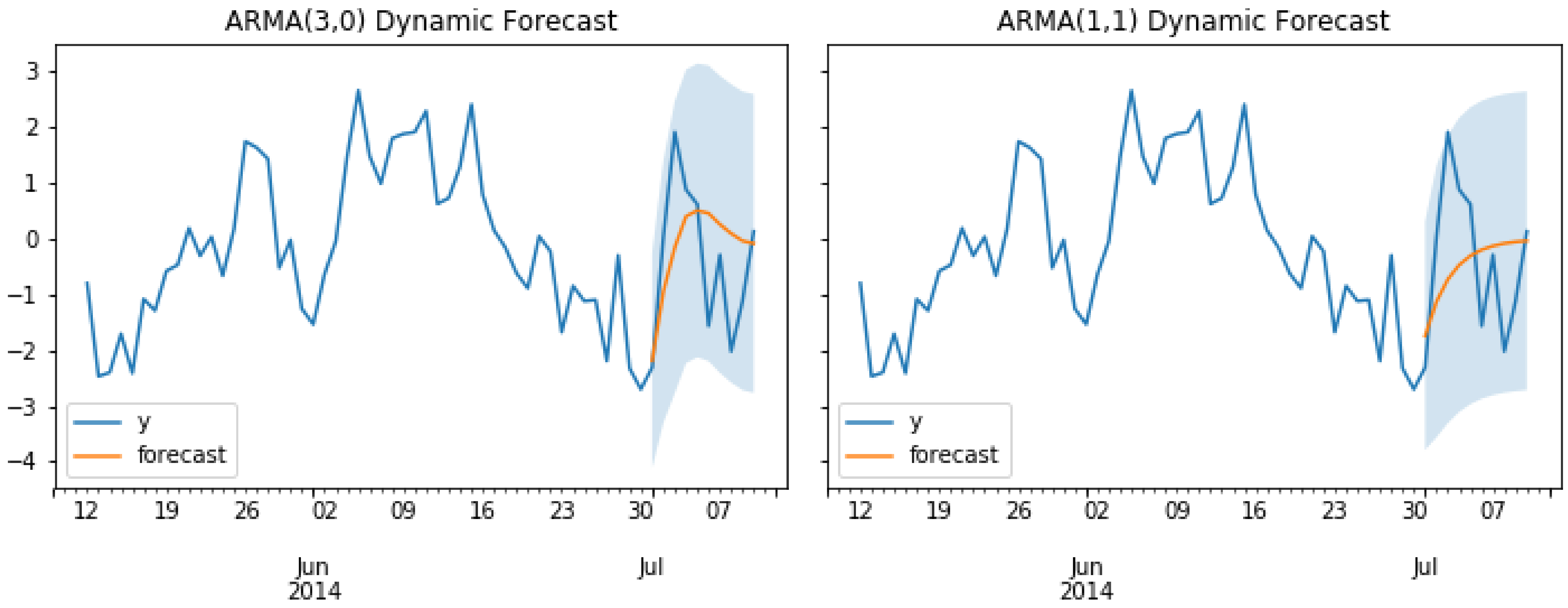
Intro to ACF and PACF

ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

Motivation



ARMA(3,0) Dynamic Forecast
ARMA(1,1) Dynamic Forecast
y
forecast
3
2
1
0
−1
−2
−3
−4
12
19
26
02
09
16
23
30
07
Jun
2014
Jul

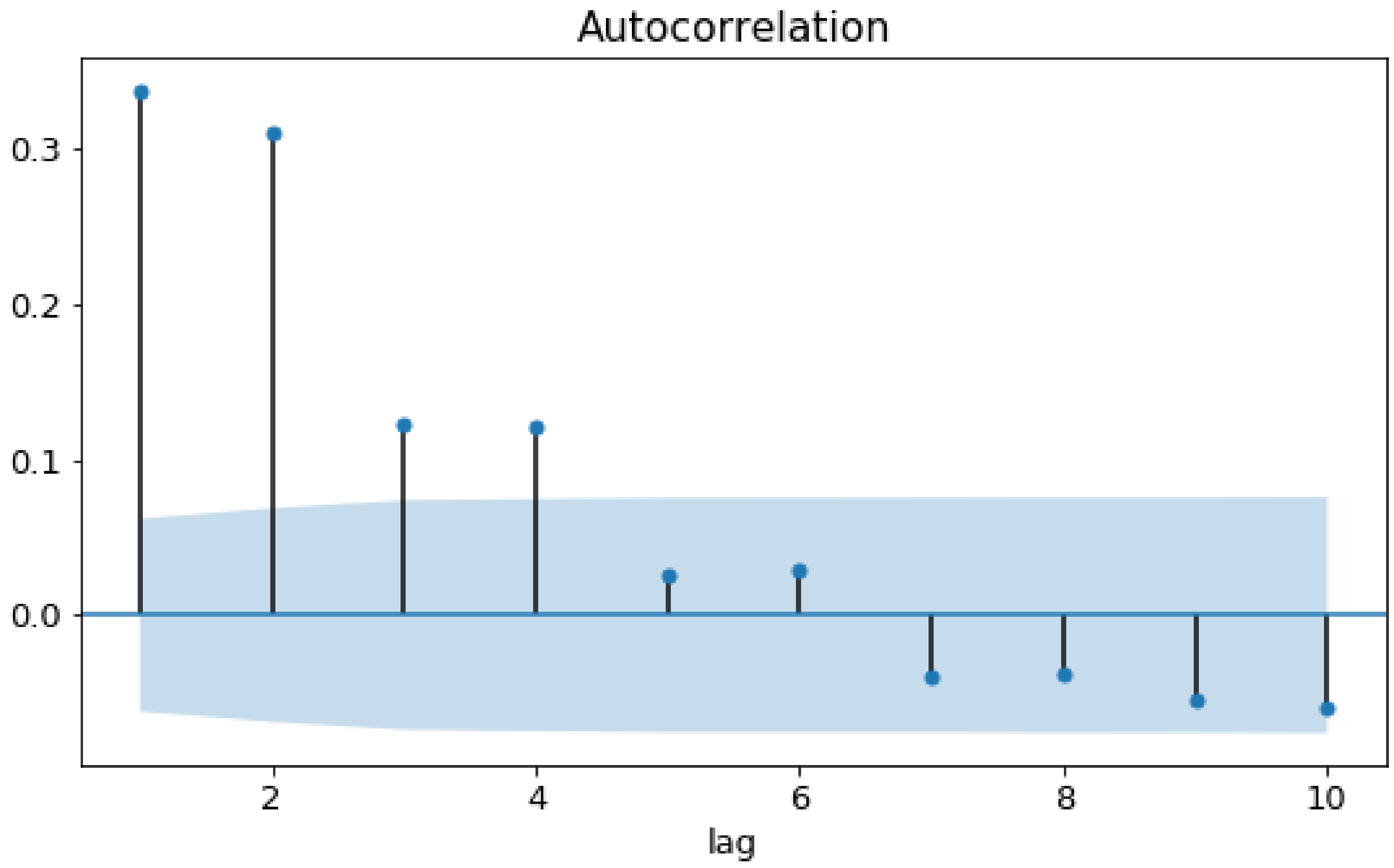
ACF and PACF

- ACF - Autocorrelation Function
- PACF - Partial autocorrelation function

What is the ACF

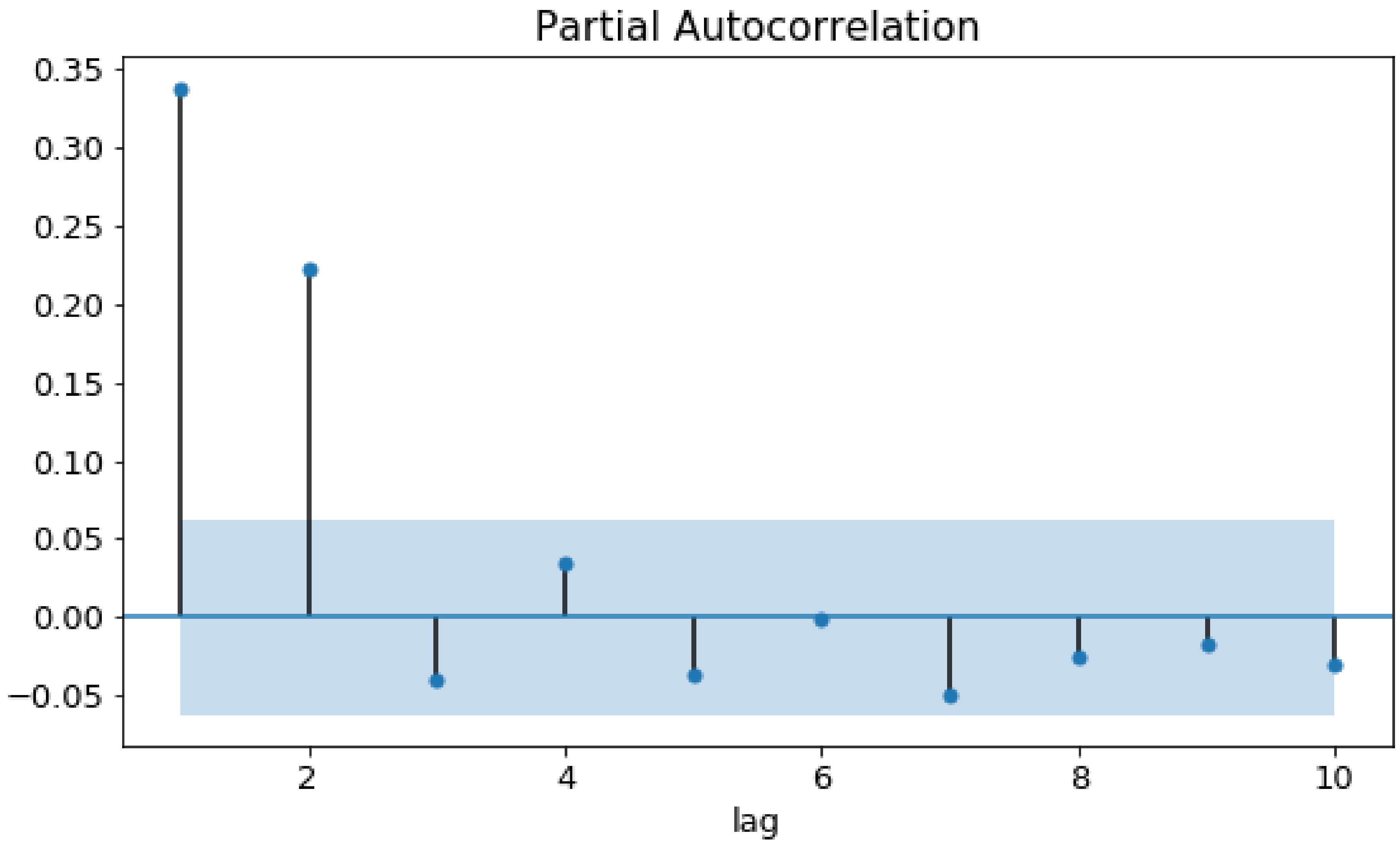
- lag-1 autocorrelation $\rightarrow corr(y_t, y_{t-1})$
- lag-2 autocorrelation $\rightarrow corr(y_t, y_{t-2})$
- ...
- lag-n autocorrelation $\rightarrow corr(y_t, y_{t-n})$

