1 Functions

1.1 Tues, Feb. 20: Domain and co-domain, graphing solutions to pairs of linear equations

A function is a defined relationship between two things. These can be inputs/outputs, two numbers, a number and an idea, etc. The first thing is called the "domain" and the second thing is the "co-domain":

$$F : A \to B \tag{1}$$

The function is said to be one-to-one (injective) if every $a \in A$ maps to a unique b $\in$ B, *i.e.* no two $a \in A$ map to the same $b \in B$, and "onto" if for every $b \in B$ there is an $a \in A$ that maps to it. The former function is said to be an "injection" and the latter a "surjection". A function that is both is a "bijection," or a correspondence.

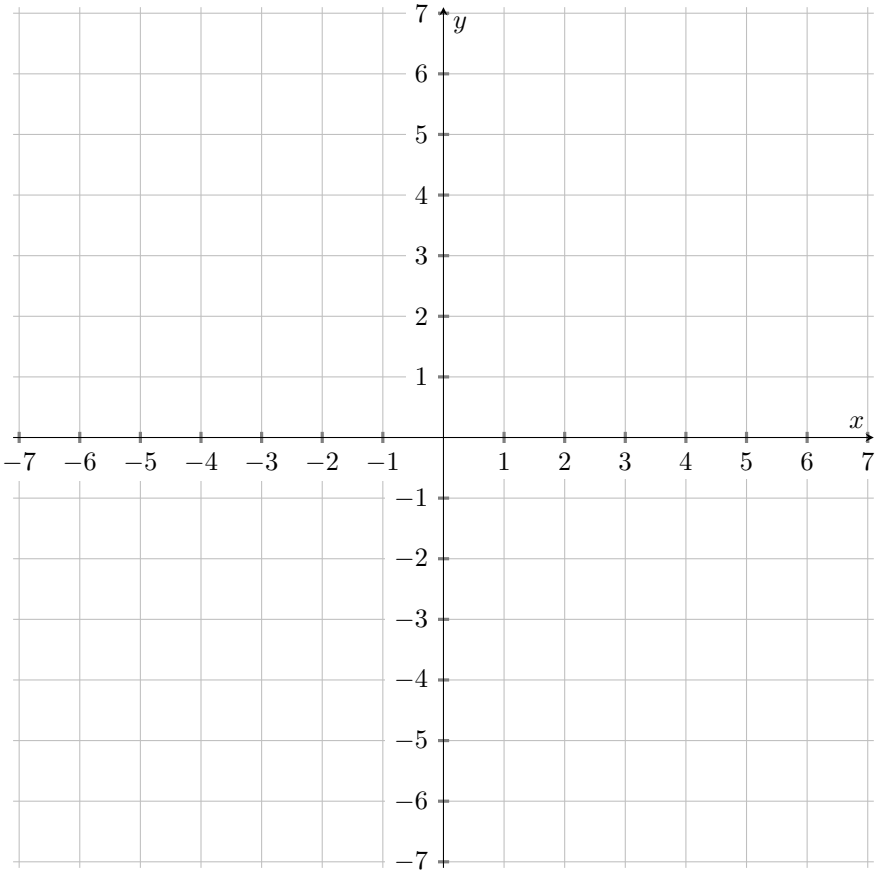
Exercise 1. *For the counting numbers* $1, 2, 3, ...$*, not including halves or fractional numbers, name a function that is one-to-one, onto, and both one-to-one and onto.*

The "linear" or line function is fundamental for relating two numbers with a constant relationship in the Cartesian plane. You probably know about the Cartesian plane from algebra... it's just the $x - y$ coordinate graph.

