Documentation: PyTropical

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1 Introduction

Tropical mathematics uses a modified form of arithmetic where the usual operations of addition and multiplication are replaced with alternative operations, typically *maximum* (or *minimum*) and *addition*, respectively [1, 2]. These operations are then called *tropical sum*, denoted by $\oplus$, and *tropical multiplication*, denoted by $\otimes$:

$$a \oplus b = \max(a, b) \quad (\text{or } \min(a, b)),$$

$$a \otimes b = a + b.$$

The set $\bar{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$ equipped with these operations form the *tropical semiring*, $(\bar{\mathbb{R}}, \oplus, \otimes)$, where $\infty = -\infty$ (if $\oplus$ is the maximum) or $\infty = +\infty$ (if $\oplus$ is the minimum). By definition, ∞ works as the additive identity and 0 as the multiplicative identity. These operations can also be extended to matrices and vectors [1].

`PyTropical` is a Python package that implements several tropical mathematics operations.

2 Installation

Install from PyPI via the `pip` command:

```
pip install pytropical
```

Or from source:

```
git clone https://github.com/heitorbaldo/PyTropical.git
cd PyTropical
pip install .
```

3 Dependencies

The following Python packages are required:

- numpy

4 Getting Started

4.1 Basic usage

```
from pytropical.tropical_algebra import MaxPlusAlgebra
from pytropical.utils import *

maxp = MaxPlusAlgebra()
inf = maxp.inf #inf is the additive identity

#tropical sum
maxp.trop_sum(2.8, 3)

#tropical multiplication
maxp.trop_mult(90, inf)

#tropical exponentiation
print("2^3=", maxp.trop_pow(2, 3))
```

5 Modules Overview

5.1 pytropical.tropical_algebra

5.1.1 Class: MaxPlusAlgebra

```
class MaxPlusAlgebra
```

Methods

- `trop_sum(a: float, b: float, symbol: Boolean)` – Returns the tropical sum (maximum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\oplus$.
- `trop_mult(a: float, b: float, symbol: Boolean)` – Returns the tropical multiplication (sum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\otimes$.
- `trop_pow(a: float, n: int)` – Returns tropical exponentiation of a.
- `trop_polynomial(A: array)` – Returns the tropical polynomial of A (A must be an array of coefficients and powers).
- `vec_trop_suml(v: array, w: array)` – Returns the tropical sum (maximum) of two vectors v and w.
- `vec_trop_scalar_mult(a: float, v: array)` – Returns the tropical scalar multiplication of a vector v by a real number a.
- `matrix_trop_sum(A: NumPy matrix, B: NumPy matrix)` – Returns the tropical sum (maximum) of two matrices A and B.
- `matrix_trop_scalar_mult(a: float, B: NumPy matrix)` – Returns the tropical scalar multiplication of a matrix A by a real number a.
- `matrix_trop_mult(A: NumPy matrix, B: NumPy matrix)` – Returns the tropical multiplication of two matrices A and B.

5.1.2 Class: MinPlusAlgebra

```
class MinPlusAlgebra
```

Methods

- `trop_sum(a: float, b: float, symbol: Boolean)` – Returns the tropical sum (minimum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\oplus$.
- `trop_mult(a: float, b: float, symbol: Boolean)` – Returns the tropical multiplication (sum) of a and b. If the parameter `symbol` is `True`, then it prints the operation using the symbol $\otimes$.

- `trop_pow(a: float, n: int)` – Returns tropical exponentiation of a.
- `trop_polynomial(A: array)` – Returns the tropical polynomial of A (A must be an array of coefficients and powers).
- `vec_trop_suml(v: array, w: array)` – Returns the tropical sum (minimum) of two vectors v and w.
- `vec_trop_scalar_mult(a: float, v: array)` – Returns the tropical scalar multiplication of a vector v by a real number a.
- `matrix_trop_sum(A: NumPy matrix, B: NumPy matrix)` – Returns the tropical sum (minimum) of two matrices A and B.
- `matrix_trop_scalar_mult(a: float, B: NumPy matrix)` – Returns the tropical scalar multiplication of a matrix A by a real number a.
- `matrix_trop_mult(A: NumPy matrix, B: NumPy matrix)` – Returns the tropical multiplication of two matrices A and B.

6 Examples

6.1 Max-Plus Algebra

```
from pytropical.tropical_algebra import MaxPlusAlgebra
from pytropical.utils import *

maxp = MaxPlusAlgebra()
inf = maxp.inf #inf is the additive identity

#tropical sum
maxp.trop_sum(2.8, 3)

#tropical multiplication
maxp.trop_mult(90, inf)

#tropical exponentiation
print("2^3=", maxp.trop_pow(2, 3))
```

6.2 Min-Plus Algebra

```
from pytropical.tropical_algebra import MinPlusAlgebra
from pytropical.utils import *

minp = MinPlusAlgebra()
inf = minp.inf #inf is the additive identity

#tropical sum
minp.trop_sum(2.8, 3)

#tropical multiplication
minp.trop_mult(90, inf)

```

```
#tropical exponentiation
print("2^3=", minp.trop_pow(2, 3))
```

7 License

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References

[1] Butkovic, P. (2010). *Max-linear Systems: Theory and Algorithms*. Springer Monographs in Mathematics.

[2] Speyer, D., & Sturmfels, B. (2009). Tropical Mathematics. Mathematics Magazine, 82(3), 163–173. https://doi.org/10.1080/0025570X.2009.11953615