

# Introduction to time series and stationarity

ARIMA MODELS IN PYTHON



**James Fulton**

Climate informatics researcher

# Motivation

Time series are everywhere

- Science
- Technology
- Business
- Finance
- Policy

# Course content

You will learn

- Structure of ARIMA models
- How to fit ARIMA model
- How to optimize the model
- How to make forecasts
- How to calculate uncertainty in predictions

# Loading and plotting

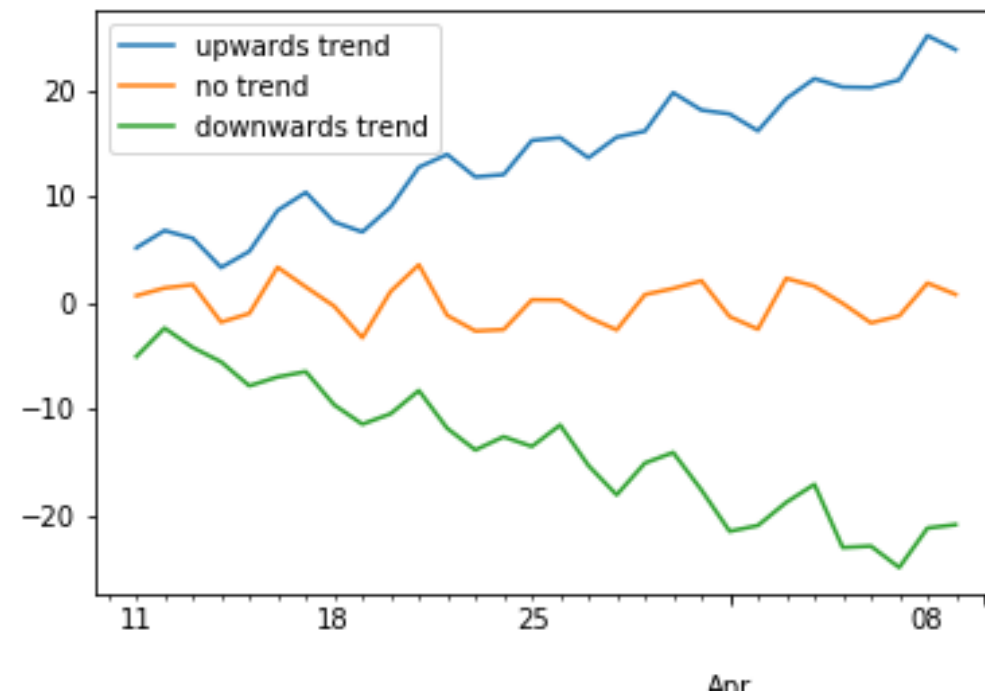
```
import pandas as pd
import matplotlib as plt

df = pd.read_csv('time_series.csv', index_col='date', parse_dates=True)
```

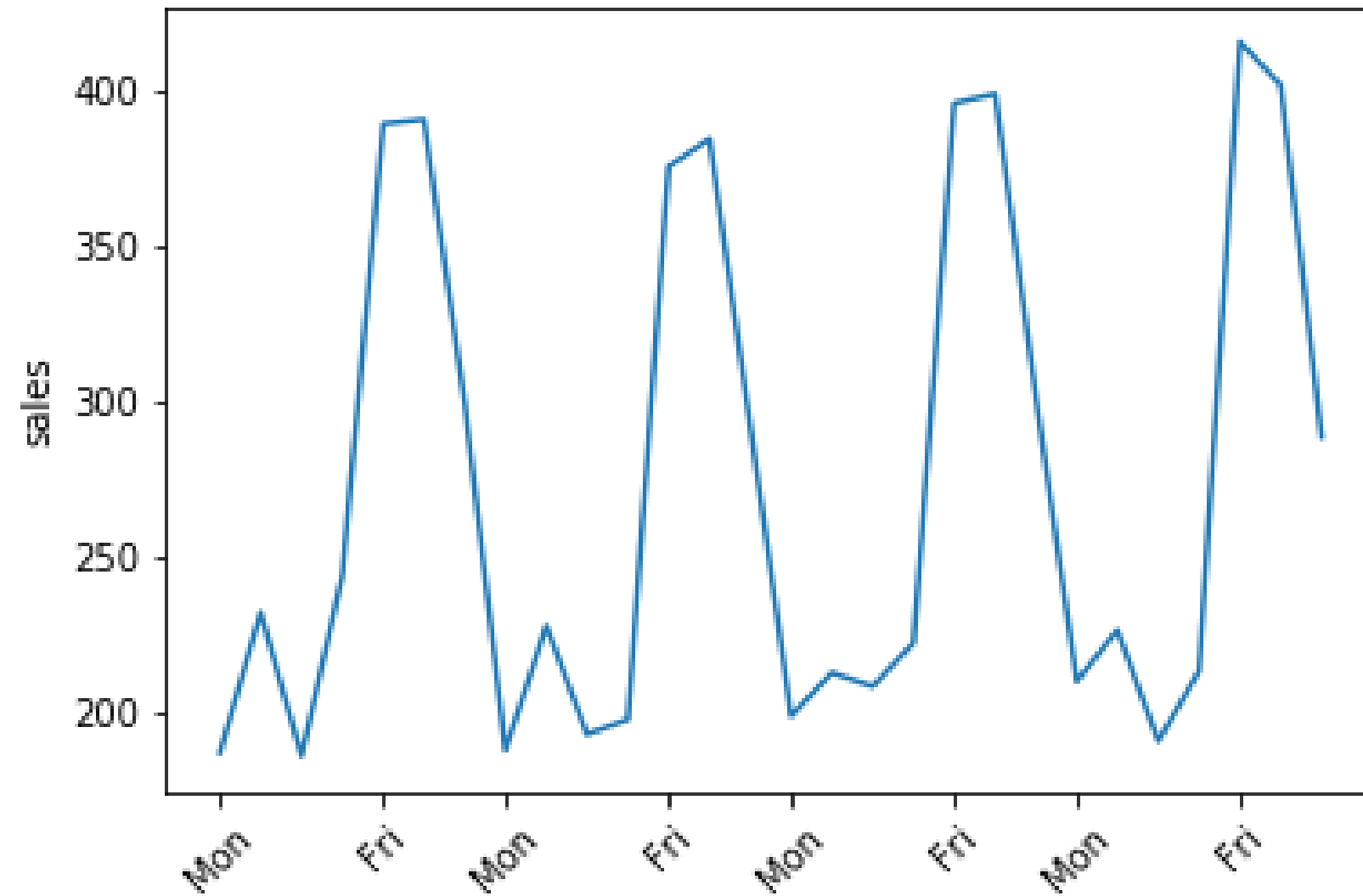
date	values
2019-03-11	5.734193
2019-03-12	6.288708
2019-03-13	5.205788
2019-03-14	3.176578

