

### 3 Trigonometry and Euclid

#### 3.1 Representative triangles

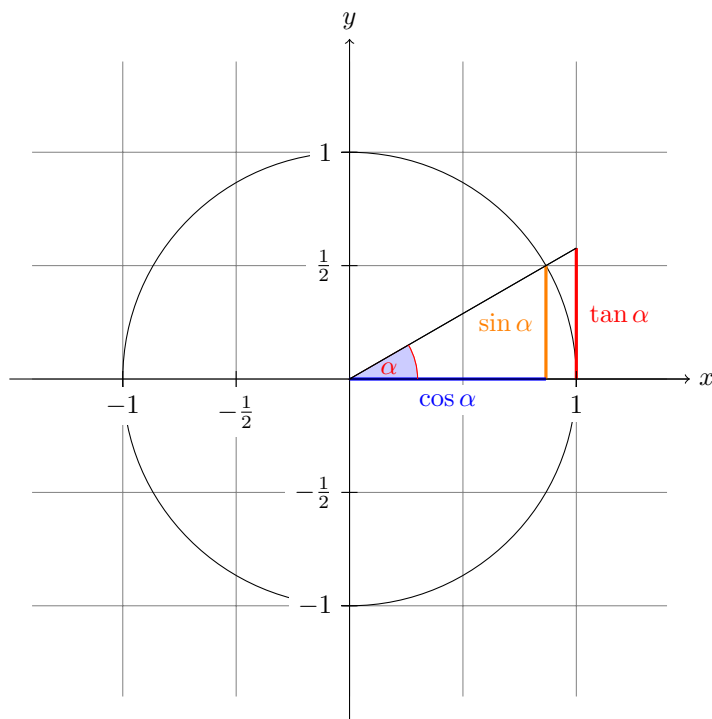
In calc we introduce radian measure and stop using degrees, to simplify our computations... because as we learned from deriving  $\pi$ , there are 6.28 radius lengths in a circle and so we have the formula:

$$s = 2\pi r \quad (41)$$

So  $\frac{2\pi}{2}$  is a half revolution, or 180 degrees. It's also useful to know that  $\frac{\pi}{6} = 30^\circ$ ,  $\frac{\pi}{4} = 45^\circ$ ,  $\frac{\pi}{2} = 90^\circ$ . We can convert from degrees to radians easily by observing:

$$360^\circ = 2\pi \text{ rad} \quad (42)$$

$\frac{\pi}{180^\circ}$  is the conversion you most often want to multiply by.



The trig functions and functions for their reciprocals are:

$$\begin{aligned} \sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc(\theta) &= \frac{h}{o} \\ \cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec(\theta) &= \frac{h}{a} \\ \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} & \cot(\theta) &= \frac{a}{o} \end{aligned}$$

**Exercise 41.** Find the sides and all six trig ratios for the representative triangles 45 – 45 – 90 and 30 – 60 – 90 in each quadrant, i.e. when  $\theta = \frac{3\pi}{4}$ ,  $\theta = \frac{5\pi}{6}$ , etc. (See the last page of this section for all the trig ratios... all you'll have to do is figure out how the signs change.)

**Exercise 42.** Compute  $\sin(\theta)$  for three  $\theta$  you choose... then compute  $\cos(\theta - 90^\circ)$  for the same three  $\theta$  values. What is their relationship? Why?

**Exercise 43.** Find  $\sin(30^\circ)$ .

**Exercise 44.** Find  $\sin(45^\circ)$ .

**Exercise 45.** Find  $\cos(270^\circ)$ .

**Exercise 46.** Find  $\tan(13^\circ)$ .

**Exercise 47.** Find  $\sin(180^\circ)$ .

**Exercise 48.** Find  $\sin(330^\circ)$ .

**Exercise 49.** Find  $\cos(30^\circ)$ .

**Exercise 50.** Find  $\cos(\pi)$ .

**Exercise 51.** Find  $\sin(\frac{\pi}{6})$ .

**Exercise 52.** Find  $\cos(\frac{\pi}{2})$ .

**Exercise 53.** Find  $\tan(\frac{5\pi}{6})$ .

**Exercise 54.** Find  $\cos(\frac{\pi}{3})$ .

There are functions that invert the regular trig functions. Instead of taking in an angle and returning a ratio of sides, they take in a ratio of sides and return an angle:

$$\begin{aligned}\arcsin\left(\frac{\textit{opposite}}{\textit{hypotenuse}}\right) &= \theta \\ \arccos\left(\frac{\textit{adjacent}}{\textit{hypotenuse}}\right) &= \theta \\ \arctan\left(\frac{\textit{opposite}}{\textit{adjacent}}\right) &= \theta\end{aligned}$$

