Introducing an AR Model

TIME SERIES ANALYSIS IN PYTHON

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Mathematical Description of AR(1) Model

$$R_t \quad = \quad \mu \quad + \quad \phi \quad R_{t-1} \quad + \quad \epsilon_t$$

- Since only one lagged value on right hand side, this is called:
  - AR model of order 1, or
  - AR(1) model
- AR parameter is ϕ
- For stationarity, $-1 < \phi < 1$

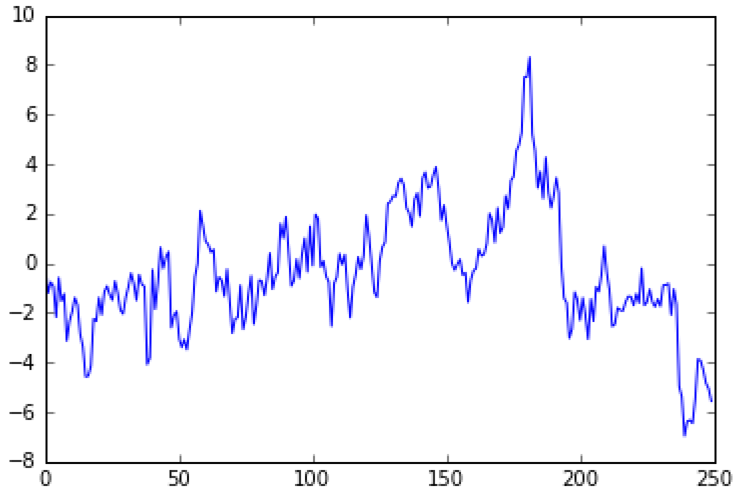
Interpretation of AR(1) Parameter

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

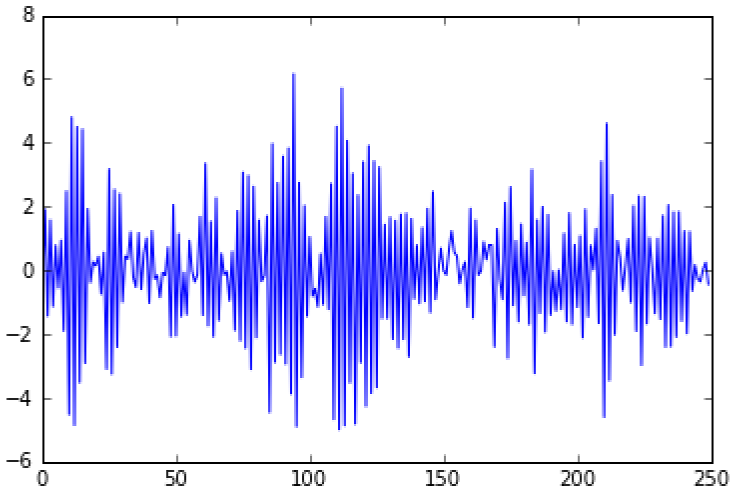
- Negative ϕ: Mean Reversion
- Positive ϕ: Momentum

