Describe Model

TIME SERIES ANALYSIS IN PYTHON

Rob Reider
Adjunct Professor, NYU-Courant
Consultant, Quantopian

Mathematical Description of MA(1) Model

$$R_t \ equals \ \mu \ + \ \epsilon_t \ + \ \theta \, \epsilon_{t-1}$$

- Since only one lagged error on right hand side, this is called:
  - MA model of order 1, or
  - MA(1) model
- MA parameter is θ
- Stationary for all values of θ

Interpretation of MA(1) Parameter

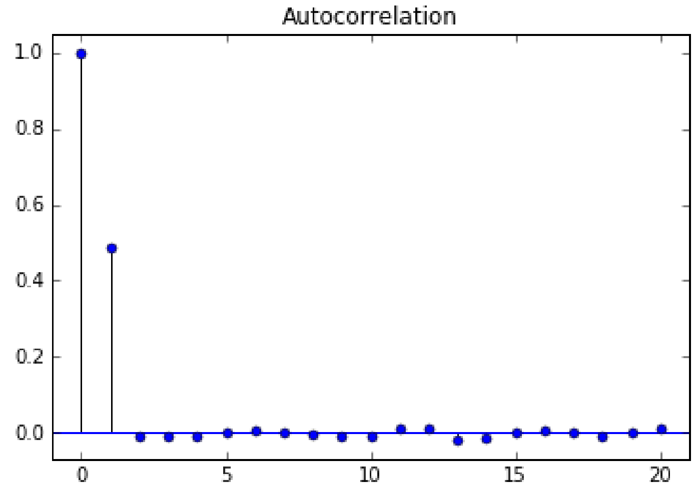
$$R_t = \mu + \epsilon_t + \theta\,\epsilon_{t-1}$$

- Negative θ: One-Period Mean Reversion
- Positive θ: One-Period Momentum
- Note: One-period autocorrelation is $\theta/(1+\theta^2)$, not θ

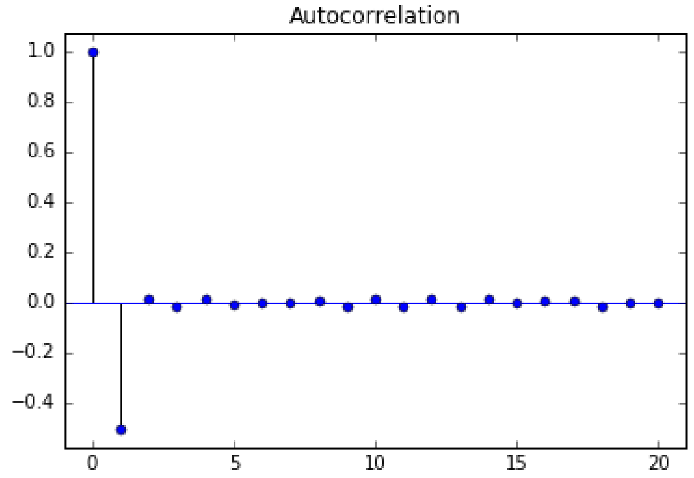
If theta is negative, a positive shock last period, represented by epsilon t-1, would have caused last period's return to be positive, but this period's return is more likely to be negative. A shock two periods ago would have no effect on today's return — only the shock now and last period.