# Trend

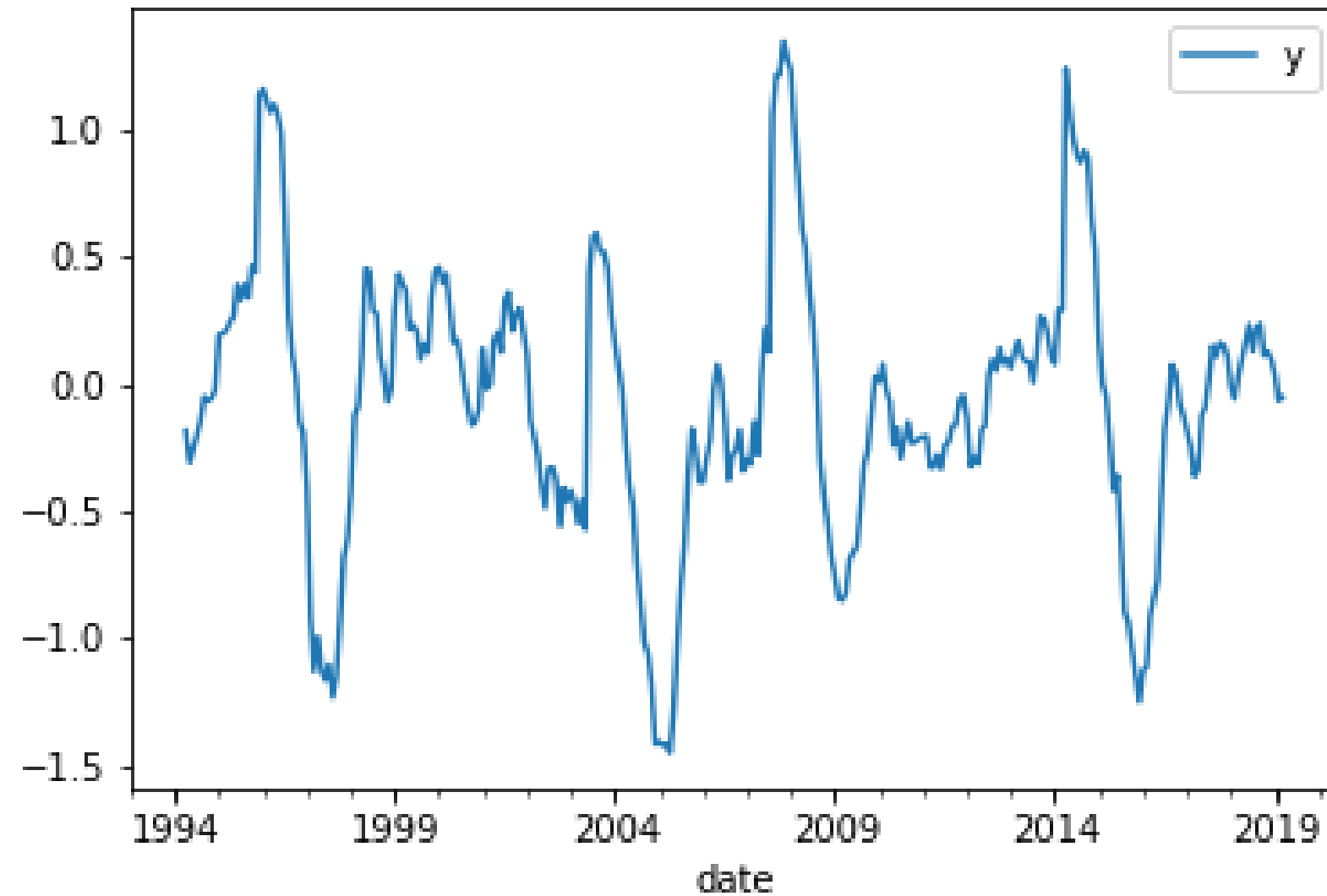
```
fig, ax = plt.subplots()
df.plot(ax=ax)
plt.show()
```



# Seasonality



# Cyclicality



# White noise

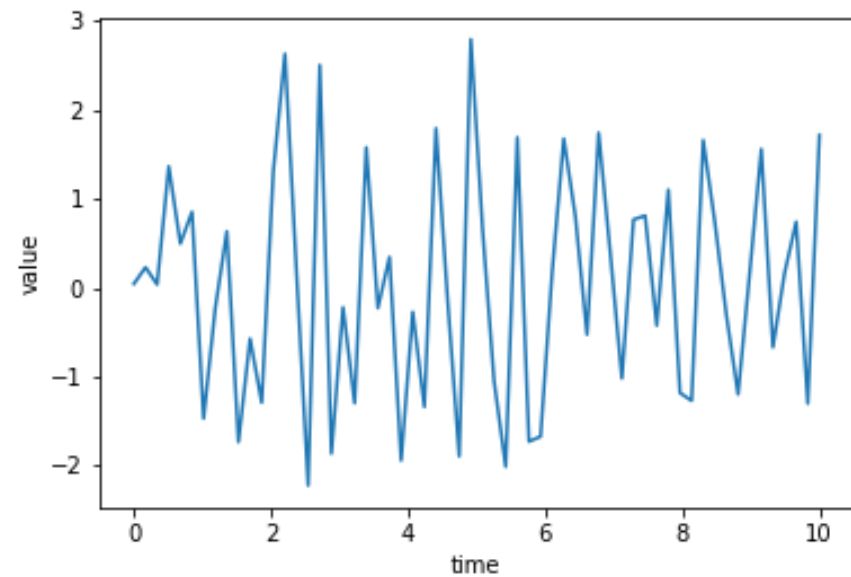
White noise series has uncorrelated values

- Heads, heads, heads, tails, heads, tails, ...
- 0.1, -0.3, 0.8, 0.4, -0.5, 0.9, ...

