# An Online Simplified Rotor Resistance Estimator for Induction Motors

Godpromesse Kenné, Rostand Sorel Simo, Françoise Lamnabhi-Lagarrigue, Amir Arzandé, and Jean Claude Vannier

***Abstract*—This brief presents an adaptive variable structure identifier that provides finite time convergent estimate of the induction motor rotor resistance under feasible persistent of excitation condition. The proposed rotor resistance scheme is based on the standard dynamic model of induction motor expressed in a fixed reference frame attached to the stator. The available variables are the rotor speed, the stator currents and voltages. Experiments show that the proposed method achieved very good estimation of the rotor resistance which is subjected to online large variation during operation of the induction motor. Also, the proposed online simplified rotor resistance estimator is robust with respect to the variation of the stator resistance, measurement noise, modeling errors, discretization effects and parameter uncertainties. Important advantages of the proposed algorithm include that it is an online method (the value of $R_r$ can be continuously updated) and it is very simple to implement in real-time (this feature distinguishes the proposed identifier from the known ones).**

***Index Terms*—Equivalent injection term, nonlinear observer, online parameter estimation.**

## I. Introduction

THE FACT that the induction motor (IM) is a multivariable, nonlinear and highly coupled process with time-varying parameters, has motivated a lot of work in the control community during the last decade [1]–[14]. The popular alternative method in many drive applications is the field-oriented control (F.O.C.) which provides a means to obtain high-performance control of an IM. But this F.O.C. methodology requires knowledge of the rotor fluxes which are not usually measured [13]. Traditionally, observers are used to estimate the rotor fluxes. However, the flux observers used in the currently IM control rely on a good knowledge of the rotor resistance. It is well known in literature (e.g., see [15], [16], [1], [2], [4], [5]) that the rotor resistance and the stator resistance may vary up to 100% and 50% of their nominal values, respectively, during operation of the IM due to rotor heating. Standard methods for the estimation of IM parameters include the blocked rotor test, the no-load test and the standstill frequency response test. But, these methods cannot be used online during normal operation of the machine. The most natural solution is to online identify the time-varying parameters. Several papers addressed the problem of the online IM parameter estimation [1]–[13], [10], [14]. In [3], [6], [7], [10], [14], the problem of IM time-varying parameter has been studied but experiments have not been carried out to verify the effectiveness of the approaches. In [8], [9], and [13], interesting algorithms for IM parameter estimation are proposed using least square technique but more sophisticated filters are required when PWM inverter is used. Moreover, online variation of the IM parameters was not investigated. In [12], a method for rotor resistance estimation for indirect field oriented control of IM based on reactive power reference model is presented under motoring and generating modes. Sensitivity of the algorithm to errors in other machines parameters is investigated but without variation of the rotor resistance. In [11], a robust nested sliding mode regulation with application to rotor flux modulus and rotor speed of IM with unknown load torque has been introduced. The variation of the stator/rotor resistance has been investigated but the estimation of these parameters was not achieved. Remarkable results have been obtained by the authors of [17] in deriving rotor resistance and load torque estimators suitable for online rotor speed and flux adaptive control. The main drawback of this approach is that the rotor resistance estimator is based on a simplify model of IM which requires the rotor speed to vary slowly.

In this brief, the rotor resistance $R_r$ is estimated and its online implementation does not require the assumption of slowly variation of the rotor speed. The effect of the stator resistance $R_s$ variation on the estimation of $R_r$ is also investigated. This brief is organized as follows.

In Section II, the IM mathematical model is recalled. The design procedure of the proposed rotor resistance identifier is described in Section III and the proof of the finite time convergent estimate to its nominal value is achieved under feasible persistent of excitation (P.E.) condition. Experimental results of online implementation are reported in Section IV and some concluding remarks are given in Section V.

## II. Induction Motor Model

According to the classical $a$-$b$ axes transformation with a fixed reference frame attached to the stator, and assuming linear magnetic behavior, the dynamic of a balanced IM is given by the following fifth-order nonlinear system [1], [18]:

$$\begin{aligned}\frac{di_{sa}}{dt} &= -\frac{R_s}{\sigma L_s}i_{sa} - \beta M\frac{R_r}{L_r}i_{sa} + \beta\frac{R_r}{L_r}\lambda_{ra} + n_p\beta\omega\lambda_{rb}\\ &\quad + \frac{1}{\sigma L_s}v_{sa} \qquad (1)\\ \frac{di_{sb}}{dt} &= -\frac{R_s}{\sigma L_s}i_{sb} - \beta M\frac{R_r}{L_r}i_{sb} + \beta\frac{R_r}{L_r}\lambda_{rb} - n_p\beta\omega\lambda_{ra}\end{aligned}$$

Manuscript received June 12, 2009; accepted September 14, 2009. Manuscript received in final form September 29, 2009. First published November 03, 2009; current version published August 25, 2010. Recommended by Associate Editor L. Dessaint.