Comparison of MA(1) Autocorrelation Functions

- $\theta = 0.9$

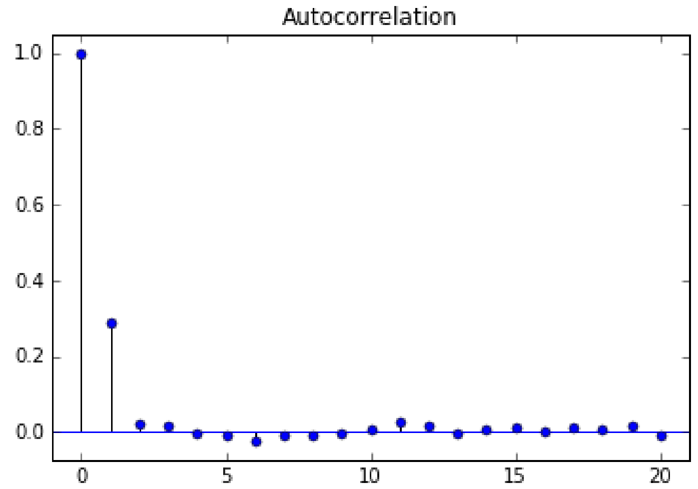


- $\theta = -0.9$

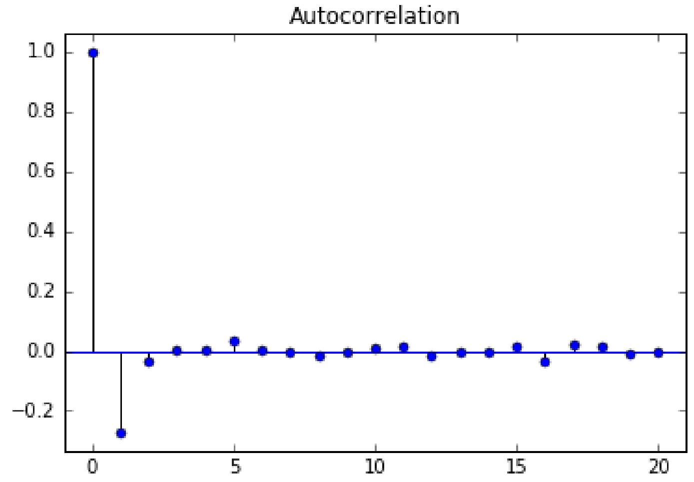


In each case, there is zero autocorrelation for an MA(1) beyond lag 1. When theta is positive, the lag 1 autocorrelation is positive.

