# Hazel PHI: 10-modules

July 2, 2021

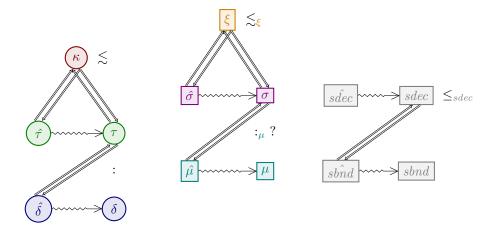
# prerequisites

- Hazel PHI: 9-type-aliases-redux
  - github
  - $\ current \ commit: \ 4410cd565ce717707e580e44f64868d3175fe2a6$
- (optional) Hazel PHI: 1-labeled-tuples
  - github
  - $\ current \ commit: \ 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1$

### how to read

800000	kinds	D08000	signature kind
008000	types (constructors)	800080	signatures
000080	terms	008080	modules

#### notes



external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet—will be left till end.

### syntax



```
HTyp
                                                                                                                          type variable
                                               t
                                                bse
                                                                                                                              base type
                                                                                                                             type binop
                                               	au_1 \oplus 	au_2
                                               [	au]
                                                                                                                                list type
                                                                                                                         type function
                                               \lambda t :: \kappa.\tau
                                                                                                                      type application
                                                                                                   labelled product type (record)
                                               \{lab_1 \hookrightarrow \tau_1, \dots \, lab_n \hookrightarrow \tau_n\}
                                                                                                             module type projection
                                                                                                                      empty type hole
                                                (|\tau|)
                                                                                                                 nonempty type hole
                base type
                                bse
                                               Int
                                               Float
                                               Bool
           HTyp BinOp
                                 \oplus
   external expression
                                               signature s = \hat{\sigma} in \hat{\delta}
                                               module m=\hat{\mu} in \hat{\delta}
                                               module m{:}_{\mu}s=\hat{\mu} in \hat{\delta}
                                               functor something = something in \hat{\delta}
                                               \hat{\mu}.lab
                                                                                                            module term projection
    internal expression
                                 δ
                                         ::=
                                               signature s = \sigma in \delta
                                               module m:_{\mu} s = \mu in \delta
                                               functor something = something in \delta
                                               \mu.lab
                                                                                                            module term projection
         signature kind
                                               SSigKind(\sigma)
                                               SigKHole
     external signature
                                                                                                                    signature variable
                                                \{sdecs\}
                                                                                                                  structure signature
                                               \Pi_{m:_{\mu}\hat{\sigma_{1}}}.\hat{\sigma_{2}}
                                                                                                                     functor signature
                                                                                                               empty signature hole
                                                (s)^{\mathbf{u}}
                                                                                                           nonempty signature hole
                signature
                                                                                                                    signature variable
                                               \{sdecs\}
                                                                                                                  structure signature
                                               \Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2}
                                                                                                                     functor signature
                                                                                                               empty signature hole
                                                (s)^{\mathbf{u}}
                                                                                                           nonempty signature hole
                                                                                                        unbound signature variable
                                                \stackrel{s}{\sim}
                  module
                                                                                                                      module variable
                                               m
                                               \{sbnds\}
                                                                                                                               structure
                                               \lambda m:_{\mu} \sigma.\mu
                                                                                                                                 functor
                                                                                                                  functor application
                                               \mu_1 \mu_2
                                               \mu.lab
                                                                                                               submodule projection
                                                                                                                  empty module hole
                                               (\mu)
                                                                                                             nonempty module hole
signature declarations
                               sdecs
                                               sdec, sdecs
 signature declaration
                               sdec
                                               type lab::\kappa
                                               {\tt val}\ lab{:}\tau
                                               module lab:_{\mu}\sigma
                                               functor lab:_{\mu}\sigma
    structure bindings sbnds ::=
```