What is the ACF



What is the PACF

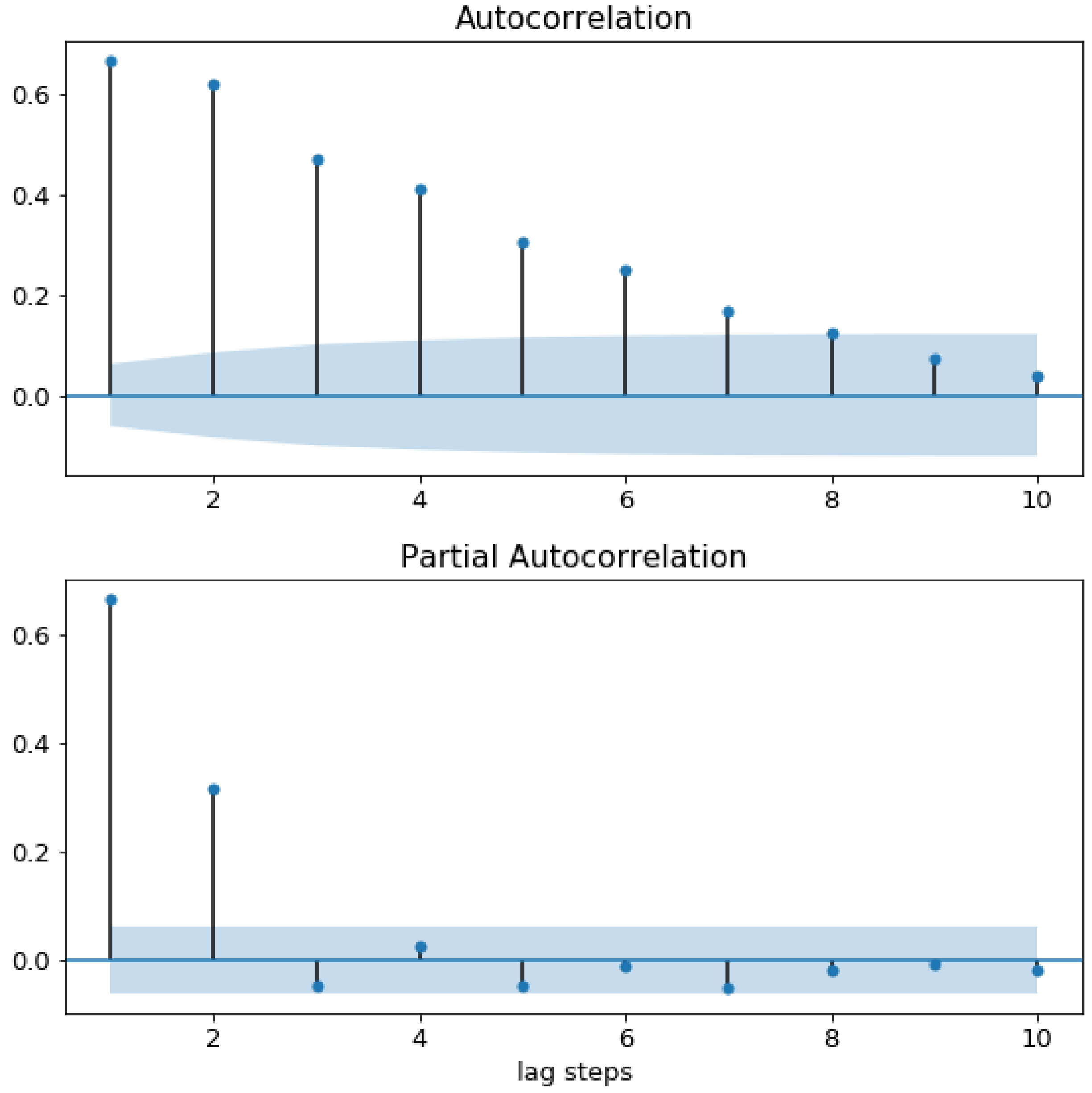
The PACF is the correlation between a time series and the lagged version of itself after we subtract the effect of correlation at smaller lags.
So it is the correlation associated with just that particular lag.



