Appendix A: Ordinary Differential Equations for Plasmid Dynamics

A1. Model Equations

The population dynamics of plasmid-free (F) and plasmid-bearing (P) bacterial cells are described by the following system of ordinary differential equations (ODEs):

$$\frac{dF}{dt} = r(1-s)\left(1-\frac{N}{K}\right)F - \mu F + \delta P - \beta FP$$

$$\frac{dP}{dt} = r(1-c)\left(1-\frac{N}{K}\right)P - \mu P - \delta P + \beta FP$$

where the total population size is $N = F + P$.

Plasmid-free cells experience an environmental selective pressure s, while plasmid-bearing cells incur a growth cost c associated with plasmid carriage. Plasmids may be lost through segregational loss at rate δ, or horizontally transferred from plasmid-bearing to plasmid-free cells at rate β.

A2. Model Parameters

Parameter	Description
r	Baseline per-capita bacterial growth rate in the absence of costs or selection
s	Selective pressure acting on plasmid-free cells
c	Growth cost associated with plasmid carriage
K	Environmental carrying capacity
μ	Baseline mortality rate
δ	Rate of plasmid segregational loss
β	Rate of plasmid transfer via conjugation

Density dependence is modelled through logistic growth, with both populations experiencing identical crowding effects.

A3. Model Assumptions

The model is constructed under the following assumptions:

- Complete protection: Plasmid-bearing cells are fully protected from the selective pressure.
- Mass-action conjugation: Horizontal plasmid transfer occurs via direct interactions between plasmid-free and plasmid-bearing cells and is modelled using mass-action kinetics.
- Logistic growth: Population growth is limited by a shared carrying capacity.
- Constant segregational loss: Plasmid-bearing cells lose plasmids at a constant per-capita rate, representing an effective approximation of plasmid partitioning at quasi-steady intracellular copy number.
- Homogeneous mixing: All cells interact uniformly.

- Time-invariant parameters: All rates are constant over the modelled timescale.