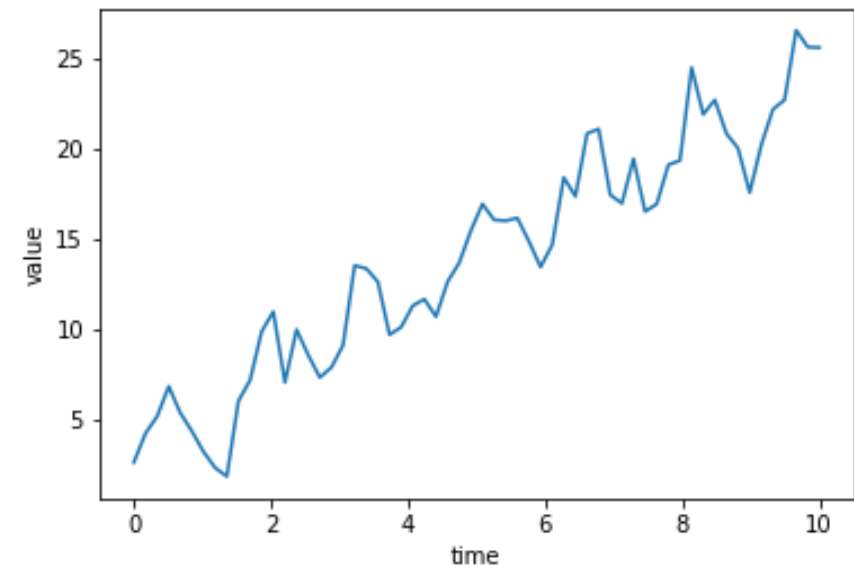


# Stationarity

Stationary



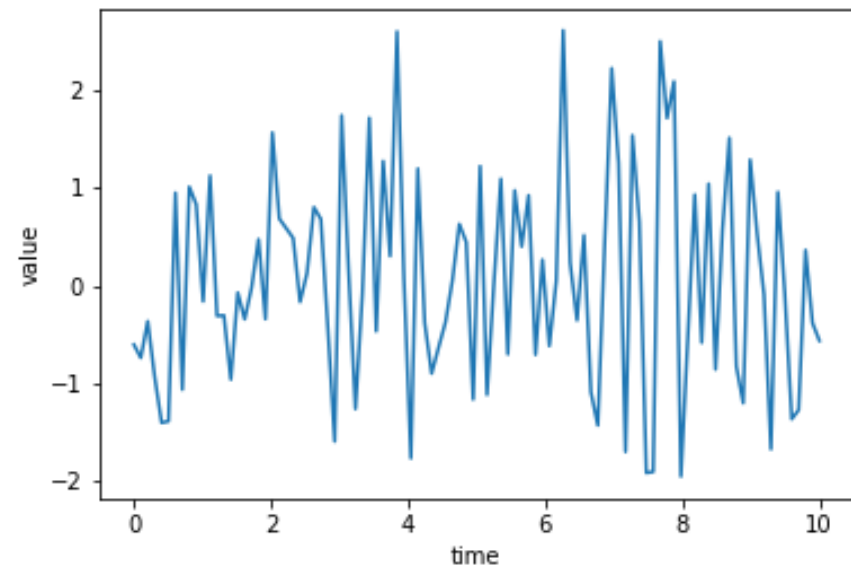
Not stationary



- Trend stationary: Trend is zero

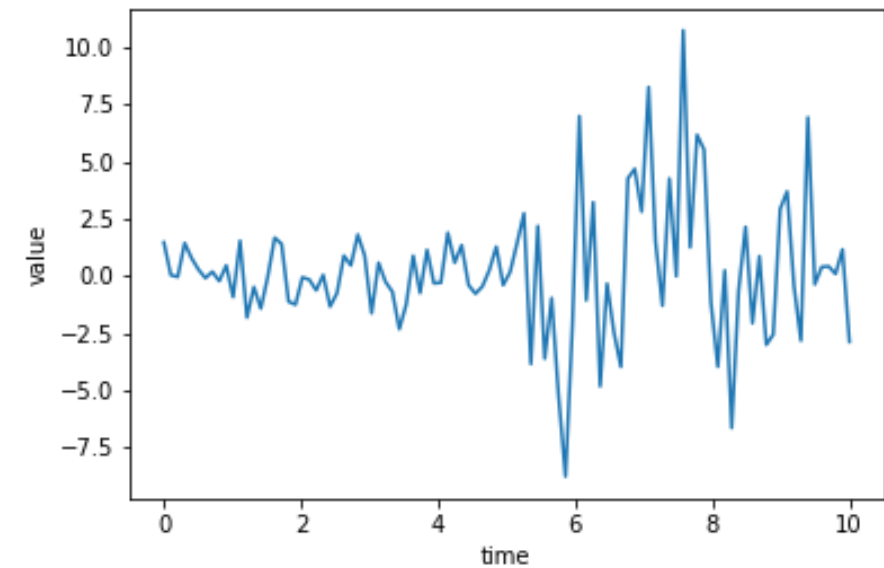
# Stationarity

Stationary



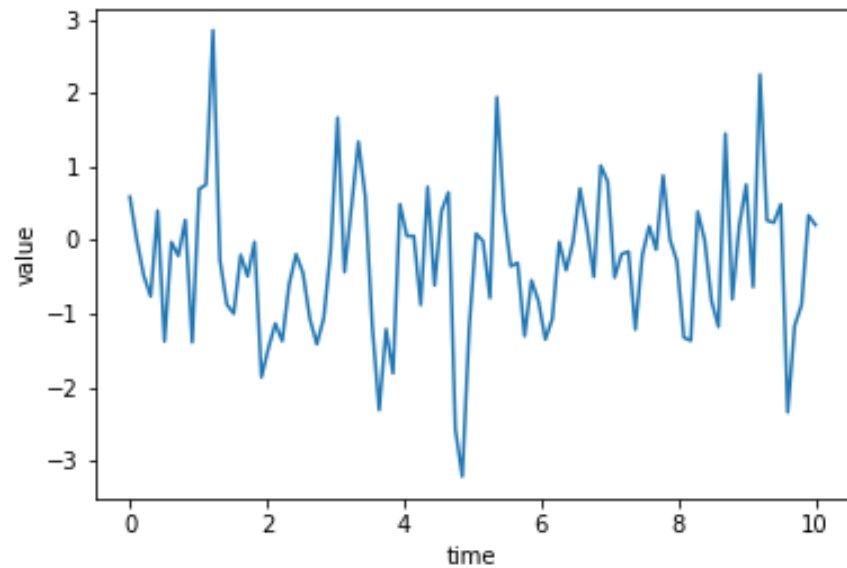
- Trend stationary: Trend is zero
- Variance is constant