- $\theta = 0.5$

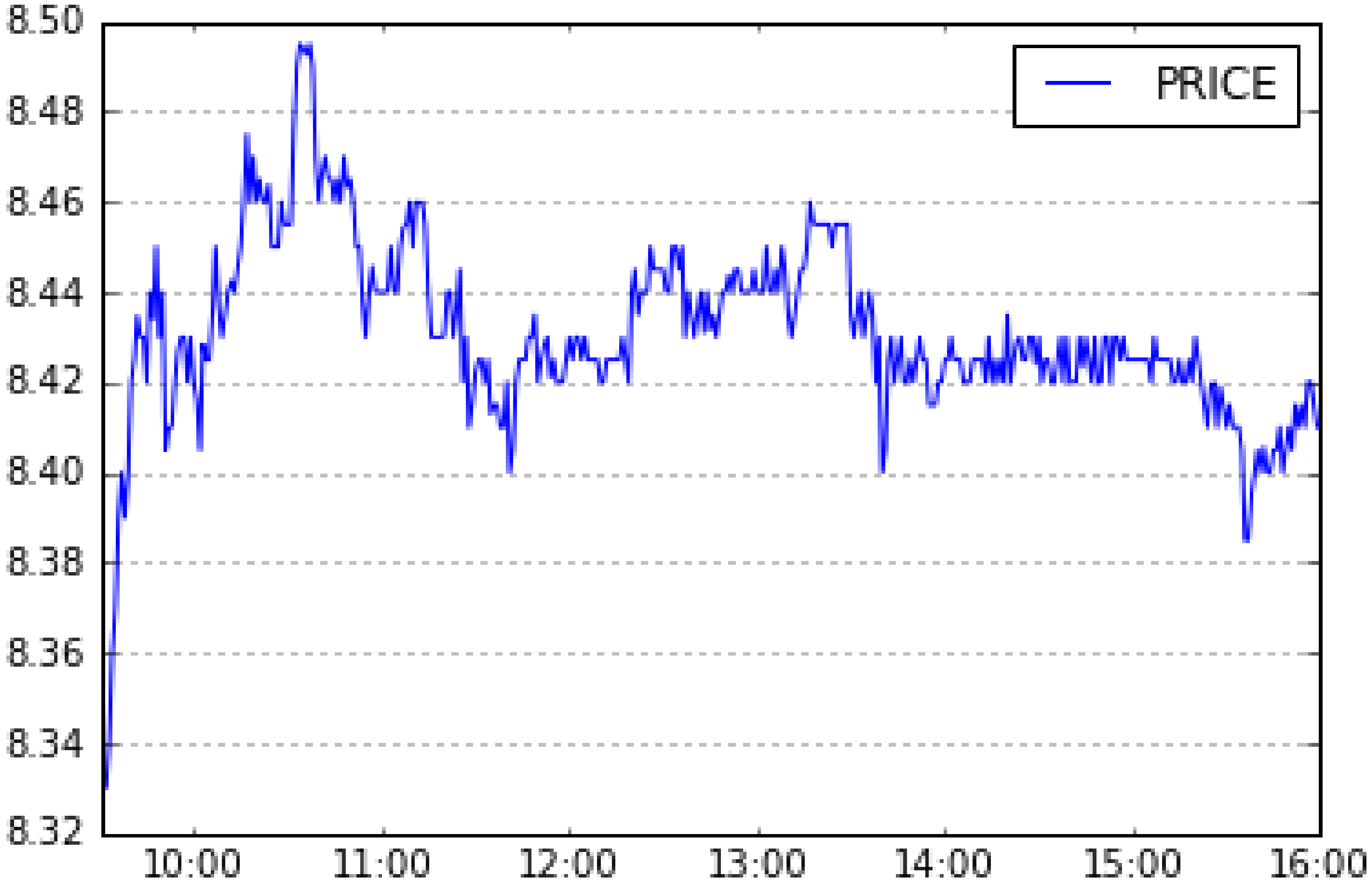


- $\theta = -0.5$



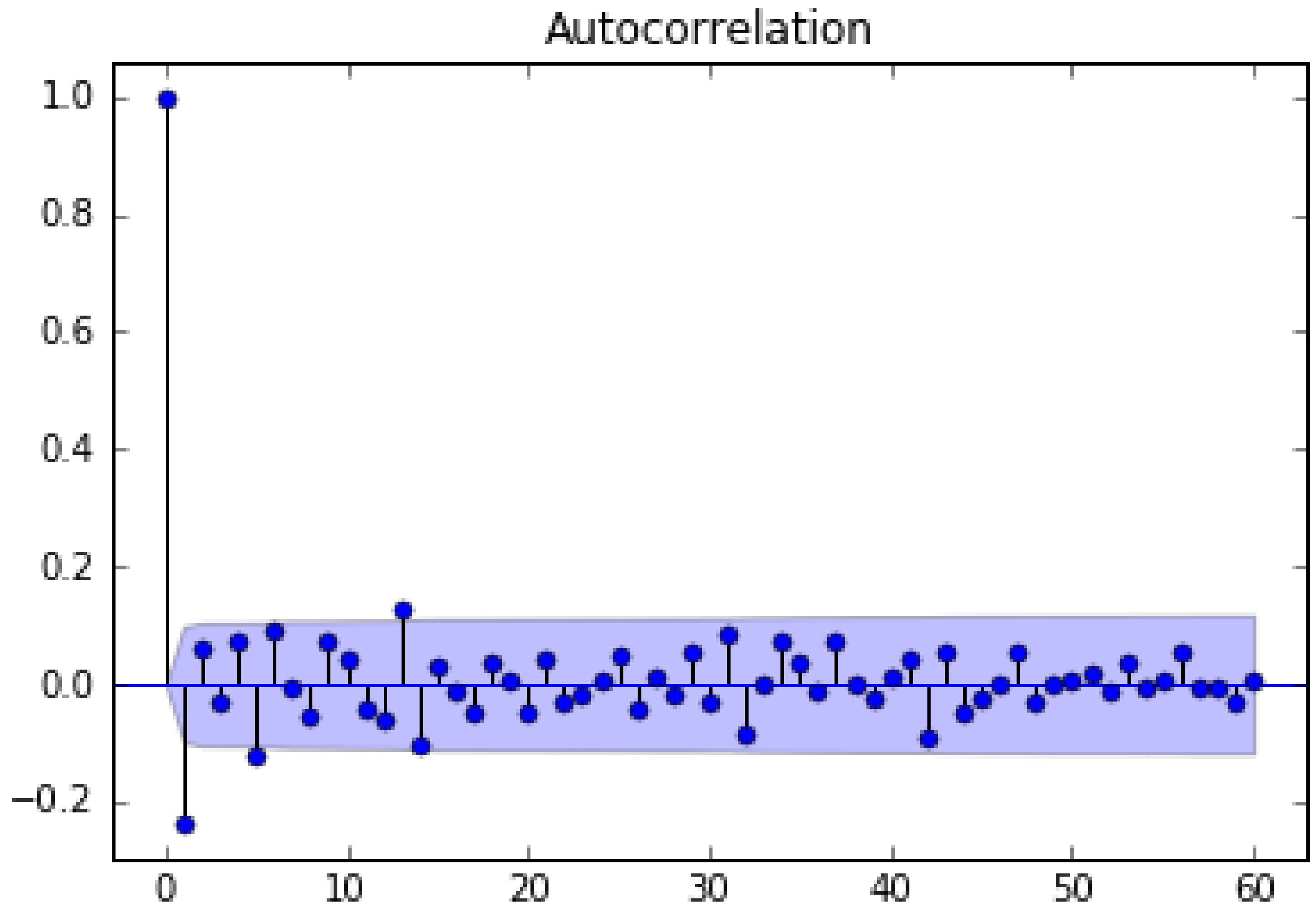
Example of MA(1) Process: Intraday Stock Returns

high frequency stock returns



Stocks trade at discrete one-cent increments rather than at continuous prices, and you can see that stock can bounce back and forth over a one-cent range for long periods of time.

Autocorrelation Function of Intraday Stock Returns



Higher Order MA Models

- MA(1)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1}$$

- MA(2)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1} - \theta_2 \, \epsilon_{t-2}$$

- MA(3)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1} - \theta_2 \, \epsilon_{t-2} - \theta_3 \, \epsilon_{t-3}$$

- ...

