

# Running Lecture Outline: Calculus of Variations

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## Purpose of class

Physics is local. This means that things tend to change other things around them and do so in predictable, if very complicated, ways. Calculus is a rigorous way of thinking about such changes. By giving engineers, economists, and you—if you want to know—the ability to predict and control physical systems, calculus, more than biology or the social sciences, is responsible for creating the modern world of the last 300 years. AI now accomplishes many of these goals of prediction and simulation, but it works together with calculus and understanding both is important. On a device as accessible as your laptop computer, you can simulate and predict non-trivial systems and even change those systems if you want.

Calculus also develops important ideas about functions and numbers, and as a well-constructed system of thought it is useful in developing your own ability to think about things consistently and energetically, whatever problems you choose to put those skills towards!

## Grades

There will be a weekly problem set. Problem sets will come out on the first day of class in a given week and be due the first class day of the following week. We will structure our in-class time around learning to do the exercises in these problem sets together. This will be the only work that is taken for a grade. But the problems will be difficult to work without practicing in class, so attendance is strongly encouraged. There will be 12 problem sets and I will drop 2. The grading will be on a curve so that  $grade = \frac{\% correct}{\% max correct}$ . I will also grade by effort if you prefer in this manner:

A	Only attainable through > 90% correct answers on the curve
B	Strong, honest effort no matter % of problems correct
C	Very little effort
F/0	No effort/not turned in

## Exceptions and other policies

Late homework can be turned in with a letter grade penalty per week and we'll find a way to work with any reasonable medical/personal issues on the honor system in a no-stress way that emphasizes learning rather than rules. I'm very open to suggestions on course content and pace. If you feel that the class is not working for you, we can try *any* combination of online/self-paced instruction or change the way class works as long as some reasonable work and learning is happening. Bring these issues to me and KAPS faculty/administration early and often. My school e-mail is [theise@kapschool.org](mailto:theise@kapschool.org).

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