Comparison of AR(1) Time Series

- $\phi = 0.9$

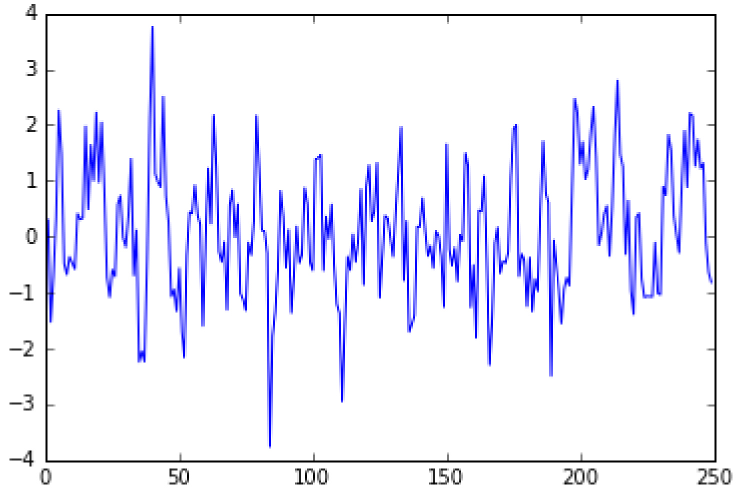


- $\phi = -0.9$

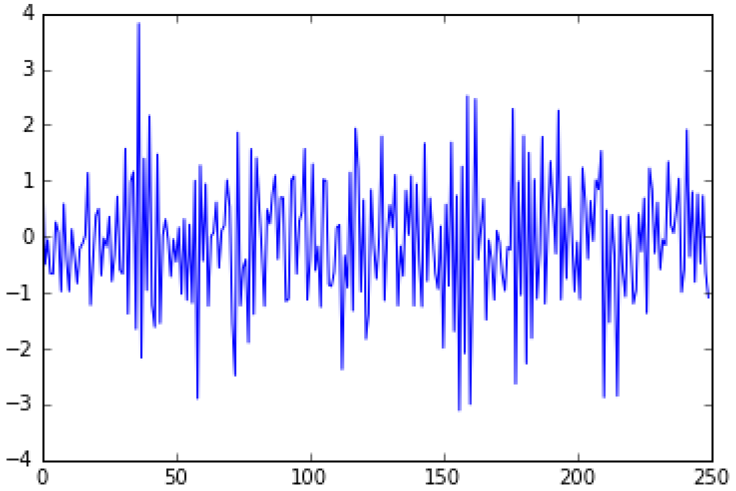


When phi equals to 0.9, it looks close to a random walk.
When phi equals to -0.9, the process looks more erratic — a large positive value is usually followed by a large negative one.

- $\phi = 0.5$



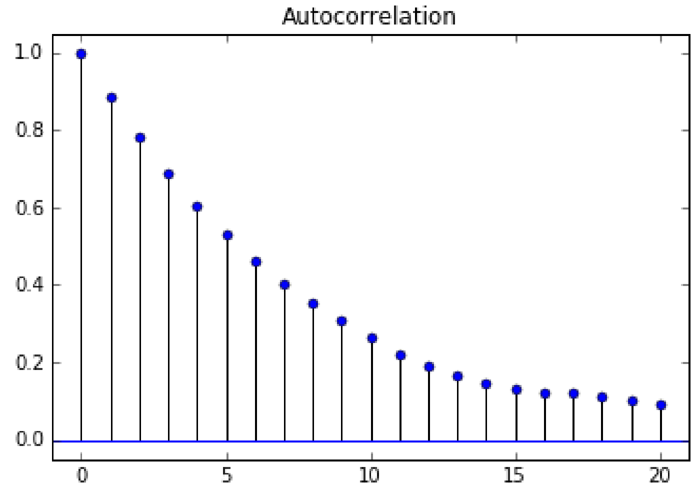
- $\phi = -0.5$



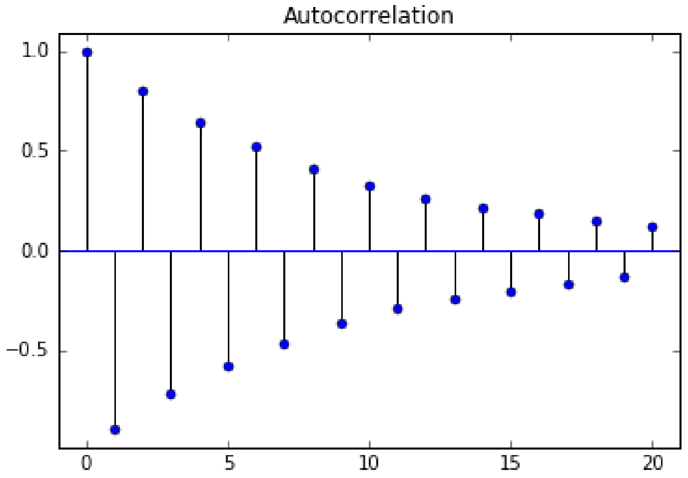
The bottom two are similar, but are less exaggerated and closer to white noise.