Using ACF and PACF to choose model order

	AR(p)
ACF	Tails off
PACF	Cuts off after lag p

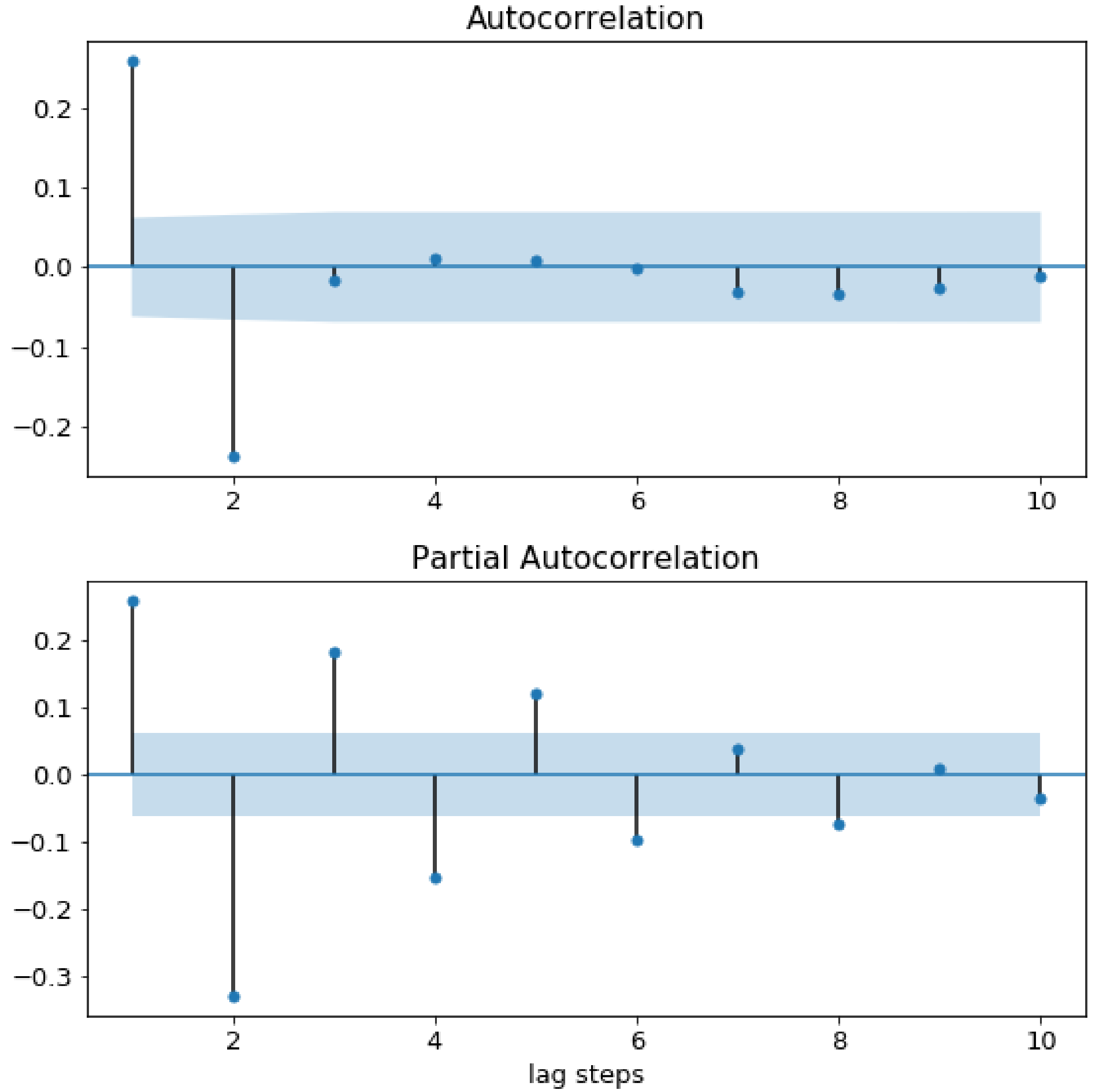
- AR(2) model $\rightarrow$



Using ACF and PACF to choose model order

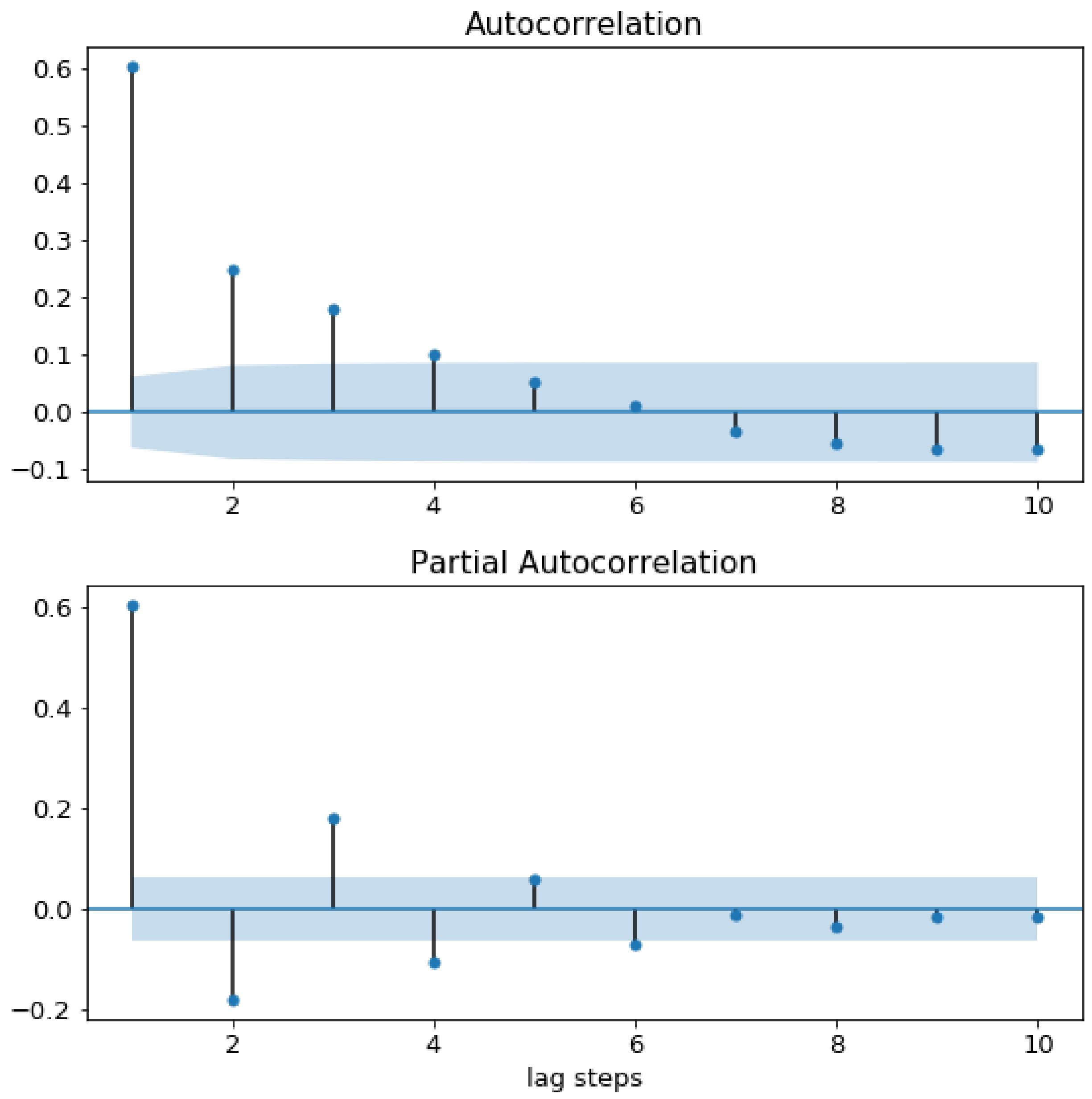
	MA(q)
ACF	Cuts off after lag q
PACF	Tails off

- MA(2) model $\rightarrow$



Using ACF and PACF to choose model order

	ARMA(p,q)
ACF	Tails off
PACF	Tails off



Using ACF and PACF to choose model order

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

Implementation in Python

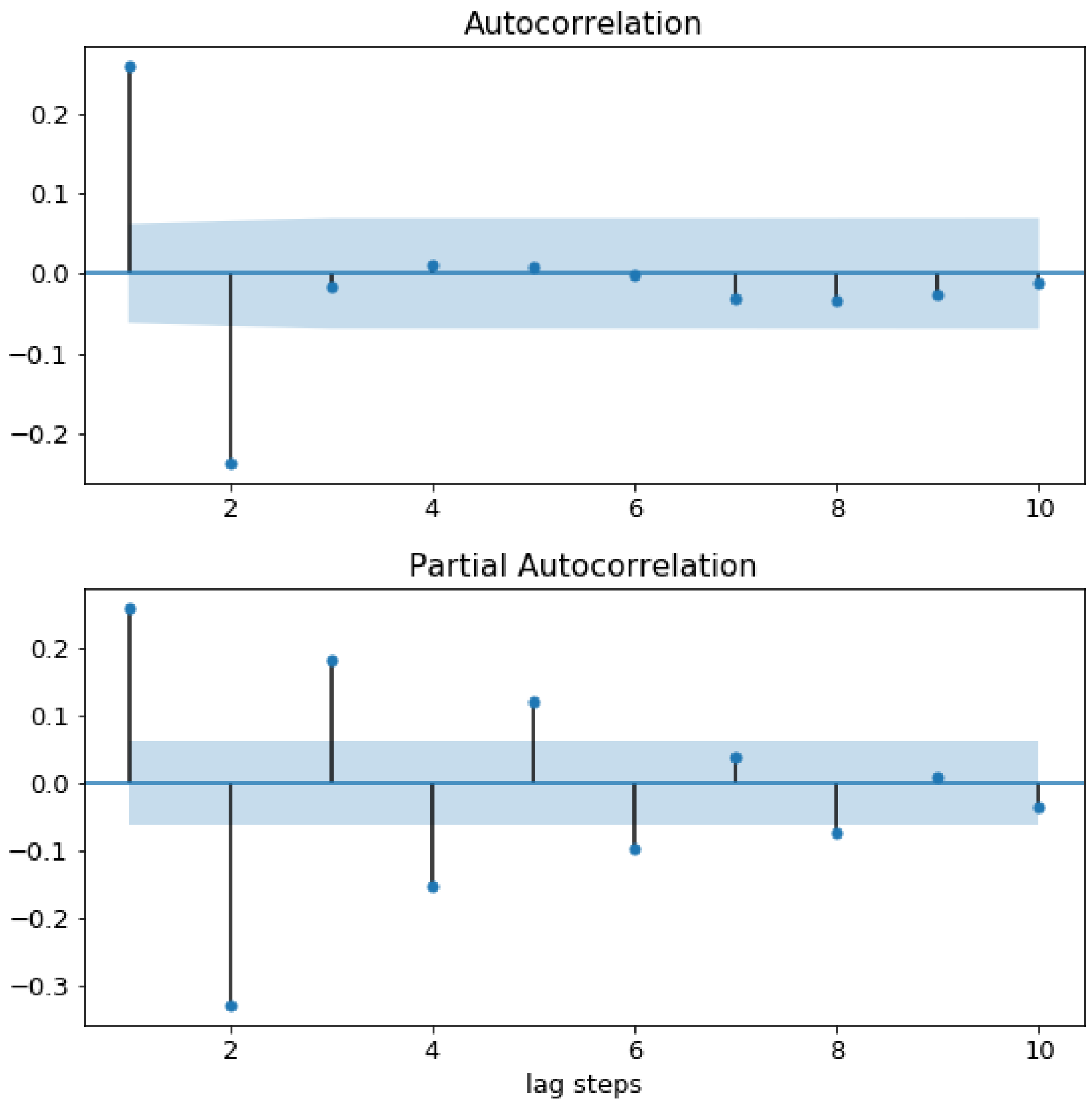
```
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

```
# Create figure
fig, (ax1, ax2) = plt.subplots(2,1, figsize=(8,8))
# Make ACF plot
plot_acf(df, lags=10, zero=False, ax=ax1)
# Make PACF plot
plot_pacf(df, lags=10, zero=False, ax=ax2)

