Appendix A: Ordinary Differential Equations for Plasmid Dynamics

A.1 Model Equations

The dynamics of plasmid-free (F) and plasmid-bearing (P) bacterial populations were modelled using the following system of ordinary differential equations (ODEs):

$$\frac{dF}{dt} = r(1-s)\left(1-\frac{N}{K}\right)F - \mu F + \delta P - \beta FP$$

$$\frac{dP}{dt} = r(1-c)\left(1-\frac{N}{K}\right)P - \mu P - \delta P + \beta FP$$

where the total population size is $N = F + P$.

A.2 Model Parameters

Parameter	Description	Notes
r	Baseline growth rate of bacteria	Represents maximum per capita growth rate in the absence of plasmid costs or selective pressures.
s	Selective pressure against plasmid-free cells	Captures fitness cost imposed by environmental conditions onto susceptible, plasmid-free cells.
c	Cost of plasmid carriage on growth	Reflects reduced growth of plasmid-bearing cells relative to plasmid-free cells.
K	Carrying capacity	Maximum population density supported by the environment; density dependence is modelled via logistic growth.
μ	Baseline mortality	Accounts for natural cell death independent of plasmid status.
δ	Rate of segregational loss	Represents the loss of plasmids during cell division, assumed to be a constant as a quasi-state equilibrium of plasmid copy number (PCN).
β	Rate of plasmid transfer	Describes horizontal plasmid transfer via conjugation, assumed to follow mass-action kinetics proportional to the product of plasmid-bearing and plasmid-free cells (density-scaled in the stochastic implementation).

A.3 Model Assumptions

1. **Full immunity**: Plasmid-bearing cells (P) are fully protected from the selective pressure s, affecting plasmid-free cells (F).
2. **Mass-action**: Plasmid transfer occurs via direct interaction between plasmid-free and plasmid-bearing cells and is modelled as βFP.

3. **Logistic growth**: Population growth is limited by carrying capacity K, with density dependence acting equally on both plasmid-free and plasmid-bearing cells through the term $1 - N/K$ where $N = F + P$.
4. **Constant segregational loss**: Plasmid-bearing cells can lose plasmids at rate δ, converting them into plasmid-free cells. This is modeled as a constant rate, justified by approximating plasmid partitioning as a Poisson series.
5. **Homogenous mixing**: All cells are equally likely to interact.
6. **Time-invariant parameters**: Growth rates, death rates, transfer rates and segregational loss are constant over the modelled timescale.