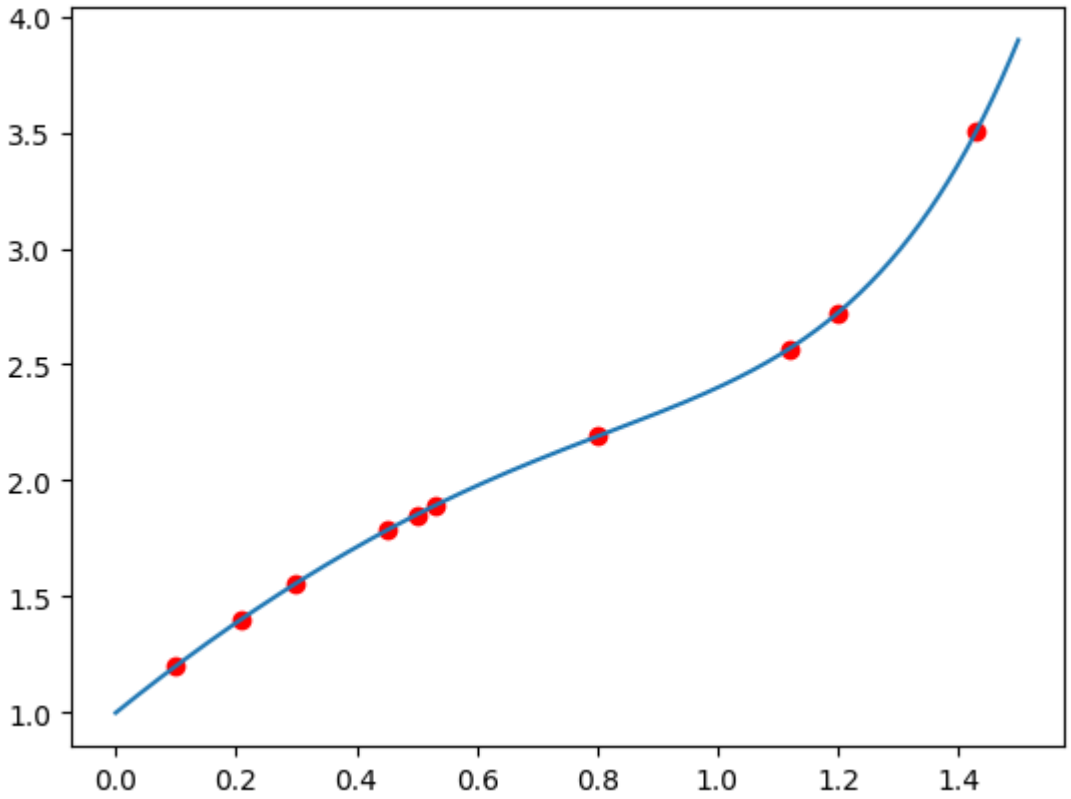
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [35]:
```
xx = np.array([0.1, 0.21, 0.3, 0.45, 0.5, 0.53, 0.8, 1.12, 1.2, 1.43])

extreme_xx = np.linspace(0.0, 1.5, 1000)
```

In [36]:
```
def f(x):
    return 1.0 + 2.1*x - 0.9*x**2 + 0.6*x**3 - 1.1*x**4 + 0.7*x**5

extreme_yy = f(extreme_xx)
yy = f(xx)

plt.plot(extreme_xx, extreme_yy)
plt.scatter(xx, yy, c='r')
```