plt.show()
```

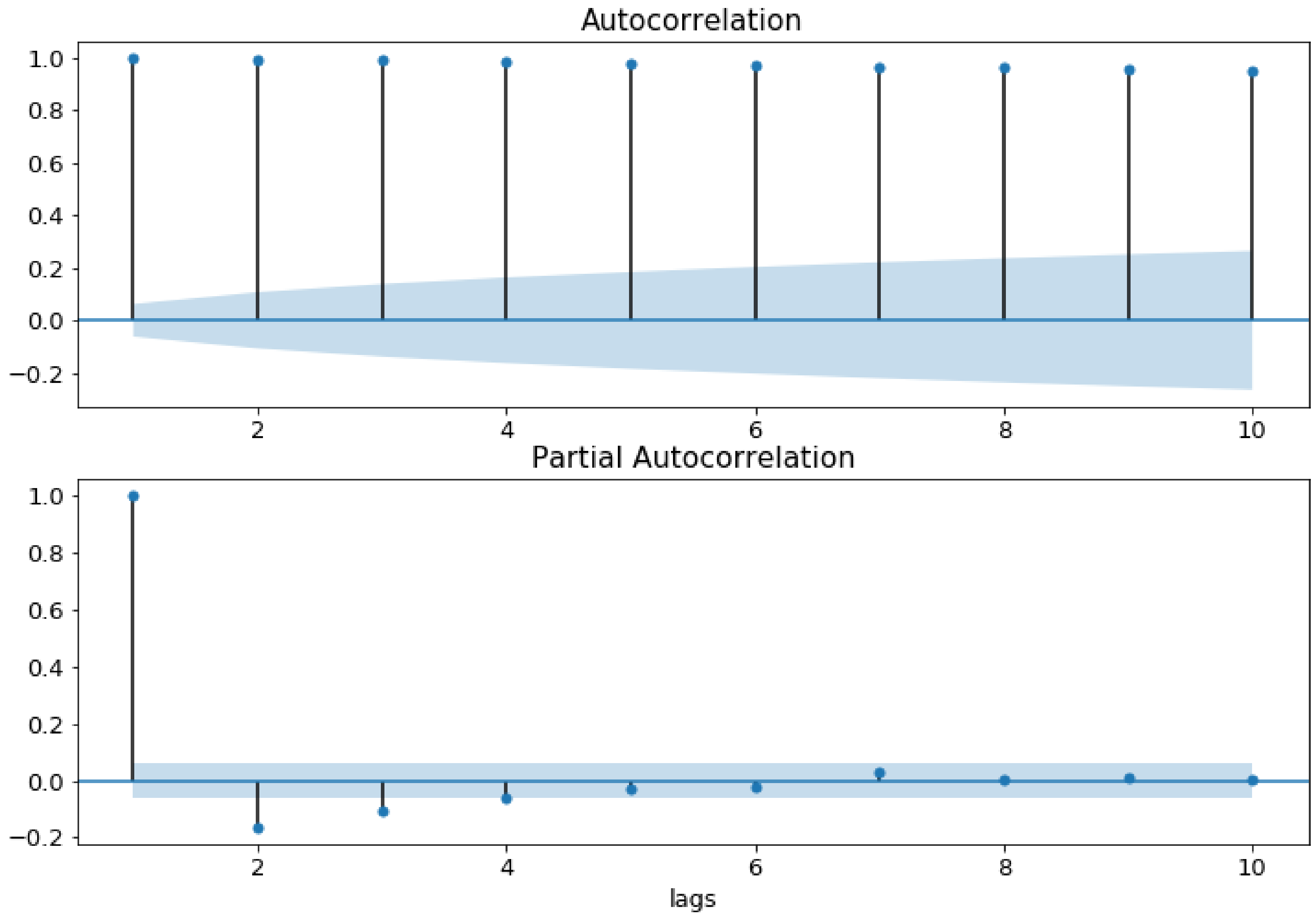
whether to show the autocorrelation at lag 0

Implementation in Python



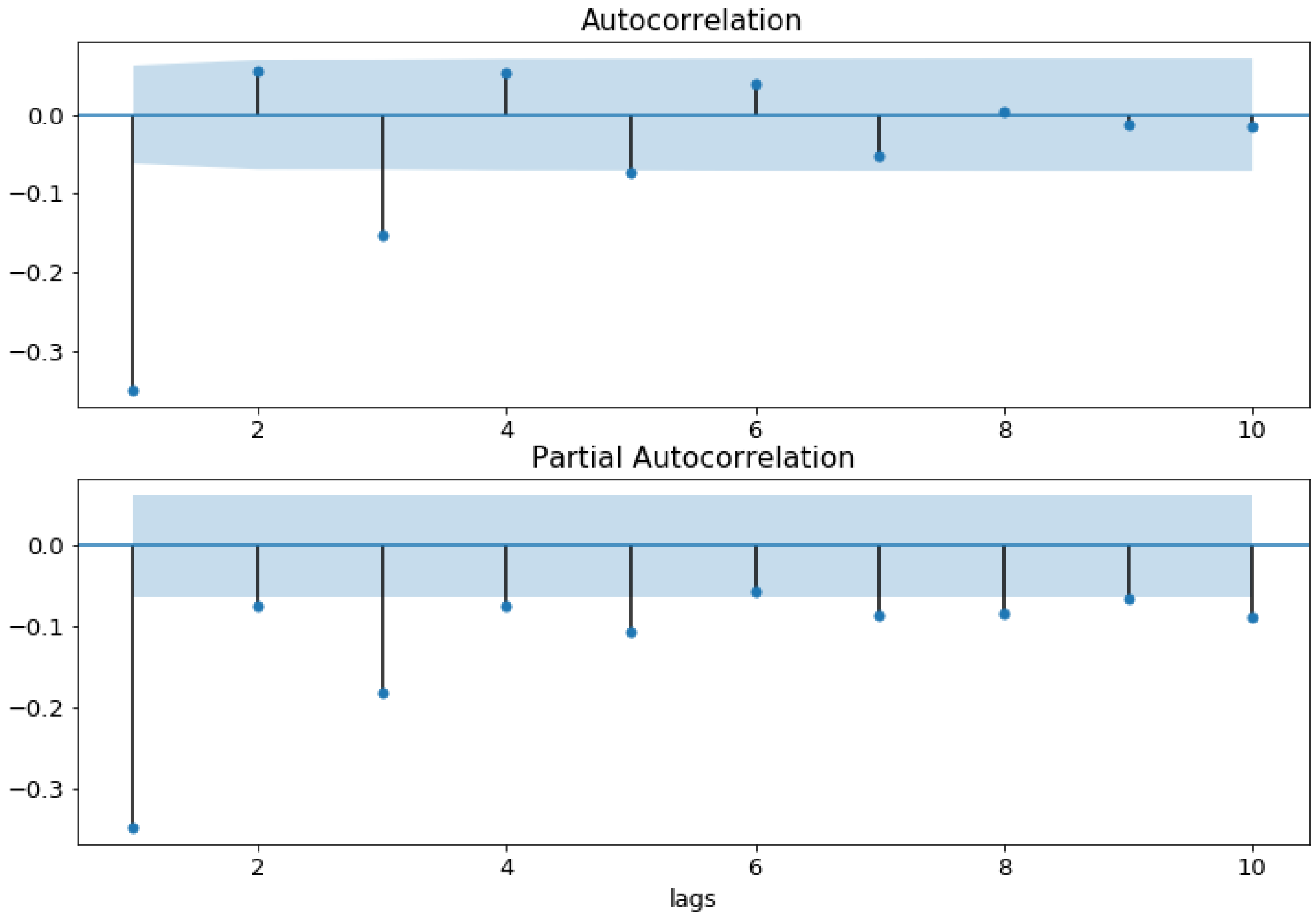
Over/under differencing and ACF and PACF

If the ACF values are high and tail off very very slowly, this is a sign that the data is non-stationarity, so it needs to be differenced.



Over/under differencing and ACF and PACF

If the autocorrelation at lag1 is very negative, this is a sign that we have taken the difference too many times.



Let's practice!

ARIMA MODELS IN PYTHON

AIC and BIC

ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

AIC - Akaike information criterion

- Lower AIC indicates a better model
- AIC likes to choose simple models with lower order

BIC - Bayesian information criterion

- Very similar to AIC
- Lower BIC indicates a better model
- BIC likes to choose simple models with lower order

AIC vs BIC

- BIC favors simpler models than AIC
- AIC is better at choosing predictive models
- BIC is better at choosing good explanatory model

AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))
# Fit model
results = model.fit()
# Print fit summary
print(results.summary())
```

```
                           Statespace Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 1000
Model:               SARIMAX(2, 0, 0)   Log Likelihood               -1399.704
Date:                Fri, 10 May 2019   AIC                           2805.407
Time:                        01:06:11   BIC                           2820.131
Sample:                    01-01-2013   HQIC                          2811.003
                         - 09-27-2015
Covariance Type:                  opg
```

AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))

# Fit model
results = model.fit()

# Print AIC and BIC
print('AIC:', results.aic)
print('BIC:', results.bic)
```

```
AIC: 2806.36
BIC: 2821.09
```

Searching over AIC and BIC

```python
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()
        # print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

```
0 0 2900.13 2905.04
0 1 2828.70 2838.52
0 2 2806.69 2821.42
1 0 2810.25 2820.06
1 1 2806.37 2821.09
1 2 2807.52 2827.15
...
```

Searching over AIC and BIC

```
order_aic_bic =[]
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()
        # Add order and scores to list
        order_aic_bic.append((p, q, results.aic, results.bic))
```

```
# Make DataFrame of model order and AIC/BIC scores
order_df = pd.DataFrame(order_aic_bic, columns=['p','q', 'aic', 'bic'])
```

Searching over AIC and BIC

```
# Sort by AIC
print(order_df.sort_values('aic'))
```

```
   p  q      aic      bic
7  2  1  2804.54  2824.17
6  2  0  2805.41  2820.13
4  1  1  2806.37  2821.09
2  0  2  2806.69  2821.42
...
```

```
# Sort by BIC
print(order_df.sort_values('bic'))
```

```
   p  q      aic      bic
3  1  0  2810.25  2820.06
6  2  0  2805.41  2820.13
4  1  1  2806.37  2821.09
2  0  2  2806.69  2821.42
...
```

Non-stationary model orders

```
# Fit model
model = SARIMAX(df, order=(2,0,1))
results = model.fit()
```

```
ValueError: Non-stationary starting autoregressive parameters
found with `enforce_stationarity` set to True.
```

This ValueError tells us that we have tried to fit a model, which would result in a non-stationary set of AR coefficients

When certain orders don't work

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):


        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()

        # Print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

When certain orders don't work

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        try:
            # Fit model
            model = SARIMAX(df, order=(p,0,q))
            results = model.fit()

            # Print the model order and the AIC/BIC values
            print(p, q, results.aic, results.bic)
        except:
            # Print AIC and BIC as None when fails
            print(p, q, None, None)
```

Let's practice!

ARIMA MODELS IN PYTHON

Model diagnostics

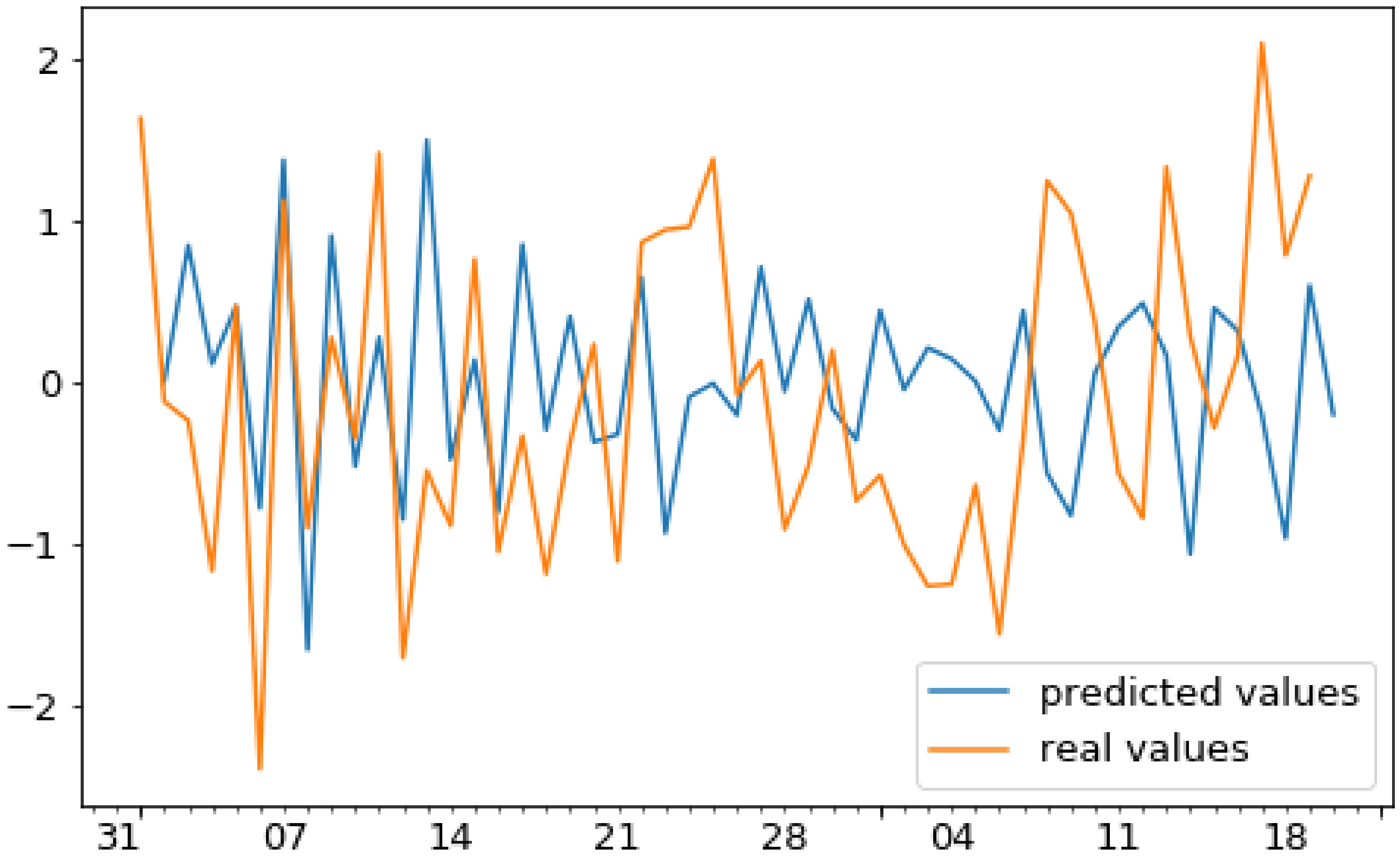
ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

Introduction to model diagnostics

- How good is the final model?

Residuals



Residuals