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Digital Object Identifier 10.1109/TCST.2009.2033790

$$+\frac{1}{\sigma L_s}v_{sb} \tag{2}$$

$$\frac{d\lambda_{ra}}{dt} = -\frac{R_r}{L_r}\lambda_{ra} - n_p\omega\lambda_{rb} + \frac{R_r}{L_r}Mi_{sa} \tag{3}$$

$$\frac{d\lambda_{rb}}{dt} = -\frac{R_r}{L_r}\lambda_{rb} + n_p\omega\lambda_{ra} + \frac{R_r}{L_r}Mi_{sb} \tag{4}$$

$$\frac{d\omega}{dt} = \frac{T_e}{m} - \frac{T_L}{m}. \tag{5}$$

In the above equations, the state variables are rotor speed $\omega$, rotor flux $(\lambda_{ra}, \lambda_{rb})$, and stator currents $(i_{sa}, i_{sb})$; the control inputs are stator voltages $(v_{sa}, v_{sb})$; the measured variables are $(\omega, i_{sa}, i_{sb})$ while $(\lambda_{ra}, \lambda_{rb})$ are not measured; the parameters are the external load torque $T_L$, total motor, and load moment of inertia $m$, rotor and stator winding resistances $(R_r, R_s)$, inductances $(L_r, L_s)$ and mutual inductance $M$; $T_e$ is the electromagnetic torque and $n_p$ is the number of pole pairs. To simplify the notations, we use $\sigma = 1 - (M^2)/(L_sL_r)$ (leakage parameter) and the constant $\beta = (M)/(\sigma L_s L_r)$.

The following assumptions will be considered until further notice:

(i) stator current and voltage are bounded signals;

(ii) rotor resistance $R_r \in \Omega_{R_r}$, where $\Omega_{R_r}$ is a compact set of $\mathbb{R}$.

Our goal is to design a rotor resistance estimation algorithm assuming that there exists a control input which can stabilizes the motor in a wide range of operating points.

## III. Rotor Resistance Adaptation Algorithm

To derive an online estimate of the rotor resistance, let us consider the following observer ($K > 0$ is a constant designed parameter):

$$\frac{d\hat{i}_{sa}}{dt} = -\frac{R_s}{\sigma L_s}i_{sa} - \beta M\frac{\hat{R}_r}{L_r}i_{sa} + \beta\frac{\hat{R}_r}{L_r}\hat{\lambda}_{ra} + n_p\beta\omega\hat{\lambda}_{rb} + \frac{1}{\sigma L_s}v_{sa} + K\text{sign}(i_{sa} - \hat{i}_{sa}) \tag{6}$$

$$\frac{d\hat{i}_{sb}}{dt} = -\frac{R_s}{\sigma L_s}i_{sb} - \beta M\frac{\hat{R}_r}{L_r}i_{sb} + \beta\frac{\hat{R}_r}{L_r}\hat{\lambda}_{rb} - n_p\beta\omega\hat{\lambda}_{ra} + \frac{1}{\sigma L_s}v_{sb} + K\text{sign}(i_{sb} - \hat{i}_{sb}) \tag{7}$$

$$\frac{d\hat{\lambda}_{ra}}{dt} = -\frac{\hat{R}_r}{L_r}\hat{\lambda}_{ra} - n_p\omega\hat{\lambda}_{rb} + \frac{\hat{R}_r}{L_r}Mi_{sa} + u_a \tag{8}$$

$$\frac{d\hat{\lambda}_{rb}}{dt} = -\frac{\hat{R}_r}{L_r}\hat{\lambda}_{rb} + n_p\omega\hat{\lambda}_{ra} + \frac{\hat{R}_r}{L_r}Mi_{sb} + u_b \tag{9}$$

where $u_a$ and $u_b$ are additional signals yet to be designed and "sign" is the well known "sign" function. The estimated quantities are shown as $\hat{x}$ while the error quantities are shown as $\tilde{x} = x - \hat{x}$ (e.g., $\tilde{i}_s = i_s - \hat{i}_s$, $\tilde{\lambda}_r = \lambda_r - \hat{\lambda}_r$, $\tilde{R}_r = R_r - \hat{R}_r$). The dynamics of the observer error can be computed using (1)–(4) and (6)–(9) as

$$\frac{d\tilde{i}_{sa}}{dt} = -K\text{sign}(\tilde{i}_{sa}) + \frac{\beta}{L_r}(R_r\lambda_{ra} - \hat{R}_r\hat{\lambda}_{ra}) + \beta n_p\omega\tilde{\lambda}_{rb} - \frac{\beta}{L_r}Mi_{sa}\tilde{R}_r \tag{10}$$

$$\frac{d\tilde{i}_{sb}}{dt} = -K\text{sign}(\tilde{i}_{sb}) + \frac{\beta}{L_r}(R_r\lambda_{rb} - \hat{R}_r\hat{\lambda}_{rb}) - \beta n_p\omega\tilde{\lambda}_{ra} - \frac{\beta}{L_r}Mi_{sb}\tilde{R}_r \tag{11}$$

$$\frac{d\tilde{\lambda}_{ra}}{dt} = -\frac{1}{L_r}(R_r\lambda_{ra} - \hat{R}_r\hat{\lambda}_{ra}) - n_p\omega\tilde{\lambda}_{rb} + \frac{M}{L_r}i_{sa}\tilde{R}_r - u_a \tag{12}$$

$$\frac{d\tilde{\lambda}_{rb}}{dt} = -\frac{1}{L_r}(R_r\lambda_{rb} - \hat{R}_r\hat{\lambda}_{rb}) + n_p\omega\tilde{\lambda}_{ra} + \frac{M}{L_r}i_{sb}\tilde{R}_r - u_b. \tag{13}$$