Simulating an MA Process

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1])
ma = np.array([1, 0.5])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

Let's practice!

Estimation and Forecasting an MA Model

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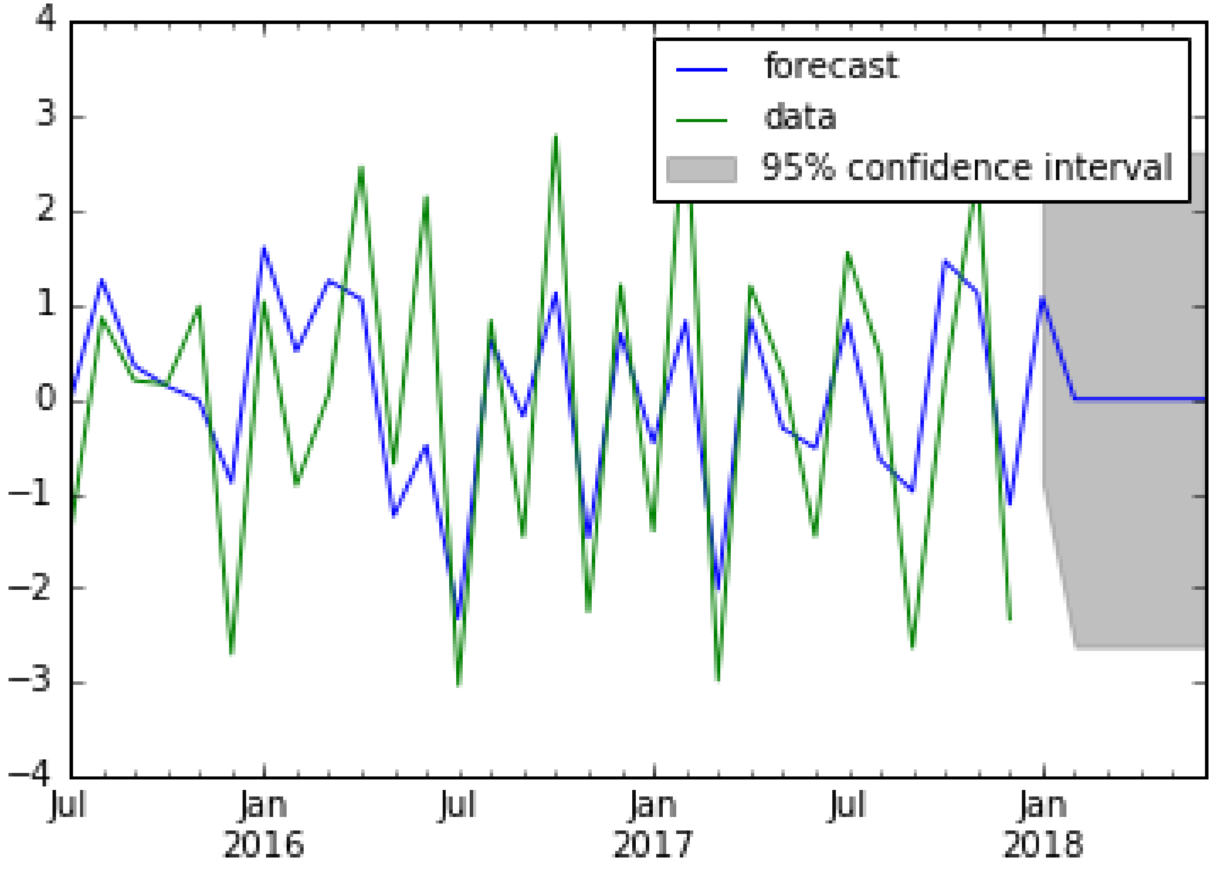
Estimating an MA Model

- Same as estimating an AR model (except `order=(0,1)`)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(0,1))
result = mod.fit()
```

Forecasting an MA Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(0,1))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```



Let's practice!

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ARMA models

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ARMA Model

- ARMA(1,1) model:

$$R_t = \mu + \phi\, R_{t-1} + \epsilon_t + \theta\, \epsilon_{t-1}$$

Converting Between ARMA, AR, and MA Models

- Converting AR(1) into an MA(infinity)

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

$$R_t = \mu + \phi(\mu + \phi R_{t-2} + \epsilon_{t-1}) + \epsilon_t$$

$$\vdots$$

$$R_t = \frac{\mu}{1-\phi} + \epsilon_t + \phi\epsilon_{t-1} - \phi^2\epsilon_{t-2} + \phi^3\epsilon_{t-3} + \ldots$$

Let's practice!

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