Comparison of AR(1) Autocorrelation Functions

- $\phi = 0.9$

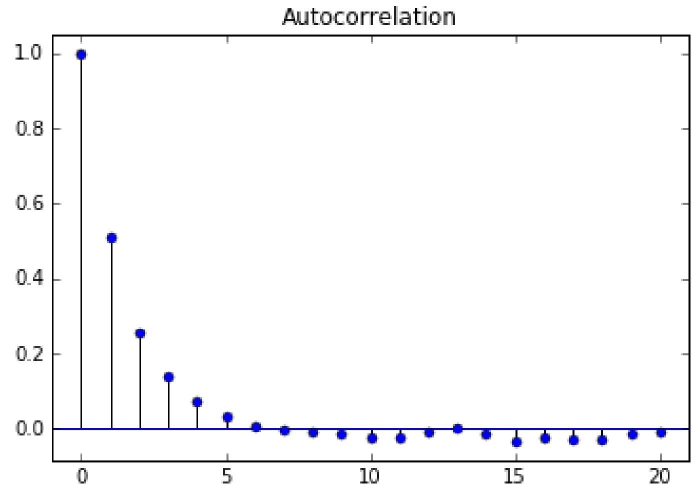


- $\phi = -0.9$

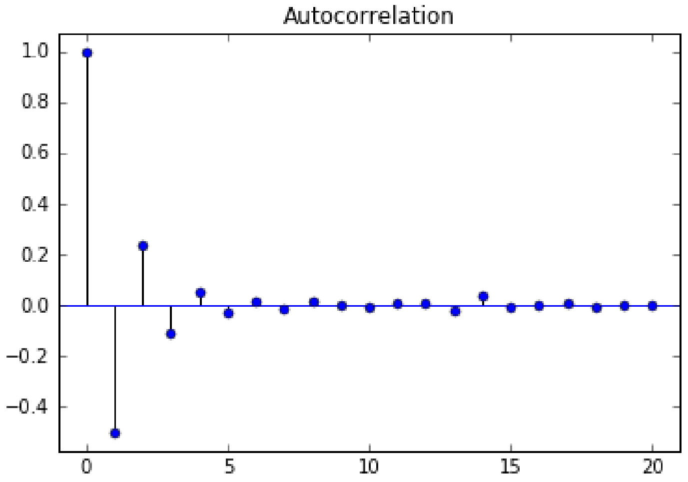


The autocorrelation decays exponentially at a rate of phi.
Therefore if phi is 0.9, the lag 1 autocorrelation is 0.9, the lag 2 autocorrelation is 0.9 squared, the lag 3 autocorrelation is 0.9 cubed, etc.
When phi is negative, the autocorrelation function still decays exponentially, but the signs of the autocorrelation function reverse at each lag.

- $\phi = 0.5$



- $\phi = -0.5$



Higher Order AR Models

- AR(1)

$$R_t = \mu + \phi_1 R_{t-1} + \epsilon_t$$

- AR(2)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \epsilon_t$$

- AR(3)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \phi_3 R_{t-3} + \epsilon_t$$

- ...

Simulating an AR Process

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1, -0.9])
ma = np.array([1])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

The convention is a little counterintuitive: You must include the zero-lag coefficient of 1, and the sign of the other coefficient is the opposite of what we have been using.
For example, for an AR(1) process with phi equal to 0.9, the second element of the array should be -0.9. This is consistent with the time series literature in the field of signal processing.

Let's practice!

TIME SERIES ANALYSIS IN PYTHON

Estimating and Forecasting an AR Model

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Estimating an AR Model

- To estimate parameters from data (simulated)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

The order (1,0) means you're fitting the data to an AR(1) model.

