# 3 Trigonometry and Euclid

## 3.1 Tues., Mar. 19: Representative triangles

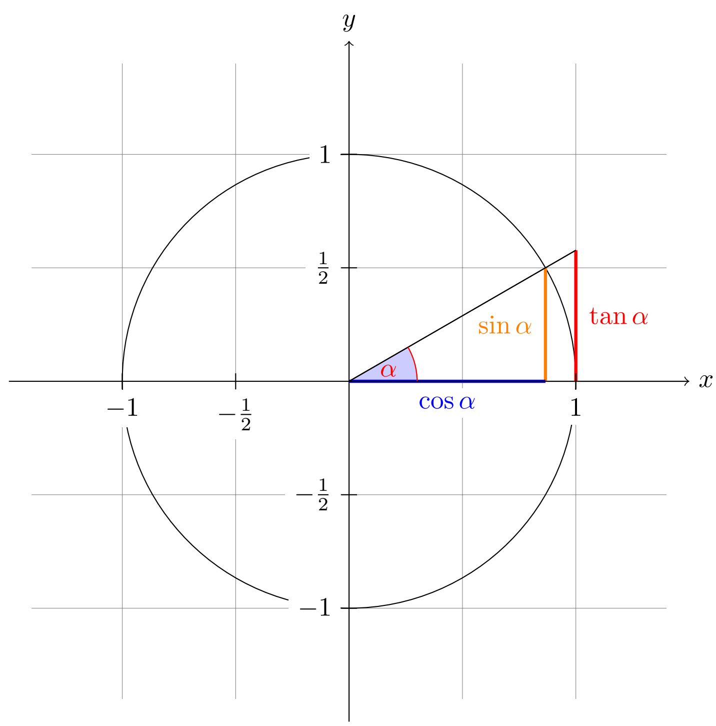
In calc we turn from degree measure to radian measure... as we learned from deriving $\pi$, there are 6.28 radius lengths in a circle and so we have the formula:

$$s = 2\pi r \tag{41}$$

So $\frac{2\pi}{2}$ is a half revolution, or 180 degrees. It's also useful to know that $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, $\frac{\pi}{2} = 90°$. We can convert from degrees to radians easily by observing:

$$360° = 2\pi \,\text{rad} \tag{42}$$

$\frac{\pi}{180°}$ is the conversion you most often want to multiply by.




The trig functions and functions for their reciprocals are:

$$\begin{array}{ll} \sin(\theta) = \frac{opposite}{hypotenuse} & \csc(\theta) = \frac{h}{o} \\ \sin(\theta) = \frac{adjacent}{hypotenuse} & \sec(\theta) = \frac{h}{a} \\ \sin(\theta) = \frac{opposite}{adjacent} & \cot(\theta) = \frac{a}{o} \end{array}$$

**Exercise 41.** *Find the sides and all six trig ratios for the representative triangles* $45 - 45 - 90$ *and* $30 - 60 - 90$ *in each quadrant,* i.e. *when* $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{6}$, *etc. (See the last page of this section for all the trig ratios... all you'll have to do is figure out how the signs change.)*

**Exercise 42.** *Compute* $\sin(\theta)$ *for three* $\theta$ *you choose... then compute* $\cos(\theta - 90°)$ *for the same three* $\theta$ *values. What is their relationship? Why?*

**Exercise 43.** *Find* $\sin(30°)$.

**Exercise 44.** *Find* $\sin(45°)$.

**Exercise 45.** *Find* $\cos(270°)$.

**Exercise 46.** *Find* $\tan(13°)$.

**Exercise 47.** *Find* $\sin(180°)$.

**Exercise 48.** *Find* $\sin(330°)$.

**Exercise 49.** *Find* $\cos(30°)$.

**Exercise 50.** *Find* $\cos(\pi)$.

**Exercise 51.** *Find* $\sin(\frac{\pi}{6})$.

**Exercise 52.** *Find* $\cos(\frac{\pi}{2})$.

**Exercise 53.** *Find* $\tan(\frac{5\pi}{6})$.

**Exercise 54.** *Find* $\cos(\frac{\pi}{3})$.

There are functions that invert the regular trig functions. Instead of taking in an angle and returning a ratio of sides, they take in a ratio of sides and return an angle:

$$\arcsin(\frac{opposite}{hypotenuse}) = \theta$$
$$\arccos(\frac{adjacent}{hypotenuse}) = \theta$$
$$\arctan(\frac{opposite}{adjacent}) = \theta$$