Not stationary

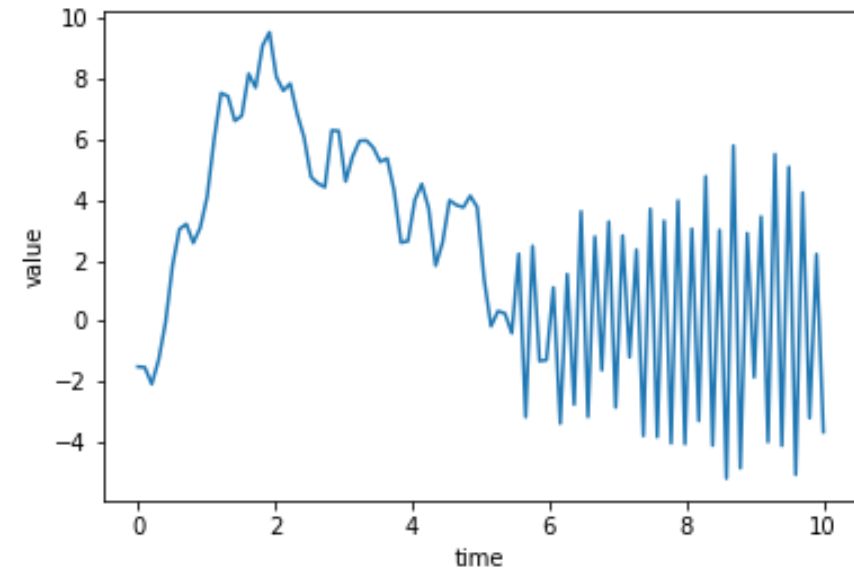


# Stationarity

Stationary



Not stationary



- Trend stationary: Trend is zero
- Variance is constant
- Autocorrelation is constant

# Train-test split

```
# Train data - all data up to the end of 2018
df_train = df.loc[:'2018']

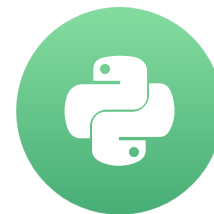
# Test data - all data from 2019 onwards
df_test = df.loc['2019':]
```

# Let's Practice!

ARIMA MODELS IN PYTHON

# Making time series stationary

ARIMA MODELS IN PYTHON



**James Fulton**

Climate informatics researcher

# Overview

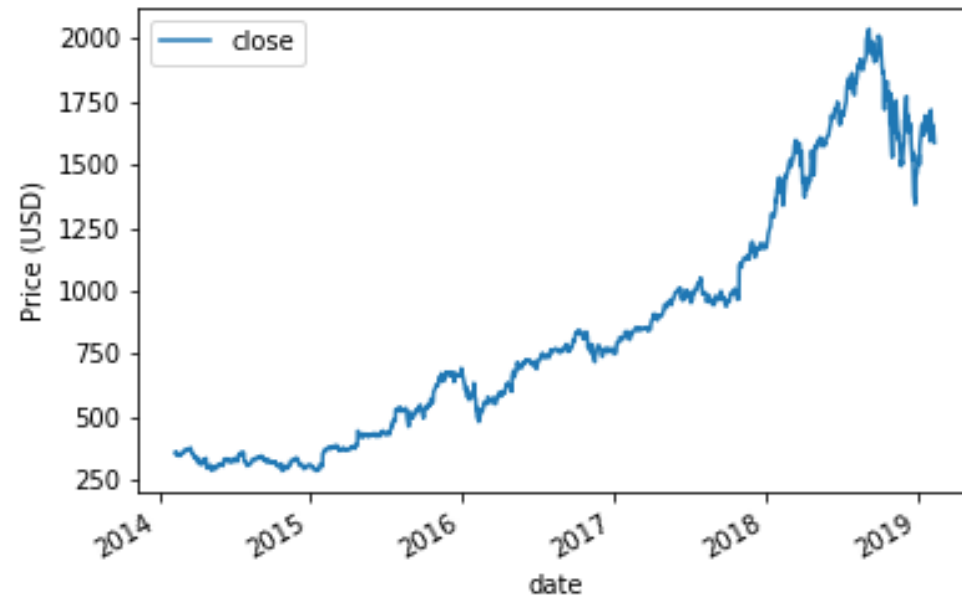
- Statistical tests for stationarity
- Making a dataset stationary

# The augmented Dicky-Fuller test

- Tests for trend non-stationarity
- Null hypothesis is time series is non-stationary



# Applying the adfuller test



```
from statsmodels.tsa.stattools import adfuller
```

```
results = adfuller(df['close'])
```

# Interpreting the test result

```
print(results)    The result is a tuple.
```

```
(-1.34, 0.60, 23, 1235, {'1%': -3.435, '5%': -2.913, '10%': -2.568}, 10782.87)
```

- 0th element is test statistic (-1.34)
  - More negative means more likely to be stationary
- 1st element is p-value: (0.60)
  - If p-value is small → reject null hypothesis. Reject non-stationary.
- 4th element is the critical test statistics

# Interpreting the test result

```
print(results)
```

```
(-1.34, 0.60, 23, 1235, {'1%': -3.435, '5%': -2.863, '10%': -2.568}, 10782.87)
```

- 0th element is test statistic (-1.34)
  - More negative means more likely to be stationary
- 1st element is p-value: (0.60)
  - If p-value is small → reject null hypothesis. Reject non-stationary.
- 4th element is the critical test statistics

<sup>1</sup> <https://www.statsmodels.org/dev/generated/statsmodels.tsa.stattools.adfuller.html>

