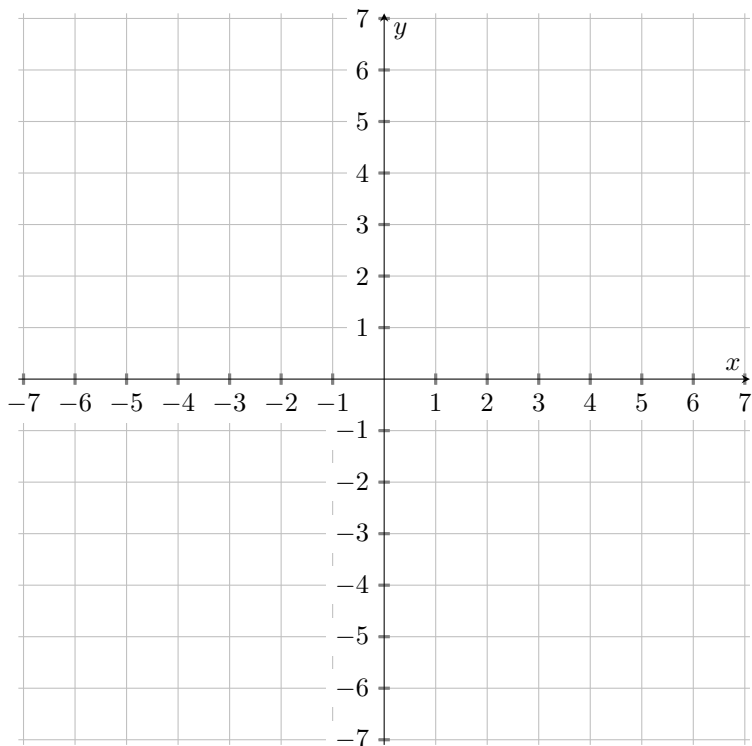$$\begin{cases} 8 = 2x + 3y \\ -2 = x \end{cases}$$

$$\begin{cases} -6x + \frac{1}{2}y = 4 \\ y = 4 \end{cases}$$

$$\begin{cases} 5x - y = 17 \\ x = y + 1 \end{cases}$$

We can solve these with substitution. But will that continue to work as the equations get more complicated? Let's consider another system of equations and solve it graphically.
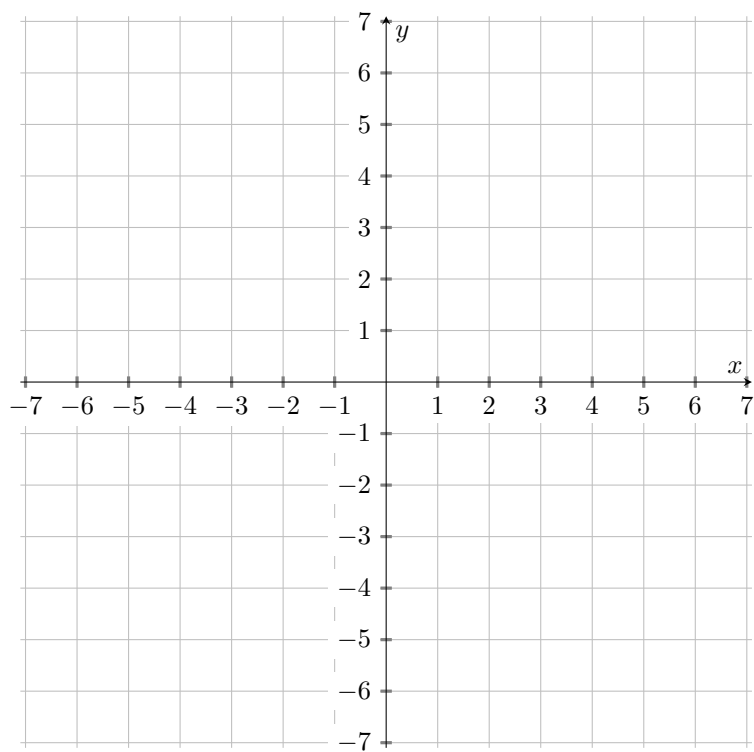
Exercise 9.