**Exercise 55.** $\arccos(\frac{1}{2}) = \theta$

**Exercise 56.** $\arccos(\frac{2\sqrt{2}}{2}) = \theta$

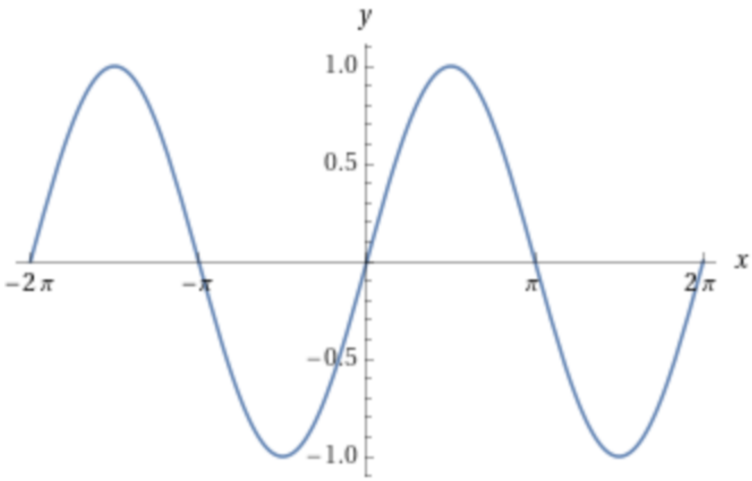
**Exercise 57.** $\arcsin(\frac{\sqrt{3}}{2}) = \theta$

**Exercise 58.** $\arctan(1) = \theta$

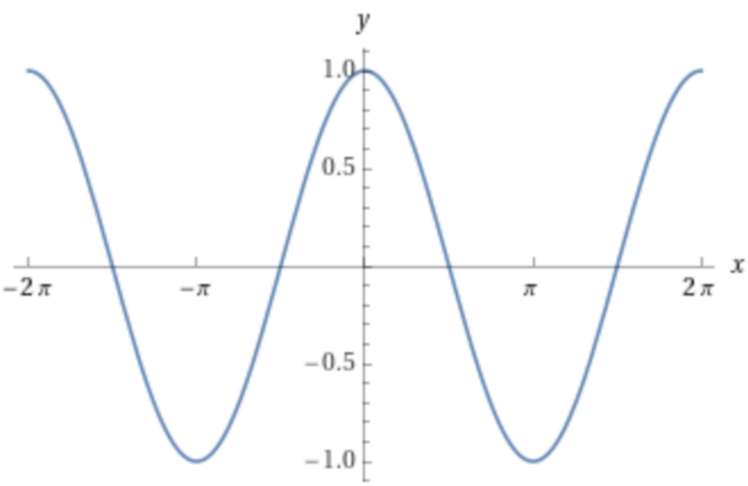
**Exercise 59.** $\arcsin(-1) = \theta$

## 3.2 Thurs., Mar. 21: Plots of trig functions

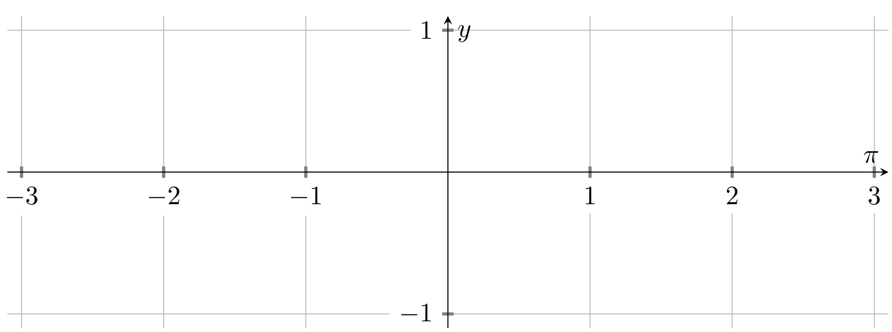
The plot of $y = sin(x)$ is:



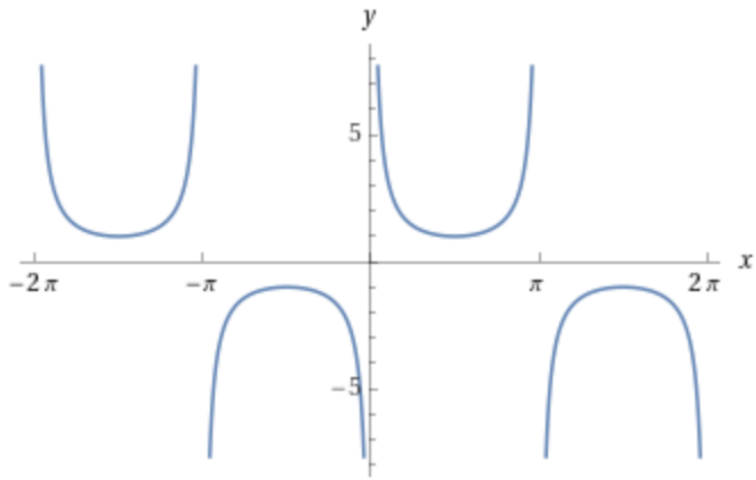

The plot of $y = cos(x)$ is:



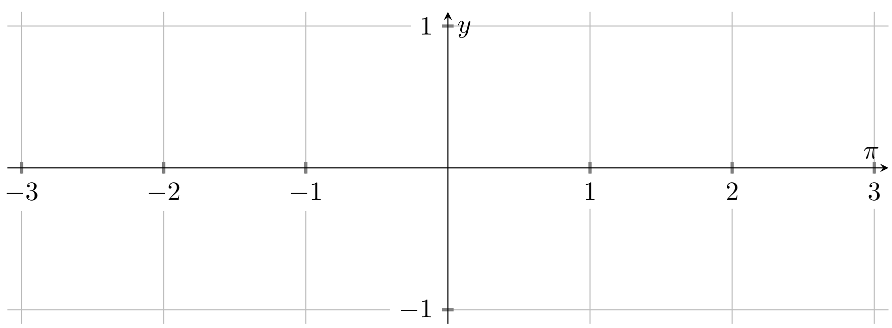

**Exercise 60.** *What is the plot of $y = tan(x)$? (Hint: use simple 'choose x, find y' but choose points strategically. You can use a calculator to evaluate the trig functions but don't use a graphing calculator.)*



**Exercise 61.** *I've given the plot of $y = csc(x)$ below. Based on what we know about these two functions, without doing any computation, what is the plot of $y = sec(x)$?*

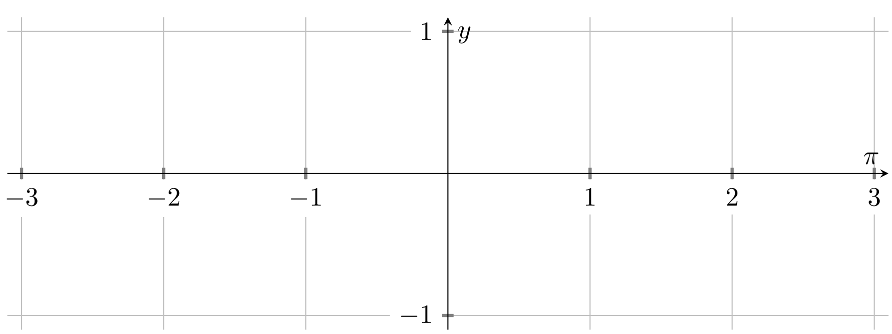



Now your turn!