The above associated error dynamics can be rewritten as

$$\frac{d\tilde{i}_{sa}}{dt} = -K\text{sign}(\tilde{i}_{sa}) + \beta n_p\omega\tilde{\lambda}_{rb} + R_r\frac{\beta}{L_r}\tilde{\lambda}_{ra} + \tilde{R}_r\frac{\beta}{L_r}(\hat{\lambda}_{ra} - Mi_{sa}) \tag{14}$$

$$\frac{d\tilde{i}_{sb}}{dt} = -K\text{sign}(\tilde{i}_{sb}) - \beta n_p\omega\tilde{\lambda}_{ra} + R_r\frac{\beta}{L_r}\tilde{\lambda}_{rb} + \tilde{R}_r\frac{\beta}{L_r}(\hat{\lambda}_{rb} - Mi_{sb}) \tag{15}$$

$$\frac{d\tilde{\lambda}_{ra}}{dt} = -u_a - \frac{R_r}{L_r}\tilde{\lambda}_{ra} - n_p\omega\tilde{\lambda}_{rb} - \frac{\tilde{R}_r}{L_r}(\hat{\lambda}_{ra} - Mi_{sa}) \tag{16}$$

$$\frac{d\tilde{\lambda}_{rb}}{dt} = -u_b - \frac{R_r}{L_r}\tilde{\lambda}_{rb} + n_p\omega\tilde{\lambda}_{ra} - \frac{\tilde{R}_r}{L_r}(\hat{\lambda}_{rb} - Mi_{sb}). \tag{17}$$

To achieve the design of the rotor resistance identifier the following additive assumption is required.

*Assumption (iii):* It is assumed that the following rotor resistance identifiability condition holds:

$$\|\lambda_r(t) - Mi_s(t)\| = \|P(t)\| \geq \delta > 0 \quad \forall t \geq 0. \tag{18}$$

*Remark 1:* The identifiability condition (18) can be replaced by the following persistency of excitation (P.E.) condition. There exists $\alpha > 0$, $T > 0$, $t_0 > 0$ such that $\forall t \geq 0$

$$\int_t^{t+T} P(s)P(s)^T ds \geq \alpha I > 0. \tag{19}$$

*Remark 2:* The persistency of excitation condition (19) is often satisfied when the IM is fed by PWM power inverter. This is the case of the control system considered in this work.

By considering the following Lyapunov candidate function:

$$V_1 = \frac{1}{2}\tilde{i}_{sa}^2 + \frac{1}{2}\tilde{i}_{sb}^2 \tag{20}$$

and computing its time-derivative along the trajectories of (14) and (15), we obtain

$$\dot{V}_1 = -K|\tilde{i}_{sa}| + R_r\frac{\beta}{L_r}\tilde{i}_{sa}\tilde{\lambda}_{ra} + \beta\omega n_p\tilde{i}_{sa}\tilde{\lambda}_{rb} + \tilde{R}_r\frac{\beta}{L_r}\tilde{i}_{sa}(\hat{\lambda}_{ra} - Mi_{sa}) - K|\tilde{i}_{sb}| + R_r\frac{\beta}{L_r}\tilde{i}_{sb}\tilde{\lambda}_{rb} - \beta\omega n_p\tilde{i}_{sb}\tilde{\lambda}_{ra} + \tilde{R}_r\frac{\beta}{L_r}\tilde{i}_{sb}(\hat{\lambda}_{rb} - Mi_{sb}). \tag{21}$$

From (21), by taking into account *assumptions (i) and (ii)*, the following inequalities hold:

$$\begin{aligned}\dot{V}_1 \leq & -|\tilde{i}_{sa}|\left\{K-\beta\left|\frac{R_r}{L_r}\tilde{\lambda}_{ra}\right.\right.\\ &\left.\left. +\omega n_p\tilde{\lambda}_{rb}+\frac{\tilde{R}_r}{L_r}(\hat{\lambda}_{ra}-Mi_{sa})\right|\right\}\\ &-|\tilde{i}_{sb}|\left\{K-\beta\left|\frac{R_r}{L_r}\tilde{\lambda}_{rb}-\omega n_p\tilde{\lambda}_{ra}\right.\right.\\ &\left.\left.+\frac{\tilde{R}_r}{L_r}(\hat{\lambda}_{rb}-Mi_{sb})\right|\right\}\\ \leq & -|\tilde{i}_{sa}|\left\{K-\beta\left[\frac{R_r}{L_r}|\tilde{\lambda}_{ra}|+\omega n_p|\tilde{\lambda}_{rb}|\right.\right.\\ &\left.\left.+\frac{|\tilde{R}_r|}{L_r}(|\hat{\lambda}_{ra}|+M|i_{sa}|)\right]\right\}\\ &-|\tilde{i}_{sb}|\left\{K-\beta\left[\frac{R_r}{L_r}|\tilde{\lambda}_{rb}|+\omega n_p|\tilde{\lambda}_{ra}|\right.\right.\\ &\left.\left.+\frac{|\tilde{R}_r|}{L_r}(|\hat{\lambda}_{rb}|+M|i_{sb}|)\right]\right\}. \end{aligned} \tag{22}$$

Assuming that the estimate $\hat{R}_r$ and $\hat{\lambda}_r$ are bounded,[1] let positive constants $\xi_a$ and $\xi_b$ be available such that

$$\begin{aligned}\frac{\xi_a}{\beta} &= \frac{R_r}{L_r}|\tilde{\lambda}_{ra}|_m+\omega n_p|\tilde{\lambda}_{rb}|_m\\ &\quad+\frac{|\tilde{R}_r|_m}{L_r}(|\hat{\lambda}_{ra}|_m+M|i_{sa}|_m)\\ \frac{\xi_b}{\beta} &= \frac{R_r}{L_r}|\tilde{\lambda}_{rb}|_m+\omega n_p|\tilde{\lambda}_{ra}|_m\\ &\quad+\frac{|\tilde{R}_r|_m}{L_r}(|\hat{\lambda}_{rb}|_m+M|i_{sb}|_m)\end{aligned} \tag{23}$$

where $|\cdot|_m$ denotes the maximum value of $|\cdot|$.