#### context definitions

```
declarative statics
          \Delta; \Phi; \Xi; \Psi; \vdash \Delta; ; \Phi; \lesssim \kappa_1 \kappa_2 \mid \kappa_1 \text{ is a consistent subkind of } \kappa_2
                                                                                                                                                   KCSubsumption
                                                                                                                                                    test
                                                                                                                                                    test
  \Delta; \Phi; \Xi; \Psi; \vdash \xi_1 \lesssim_{\xi} \xi_2 \mid \xi_1 \text{ is a consistent sub signature kind of } \xi_2
                                         nameMe
                                                             \exists sdec_x \in sdecs_1 \ st \ \Delta; ; \Phi; ; \Xi; ; \Psi; \vdash SSigKind(\{sdec_x\}) \lesssim_{\varepsilon} SSigKind(\{sdec_2\})
                                                      \Delta; \Phi; , type(\Delta; \Phi; \Xi; \Psi; , sdec_2); \Xi; , submodule(sdec_2); \Psi; \vdash {sdecs_1} \lesssim_{\xi} {sdecs_2}
                                          \overline{\Delta}; \Phi; \overline{\Xi}; \Psi; \vdash \underline{\mathsf{SSigKind}}(\{sdec_{11}, sdec_{12}, sdecs_{13} \text{ as } sdecs_1\}) \lesssim_{\varepsilon} \underline{\mathsf{SSigKind}}(\{sdec_2, sdecs_2\})
          single
          \frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash sdec_1 \leq_{sdec} sdec_2}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathsf{SSigKind}(\{sdec_1\}) \lesssim_{\xi} \mathsf{SSigKind}(\{sdec_2\})} \qquad \frac{\mathtt{nil}}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathsf{SSigKind}(\{sdec_s\}) \lesssim_{\xi} \mathsf{SSigKind}(\{\cdot\})}
                                      \begin{array}{ll} \operatorname{varprop} & \operatorname{nameMe?delete?} \\ \underline{s::_{\sigma}\xi \in \Psi;} & \underline{\sigma_1 \neq s} \quad \Delta;; \Phi;; \Xi;; \Psi; \vdash \sigma_1 \; \Leftarrow \; \operatorname{SSigKind}(\sigma_2) \\ \overline{\Delta;; \Phi;; \Xi;; \Psi; \vdash \operatorname{SSigKind}(s) \lesssim_{\xi} \xi} & \overline{\Delta;; \Phi;; \Xi;; \Psi; \vdash \operatorname{SSigKind}(\sigma_1) \lesssim_{\xi} \operatorname{SSigKind}(\sigma_2)} \end{array}
                funct
                \frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \operatorname{SSigKind}(\sigma_{21}) \lesssim_{\xi} \operatorname{SSigKind}(\sigma_{11})}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \operatorname{SSigKind}(\Pi_{m:_{\mu}\sigma_{11}}, \sigma_{12}) \lesssim_{\xi} \operatorname{SSigKind}(\Pi_{m:_{\mu}\sigma_{21}}, \sigma_{22})}
           CSubSigKindHoleR
                                                                                                                      \overline{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \xi \lesssim_{\varepsilon} SigKHole
 \Delta; \Phi; \Xi; \Psi; \vdash \sigma \Rightarrow \xi \mid \sigma \text{ synthesizes signature kind } \xi
             SynSigKndVar
             \frac{s ::_{\sigma} \xi \in \Psi;}{\Delta;; \Phi;; \Xi;; \Psi; \vdash s \Rightarrow \frac{\text{SSigKind}(s)}{\text{SynSigKndvarrall}}} \frac{s \notin \text{dom}(\Psi;)}{\Delta;; \Phi;; \Xi;; \Psi; \vdash s \Rightarrow \frac{\text{SigKHole}}{\text{SigKind}(\{s decs\})}} \frac{\{s decs\} well formed?}{\vdash \{s decs\}} \Rightarrow \frac{\text{SSigKind}(\{s decs\})}{\text{SSigKind}(\{s decs\})}
                                                                                                                     SynSigKndVarFail
\frac{\Delta;;\Phi;;\Xi;,m:_{\mu}\sigma_{1};\Psi;\vdash\sigma_{2}\Rightarrow \pmb{\xi}}{\Delta;;\Phi;;\Xi;;\Psi;\vdash\Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2}\Rightarrow \mathbf{SSigKind}(\Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2})} \qquad \frac{u::_{\sigma}\pmb{\xi}\in\Delta;}{\Delta;;\Phi;;\Xi;;\Psi;\vdash (\!\!\|)^{\mathbf{u}}\Rightarrow \pmb{\xi}} \qquad \frac{u::_{\sigma}\pmb{\xi}\in\Delta;}{\Delta;;\Phi;;\Xi;;\Psi;\vdash (\!\!\|)^{\mathbf{u}}\Rightarrow \pmb{\xi}} \qquad \frac{u::_{\sigma}\pmb{\xi}\in\Delta;}{\Delta;;\Phi;;\Xi;;\Psi;\vdash (\!\!\|)^{\mathbf{u}}\Rightarrow \pmb{\xi}}
```

```
\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \sigma \Leftarrow \xi
                                                                                                               \sigma analyzes against signature kind \xi
                                                                                                                                                                          \frac{\Delta;;\Phi;;\Xi;;\Psi;\vdash\sigma\Rightarrow \underline{\boldsymbol{\xi_1}} \qquad \Delta;;\Phi;;\Xi;;\Psi;\vdash\underline{\boldsymbol{\xi_1}}\lesssim_{\underline{\boldsymbol{\xi}}}\underline{\boldsymbol{\xi}}}{\Delta;;\Phi;;\Xi;;\Psi;\vdash\sigma\Leftarrow\underline{\boldsymbol{\xi}}}