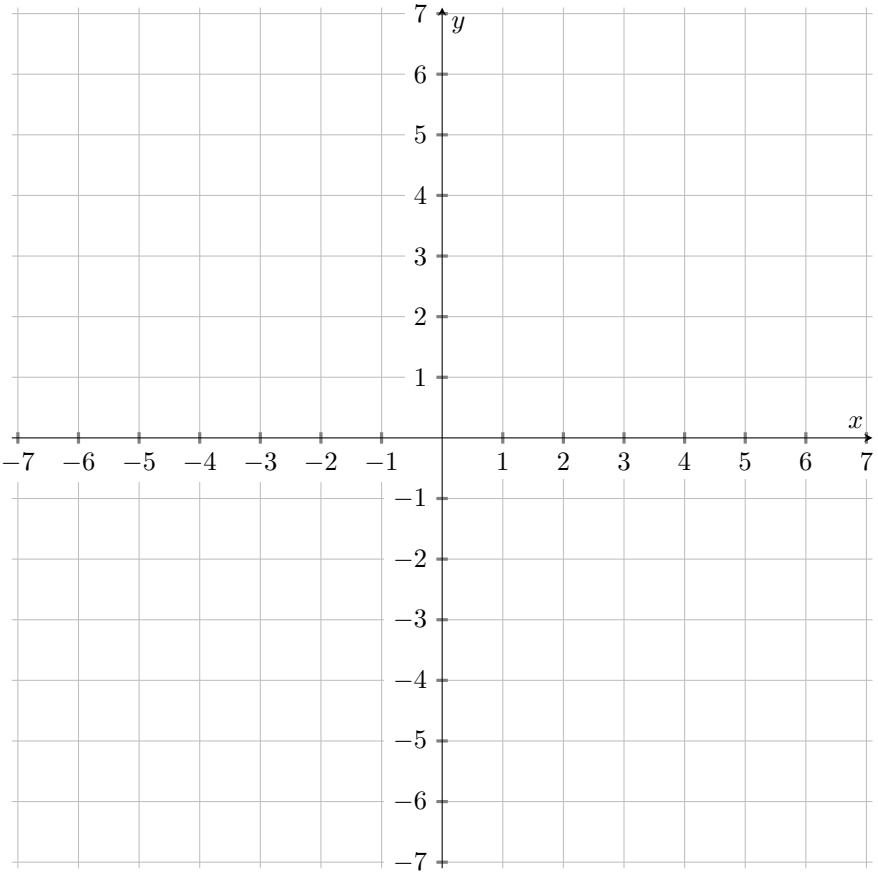
Pairs of these functions can be solved visually with graphing or with algebraic rearrangement. Each pair of linear equations will have exactly one solution (x, y) that satisfies both equations, and this point will be the intersection of the two lines, whereas many (x, y) will solve a single linear equation in the form:

$$y = f(x) = mx + b \tag{2}$$

Exercise 2. *Graph the points* $(6, 7)$ *and* $(-4, -8)$*. Find the slope of the line that runs through these points and its equation. Find another point on this line. Find its intercepts. Find the equation of a line perpendicular to this line, and the equation for a line parallel to it.*