```
# Fit model
model = SARIMAX(df, order=(p,d,q))
results = model.fit()

# Assign residuals to variable
residuals = results.resid
```

```
2013-01-23     1.013129
2013-01-24     0.114055
2013-01-25     0.430698
2013-01-26    -1.247046
2013-01-27    -0.499565
       ...          ...
```

Mean absolute error

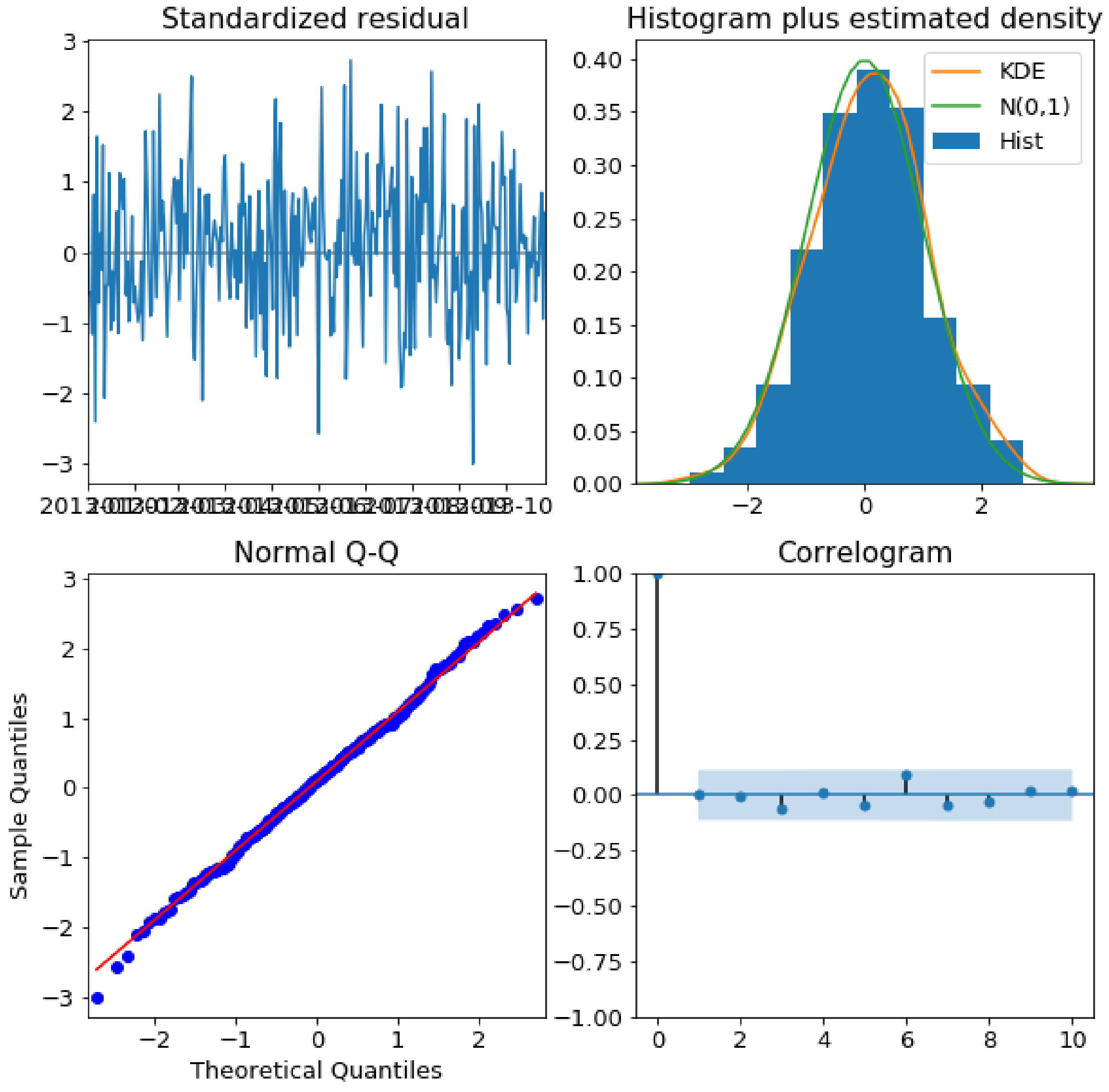
How far our the predictions from the real values?