   \Delta; ; \Phi; ; \Xi; ; \Psi; \vdash sdec_1 \leq_{sdec} sdec_2
                                                                                                                                                           sdec_1 is a subsdec of sdec_2
                                                        singleType
                                                                                                                                                                                                                                                                                                                                       singleVa
                                                         \frac{\Delta;\Phi;\Xi;\Psi;\vdash\lesssim\kappa_{1}\kappa_{2}}{\Delta;;\Phi;;\Xi;;\Psi;\vdash\mathsf{type}\;lab::\kappa_{1}\leq_{sdec}\mathsf{type}\;lab::\kappa_{2}}
                                                                                                                                                                                                                                                                                            \frac{\Delta; \Phi; \Xi; \Psi; \vdash \Delta; ; \Phi; ; \Xi; ; \Psi; \equiv \tau_1 \tau_2}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathsf{val} \ \mathit{lab} : \tau_1 \leq_\mathit{sdec} \ \mathsf{val} \ \mathit{lab} : \tau_2}
                                                                                                                                                                        singleMod
                                                                                                                                                                                                             \Delta; \Phi; \Xi; \Psi; \vdash \sigma_1 \Leftarrow SSigKind(\sigma_2)
                                                                                                                                                                         \frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \text{ module } lab:_{\mu} \sigma_{1} \leq_{sdec} \text{ module } lab:_{\mu} \sigma_{2}}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \text{ module } lab:_{\mu} \sigma_{1} \leq_{sdec} \text{ module } lab:_{\mu} \sigma_{2}}
elaboration
                  \Gamma; \Phi; \Xi; \vdash \hat{\delta} \Rightarrow \tau \leadsto \delta \dashv \Delta; \mid \hat{\delta} \text{ synthesizes type } \tau \text{ and elaborates to } \delta \text{ with hole context } \Delta;
                                                                                                                                                                                    SynElabLetMod
                                                                                                                                                                                    \frac{\Gamma; ; \Phi; ; \Xi; \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta;_1}{\Gamma; ; \Phi; ; \Xi; \vdash \text{module } m = \hat{\mu} \text{ in } \hat{\delta} \Rightarrow \tau \leadsto \text{module } m = \mu \text{ in } \delta \dashv \Delta;_1 \cup \Delta;_2}
                                                                    SynElabLetModAnn
                                                                     \Phi;;\Xi;\vdash\hat{\sigma} \ \Rightarrow \ {\color{red}\xi} \leadsto \sigma\dashv \Delta;_1 \qquad \Gamma;;\Phi;;\Xi;\vdash\hat{\mu} \ \Leftarrow \ \sigma\leadsto \mu\dashv \Delta;_2 \qquad \Gamma;;\Phi;;\Xi;, m:_{\mu}\sigma\vdash\hat{\delta} \ \Rightarrow \ \tau\leadsto \delta\dashv \Delta;_3
                                                                                                                 \Gamma; ; \Phi; ; \Xi; \vdash \texttt{module} \ m :_{\mu} \hat{\sigma} = \hat{\mu} \ \texttt{in} \ \hat{\delta} \ \Rightarrow \ \tau \leadsto \texttt{module} \ m :_{\mu} \sigma = \mu \ \texttt{in} \ \delta \dashv \Delta;_{1} \cup \Delta;_{2} \cup \Delta;_{3}
                                                                                                                                                                                                SynElabModTermPrj
                                                                                                                                                                                                \Gamma; ; \Phi; ; \Xi; \vdash \hat{\mu} \ \Rightarrow \ \sigma \leadsto \mu \dashv \Delta; \qquad \Phi; ; \Xi; \vdash \sigma \ \Rightarrow \ \xi
                                                                                                                                                                                                                               \Gamma; ; \Phi; ; \Xi; \vdash \hat{\mu}.lab \Rightarrow \tau \leadsto \mu.lab \dashv \Delta:
   \Phi; \Xi; \vdash \hat{\tau} \Rightarrow \kappa \leadsto \tau \dashv \Delta; \mid \hat{\tau} \text{ synthesizes kind } \kappa \text{ and elaborates to } \tau \text{ with hole context } \Delta;
                                                                                                                                                                                                                                              SynElabModTypPrj
                                                                                                                                                                                                                                               \Phi; \Xi; \vdash m \Rightarrow \sigma \leadsto m \dashv \Delta; \quad something \sigma \kappa
                                                                                                                                                                                                                                                                         \Phi::\Xi:\vdash m.lab \Rightarrow \kappa \leadsto m.lab \dashv \Delta:
  \Phi; \Xi; \vdash \hat{\tau} \leftarrow \kappa \leadsto \tau \dashv \Delta; \mid \hat{\tau} \text{ analyzes against kind } \kappa \text{ and elaborates to } \tau \text{ with hole context } \Delta;
  \Gamma; \varphi; \Xi; \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta; \mid \hat{\mu} \text{ synthesizes signature } \sigma \text{ and elaborates to } \mu \text{ with hole context } \Delta;
                                                                                                     SynElabModVar
                                                                                                                                                                                                                                                                                                                       SynElabModVarFail
                                                                                                       \frac{m:_{\mu}\sigma\in\Xi;}{\Gamma;;\Phi;;\Xi;\vdash m\ \Rightarrow\ \sigma\leadsto m\dashv\cdot}
                                                                                                                                                                                                                                                                                                                       \frac{m \notin \mathsf{dom}(\Xi;)}{\Gamma; ; \Phi; ; \Xi; \vdash m \Rightarrow () \rightsquigarrow (m)^{\mathbf{u}} \dashv u :_{u} ())}
                                                  SynElabConsStruct
                                                                                                                                                                                                        \Gamma; \Phi; \Xi; \vdash \hat{sbnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta;_1
                                                  \Gamma;, \mathsf{val}(\mathit{sdec}); \Phi;, \mathsf{type}(\Delta;_1; \Phi;; \Xi;; \Psi;, \mathit{sdec}); \Xi;, \underline{\mathsf{submodule}(\mathit{sdec})} \vdash \{\mathit{sbnds}\} \ \Rightarrow \ \{\mathit{sdecs}\} \leadsto \{\mathit{sbnds}\} \dashv \Delta;_2 \vdash \{\mathit{sbnds}\} \vdash \{\mathit{s
                                                                                                                             \Gamma; \Phi; \Xi; \vdash \{\hat{sbnd}, \hat{sbnds}\} \Rightarrow \{\hat{sdec}, \hat{sdecs}\} \leadsto \{\hat{sbnd}, \hat{sbnds}\} \dashv \Delta;_1 \cup \Delta;_2
              SynElabNilStruct
                                                                                                                                                                                                 SynElabEmptyModHole
                                                                                                                                                                                                                                                                                                                                                                                                  SynElabNonemptyModHole
              \overline{\Gamma;;\Phi;;\Xi;\vdash\{\cdot\}\ \Rightarrow\ \{\cdot\}\ \leadsto\{\cdot\}\ \dashv\cdot}\qquad \overline{\Gamma;;\Phi;;\Xi;\vdash(\mathbb{D}^{\mathbf{u}}\ \Rightarrow\ (\mathbb{D}^{\mathbf{u}}\ \dashv\ u:_{\mu}(\mathbb{D})}\qquad \overline{\Gamma;;\Phi;;\Xi;\vdash(m)^{\mathbf{u}}\ \Rightarrow\ (\mathbb{D}^{\mathbf{u}}\ \dashv\ u:_{\mu}(\mathbb{D})}
                                                                                                                                                                                                                                                                    functor stuff
```