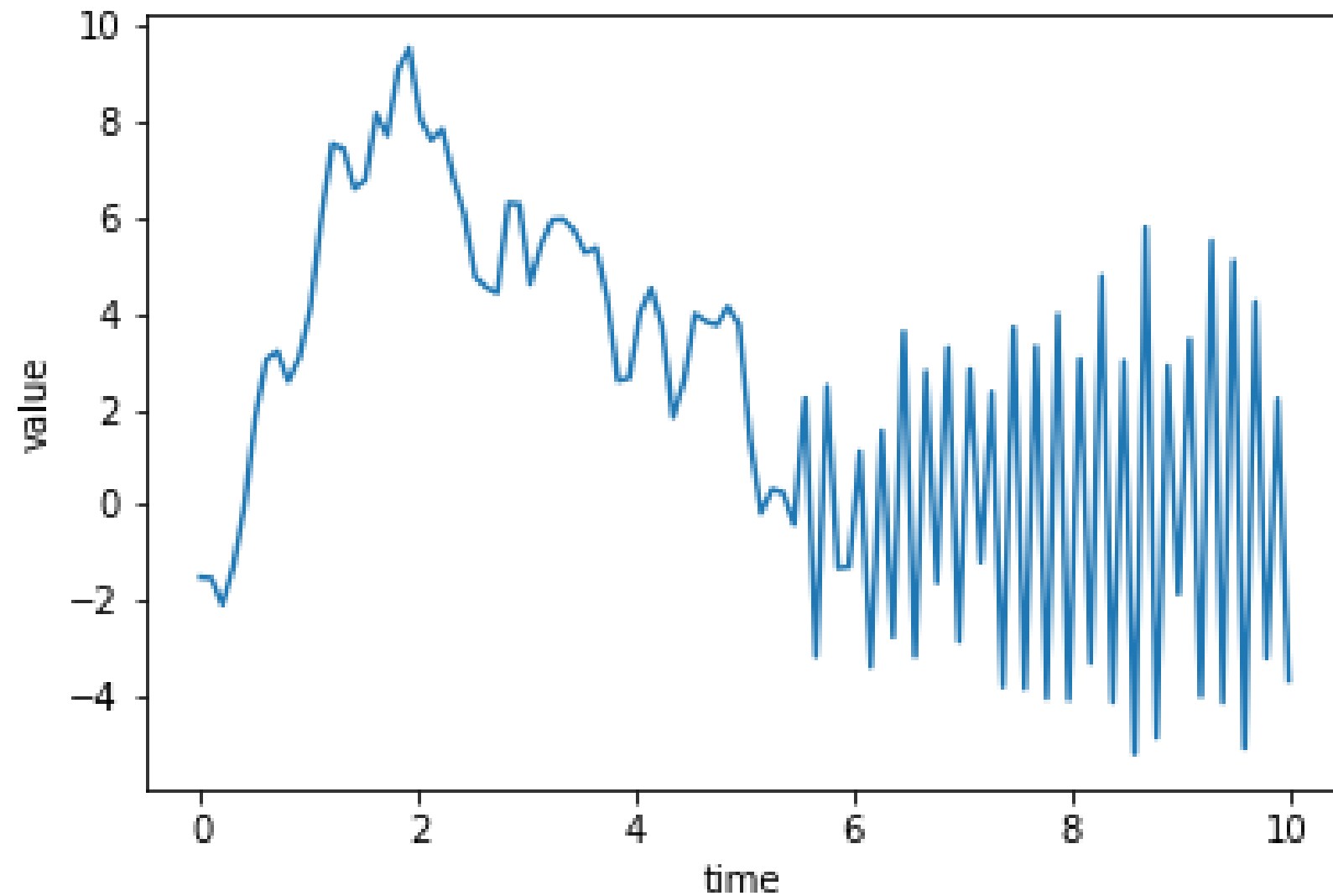
# The value of plotting

- Plotting time series can stop you making wrong assumptions

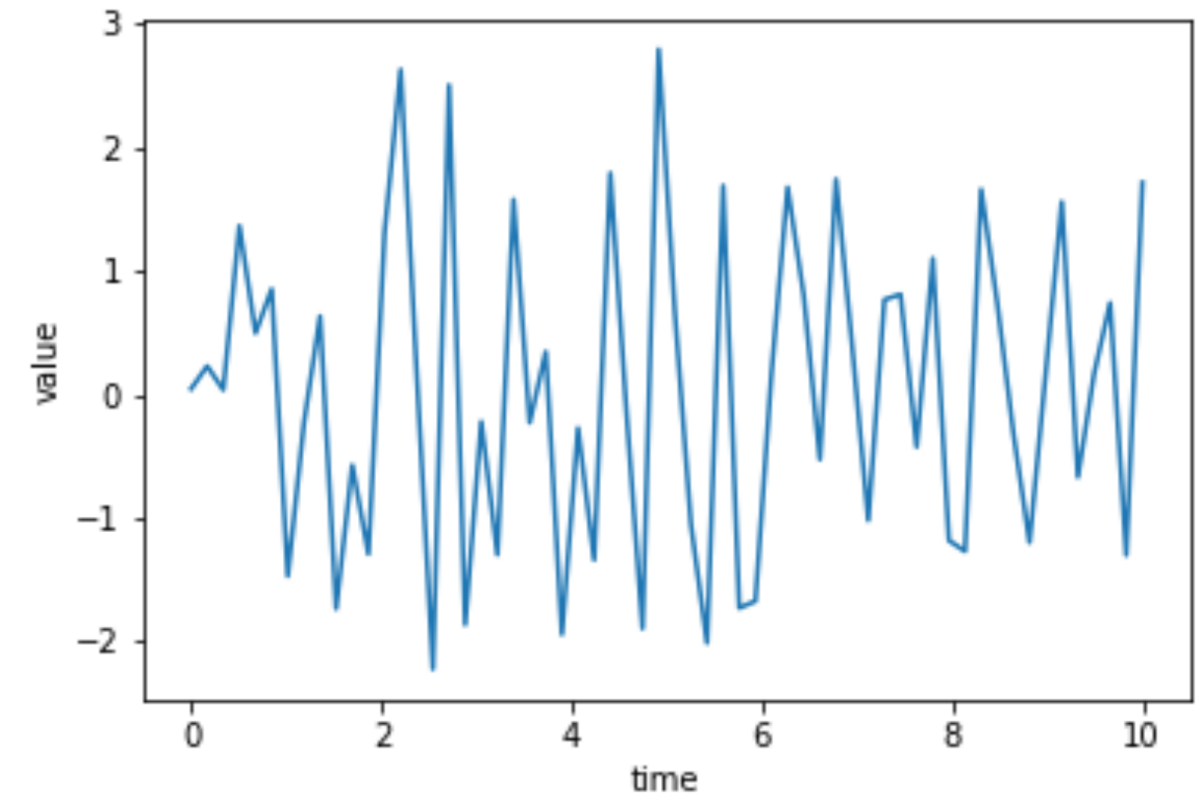
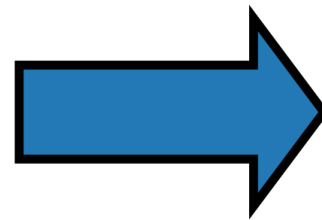
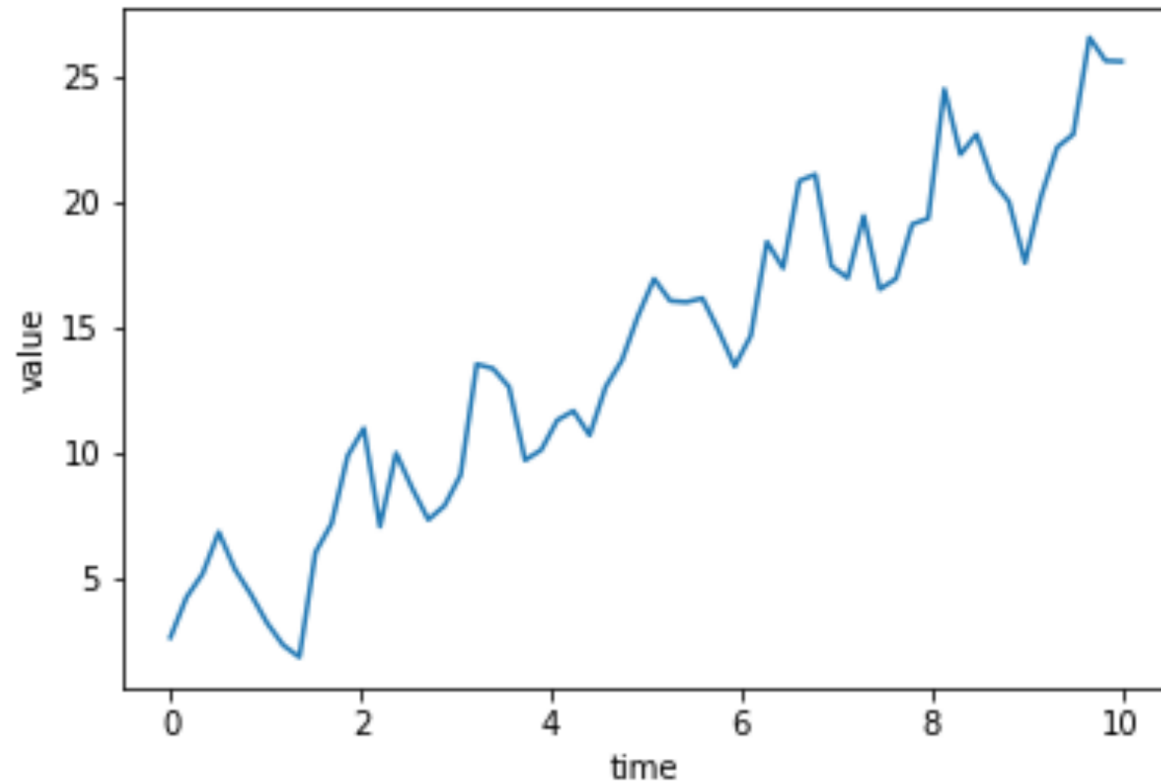
# The value of plotting