Estimating an AR Model

- Full output (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 5000
Model:                     ARMA(1, 0)   Log Likelihood               -7178.386
Method:                       css-mle   S.D. of innovations              1.017
Date:                Fri, 01 Dec 2017   AIC                          14362.772
Time:                        15:34:50   BIC                          14382.324
Sample:                             0   HQIC                         14369.625

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const         -0.0361      0.152     -0.238      0.812        -0.333     0.261
ar.L1.y        0.9054      0.006    151.020      0.000         0.894     0.917
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           1.1045           +0.0000j            1.1045            0.0000
-----------------------------------------------------------------------------
```

Estimating an AR Model

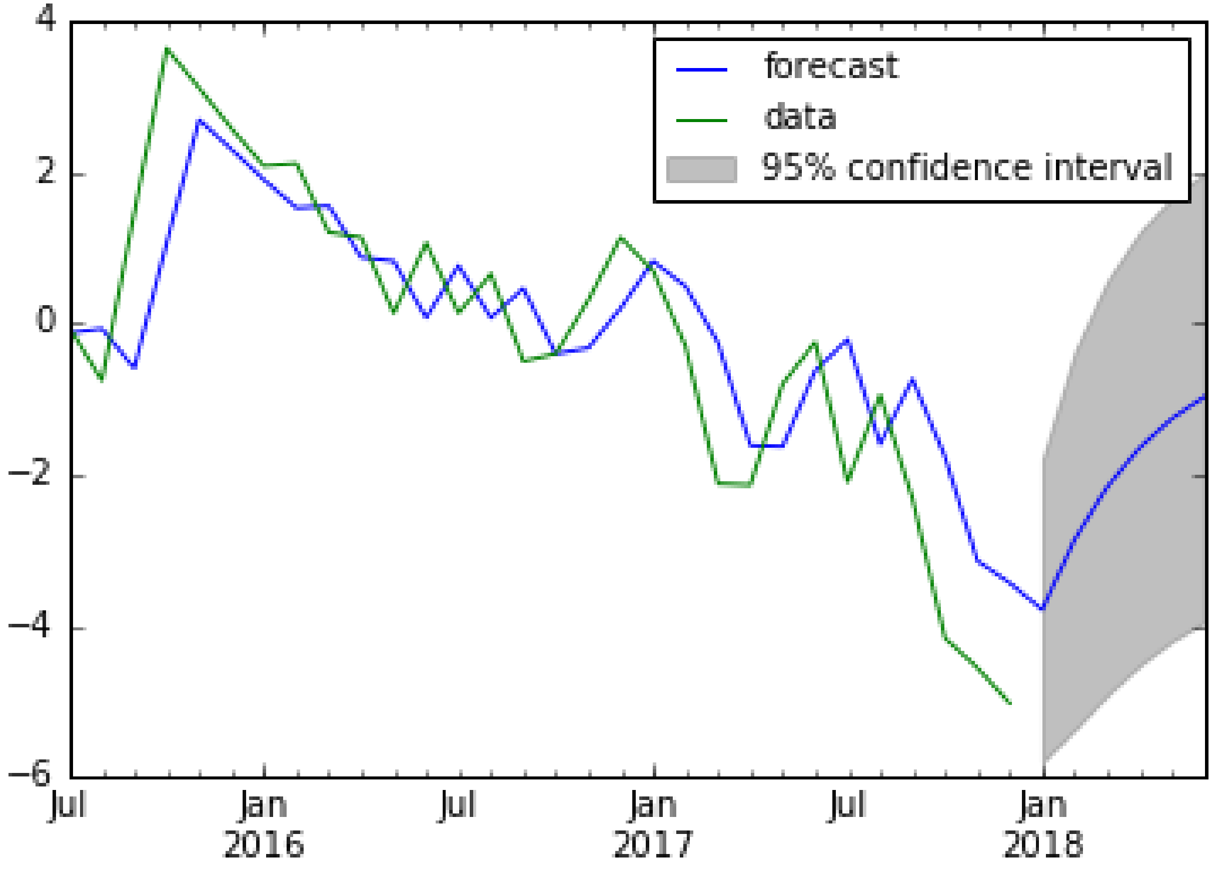
- Only the estimates of μ and ϕ (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.params)
```

```
array([-0.03605989,  0.90535667])
```