 $\Gamma; ; \Phi; ; \Xi; \vdash \hat{\mu} \Leftarrow \sigma \leadsto \mu \dashv \Delta;$  $\hat{\mu}$  analyzes against signature  $\sigma$  and elaborates to  $\mu$  with hole context  $\Delta$ ;

AnaElabModSubsumption

$$\frac{\Gamma; ; \Phi; ; \Xi; \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta;}{\Gamma; : \Phi; ; \Xi; \vdash \hat{\mu} \Leftarrow \sigma \leadsto \mu \dashv \Delta;}$$

 $\Gamma; ; \Phi; ; \Xi; \vdash \hat{sbnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta;$ shind synthesizes declaration sdec and elaborates to shind with hole context  $\Delta$ ;

SynElabTypeSbnd

$$\Phi \cdot \cdot \Xi \cdot \vdash \hat{\tau} \rightarrow \kappa \sim \tau \rightarrow \Lambda$$

 $\frac{\Phi;;\Xi;\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta;}{\Gamma;;\Phi;;\Xi;\vdash\text{type }t=\hat{\tau}\ \Rightarrow\ \text{type }t::\kappa\leadsto\text{type }t=\tau\dashv\Delta;} \qquad \frac{\Phi;;\Xi;\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta;_1}{\Gamma;;\Phi;;\Xi;\vdash\text{let }x:\hat{\tau}=\hat{\delta}\ \Rightarrow\ \text{val }x:\tau\leadsto\text{let }x:\tau=\delta\dashv\Delta;_2}{\Gamma;;\Phi;;\Xi;\vdash\text{let }x:\hat{\tau}=\hat{\delta}\ \Rightarrow\ \text{val }x:\tau\leadsto\text{let }x:\tau=\delta\dashv\Delta;_1\cup\Delta;_2}$ 

$$\Gamma: \Phi: \Xi: \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta$$

 $\frac{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta;}{\Gamma;;\Phi;;\Xi;\vdash \mathtt{module} \ m = \hat{\mu} \Rightarrow \mathtt{module} \ m{:}_{\mu}\sigma \leadsto \mathtt{module} \ m{:}_{\mu}\sigma = \mu \dashv \Delta;}$ 

SynElabModAnnSbnd

 $\frac{\Phi;;\Xi;\vdash\hat{\sigma}\ \Rightarrow\ \xi\leadsto\sigma_1\dashv\Delta;_1\qquad \Gamma;;\Phi;;\Xi;\vdash\hat{\mu}\ \Rightarrow\ \sigma_2\leadsto\mu\dashv\Delta;_2\qquad \Phi;;\Xi;;\Psi;\vdash\sigma_2\ \Leftarrow\ \xi}{\Gamma;;\Phi;;\Xi;\vdash \mathsf{module}\ m:_{\mu}\hat{\sigma}=\hat{\mu}\ \Rightarrow\ \mathsf{module}\ m:_{\mu}\sigma_1\leadsto \mathsf{module}\ m:_{\mu}\sigma_1=\mu\dashv\Delta;_1\cup\Delta;_2}$ 

 $\Gamma; \Phi; \Xi; \vdash \hat{sbnd} \Leftarrow sdec \leadsto \hat{sbnd} \dashv \Delta; \mid \hat{sbnd} \text{ analyzes against declaration } sdec \text{ and elaborates to } \hat{sbnd} \text{ with hole context } \Delta;$ 

subsump

 $\underline{\Gamma;;\Phi;;\Xi;;l\Psi;\vdash s\^{bnd} \Rightarrow sdec_1 \leadsto sbnd} \dashv \Delta; \qquad \Delta;;\Phi;;\Xi;;\Psi;\vdash sdec_1 \leq_{sdec} sdec_1 \Leftrightarrow sdec_2 \Leftrightarrow sdec_1 \Leftrightarrow sdec_2 \Leftrightarrow sdec_$ 

 $\Gamma::\Phi::\Xi::\Psi:\vdash \hat{sbnd} \iff sdec \leadsto sbnd \dashv \Delta:$ 

