Documentation: PyTropical

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1 Introduction

Tropical mathematics uses a modified form of arithmetic where the usual operations of addition and multiplication are replaced with alternative operations, typically *maximum* (or *minimum*) and *addition*, respectively [1, 2]. These operations are then called *tropical sum*, denoted by $\oplus$, and *tropical multiplication*, denoted by $\otimes$:

$$a \oplus b = \max(a, b) \quad (\text{or } \min(a, b)),$$

$$a \otimes b = a + b.$$

The set $\bar{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$ equipped with these operations form the *tropical semiring*, $(\bar{\mathbb{R}}, \oplus, \otimes)$, where $\infty = -\infty$ (if $\oplus$ is the maximum) or $\infty = +\infty$ (if $\oplus$ is the minimum). By definition, ∞ works as the additive identity and 0 as the multiplicative identity. These operations can also be extended to matrices and vectors [1].

`PyTropical` is a Python package that implements several tropical mathematics operations.

2 Installation

Install from PyPI via the `pip` command:

```
1  pip install pytropical
```

Or from source:

```
1  git clone https://github.com/heitorbaldo/PyTropical.git
2  cd PyTropical
3  pip install .
```

3 Getting Started

3.1 Basic usage

```
1  from pytropical.tropical_algebra import MaxPlusAlgebra
2  from pytropical.utils import *
3
4  maxp = MaxPlusAlgebra()
5  inf = maxp.inf #inf is the additive identity
6
7  #tropical sum
8  maxp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 maxp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", maxp.trop_pow(2, 3))
```

4 Modules Overview

4.1 `pytropical.tropical_algebra`

4.1.1 Class: `MaxPlusAlgebra`

```
1 class MaxPlusAlgebra
```

Methods

- `trop_sum(a:  float, b:  float, symbol:  Boolean)` – Returns the tropical sum (maximum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\oplus$.

- `trop_mult(a:  float, b:  float, symbol:  Boolean)` – Returns the tropical multiplication (sum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\otimes$.

- `trop_pow(a:  float, n:  int)` – Returns tropical exponentiation of a.

- `trop_polynomial(A: array)` – Returns the tropical polynomial of A (A must be an array of coefficients and powers).

4.1.2 Class: `MinPlusAlgebra`

```
1 class MinPlusAlgebra
```

Methods

- `trop_sum(a:  float, b:  float, symbol:  Boolean)` – Returns the tropical sum (minimum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\oplus$.

- `trop_mult(a:  float, b:  float, symbol:  Boolean)` – Returns the tropical multiplication (sum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\otimes$.

- `trop_pow(a:  float, n:  int)` – Returns tropical exponentiation of a.

- `trop_polynomial(A: array)` – Returns the tropical polynomial of A (A must be an array of coefficients and powers).

5 Examples

5.1 Max-Plus Algebra

```python
1 from pytropical.tropical_algebra import MaxPlusAlgebra
2 from pytropical.utils import *
3
4 maxp = MaxPlusAlgebra()
5 inf = maxp.inf #inf is the additive identity
```

```
6
7  #tropical sum
8  maxp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 maxp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", maxp.trop_pow(2, 3))
```

5.2 Min-Plus Algebra

```
1  from pytropical.tropical_algebra import MinPlusAlgebra
2  from pytropical.utils import *
3
4  minp = MinPlusAlgebra()
5  inf = minp.inf #inf is the additive identity
6
7  #tropical sum
8  minp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 minp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", minp.trop_pow(2, 3))
```

6 License

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References

[1] Butkovic, P. (2010). *Max-linear Systems: Theory and Algorithms*. Springer Monographs in Mathematics.

[2] Speyer, D., & Sturmfels, B. (2009). Tropical Mathematics. Mathematics Magazine, 82(3), 163–173. https://doi.org/10.1080/0025570X.2009.11953615