The Dicky-Fuller only tests for trend stationarity.

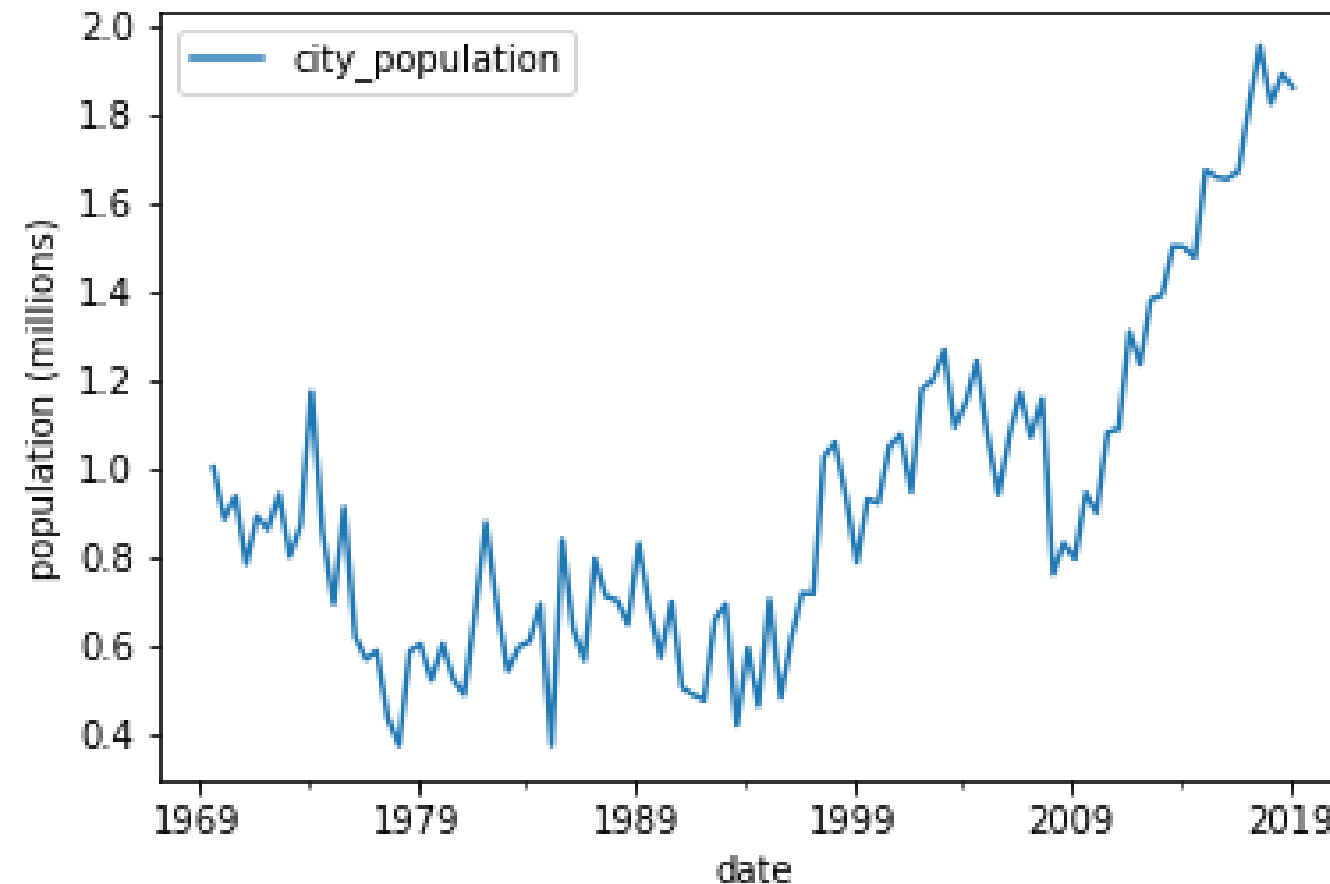
In this example, although the time series behavior clearly changes, and is non-stationary, it passes the Dicky-Fuller test.