*Remark 3:* The values of the constants $\xi_a$ and $\xi_b$ can be evaluated for any given operating condition on the IM by using the nominal values of the rotor resistance and inductance (to compute the value of $(R_r)/(L_r)$) and the maximum admissible values of the rotor resistance and rotor flux estimation errors in transient period. Other offline methods can be exploited to evaluate the nominal value of the rotor time constant without using the nominal values of $R_r$ and $L_r$ in the case of squirrel IM.

By choosing

$$K > \sup(\xi_a, \xi_b) \tag{24}$$

the derivative of $V_1$ will be negative definite $\forall \tilde{i}_{sa} \neq 0$ and $\forall \tilde{i}_{sb} \neq 0$. Therefore, the observer errors $\tilde{i}_{sa}$ and $\tilde{i}_{sb}$ converge to 0 in finite time if $K$ is chosen such that condition (24) is satisfied.

[1]The proof of the boundness and convergence will given later.

We now consider the following quadratic function of the rotor flux observer error and rotor resistance estimation error:

$$V_2 = \frac{1}{2}\tilde{\lambda}_{ra}^2+\frac{1}{2}\tilde{\lambda}_{rb}^2+\frac{1}{2}\tilde{R}_r^2. \tag{25}$$

Its time-derivative along the trajectories of (16) and (17) yields

$$\begin{aligned}\dot{V}_2 =& -\frac{R_r}{L_r}\tilde{\lambda}_{ra}^2-\tilde{\lambda}_{ra}u_a+\frac{\tilde{R}_r}{L_r}\tilde{\lambda}_{ra}(Mi_{sa}-\hat{\lambda}_{ra})-\frac{R_r}{L_r}\tilde{\lambda}_{rb}^2-\tilde{\lambda}_{rb}u_b\\ &+\frac{\tilde{R}_r}{L_r}\tilde{\lambda}_{rb}(Mi_{sb}-\hat{\lambda}_{rb})+\tilde{R}_r\dot{\tilde{R}}_r.\end{aligned} \tag{26}$$

If we choose $u_a, u_b$, and $\dot{\tilde{R}}_r$ as follows ($k_{R_r} > 0$ is a designed parameter):

$$\begin{aligned}u_a &= \frac{\tilde{R}_r}{L_r}(Mi_{sa}-\hat{\lambda}_{ra})\\ u_b &= \frac{\tilde{R}_r}{L_r}(Mi_{sb}-\hat{\lambda}_{rb})\\ \dot{\tilde{R}}_r &= -k_{R_r}\mathrm{sign}(\tilde{R}_r).\end{aligned} \tag{27}$$

$\dot{V}_2$ becomes

$$\dot{V}_2 = -\frac{R_r}{L_r}(\tilde{\lambda}_{ra}^2+\tilde{\lambda}_{rb}^2)-k_{R_r}|\tilde{R}_r|. \tag{28}$$

Consequently, under P.E. (19) and if the are auxiliaries variables $u_a$, $u_b$, and $\dot{\tilde{R}}_r$ are chosen as in (27), $\dot{V}_2$ will be negative definite $\forall\tilde{\lambda}_{ra} \neq 0$, $\quad \tilde{\lambda}_{rb} \neq 0$ and $\forall \tilde{R}_r \neq 0$. Thus, $\hat{\lambda}_r$ and $\hat{R}_r$ converge in finite time to their nominal values $\lambda_r$ and $R_r$ with the convergence rate $1/T_r = R_r/L_r$ and $k_{R_r}$, respectively.

*Remark 4:* If $k_{R_r} > 1/T_r$, the rotor resistance convergence will be faster than that of the rotor flux. In contrary, if $k_{R_r} < 1/T_r$, the rotor flux convergence will be faster than that of the rotor resistance. The case $k_{R_r} = 1/T_r$ is difficult to implement in practice since $T_r$ is assumed to be unknown and is time-varying but verified in normal operation of the IM $T_{r\,\min} \leq T_r \leq T_{r\,\max}$.

To achieve the design of the rotor resistance estimator, implementable expression for $\tilde{R}_r$ is required. Under condition (24), a sliding-mode occurs in finite time on the 2-D manifold

$$\tilde{i}_{sa} = i_{sa}-\hat{i}_{sa} = 0, \quad \tilde{i}_{sb} = i_{sb}-\hat{i}_{sb} = 0. \tag{29}$$

The equivalent injection terms [19] can be computed by solving the equation

$$\dot{\tilde{i}}_{sa} = 0, \quad \dot{\tilde{i}}_{sb} = 0. \tag{30}$$

Consequently, (14) and (15) can be rewritten as

$$-W_{aeq}+\beta n_p\omega\tilde{\lambda}_{rb}+R_r\frac{\beta}{L_r}\tilde{\lambda}_{ra}+\tilde{R}_r\frac{\beta}{L_R}(\hat{\lambda}_{ra}-Mi_{sa}) = 0 \tag{31}$$

$$-W_{beq}-\beta n_p\omega\tilde{\lambda}_{ra}+R_r\frac{\beta}{L_r}\tilde{\lambda}_{rb}+\tilde{R}_r\frac{\beta}{L_r}(\hat{\lambda}_{rb}-Mi_{sb}) = 0 \tag{32}$$

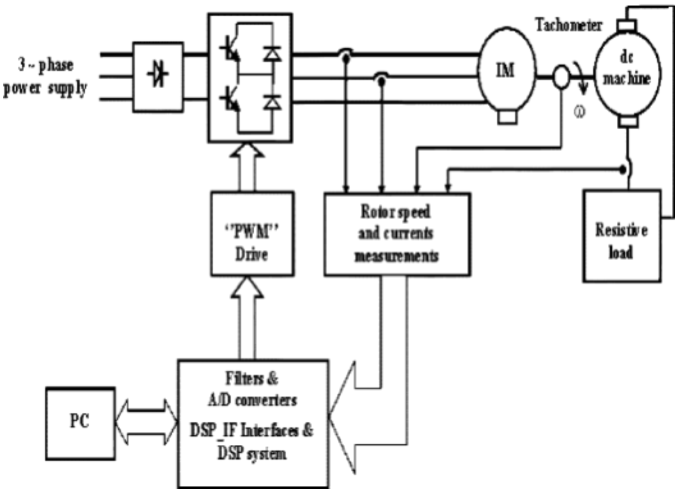


Fig. 1. Block diagram of the experimental setup.