```
mae = np.mean(np.abs(residuals))
```

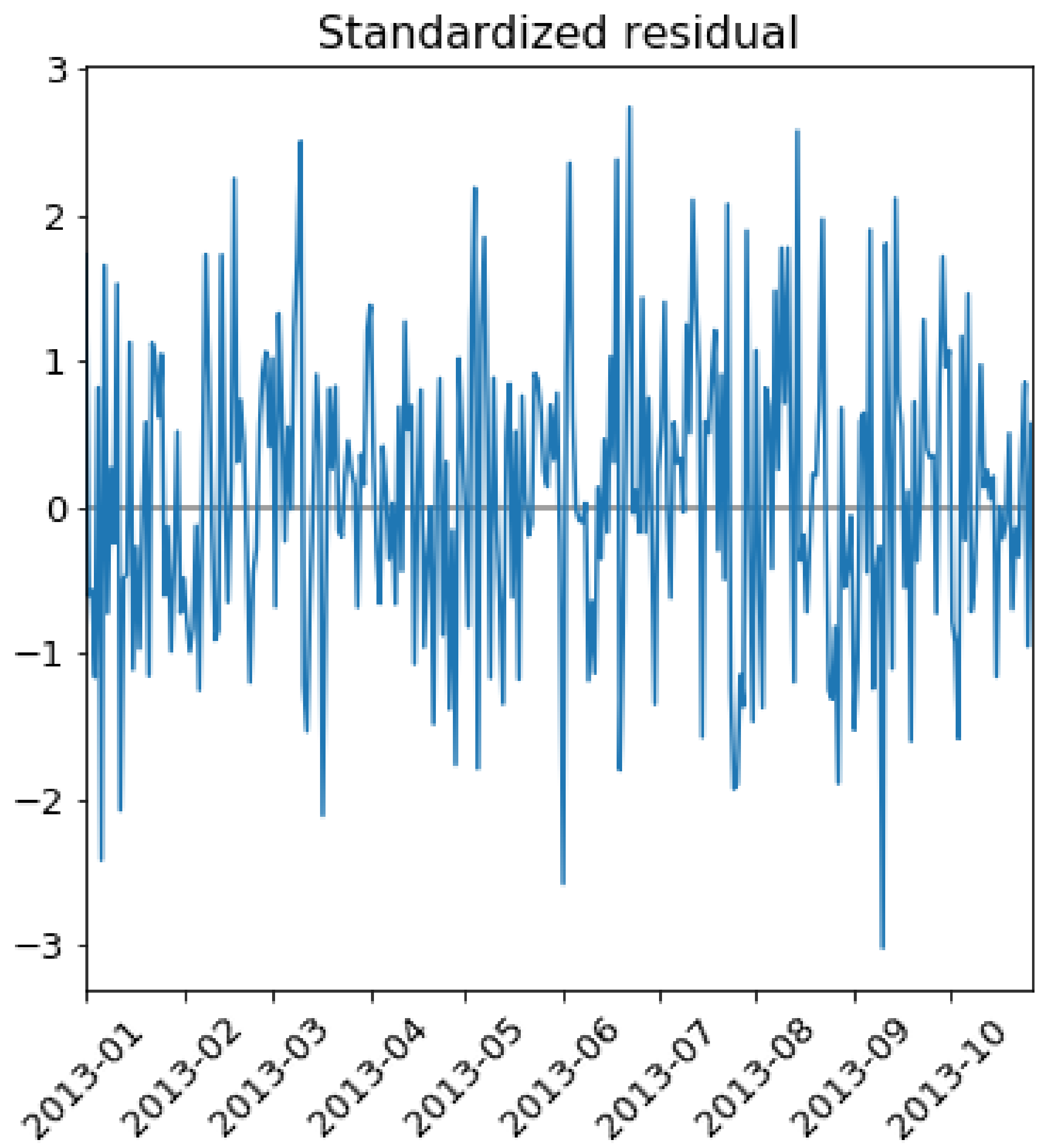
Plot diagnostics

If the model fits well the residuals will be white Gaussian noise

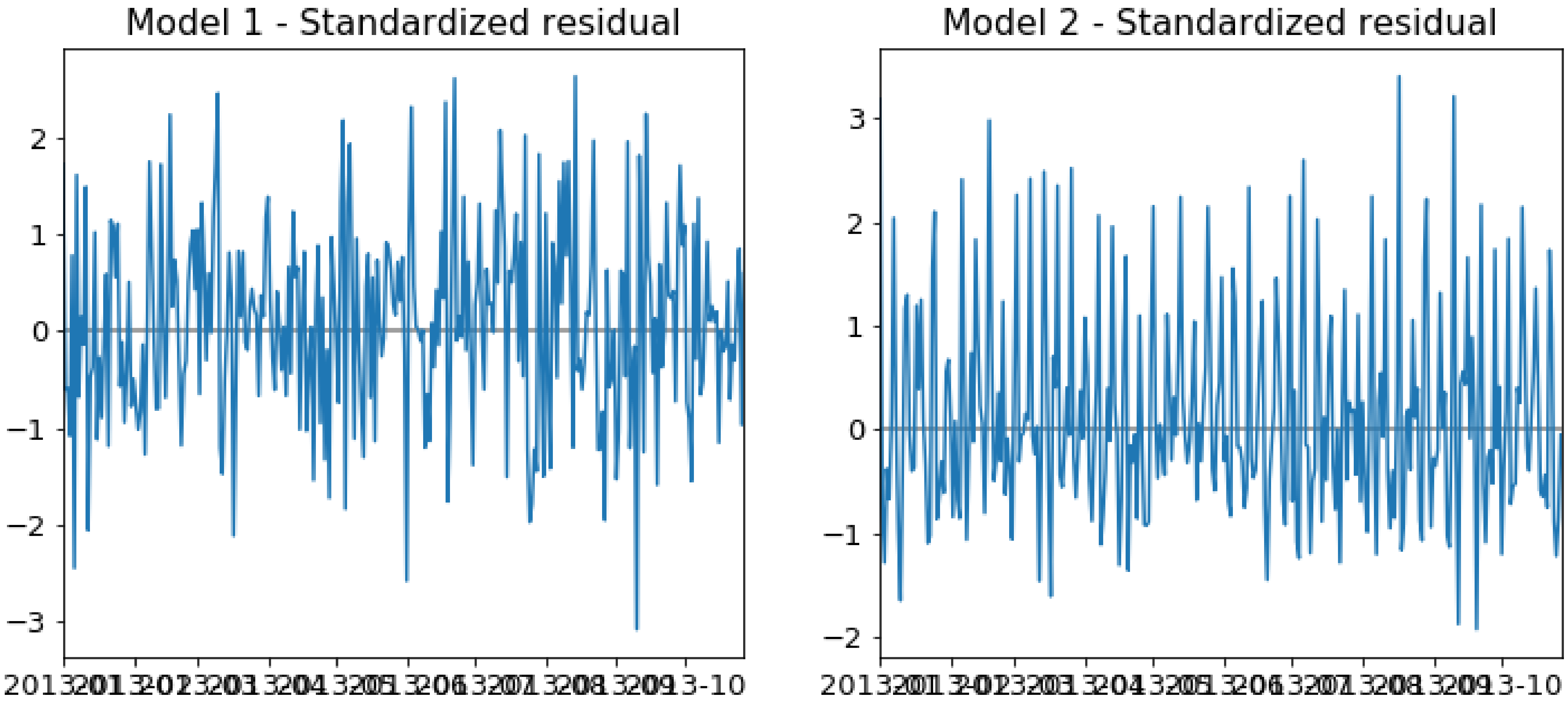
```
# Create the 4 diagostics plots
results.plot_diagnostics()
plt.show()
```



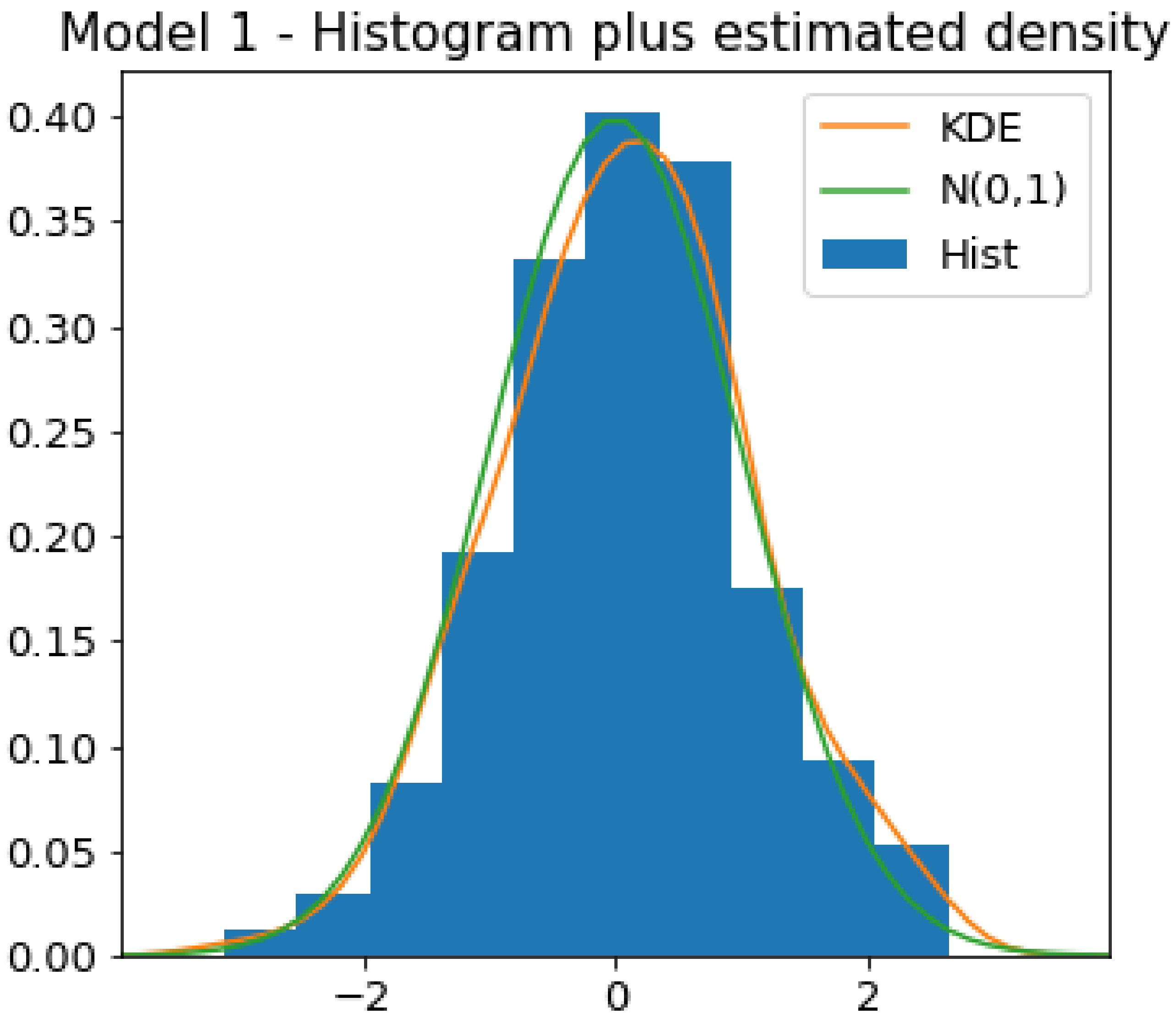
Residuals plot



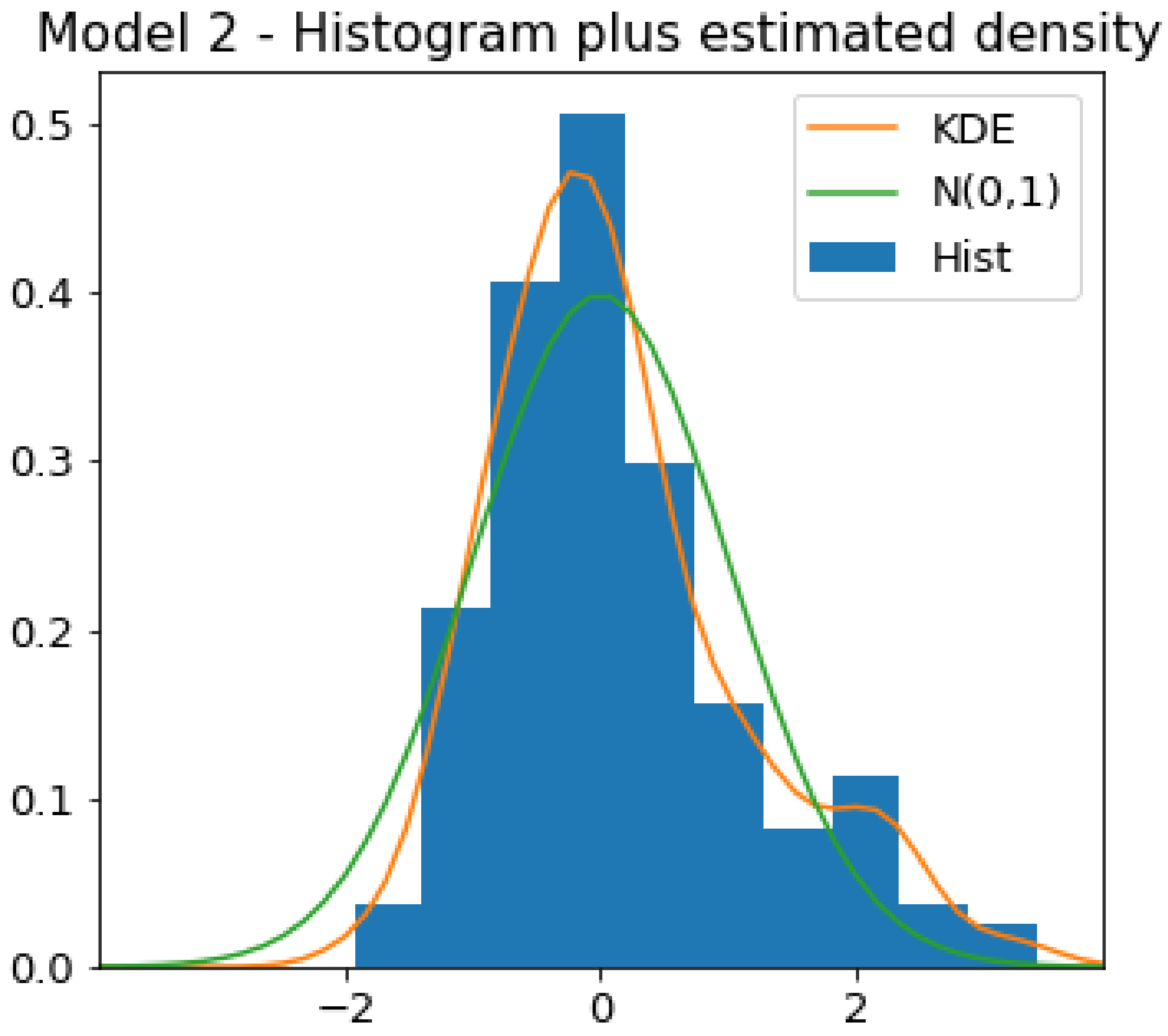
Residuals plot



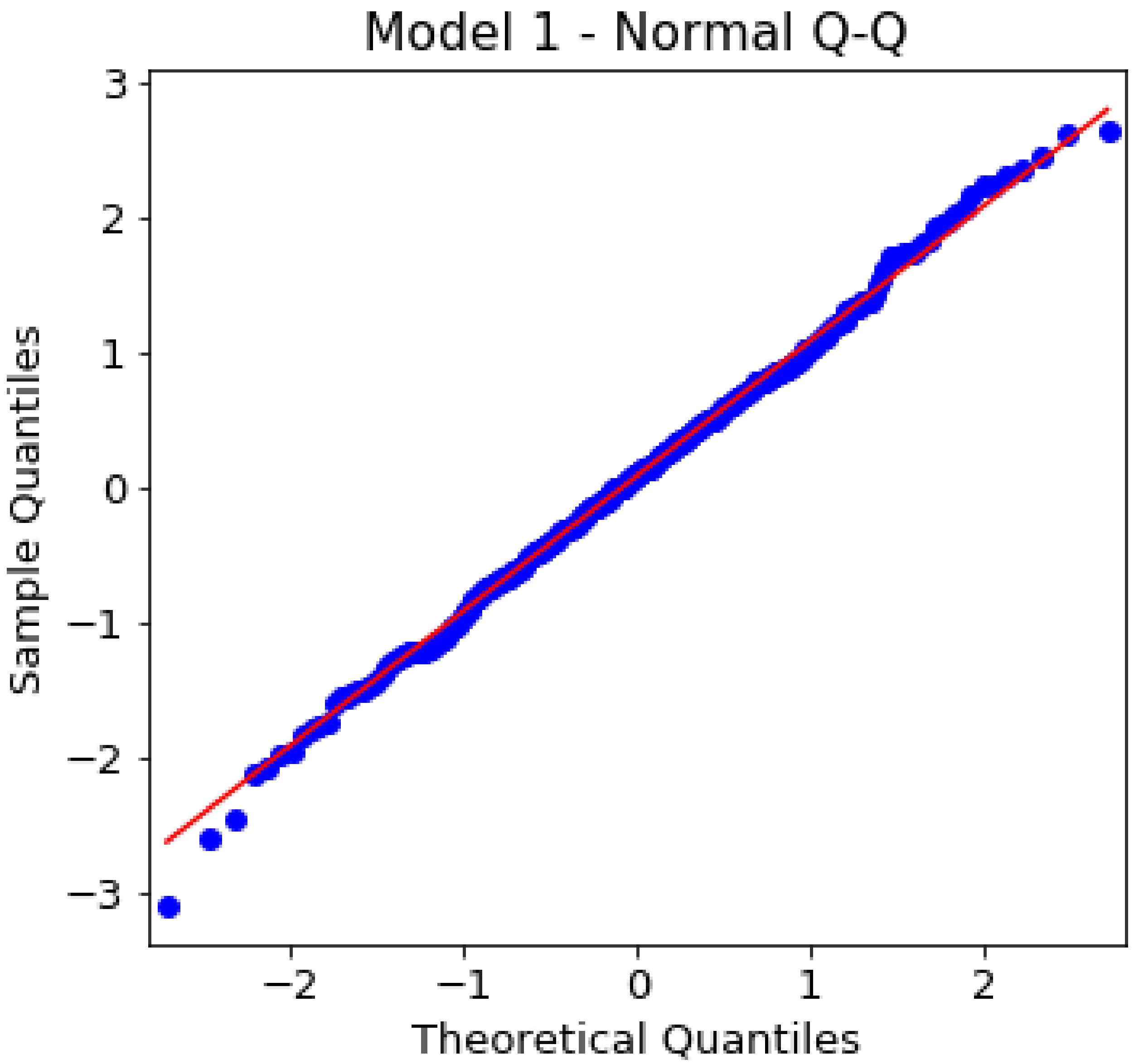
Histogram plus estimated density