Forecasting an AR Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```



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Choosing the Right Model

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Identifying the Order of an AR Model

- The order of an AR(p) model will usually be unknown
- Two techniques to determine order
  - Partial Autocorrelation Function
  - Information criteria

Partial Autocorrelation Function (PACF)

$$R_t = \phi_{0,1} + \boxed{\phi_{1,1}} R_{t-1} + \epsilon_{1t}$$

$$R_t = \phi_{0,2} + \phi_{1,2} R_{t-1} + \boxed{\phi_{2,2}} R_{t-2} + \epsilon_{2t}$$

$$R_t = \phi_{0,3} + \phi_{1,3} R_{t-1} + \phi_{2,3} R_{t-2} + \boxed{\phi_{3,3}} R_{t-3} + \epsilon_{3t}$$

$$R_t = \phi_{0,4} + \phi_{1,4} R_{t-1} + \phi_{2,4} R_{t-2} + \phi_{3,4} R_{t-3} + \boxed{\phi_{4,4}} R_{t-4} + \epsilon_{4t}$$

$$\vdots$$

Partial Autocorrelation Function measures the incremental benefit of adding another lag.

Plot PACF in Python

- Same as ACF, but use `plot_pacf` instead of `plt_acf`
- Import module

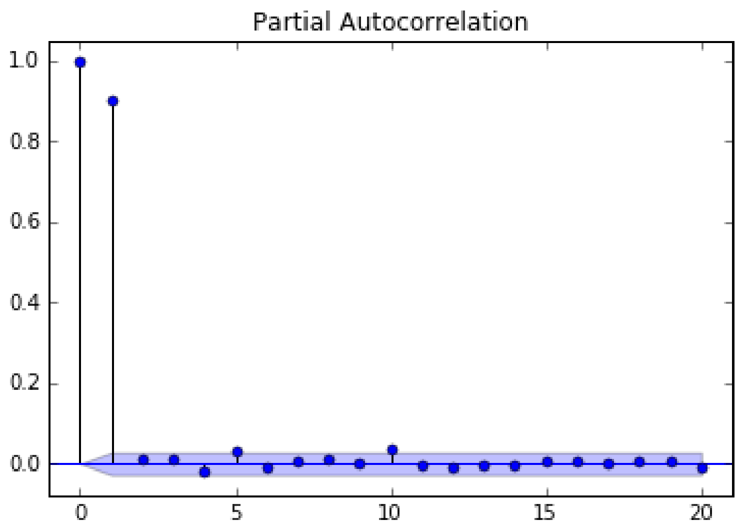
```
from statsmodels.graphics.tsaplots import plot_pacf
```

- Plot the PACF

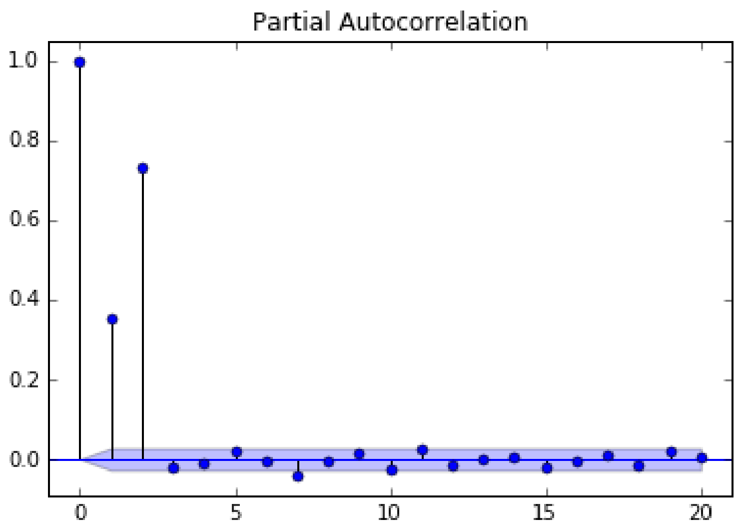
```
plot_pacf(x, lags= 20, alpha=0.05)
```