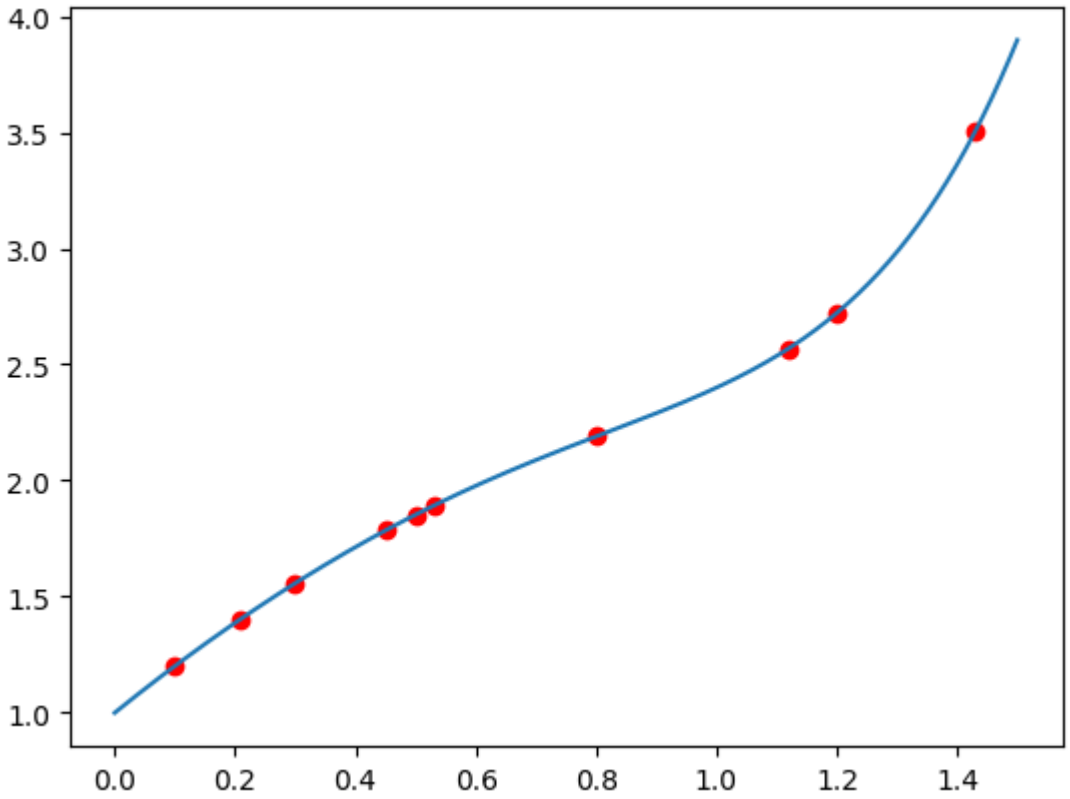
Out[36]: <matplotlib.collections.PathCollection at 0x7efde0b1b9a0>



In [38]:
```
V = np.zeros((10,10))
for k in [0,1,2,3,4,5,6,7,8,9]:
    V[:,k] = xx**k

cc = np.linalg.solve(V, yy)

print('cc=', cc)

def f2(x):
    return cc[0] + cc[1]*x + cc[2]*x**2 + cc[3]*x**3  + cc[4]*x**4 + cc[
```