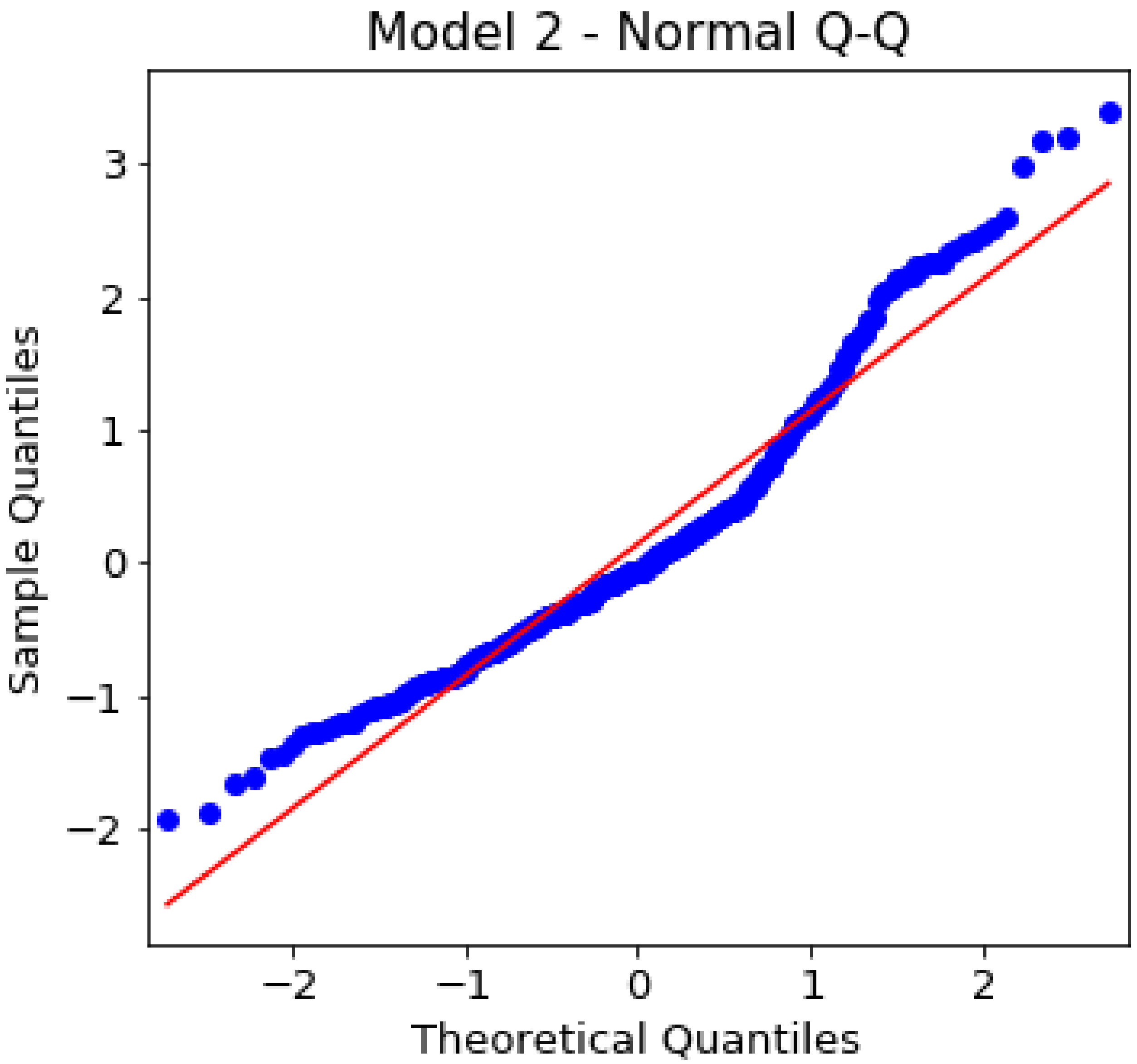
If a model is good these two lines should be almost the same.



Normal Q-Q

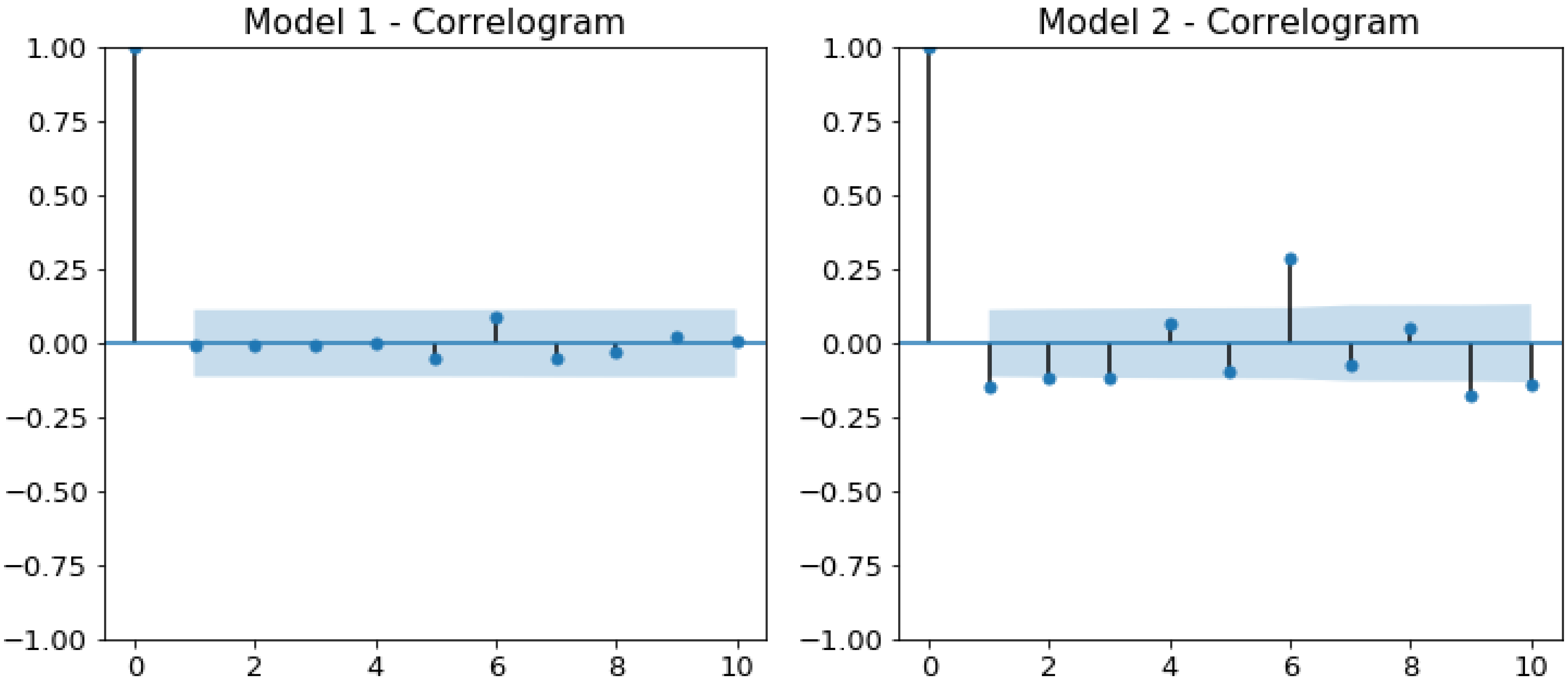


Model 1 - Normal Q-Q
Sample Quantiles
Theoretical Quantiles
3
2
1
0
−1
−2
−3
−2
−1
0
1
2



Model 2 - Normal Q-Q
Sample Quantiles
Theoretical Quantiles
3
2
1
0
−1
−2
−2
−1
0
1
2

Correlogram



ACF plot of the residuals
95% of the correlations for lag greater than zero should not be significant.

Summary statistics