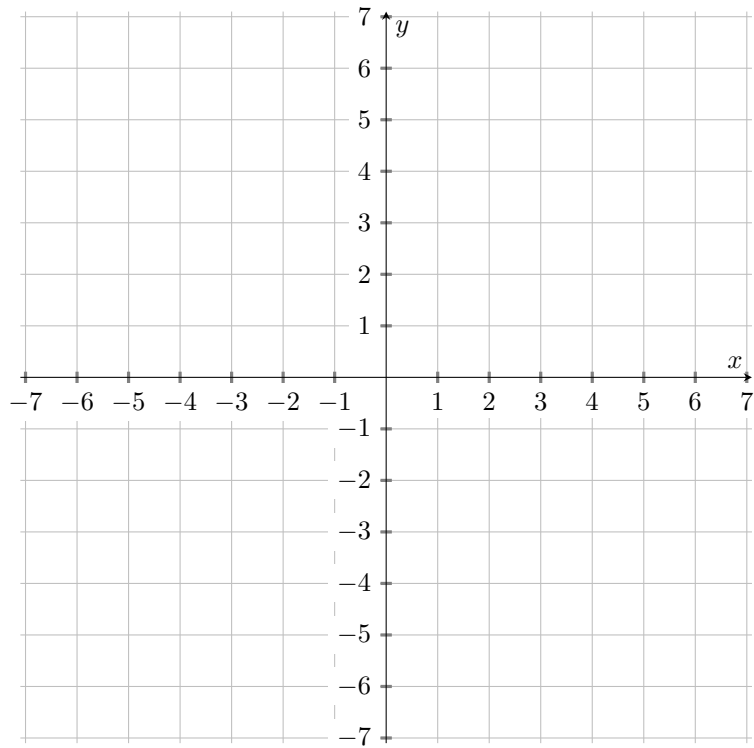$$\begin{cases} y = -x^2 + 2x + 8 \\ y = 3x + 2 \end{cases}$$

Exercise 10.

$$\begin{cases} y = x^2 - 3x - 4 \\ y = x - 8 \end{cases}$$

Exercise 11.

$$\begin{cases} y = 2x^2 + 4x + 3 \\ y = 4x - 1 \end{cases}$$

4.2 Thurs., Mar. 21: Using elimination to solve systems of equations in three unknowns

We now consider a more challenging case, where there may be three equations in three unknowns!

Exercise 12. *Let's try to solve the equation below with substitution:*

$$\begin{cases} x + y + z = -2 \\ 2x - 3y + 2z = -14 \\ 4x + 3y - z = 5 \end{cases}$$

Exercise 13. *Let's try to solve the equation below with substitution:*

$$\begin{cases} 2x - y - 2z = 3 \\ 3x + y - 2z = 11 \\ -2x - y + z = -8 \end{cases}$$

Exercise 14. *Let's try to solve the equation below with elimination:*

$$\begin{cases} x + y + z = -2 \\ 2x - 3y + 2z = -14 \\ 4x + 3y - z = 5 \end{cases}$$

Exercise 15. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 2x - 3y = 7 \\ y + z = -5 \\ x + 2y + 4z = -17 \end{cases}$$

Exercise 16. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 5x + y3z = 9 \\ -x - 2y - z = -16 \\ 2x + 4y + 2z = -30 \end{cases}$$

Exercise 17. *Let's try to solve the equation below with elimination:*

$$\begin{cases} x - 4y + 3z = -7 \\ 2x + 3y - 5z = 19 \\ 4x + y - z = 17 \end{cases}$$
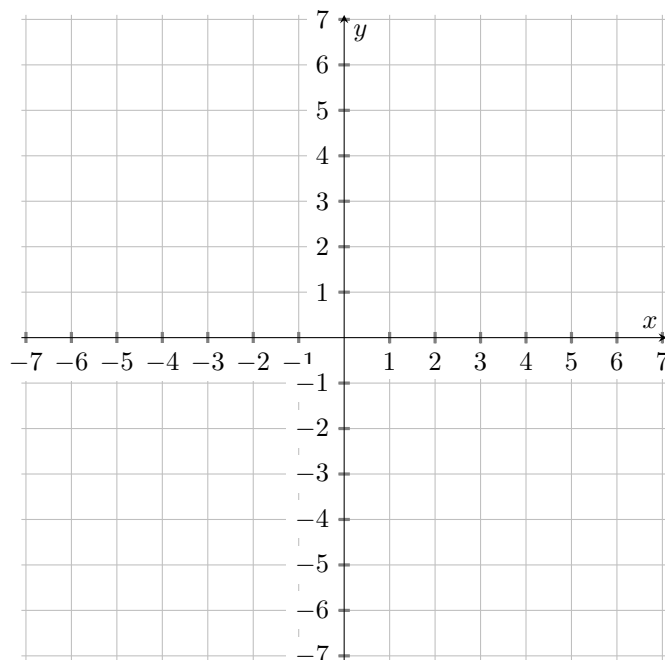
Exercise 18. *Let's try to solve the equation below with elimination:*

$$\begin{cases} 2x - 3z = 4 \\ 2x + y - 5z = -1 \\ 3y - 4z = 2 \end{cases}$$

4.2.1 Applying this to inequalities

Exercise 19. *We can also think about graphing systems of inequalities. Graph the lines and shade the region defined in this system of inequalities:*

$$\begin{cases} y > x^2 - 3x + 4 \\ y < x + 1 \end{cases}$$

Exercise 20. *We can also think about graphing systems of inequalities. Graph the lines and shade the region defined in this system of inequalities:*

$$\begin{cases} y < -2x^2 - x - \frac{1}{2} \\ y > \frac{1}{4}x - 1 \end{cases}$$