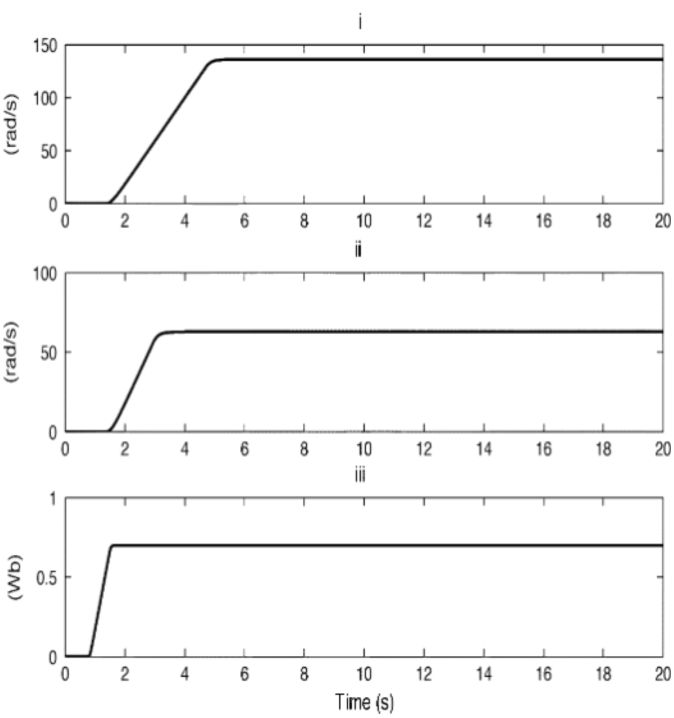


Fig. 2. Speed and flux reference signals. (i) Speed reference in experiments 1 and 3. (ii) Speed reference in experiment 2. (iii) Rotor flux reference.

where $W_{aeq} = [K\text{sign}(\tilde{i}_{sa})]_{\text{eq}}$ and $W_{beq} = [K\text{sign}(\tilde{i}_{sb})]_{\text{eq}}$. The expressions of the equivalent injection terms $W_{aeq}$ and $W_{beq}$ can be deduced from (31) and (32) but these expressions cannot be implemented in practice since $\tilde{R}_r$ and $\tilde{\lambda}_r$ are not available.[2] To overcome this problem, we approximated the equivalent injection terms $W_{aeq}$ and $W_{beq}$ by using first order low-pass filters as in [19].

If the design parameter $k_{R_r}$ is chosen such that

$$0 < k_{R_r} < 1/T_{rn} \quad \text{with} \quad T_{rn} = L_{rn}/R_{rn} \tag{33}$$

where $L_{rn}$ and $R_{rn}$ are the nominal values of the rotor inductance and rotor resistance and $T_{rn}$ is the nominal rotor time-constant, the rotor flux convergence will be faster than that of the rotor resistance. Other available offline methods can be exploited to evaluate the nominal value of the rotor time constant $T_{rn}$ without using the nominal values of $R_r$ and $L_r$ in the case of squirrel IM. Under this assumption and (P.E.) condition (19), the implementable expression of the rotor resistance estimation error $\tilde{R}_r$ can be derived from (31) and (32) by neglecting the terms containing the rotor flux estimation error. We then obtain

$$\tilde{R}_r = \frac{L_r}{\beta} \frac{(\hat{\lambda}_r - Mi_s)^T W_{\text{eq}}}{\|\hat{\lambda}_r - Mi_s\|^2}$$

with

$$W_{\text{eq}}^T = (W_{aeq}, W_{beq}). \tag{34}$$

*Remark 5:* The denominator of (34) can become zero in transient periods since the identifiability condition (18) or (P.E.) condition (19) is based on the real value of the flux and not on the estimated value. However, this singularity cannot affect significantly the estimate value of the rotor resistance since the adaptation law (27) uses the "sign" function. A singularity detector can also be used and such algorithm can provide as output the nominal value of the rotor resistance when the singularity is detected.

Finally, the overall simplified rotor resistance estimator can be summarized as follows:

$$\begin{aligned}
\frac{d\hat{i}_{sa}}{dt} &= -\frac{R_s}{\sigma L_s} i_{sa} - \beta M \frac{\hat{R}_r}{L_r} i_{sa} + \beta \frac{\hat{R}_r}{L_r}\hat{\lambda}_{ra} + n_p \beta \omega \hat{\lambda}_{rb} \\
&\quad + \frac{1}{\sigma L_s} v_{sa} + K\text{sign}(i_{sa} - \hat{i}_{sa}) \\
\frac{d\hat{i}_{sb}}{dt} &= -\frac{R_s}{\sigma L_s} i_{sb} - \beta M \frac{\hat{R}_r}{L_r} i_{sb} + \beta \frac{\hat{R}_r}{L_r}\hat{\lambda}_{rb} - n_p \beta \omega \hat{\lambda}_{ra} \\
&\quad + \frac{1}{\sigma L_s} v_{sb} + K\text{sign}(i_{sb} - \hat{i}_{sb}) \\
\frac{d\hat{\lambda}_{ra}}{dt} &= -\frac{\hat{R}_r}{L_r}\hat{\lambda}_{ra} - n_p\omega\hat{\lambda}_{rb} + \frac{\hat{R}_r}{L_r} M i_{sa} + u_a \\
\frac{d\hat{\lambda}_{rb}}{dt} &= -\frac{\hat{R}_r}{L_r}\hat{\lambda}_{rb} + n_p\omega\hat{\lambda}_{ra} + \frac{\hat{R}_r}{L_r} M i_{sb} + u_b \\
u_a &= \frac{\tilde{R}_r}{L_r}(Mi_{sa} - \hat{\lambda}_{ra}), \quad u_b = \frac{\tilde{R}_r}{L_r}(Mi_{sb} - \hat{\lambda}_{rb}) \\
\dot{\hat{R}}_r &= k_{R_r}\text{sign}(\tilde{R}_r) = k_{R_r}\text{sign}\left(\frac{L_r}{\beta}\frac{(\hat{\lambda}_r - Mi_s)^T W_{\text{eq}}}{\|\hat{\lambda}_r - Mi_s\|^2}\right).
\end{aligned} \tag{35}$$

## IV. Experimental Results

The effectiveness of the proposed algorithm combined with a nonlinear controller which stabilizes the rotor flux magnitude and the rotor speed to references values with adaptation of the rotor resistance and load torque (see [17] for more details) has been verified experimentally in various operating conditions.

*Remark 6:* The combination of both estimation algorithms (rotor resistance and load torque estimators) still converges since it has been proved in [17] that the proposed nonlinear controller can stabilize the IM to reference trajectories when the estimated values of $R_r$ and $T_L$ are bounded in the operational domain and the (P.E.) condition satisfied.

[2] $R_r$ is assumed to be unknown and $\lambda_r$ is often not measurable.

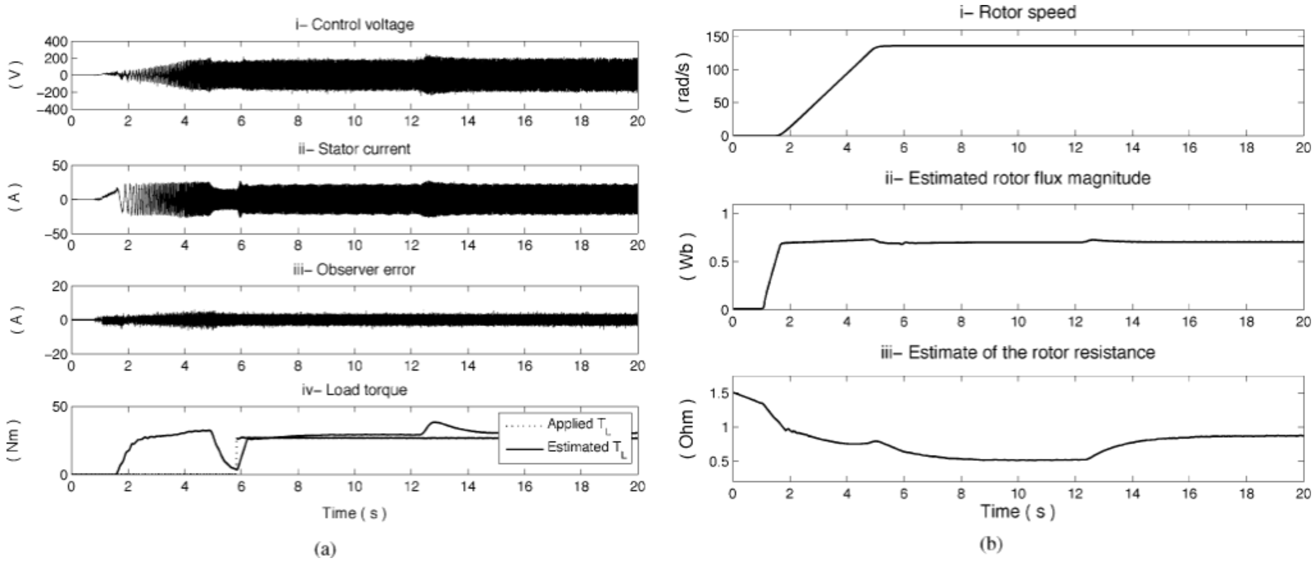


Fig. 3. First Experiment: Tracking performance of the proposed method with respect to online variation of the rotor resistance. (a) Control voltage, stator current, observer error and applied load torque: (i) control voltage; (ii) stator current; (iii) observer error; (iv) load torque. (b) Rotor speed, rotor flux magnitude, and estimate of the rotor resistance: (i) rotor speed; (ii) estimated rotor flux magnitude; (iii) estimate of the rotor resistance.

The experimental setup is illustrated by the block diagram of Fig. 1 which includes a development system DSP1103, an input/output electronics board (for analog/digital conversions) and a Personal Computer (PC). A 5-kW induction motor whose data are reported in the Appendix has been used. A PWM power converter with switching frequency of 10 kHz is controlled by a DSP. The external load torque $T_L$ is produced by a loaded dc generator. The motor instantaneous speed is measured by an optical incremental encoder with 1024 lines per revolution. The stator currents are measured by Hall-type sensors. All measured electrical parameters are converted by 16-b analog-to-digital (A/D) converter channels with 1 $\mu$s conversion time. A DSP1103 performs data acquisition and implements in real-time within the MATLAB/Simulink environment software with sampling time of 150 $\mu$s.