Comparison of PACF for Different AR Models

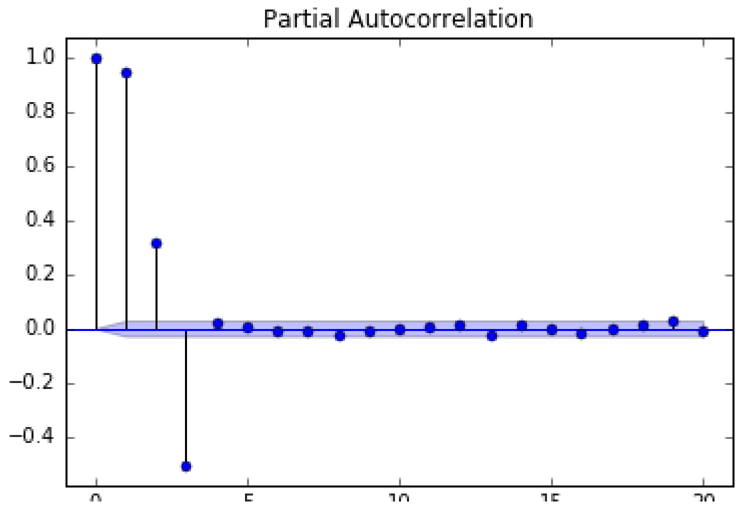
- AR(1)



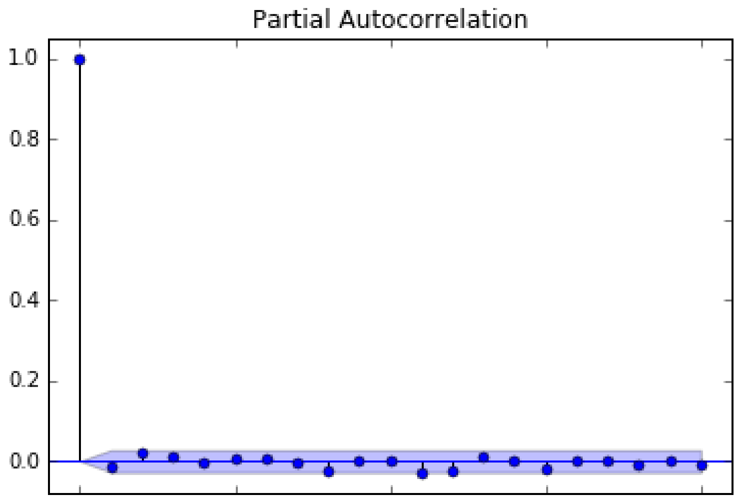
- AR(2)



- AR(3)



- White Noise



Information Criteria

- Information criteria: adjusts goodness-of-fit for number of parameters
- Two popular adjusted goodness-of-fit measures
  - AIC (Akaike Information Criterion)
  - BIC (Bayesian Information Criterion)

Information Criteria

- Estimation output

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2500
Model:                     ARMA(2, 0)   Log Likelihood               -3536.481
Method:                       css-mle   S.D. of innovations              0.996
Date:                Fri, 29 Dec 2017   AIC                           7080.963
Time:                        22:53:24   BIC                           7104.259
Sample:                             0   HQIC                          7089.420

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const          0.0054      0.010      0.517      0.605        -0.015     0.026
ar.L1.y       -0.6130      0.019    -32.243      0.000        -0.650    -0.576
ar.L2.y       -0.3109      0.019    -16.351      0.000        -0.348    -0.274
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1          -0.9859           -1.4982j            1.7935           -0.3426
AR.2          -0.9859           +1.4982j            1.7935            0.3426
-----------------------------------------------------------------------------
```

Getting Information Criteria From `statsmodels`

- You learned earlier how to fit an AR model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

- And to get full output