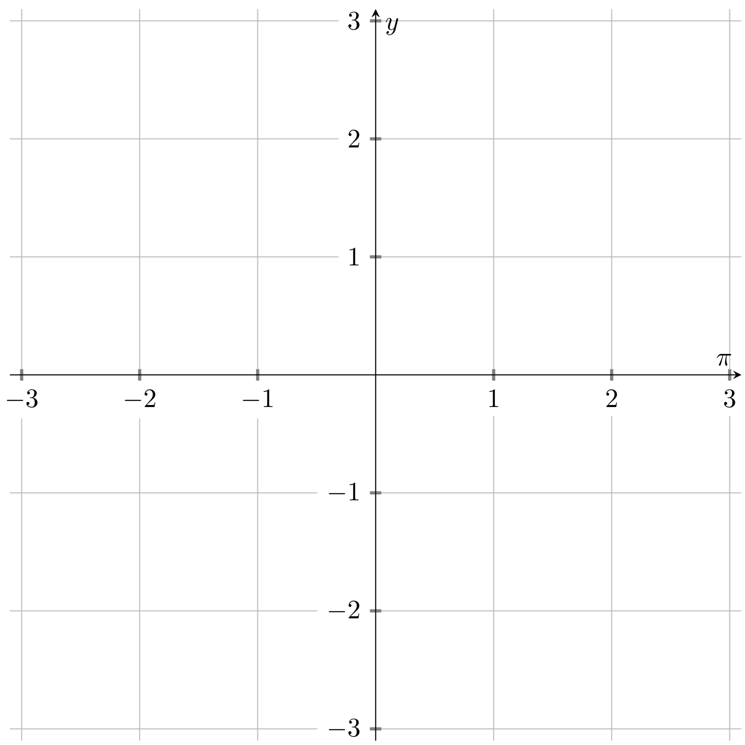



## 3.3 Fri., Mar. 22: Dilation and shifting of the trig functions!

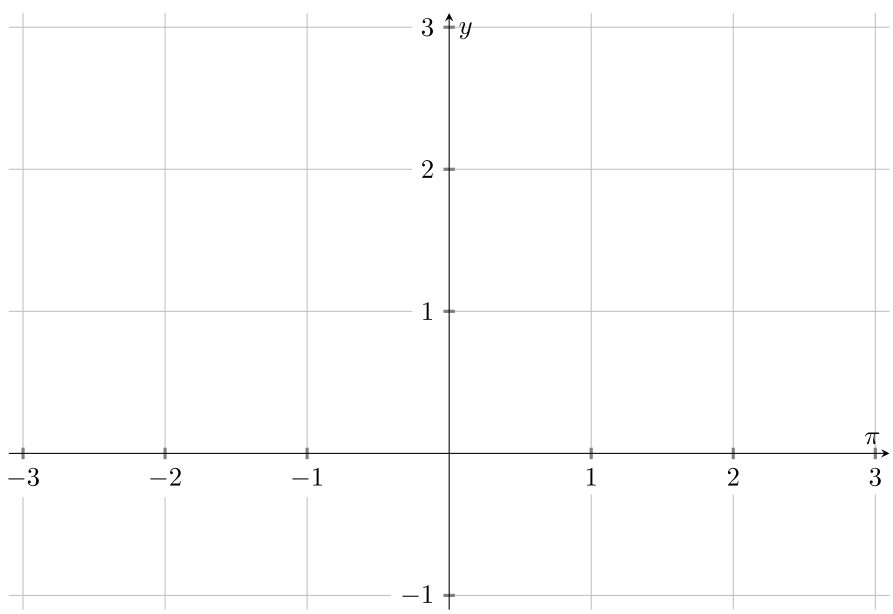
**Exercise 62.** *What is the plot of $y = sin(\frac{x}{3})$?*



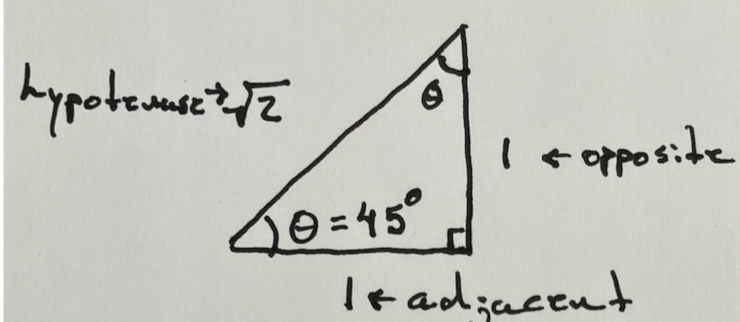
**Exercise 63.** *What is the plot of $y = 3sin(x)$?*

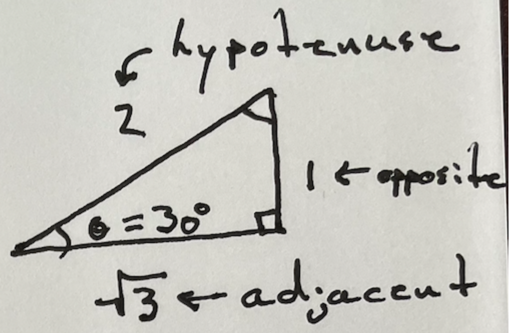



**Exercise 64.** *What is the plot of $y = sin(x) + 2$?*



Here are the values for several common trig functions as a reference:




So:

$$\sin(45^\circ) = \frac{o}{h} = \frac{1}{\sqrt{2}} = \blacksquare\,\frac{\sqrt{2}}{2} = .71$$

$$\cos(45^\circ) = \frac{a}{h} = \frac{1}{\sqrt{2}} = \blacksquare\,\frac{\sqrt{2}}{2} = .71$$

$$\tan(45^\circ) = \frac{o}{a} = \frac{1}{1} = 1$$

$$\sin(30^\circ) = \frac{o}{h} = \frac{1}{2} = .5$$

$$\cos(30^\circ) = \frac{a}{h} = \frac{\sqrt{3}}{2} = .87$$

$$\tan(30^\circ) = \frac{o}{a} = \frac{1}{\sqrt{3}} = \blacksquare\,\frac{\sqrt{3}}{3} = .58$$

$\blacksquare$ Note: this is a simplification to eliminate $\sqrt{\phantom{x}}$ in denominator:

$$\text{b/c}\quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$$