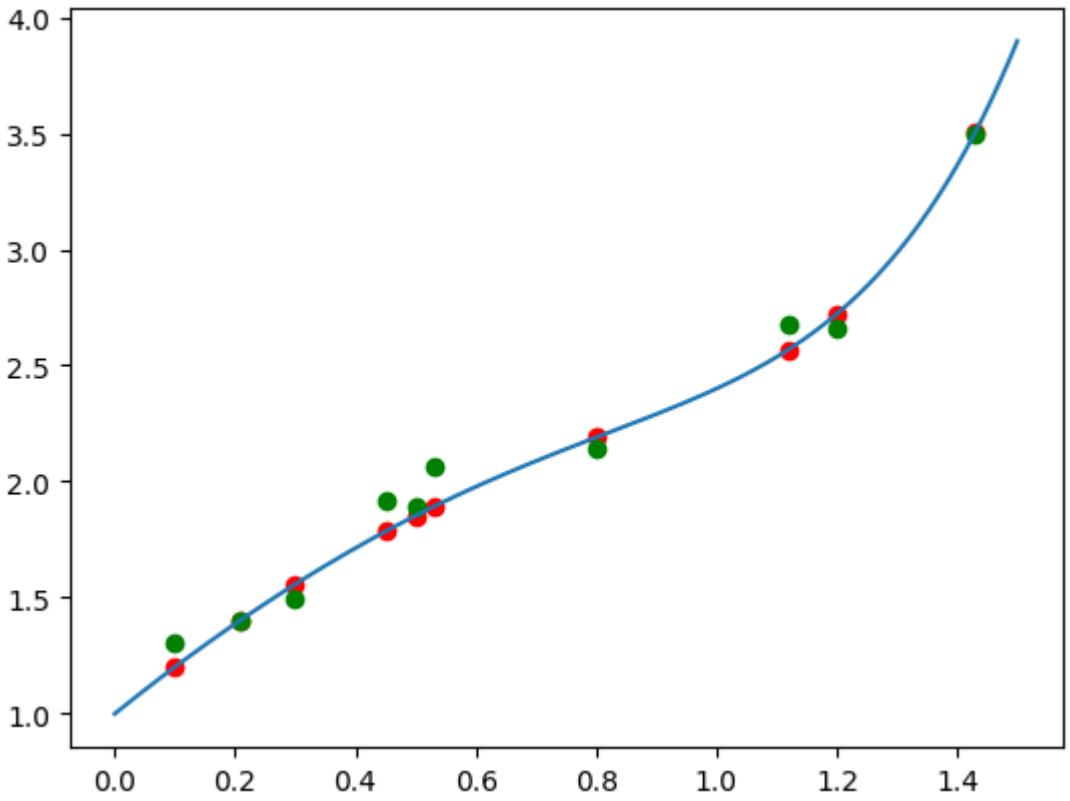
```
cc= [ 1.00000000e+00  2.10000000e+00 -9.00000000e-01  6.00000000e-01
 -1.10000000e+00  7.00000003e-01 -3.38567659e-09  2.39936417e-09
 -9.31924761e-10  1.52098872e-10]
```

Out[38]: <matplotlib.collections.PathCollection at 0x7efde0a4bd00>



In [51]:
```
noise = 0.1*np.random.randn(10)
yy_noisy = yy + noise

plt.plot(extreme_xx, extreme_yy)
plt.scatter(xx, yy, c='r')
plt.scatter(xx, yy_noisy, c='g')
```

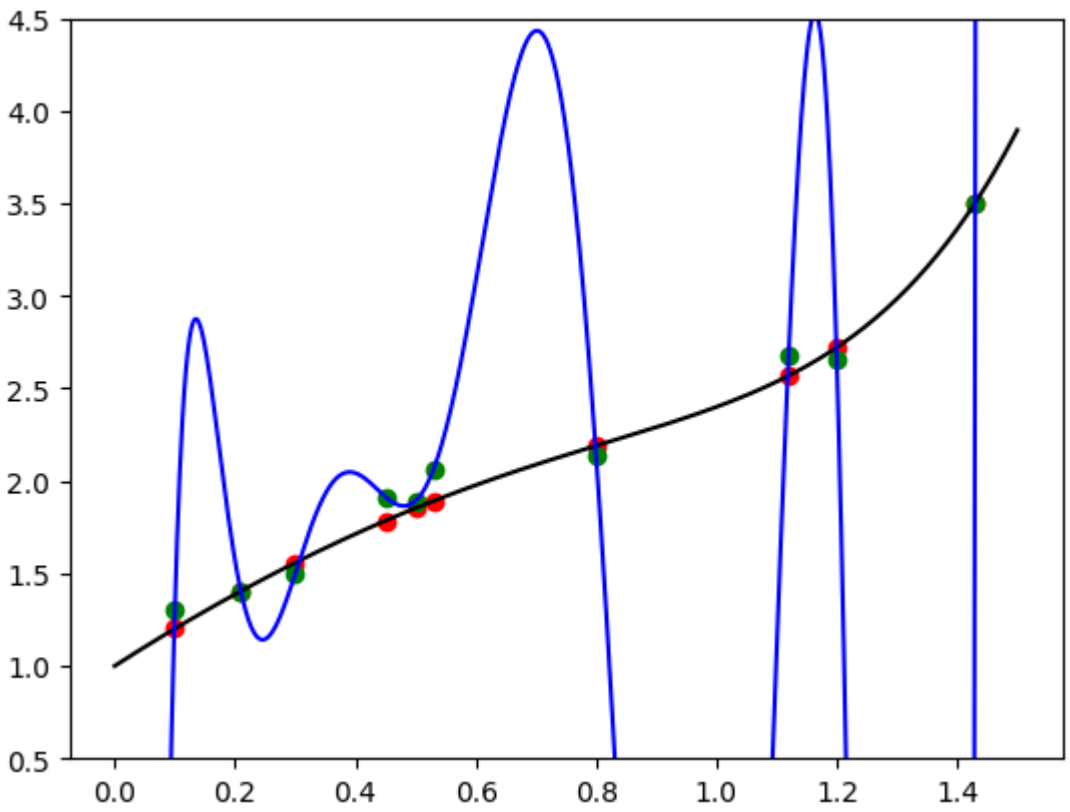
Out[51]: <matplotlib.collections.PathCollection at 0x7efde09e4ac0>