**Exercise 55.**  $\arccos(\frac{1}{2}) = \theta$

**Exercise 56.**  $\arccos(\frac{2\sqrt{2}}{2}) = \theta$

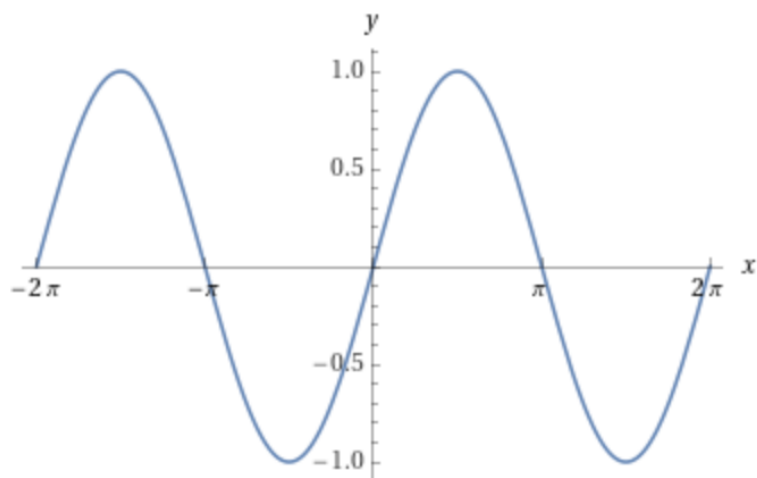
**Exercise 57.**  $\arcsin(\frac{\sqrt{3}}{2}) = \theta$

**Exercise 58.**  $\arctan(1) = \theta$

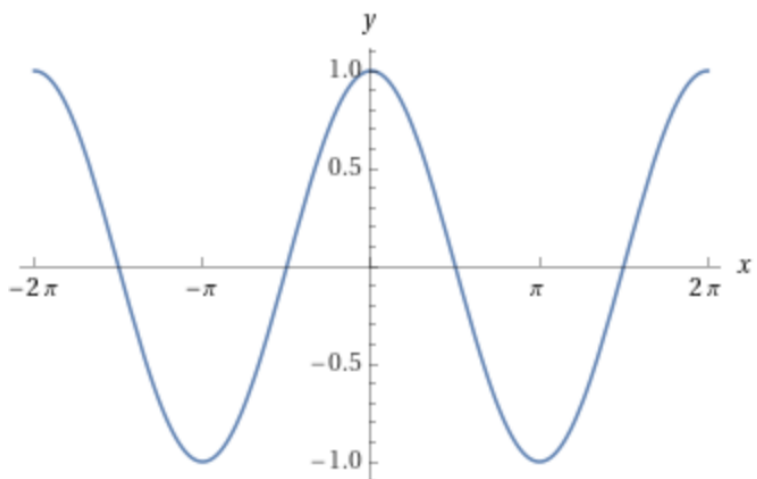
**Exercise 59.**  $\arcsin(-1) = \theta$

## 3.2 Plots of trig functions

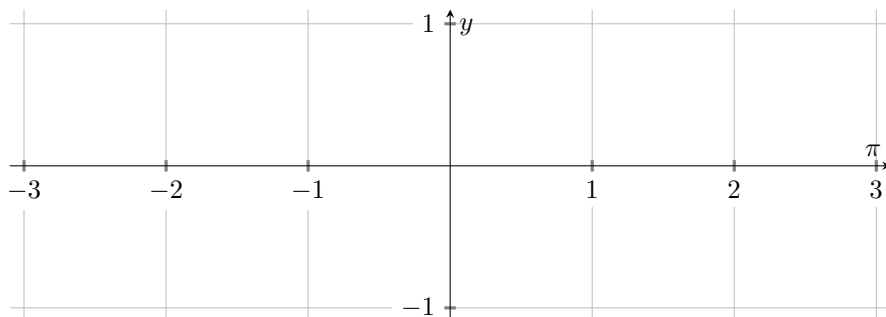
The plot of  $y = \sin(x)$  is:



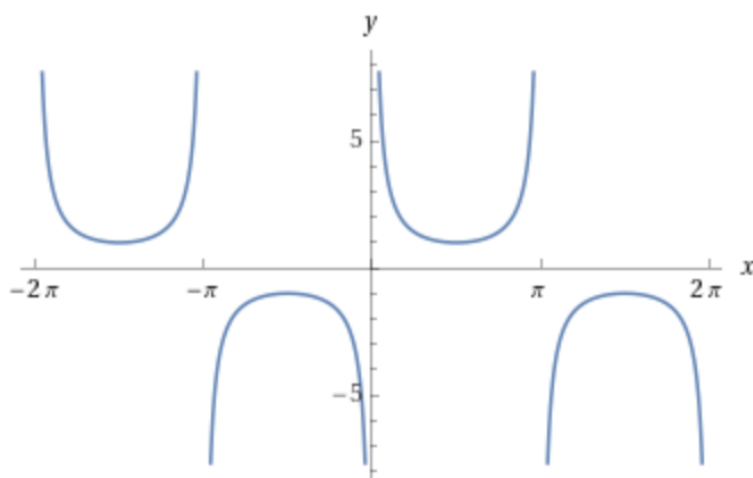
The plot of  $y = \cos(x)$  is:



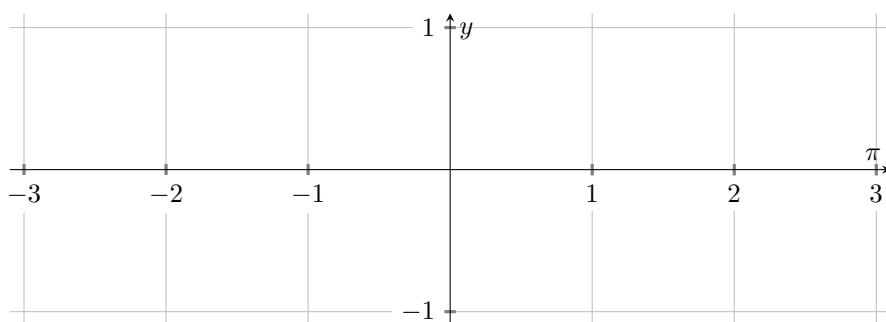
**Exercise 60.** What is the plot of  $y = \tan(x)$ ? (Hint: use simple 'choose  $x$ , find  $y$ ' but choose points strategically. You can use a calculator to evaluate the trig functions but don't use a graphing calculator.)



**Exercise 61.** I've given the plot of  $y = \csc(x)$  below. Based on what we know about these two functions, without doing any computation, what is the plot of  $y = \sec(x)$ ?

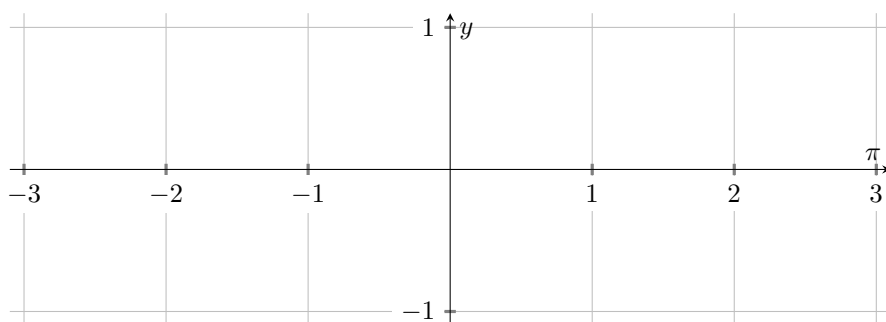


Now your turn!