```
result.summary()
```

- Or just the parameters

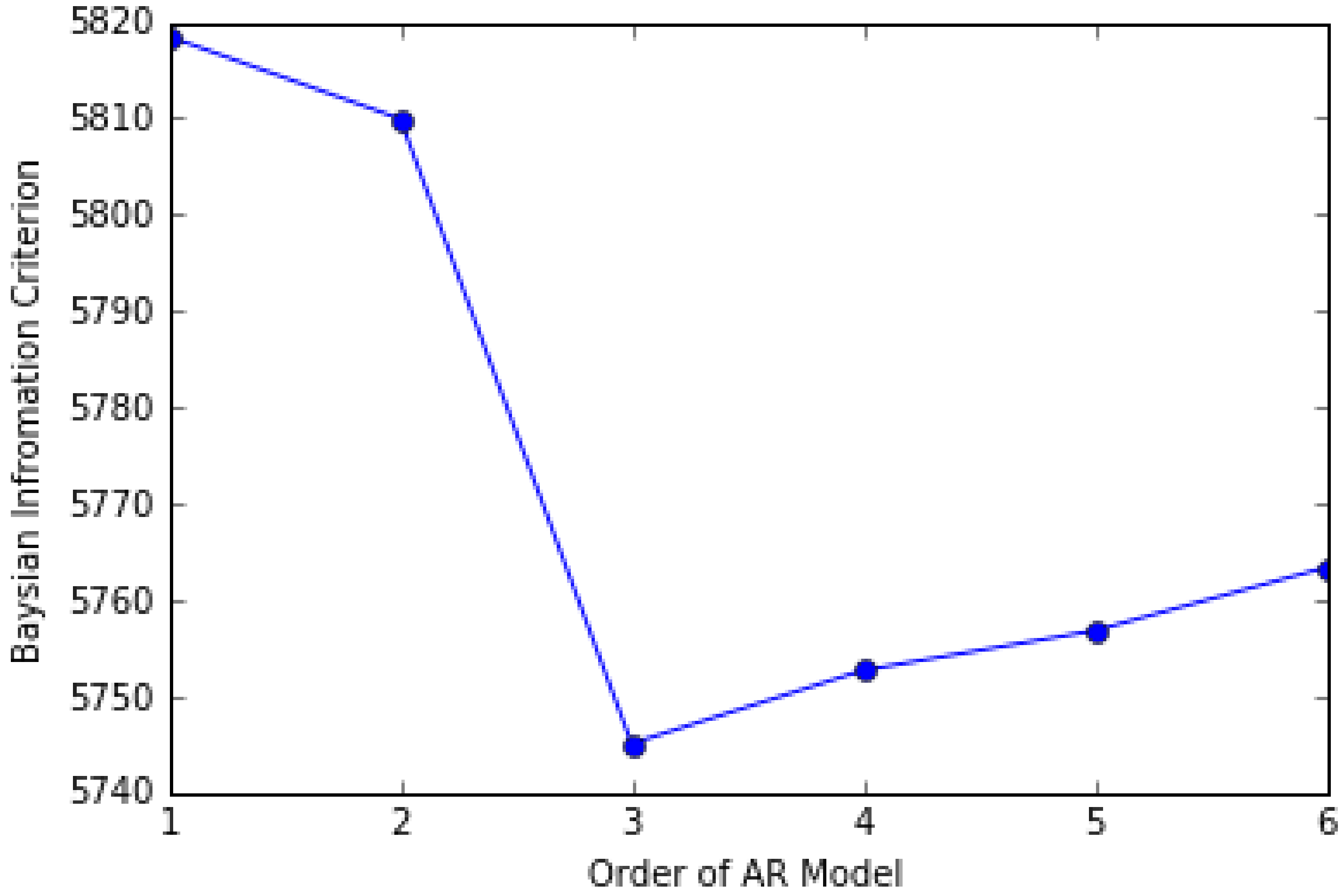
```
result.params
```

- To get the AIC and BIC

```
result.aic
result.bic
```

Information Criteria

- Fit a simulated AR(3) to different AR(p) models
- Choose p with the lowest BIC



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