# Making a time series stationary



# Taking the difference



Difference:  $\Delta y_t = y_t - y_{t-1}$

# Taking the difference

```
df_stationary = df.diff()
```

```
      city_population
date
1969-09-30         NaN
1970-03-31    -0.116156
1970-09-30     0.050850
1971-03-31    -0.153261
1971-09-30     0.108389
```

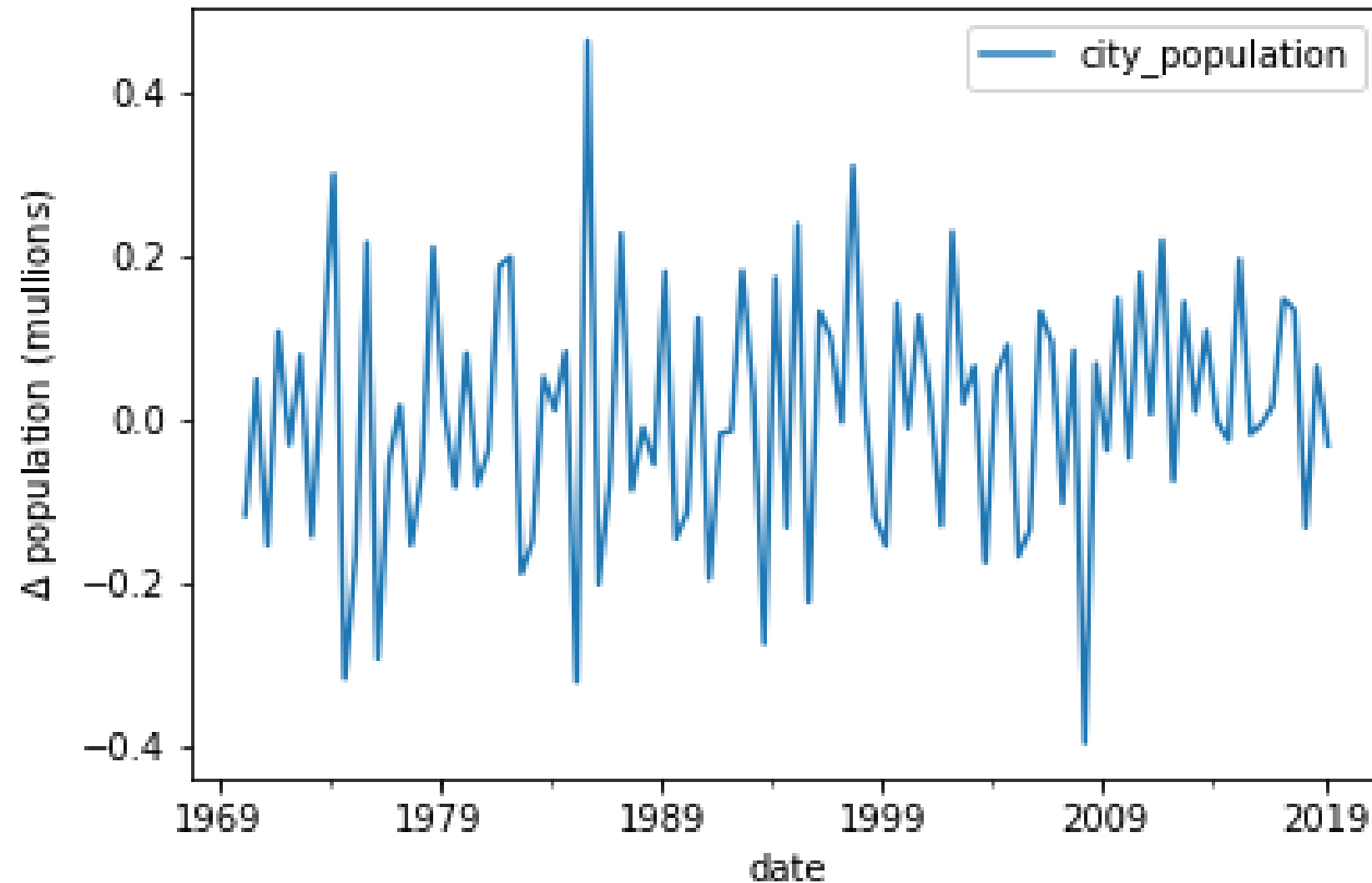


# Taking the difference

```
df_stationary = df.diff().dropna()
```

	city_population
date	
1970-03-31	-0.116156
1970-09-30	0.050850
1971-03-31	-0.153261
1971-09-30	0.108389
1972-03-31	-0.029569

# Taking the difference



# Other transforms

## Examples of other transforms

- Take the log
  - `np.log(df)`
- Take the square root
  - `np.sqrt(df)`
- Take the proportional change
  - `df.shift(1)/df`

# Let's practice!

ARIMA MODELS IN PYTHON

# Intro to AR, MA and ARMA models

ARIMA MODELS IN PYTHON



**James Fulton**

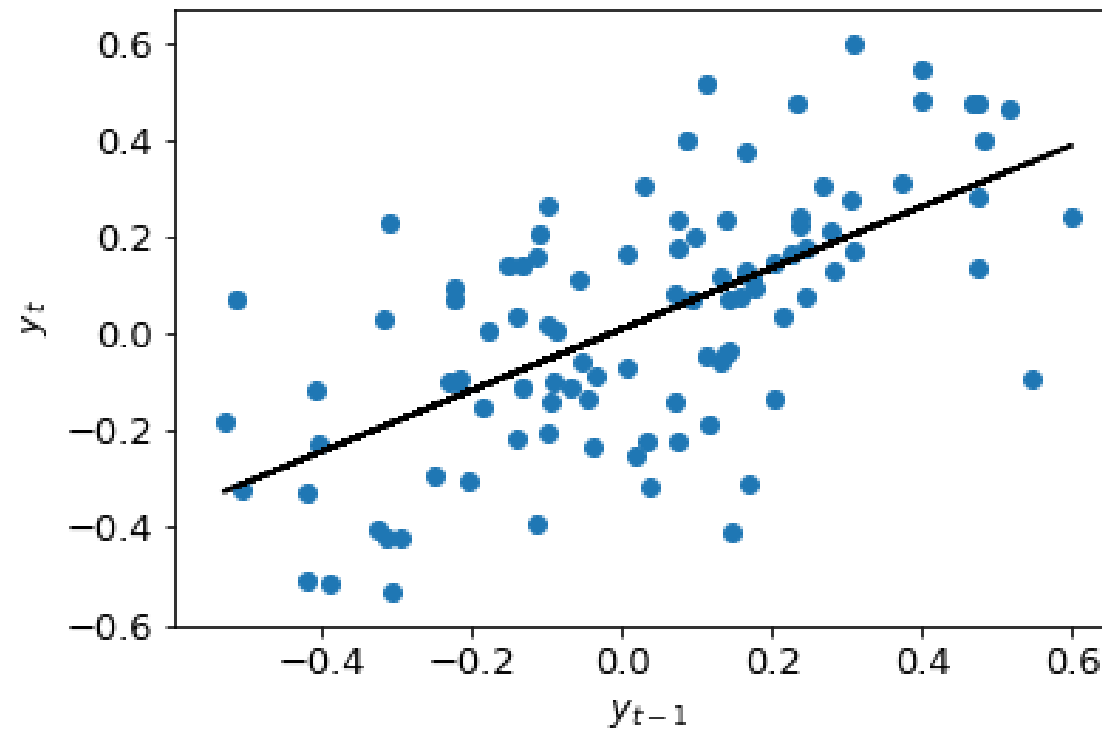
Climate informatics researcher

# AR models

Autoregressive (AR) model

AR(1) model:

$$y_t = a_1 y_{t-1} + \epsilon_t$$



# AR models

Autoregressive (AR) model

AR(1) model :

$$y_t = a_1 y_{t-1} + \epsilon_t$$

AR(2) model :