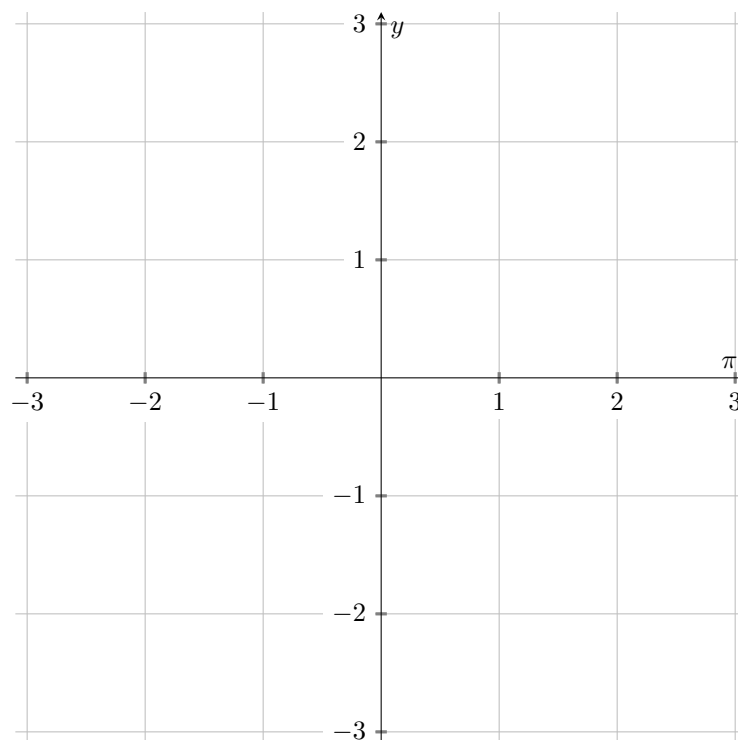


### 3.3 Dilation and shifting of the trig functions!

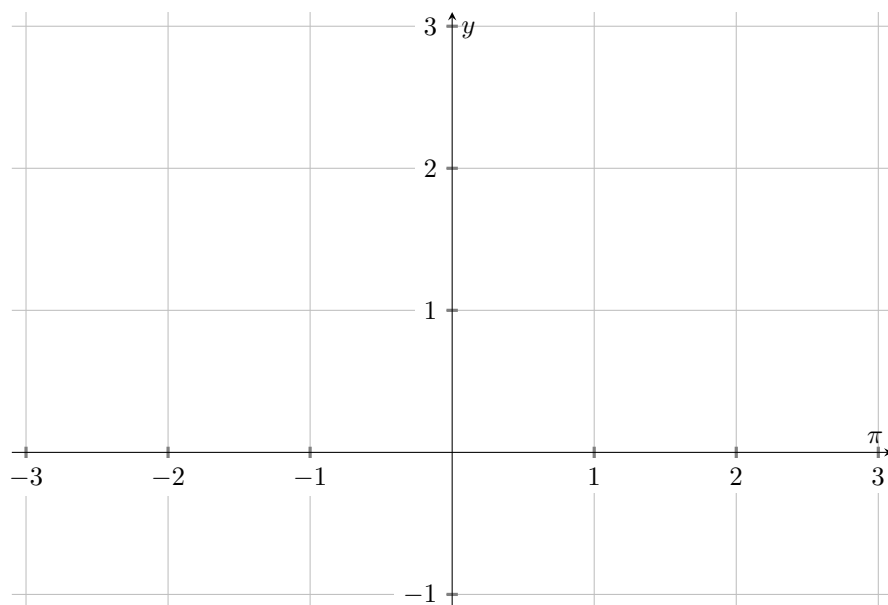
**Exercise 62.** What is the plot of  $y = \sin(\frac{x}{3})$ ?



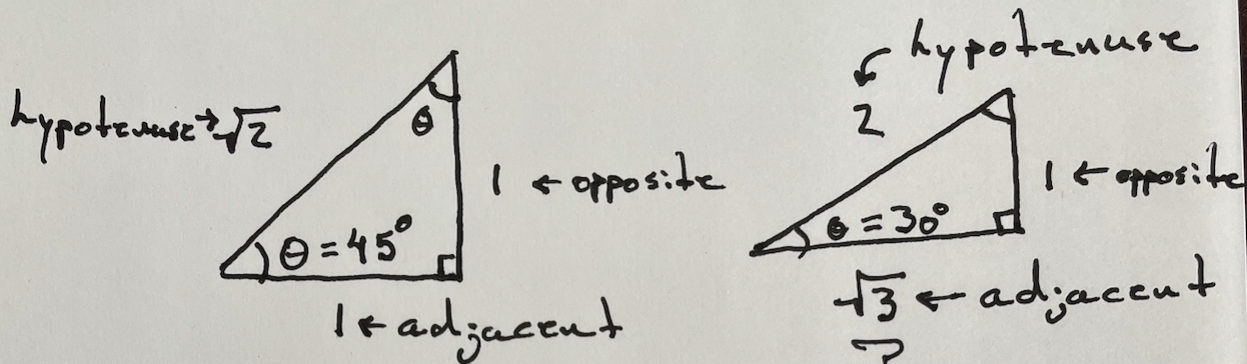
**Exercise 63.** What is the plot of  $y = 3\sin(x)$ ?



**Exercise 64.** What is the plot of  $y = \sin(x) + 2$ ?



Here are the values for several common trig functions as a reference:



$$\text{so: } \sin(45^\circ) = \frac{o}{h} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = .71$$

$$\cos(45^\circ) = \frac{a}{h} = \frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2} = .71$$

$$\tan(45^\circ) = \frac{o}{a} = \frac{1}{1} = 1$$

$$\sin(30^\circ) = \frac{o}{h} = \frac{1}{2} = .5$$

$$\cos(30^\circ) = \frac{a}{h} = \frac{\sqrt{3}}{2} = .87$$

$$\tan(30^\circ) = \frac{o}{a} = \frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} = .58$$

Note: this is a simplification to eliminate  $\sqrt{\quad}$  in denominator:

$$\text{b/c } \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$