

# Documentation: PyTropical

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# 1 Introduction

Tropical mathematics uses a modified form of arithmetic where the usual operations of addition and multiplication are replaced with alternative operations, typically *maximum* (or *minimum*) and *addition*, respectively [1, 2]. These operations are then called *tropical sum*, denoted by  $\oplus$ , and *tropical multiplication*, denoted by  $\otimes$ :

$$a \oplus b = \max(a, b) \quad (\text{or } \min(a, b)),$$

$$a \otimes b = a + b.$$

The set  $\bar{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$  equipped with these operations form the *tropical semiring*,  $(\bar{\mathbb{R}}, \oplus, \otimes)$ , where  $\infty = -\infty$  (if  $\oplus$  is the maximum) or  $\infty = +\infty$  (if  $\oplus$  is the minimum). By definition,  $\infty$  works as the additive identity and 0 as the multiplicative identity. These operations can also be extended to matrices and vectors [1].

PyTropical is a Python package that implements several tropical mathematics operations.

## 2 Installation

Install from PyPI via the `pip` command:

```
1 pip install pytropical
```

Or from source:

```
1 git clone https://github.com/heitorbaldo/PyTropical.git
2 cd PyTropical
3 pip install .
```

## 3 Dependencies

The following Python packages are required:

- numpy

## 4 Getting Started

### 4.1 Basic usage

```
1 from pytropical.tropical_algebra import MaxPlusAlgebra
2 from pytropical.utils import *
3
4 maxp = MaxPlusAlgebra()
5 inf = maxp.inf #inf is the additive identity
6
7 #tropical sum
8 maxp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 maxp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", maxp.trop_pow(2, 3))
```

## 5 Modules Overview

### 5.1 `pytropical.tropical_algebra`

#### 5.1.1 Class: `MaxPlusAlgebra`

```
1 class MaxPlusAlgebra
```

##### Methods

- `trop_sum(a: float, b: float, symbol: Boolean)` – Returns the tropical sum (maximum) of `a` and `b`. If the parameter `symbol` is `True`, then it prints the operation using the symbol  $\oplus$ .
- `trop_mult(a: float, b: float, symbol: Boolean)` – Returns the tropical multiplication (sum) of `a` and `b`. If the parameter `symbol` is `True`, then it prints the operation using the symbol  $\otimes$ .
- `trop_pow(a: float, n: int)` – Returns tropical exponentiation of `a`.
- `trop_polynomial(A: array)` – Returns the tropical polynomial of `A` (`A` must be an array of coefficients and powers).

#### 5.1.2 Class: `MinPlusAlgebra`

```
1 class MinPlusAlgebra
```

##### Methods

- `trop_sum(a: float, b: float, symbol: Boolean)` – Returns the tropical sum (minimum) of `a` and `b`. If the parameter `symbol` is `True`, then it prints the operation using the symbol  $\oplus$ .
- `trop_mult(a: float, b: float, symbol: Boolean)` – Returns the tropical multiplication (sum) of `a` and `b`. If the parameter `symbol` is `True`, then it prints the operation using the symbol  $\otimes$ .
- `trop_pow(a: float, n: int)` – Returns tropical exponentiation of `a`.
- `trop_polynomial(A: array)` – Returns the tropical polynomial of `A` (`A` must be an array of coefficients and powers).

## 6 Examples

### 6.1 Max-Plus Algebra

```
1 from pytropical.tropical_algebra import MaxPlusAlgebra
2 from pytropical.utils import *
3
4 maxp = MaxPlusAlgebra()
5 inf = maxp.inf #inf is the additive identity
```

```

6
7 #tropical sum
8 maxp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 maxp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", maxp.trop_pow(2, 3))

```

## 6.2 Min-Plus Algebra

```

1 from pytropical.tropical_algebra import MinPlusAlgebra
2 from pytropical.utils import *
3
4 minp = MinPlusAlgebra()
5 inf = minp.inf #inf is the additive identity
6
7 #tropical sum
8 minp.trop_sum(2.8, 3)
9
10 #tropical multiplication
11 minp.trop_mult(90, inf)
12
13 #tropical exponentiation
14 print("2^3=", minp.trop_pow(2, 3))

```

## 7 License

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## References

- [1] Butkovic, P. (2010). *Max-linear Systems: Theory and Algorithms*. Springer Monographs in Mathematics.
- [2] Speyer, D., & Sturmfels, B. (2009). Tropical Mathematics. *Mathematics Magazine*, 82(3), 163–173. <https://doi.org/10.1080/0025570X.2009.11953615>