7 What is slope anyway?

7.1 Reviewing slope, the m in $y = mx + b$

We find slope by finding the ratio of inputs to outputs, and set $y_1 = f(x_1)$ and $y_2 = f(x_2)$:
This gives:

$$slope = \frac{y_2 - y_1}{x_2 - x_1} \tag{62}$$

But $y = f(x)$... so we can rewrite this as:

$$slope = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \tag{63}$$

7.1.1 Let's work probelms from algebra but pay close attention to how slope is formulated!

Exercise 153. *Find the slope between (-2, -10) and (3,5) with* $\frac{y_2-y_1}{x_2-x_1}$. *Verify that this is the slope with* $\frac{f(x_2)-f(x_1)}{x_2-x_1}$.

We use

$$slope = \frac{y_2 - y_1}{x_2 - x_1} \tag{64}$$

so for (-2, -10) and (3,5) we can say...

$$slope = \frac{5 - (-10)}{3 - -2} \tag{65}$$

Exercise 154. *Find the slope between (-10, -2) and (2,4).* Work this problem and all the rest in this section in the same manner as the first...

Exercise 155. *Find the slope between (5, 5) and (30,15).*

Exercise 156. *Find the slope between (0, 0) and (1,1).*

7.1.2 This also works on more complicated functions!

Exercise 157. *Find the approximate slope of* $f(x) = x^2 + x + 2$ *between* $x_1 = 0$ *and* $x_2 = 3$. *Verify this is true with computer graphing and sketching.*

Exercise 158. *Find the approximate slope of* $f(x) = sin(x)$ *between* $x_1 = 0$ *and* $x_2 = 2\pi$.

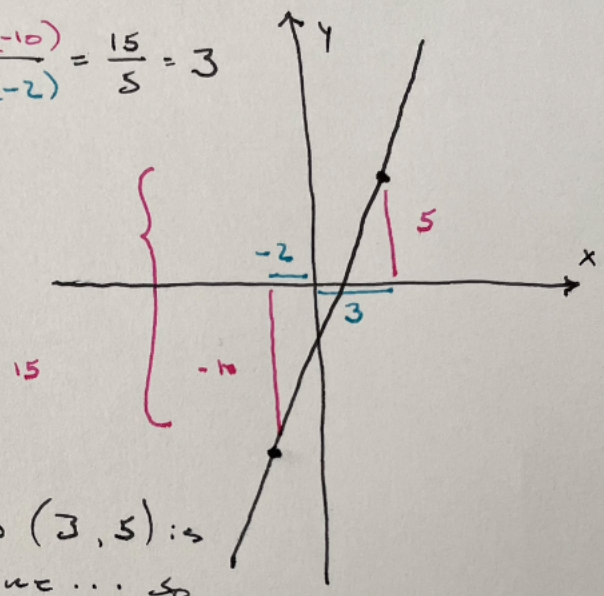
Exercise 159. *Find the approximate slope of* $f(x) = 10x^3e^{-x^2}$ *between* $x_1 = -1$ *and* $x_2 = 1$.

Exercise 160. *Find the approximate slope of* $f(x) = sin(x) + x$ *between* $x_1 = 0$ *and* $x_2 = 10$.

7.2 Answers to 7.1

153. OK we've got two points at $(-2, -10)$ (x_1, y_1) & $(3, 5)$ (x_2, y_2) Where are they?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-10)}{3 - (-2)} = \frac{15}{5} = 3$$



so $y = 3x + b$

and we know $(3, 5)$ is on this line ... so

$(5) = 3(3) + b$

and $b = -4 \rightarrow y = 3x - 4$

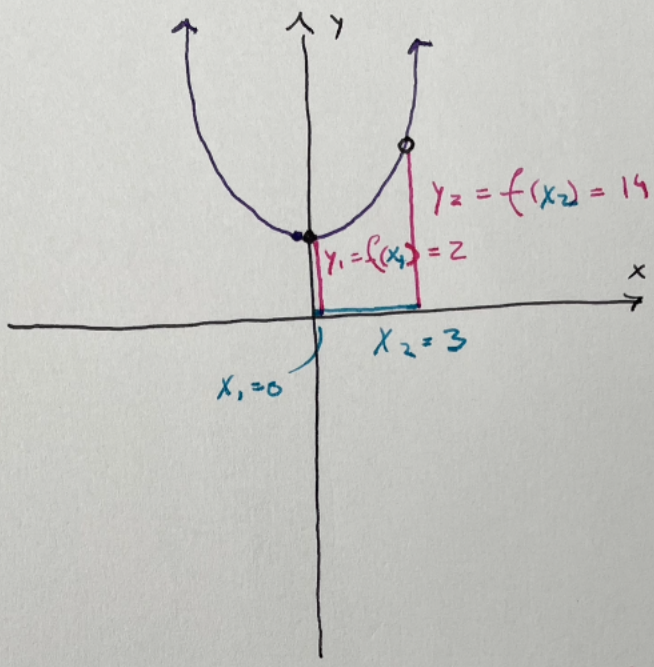
we also know $y = f(x) = 3x - 4$ so

slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ can be written as

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{(f(3) = 3 \cdot 3 - 4) - (f(-2) = 3 \cdot (-2) - 4)}{3 - (-2)}$$

$$= \frac{5 - (-10)}{3 - (-2)} = \frac{15}{5} = 3$$

157. If $f(x) = x^2 + x + 2$ what is slope from $x_1 = 0$ to $x_2 = 3$?



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{(3^2 + 3 + 2) - (0^2 + 0 + 2)}{3 - 0}$$

$$= \frac{14 - 2}{3} = 4$$

158. $f(x) = \sin(x)$. What is slope from $x = 0, 2\pi$?

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) = \sin(2\pi) - f(x_1) = \sin(0)}{2\pi - 0}$$

$$= \frac{0 - 0}{2\pi - 0} = 0$$

159. $f(x) = 10x^3 e^{-x^2}$. Slope from $x = -1, 1$ is:

$$\frac{\Delta y}{\Delta x} = \frac{10(1)^3 e^{-(1)^2} - 10(-1)^3 e^{-(-1)^2}}{1 - -1}$$

$$= \frac{10/e + 10/e}{2} = 10/e \approx 3$$

160. $f(x) = \sin(x) + x$ from $x = 0$ to 10

so $$\frac{\sin(10) + 10 - \sin(0) + 0}{10 - 0}$$

$$= \frac{10}{10} = 1$$

overall, remember! so slope is $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$