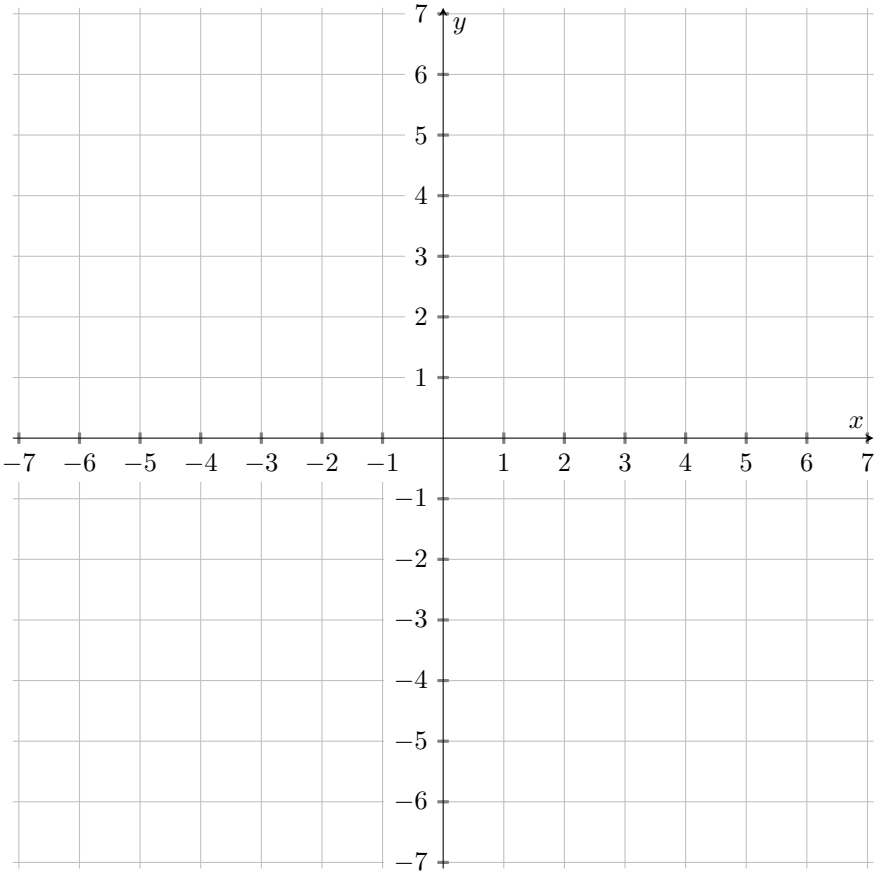


Exercise 3. *Graph $f(x) = 1/x$. Where is the function undefined? What is the behavior of the function at its big values of x and y. Find the inverse of this function. (This might be a trick question. What is the way to find a function's inverse in general?)*



Exercise 4. *Find the distance between the points $(-2, -3)$ and $(3, 5)$ with the Pythagorean theorem. Write this down for your notes as the "distance formula". Find their midpoint. Write down the midpoint formula for your notes. (Hint: it is like taking an average.)*

Exercise 5. *Graph $x^2 - 5x - 6 = 0$. Then solve. Solve first with guessing, then with completing the square, then with the quadratic formula.*



Exercise 6. *Write in standard form:*

$$\frac{2+3i}{4-2i} \tag{3}$$

1.2 Thurs, Feb. 22: Shapes of common functions

Consider the functions:

$$f(x) = c \tag{4}$$

$$f(x) = x^2 \tag{5}$$

$$f(x) = x^3 \tag{6}$$

$$x^2 + y^2 = 1 \tag{7}$$

(Note: You may have heard of the vertical line test. Does this pass the vertical line test? If not a function, what is it? Are there other ways we could describe a circle that would pass the test?)

$$f(x) = |x| \tag{8}$$

$$f(x) = \sqrt{x} \tag{9}$$

$$f(x) = \frac{1}{x} \tag{10}$$

$$f(x) = \frac{1}{x^2} \tag{11}$$

What is the basic shape of each of these functions? How do their compositions with other functions affect their shapes?

Exercise 7. *Let's invent some points and functions and work on translating between the two. Use the method of* 2^n *steps to find points on the graph with the brute force method.*

1.3 Fri, Feb. 23: Shapes... derivation of π

Exercise 8. *What is the perimeter of a circle with radius 4?*

Exercise 9. *What is the volume of a sphere with diameter 6?*

Exercise 10. *What is the volume of a regular pyramid with side length 3?*

Exercise 11. *Imagine a sphere inside a cube. The sphere touches the cube once on all six sides. What is the volume between the shapes?*

Exercise 12. *Why is* π 3.14*? Can you come up with a proof for the value for* π*? Can you come up with a proof for the area of a circle?*

Exercise 13. *Write down all the shape equations for surface and volume for future reference... (You can look these up on the internet. Try to have about 10.)*