‘modal logic’

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A judgmental reconstruction of modal logic

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Outline

- prior

- hilbert proofs

Definition

$\mathscr{M} = \langle \mathscr{W} : Set, R : \mathscr{W} \times \mathscr{W} \to 2, V : \omega \to \mathscr{W} \to 2 \rangle$

Definition

$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} P_n \iff V(n)(\alpha)$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} \bot \iff false$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} A \wedge B \iff \vDash^{\mathscr{M}}_{\alpha} A \; and \; \vDash^{\mathscr{M}}_{\alpha} B$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} A \vee B \iff \vDash^{\mathscr{M}}_{\alpha} A \; or \; \vDash^{\mathscr{M}}_{\alpha} B$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} A \to B \iff \vDash^{\mathscr{M}}_{\alpha} A \implies \vDash^{\mathscr{M}}_{\alpha} B$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} \Box A \iff \forall \beta \in \mathscr{W} . \alpha R \beta \implies \vDash^{\mathscr{W}}_{\beta} A$$
$$\vDash^{\mathscr{M}}_{\alpha \in \mathscr{W}} \Diamond A \iff \exists \beta \in \mathscr{W} . \alpha R \beta \wedge \vDash^{\mathscr{W}}_{\beta} A$$

Definition

$\mathcal{M} = \langle \mathcal{W} : Set, \leq, V : \omega \to \mathcal{W} \to 2 \rangle$
$\alpha \leq \beta \Longrightarrow \vDash_{\alpha}^{\mathcal{M}} P_n \Longrightarrow \vDash_{\beta}^{\mathcal{M}} P_n$

Definition

$$\vDash_{\alpha}^{\mathcal{M}} A \to B \Longleftrightarrow \forall \beta . \alpha \leq \beta \Longrightarrow \vDash_{\beta}^{\mathcal{M}} A \Longrightarrow \vDash_{\beta}^{\mathcal{M}} B$$

$\Box(A \to B)$

Judgment

- "Of course, there would be needed here an analysis of what is understood by an expression, but that is a comparatively trivial matter, as compared with explaining the notion of proposition and judgement. An expression in the most general sense of the word is nothing but a form, that is, something that we can passively recognize as the same in its manifold occurrences and actively reproduce in many copies. But I think that I shall have to rely here upon an agreement that we have such a general notion of expression, which is formal in character, so that the rule can now count as a formal rule."

Local Soundness/Completeness

References I

[1] TODO: ask me