In [52]:
```
cc_noisy = np.linalg.solve(V, yy_noisy)

print('cc=', cc)
print('cc_noisy=', cc_noisy)

def f2(x):
    return cc_noisy[0] + cc_noisy[1]*x + cc_noisy[2]*x**2 + cc_noisy[3]*

extreme_yy2 = f2(extreme_xx)
yy2 = f2(xx)

plt.plot(extreme_xx, extreme_yy, 'k')
plt.scatter(xx, yy, c='r')
plt.scatter(xx, yy2, c='g')
plt.plot(extreme_xx, extreme_yy2, c='b')
plt.ylim(0.5, 4.5)
```

```
cc= [ 1.00000000e+00  2.10000000e+00 -9.00000000e-01  6.00000000e-01
 -1.10000000e+00  7.00000003e-01 -3.38567659e-09  2.39936417e-09
 -9.31924761e-10  1.52098872e-10]
cc_noisy= [-7.07813209e+01  1.85863121e+03 -1.87324042e+04  9.90159806
e+04
 -3.06554278e+05  5.82353155e+05 -6.84179302e+05  4.82721533e+05
 -1.86823035e+05  3.04049469e+04]
```

Out[52]: (0.5, 4.5)



In [48]:
```
ss = np.linalg.svd(V)[1]
print('ss=', ss)
```

```
ss= [3.64554098e+01 4.18605771e+00 2.14387431e+00 5.04345154e-01
 1.19949839e-01 2.06993876e-02 4.20986816e-03 2.79460716e-04
 1.26120964e-05 9.41772881e-08]
```

In [63]:
```
alpha = 0.0001

cc_noisy = np.linalg.solve(V.T @ V + alpha*np.eye(10), V.T @ yy_noisy)

print('cc=', cc)
print('cc_noisy=', cc_noisy)

def f2(x):
    return cc_noisy[0] + cc_noisy[1]*x + cc_noisy[2]*x**2 + cc_noisy[3]*

extreme_yy2 = f2(extreme_xx)
yy2 = f2(xx)

plt.plot(extreme_xx, extreme_yy, 'k')
plt.scatter(xx, yy, c='r')
plt.scatter(xx, yy_noisy, c='g')
plt.plot(extreme_xx, extreme_yy2, c='b')
plt.ylim(0.5, 4.5)
```

```
cc= [ 1.00000000e+00  2.10000000e+00 -9.00000000e-01  6.00000000e-01
 -1.10000000e+00  7.00000003e-01 -3.38567659e-09  2.39936417e-09
 -9.31924761e-10  1.52098872e-10]
cc_noisy= [ 1.14852333  0.77162759  3.63045943 -3.5821461  -2.70143427
1.70497761
  3.2970558   0.29204377 -3.59333633  1.45816985]
```

Out[63]: (0.5, 4.5)