Three sets of experiments have been carried out. In all cases, experiments have been performed during motor startup and after the motor is operated under load torque. After the motor startup and in all experiments, the applied external unknown load torque $T_L$ is estimated by using the method described in [17]. In all experiments, the parameters of the rotor resistance identifier (35) were chosen as follows.

$K = 30\,000$, $k_{R_r} = 0.6$. The equivalent injection terms $W_{aeq}$ and $W_{beq}$ has been approximated using first order low-pass filter with time-constant of 5 ms. Note that the value of $k_{R_r}$ verifies condition (33) since $1/T_{rn} = 10.08\ \mathrm{s}^{-1}$. Using expression (23), the value of the constant $\xi_a$ or $\xi_b$ is approximately 5500. Therefore, the value of $K$ also verifies condition (24). Both speed and flux reference signals used in all experiments are given in Fig. 2.

In the first experiment, the performance of the algorithm to track the variation of the rotor resistance has been investigated. In this case, the online variation of the rotor resistance has been carried out using a three-phase variable rheostat and the value of the corresponding additional resistance was 0.36 $\Omega$. The results obtained in this case are reported in Fig. 3. These results demonstrated that the proposed algorithm has a powerful approach to track the variation of the rotor resistance.

The second experiment has been performed to verify the robustness property of the proposed method with respect to the variation of the stator resistance when the motor operates at relatively low speed. The performance of the proposed method in this case is given in Fig. 4. These results show that there is no significant effect on the rotor resistance estimate for a wide range of variation of the stator resistance (up to 100%).

Note that the assumption of non-saturated condition is often made in the literature on induction motor control. But under nominal operating condition, the induction machine will generally enter the saturation region. Therefore, the assumption that mutual inductance is constant is good only if the flux level of the machine is maintained constant and the machine operating condition is non-saturated. But in certain cases, the flux level of the machine can be varied to get better performance such as efficiency improvement or field-weakening control for higher speed operation.

By taking into account this remark, the variation effect of the machine mutual inductance has been investigated. From the fact that the leakage parameter $\sigma$ is a strictly positive constant, the mutual inductance variation $\Delta M$ should be chosen such that

$$\Delta M < \frac{\sqrt{L_{sN} L_{rN}} - M_N}{M_N}$$

or

$$\Delta M < 4.6\%$$

assuming that the inductances of the stator and rotor circuits $L_{sN}$ and $L_{rN}$ are constant parameters (see the Appendix). The results obtained in this case are depicted in Fig. 5. As it can be seen, the proposed rotor resistance estimation algorithm is more sensitive to the variation of the machine mutual inductance than that of the stator resistance.

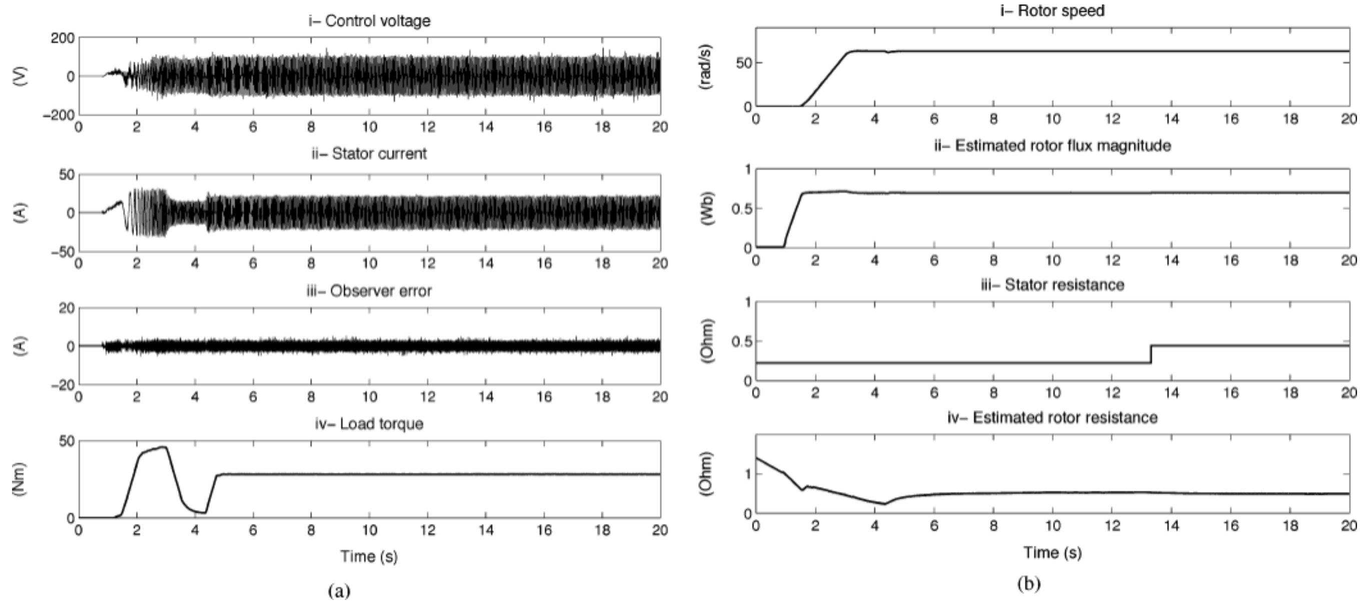


Fig. 4. Second Experiment: Performance of the proposed method at relatively low speed (63 rad/s) with 100% variation of the stator resistance $R_s$. (a) Control voltage, stator current, observer error, and load torque: (i) control voltage; (ii) stator current; (iii) observer error; (iv) load torque. (b) Rotor speed, rotor flux magnitude, stator resistance, and estimate of the rotor resistance: (i) rotor speed; (ii) estimated rotor flux magnitude; (iii) stator resistance; (iv) estimated rotor resistance.