```
print(results.summary())
```

```
...
===================================================================================
Ljung-Box (Q):                    32.10   Jarque-Bera (JB):                   0.02
Prob(Q):                           0.81   Prob(JB):                           0.99
Heteroskedasticity (H):            1.28   Skew:                              -0.02
Prob(H) (two-sided):               0.21   Kurtosis:                           2.98
===================================================================================
```

- `Prob(Q)` - p-value for null hypothesis that residuals are uncorrelated
- `Prob(JB)` - p-value for null hypothesis that residuals are normal

Let's practice!

ARIMA MODELS IN PYTHON

Box-Jenkins method

ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

The Box-Jenkins method

From raw data $\rightarrow$ production model

- identification
- estimation
- model diagnostics

Identification

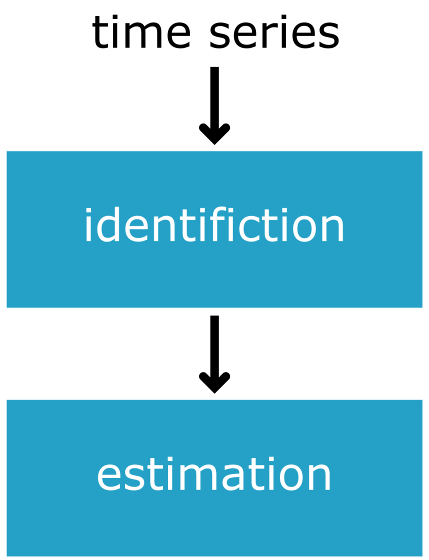
- Is the time series stationary?
- What differencing will make it stationary?
- What transforms will make it stationary?
- What values of p and q are most promising?

Identification tools

- Plot the time series
  - `df.plot()`
- Use augmented Dicky-Fuller test
  - `adfuller()`
- Use transforms and/or differencing
  - `df.diff()` , `np.log()` , `np.sqrt()`
- Plot ACF/PACF
  - `plot_acf()` , `plot_pacf()`

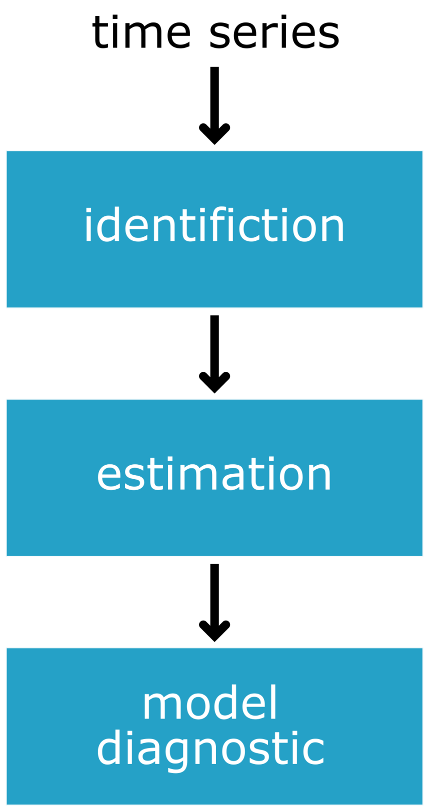
Estimation

- Use the data to train the model coefficients
- Done for us using `model.fit()`
- Choose between models using AIC and BIC
    - `results.aic` , `results.bic`

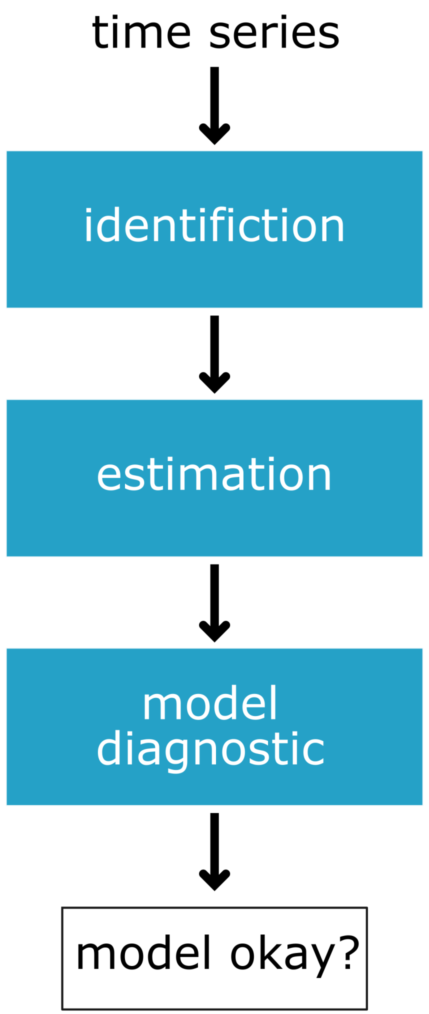


Model diagnostics

- Are the residuals uncorrelated
- Are residuals normally distributed
  - `results.plot_diagnostics()`
  - `results.summary()`

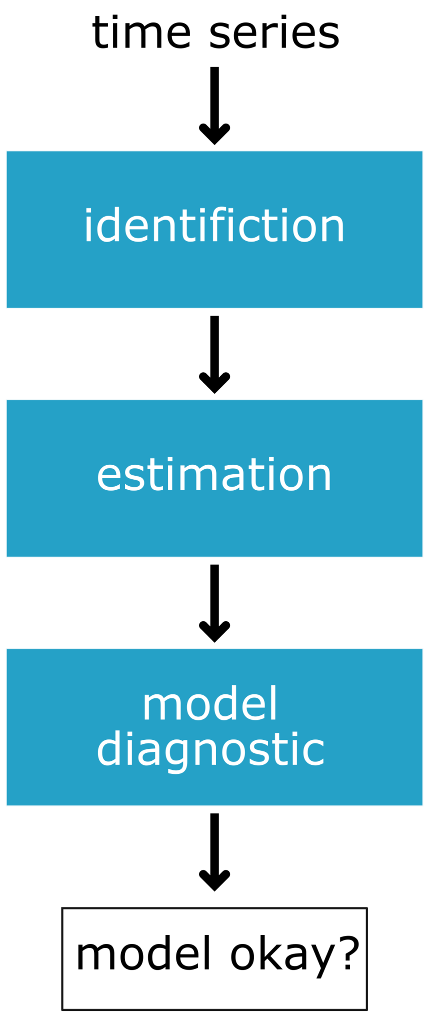


Decision


time series
identifiction
estimation
model
diagnostic
model okay?

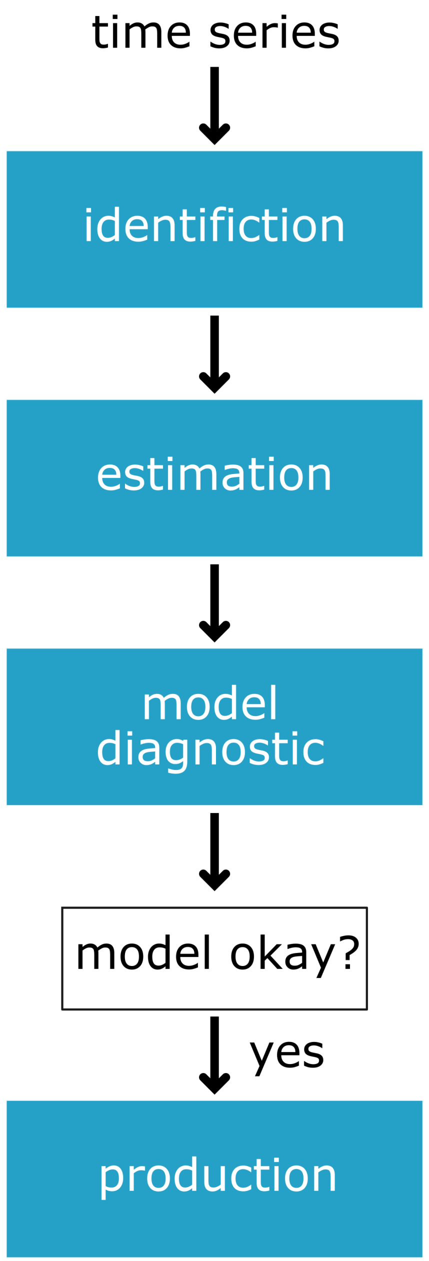
Repeat

- We go through the process again with more information
- Find a better model

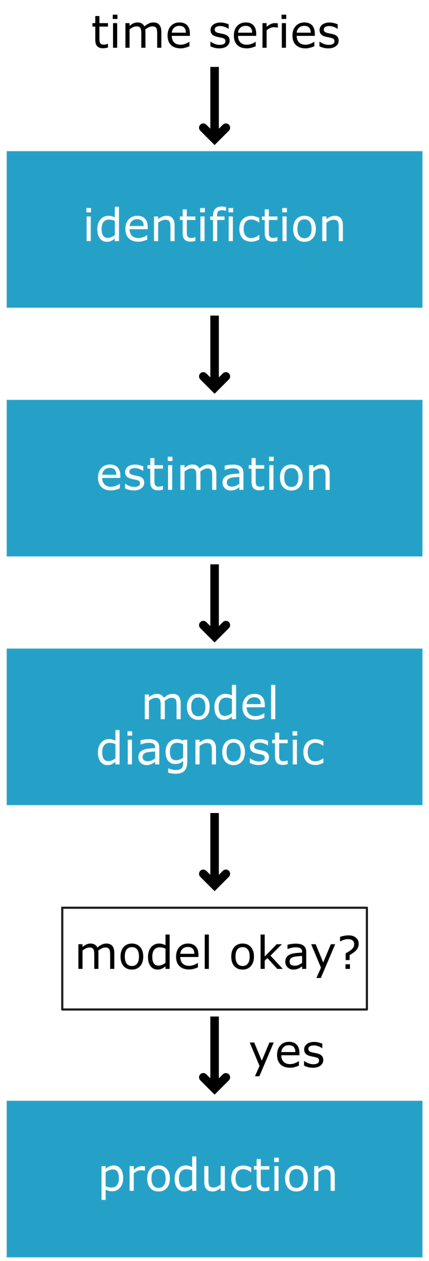


Production

- Ready to make forecasts
  - `results.get_forecast()`



Box-Jenkins


time series
identifiction
estimation
model
diagnostic
model okay?
yes
production

Let's practice!

ARIMA MODELS IN PYTHON