 $\Gamma; \Phi; \Xi; \Psi; \vdash s \hat{dec} \leadsto s dec \dashv \Delta; \mid s \hat{dec} \text{ elaborates to } s dec \text{ with hole context } \Delta;$ 

 $\frac{\Gamma;;\Phi;;\Xi;;\Psi;\vdash\hat{\tau} \Rightarrow \kappa \leadsto \tau \dashv \Delta;}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \mathsf{type}\;lab::\hat{\tau} \leadsto \mathsf{type}\;lab::\kappa \dashv \Delta;}$ 

 $\overline{\Gamma; ; \Phi; ; \Xi; ; \Psi; \vdash \mathsf{type}\; lab \leadsto \mathsf{type}\; lab::\mathsf{Type}\; \dashv}$ 

 $\begin{array}{c} \operatorname{val} & \operatorname{mod} \\ \Gamma; ; \Phi; ; \Xi; ; \Psi; \vdash \hat{\tau} \ \Rightarrow \ \kappa \leadsto \tau \dashv \Delta; \\ \hline \Gamma; ; \Phi; ; \Xi; ; \Psi; \vdash \operatorname{val} \ \mathit{lab} : \hat{\tau} \leadsto \operatorname{val} \ \mathit{lab} : \tau \dashv \Delta; \\ \end{array} \qquad \begin{array}{c} \operatorname{mod} \\ \Gamma; ; \Phi; ; \Xi; ; \Psi; \vdash \hat{\sigma} \ \Rightarrow \ \xi \leadsto \sigma \dashv \Delta; \\ \hline \Gamma; ; \Phi; ; \Xi; ; \Psi; \vdash \operatorname{module} \ \mathit{lab} :_{\mu} \hat{\sigma} \leadsto \operatorname{module} \ \mathit{lab} :_{\mu} \sigma \dashv \Delta; \end{array}$ 

we're going to need HOFunctors so we don't need to preclude users from typing a functor into a module and vice versa

 $\Phi; \Xi; \Psi; \vdash \hat{\sigma} \Rightarrow \xi \leadsto \sigma \dashv \Delta; \mid \hat{\sigma} \text{ synthesizes signature kind } \xi \text{ and elaborates to } \sigma \text{ with hole context } \Delta;$ 

SynSigEmptyHole

SynSigNonEmptyHole

$$\overline{\Phi;;\Xi;;\Psi;\vdash (\!\!\!)^{\mathbf{u}} \Rightarrow \mathtt{SigKHole} \leadsto (\!\!\!)^{\mathbf{u}}\dashv u ::_{\sigma}\mathtt{SigKHole}}$$

 $\Phi_{;;\Xi;\vdash\hat{\sigma}} \leftarrow \xi \leadsto \sigma \dashv \Delta;$   $\hat{\sigma}$  analyzes against signature kind  $\xi$  and elaborates to  $\sigma$  with hole context  $\Delta_{;\sigma}$ 

#### misc functions

$$\begin{aligned} \operatorname{val}(sdec) &= \begin{cases} lab{:}\tau & sdec = \operatorname{val}\ lab{:}\tau \\ \cdot & \text{otherwise} \end{cases} \\ \operatorname{type}(cntxts, sdec) &= \begin{cases} lab{:}\iota\kappa & sdec = \operatorname{type}\ lab{:}\iota\kappa \\ \cdot & \text{otherwise} \end{cases} \\ \operatorname{submodule}(sdec) &= \begin{cases} lab{:}_{\mu}\sigma & sdec = \operatorname{module}\ lab{:}_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{split} \mathbf{S}_{\texttt{Type}}(\tau) &:= \mathbf{S}(\tau) \\ \mathbf{S}_{\mathbf{S}(\tau_I)}(\tau) &:= \mathbf{S}(\tau) \\ \mathbf{S}_{\texttt{KHole}}(\tau) &:= \texttt{KHole} \end{split}$$

## theorems

Kind Synthesis Precision

```
If \operatorname{cntxts} \vdash \tau \ \Rightarrow \ \kappa \ \operatorname{then} \ \forall \kappa_1.\Delta; \Phi; \Xi; \Psi; \vdash \operatorname{cntxts} :: \tau \kappa_1 \ \Longrightarrow \ \Delta; \Phi; \Xi; \Psi; \vdash \operatorname{cntxts} \lesssim \kappa \kappa_1
```