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \epsilon_t$$

AR(p) model :

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_p y_{t-p} + \epsilon_t$$

# MA models

Moving average (MA) model

MA(1) model :

$$y_t = m_1 \epsilon_{t-1} + \epsilon_t \quad \text{a shock term}$$

MA(2) model :

$$y_t = m_1 \epsilon_{t-1} + m_2 \epsilon_{t-2} + \epsilon_t$$

MA(q) model :

$$y_t = m_1 \epsilon_{t-1} + m_2 \epsilon_{t-2} + \dots + m_q \epsilon_{t-q} + \epsilon_t$$



# ARMA models

Autoregressive moving-average (ARMA) model

- ARMA = AR + MA

ARMA(1,1) model:

$$y_t = a_1 y_{t-1} + m_1 \epsilon_{t-1} + \epsilon_t$$

ARMA(p, q)

- p is order of AR part
- q is order of MA part

The p tells us the order of the autoregressive part of the model and the q tells us the order of the moving-average part.

# Creating ARMA data

$$y_t = a_1 y_{t-1} + m_1 \epsilon_{t-1} + \epsilon_t$$

# Creating ARMA data

$$y_t = 0.5y_{t-1} + 0.2\epsilon_{t-1} + \epsilon_t$$

```
from statsmodels.tsa.arima_process import arma_generate_sample  
  
ar_coefs = [1, -0.5]  
ma_coefs = [1, 0.2]  
  
y = arma_generate_sample(ar_coefs, ma_coefs, nsample=100, sigma=0.5)
```

You can use the `arma_generate_sample()` function available in your workspace to generate time series using different AR and MA coefficients.

Remember for any model ARMA(p,q):

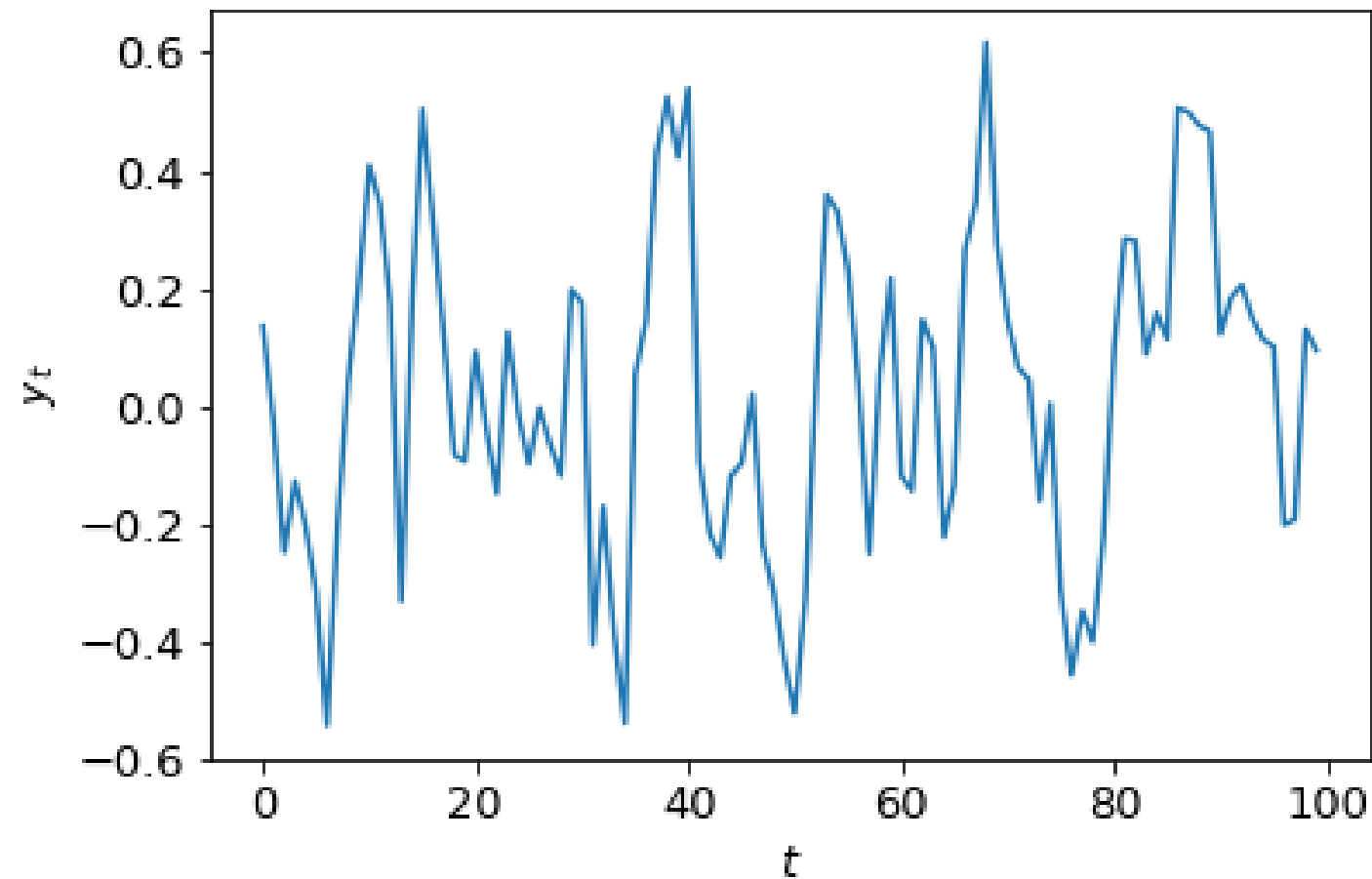
The list `ar_coefs` has the form `[1, -a_1, -a_2, ..., -a_p]`.

The list `ma_coefs` has the form `[1, m_1, m_2, ..., m_q]`,

where `a_i` are the lag-`i` AR coefficients and `m_j` are the lag-`j` MA coefficients.

# Creating ARMA data

$$y_t = 0.5y_{t-1} + 0.2\epsilon_{t-1} + \epsilon_t$$



# Fitting and ARMA model

```
from statsmodels.tsa.arima_model import ARMA

# Instantiate model object
model = ARMA(y, order=(1,1))

# Fit model
results = model.fit()
```

# Let's practice!

ARIMA MODELS IN PYTHON