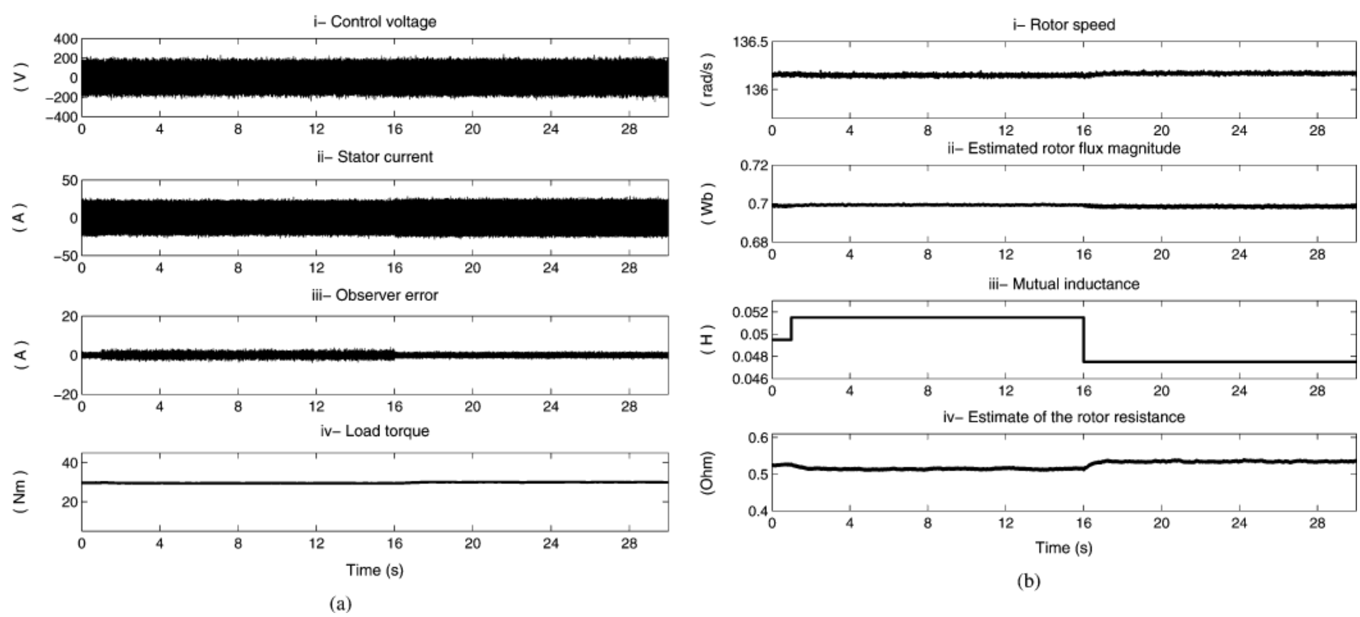


Fig. 5. Third Experiment: Investigation of the variation effect of the machine mutual inductance when the loaded motor is in the steady-state period. (a) Control voltage, stator current, observer error, and applied load torque: (i) control voltage; (ii) stator current; (iii) observer error; (iv) load torque (b) Rotor speed, rotor flux magnitude, mutual inductance, and estimate of the rotor resistance: (i) rotor speed; (ii) estimated rotor flux magnitude; (iii) mutual inductance; (iv) estimate of the rotor resistance.

In all cases, the estimate of the rotor resistance is very accurate and exhibits a short convergence transient. The steady-state error between the estimated rotor resistance and its nominal value is due to the measurement noise, mismatching between the motor and the model parameters, ohmic heating during experiments, and unmodeled dynamics.

## V. Conclusion

In this brief, a simple structure has been designed to estimate the rotor resistance of induction motors. The proposed method has been tested in closed-loop configuration by using a nonlinear controller which has been made adaptive with respect to the rotor resistance (35). The finite time convergence of the rotor resistance estimate to its nominal value has been achieved under mild P.E. requirements which can be fulfilled easily during normal operating conditions of the IM. Experimental results with online variation of the rotor resistance show that the proposed algorithm gives very satisfactory performance. The proposed online simplified rotor resistance estimator has also presented very interesting robustness properties with respect to the variation of the stator resistance, measurement noise, modeling errors, discretization effects and parameter uncertainties. Important advantages of the proposed algorithm include that it is an online method (the value of $R_r$ can be continuously updated) and it is very simple to implement in real-time (this feature distinguishes the proposed identifier from the known ones). The extension of the proposed technique in speed sensorless adaptive control of IM is yet to be done.

## APPENDIX A
## INDUCTION MOTOR DATA

| | |
|---|---|
| Rated power | 5 kW. |
| Rated torque | 32 Nm. |
| Rated frequency | 50 Hz. |
| Rated current | 22.9 A. |
| Stator resistance | $R_{sN} = 0.22\ \Omega$. |
| Rotor resistance | $R_{rN} = 0.52\ \Omega$. |
| Stator inductance | $L_{sN} = 0.052$ H. |
| Rotor inductance | $L_{rN} = 0.0516$ H. |
| Mutual inductance | $M_N = 0.0495$ H. |
| Number of pole pairs | $n_p = 2$. |
| Motor-load inertia | $m = 0.12\ \text{kg} \cdot \text{m}^2$. |

## ACKNOWLEDGMENT

The main part of the experimental setup used in this work has been supported by the "Département Energie, Ecole Supérieure d'Electricité, Gif-sur-Yvette, Paris, France".

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