5 Probability

5.1 Mon., Mar. 25:

What is probability? There's an interesting philosophical question in why we have *probability* at all. If the universe is just physical objects moving in predictable ways, why is there uncertainty? We can think about a few situations that might give rise to probability:

- Complexity *i.e.* two-joint pendulum
- Lack of knowledge
- Inherent randomness *i.e.* sub-atomic
- Prediction

We begin thinking about probability by examining the number of *ways something can happen*.

5.1.1 Permutations with repetition

If we are allowed to make a choice several times, and continue choosing things we've chosen in the past, we call this a "permutation where repetition is allowed". Think of choosing the combination for a padlock. We may have three numbers, each ranging from 0 to 9 that we are allowed to use in setting our combination. Here the ways we can choose is given as:

$$10 \ \cdot 10 \ \cdot 10 \ \cdot 10 = 10,000 \tag{1}$$

So if we choose from n things k times, the number of ways we can make this selection is:

$$n \cdot n \cdot n ... = n^k \tag{2}$$

What if we don't allow repetition? Or if the pool of available things to be selected is depleted with each selection? Then we would have:

$$(n-1) \cdot (n-2) \cdot ...(n-r) = \frac{n!}{n-k!} =_n P_k \tag{3}$$

Exercise 1. *How many ways can you set a locker room combination lock where there are three two-digit numbers that can each be chosen from 0 to 99?*

In this case, the order the numbers are chosen in matters... because 21-11-57 is different from 11-21-57. But what if we're choosing teammates for a soccer match where the team "Ralph, Spud, Skeeter" is no different from the team "Spud, Skeeter, Ralph"?

This is expressed as a "combination" and the formula is:

$$\binom{n}{k} =_n C_k = \frac{n!}{k!(n-k)!} \tag{4}$$

Why? Let's derive this...

Exercise 2. *Write down all ways of combining A, B, and C in groups of 2 where repetition is not allowed and order matters. Then cross out all those that have the same combination of letters. What is the ratio of the new to the original number of groups?*

AB	*AC*
BA	*BC*
CA	*CB*

So we can take number of permutations, and divide by the factorial of the number of groups we have to get the number of *unique* combinations.