Hazel PHI: 10-modules

July 2, 2021

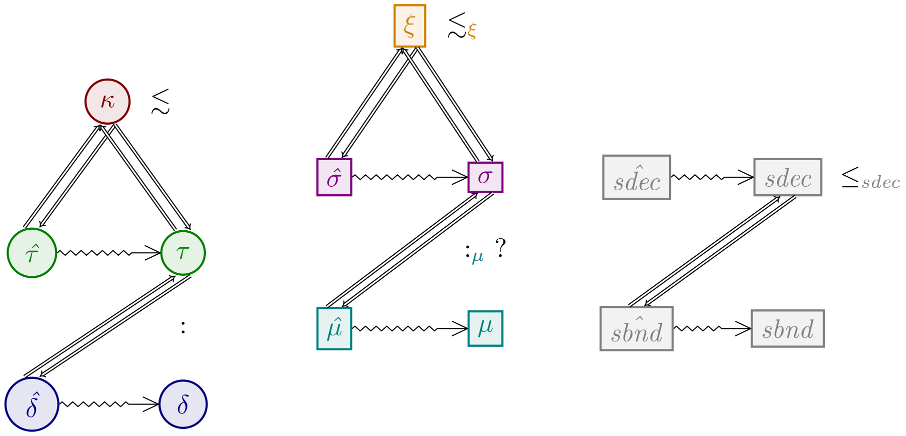
prerequisites

- Hazel PHI: 9-type-aliases-redux
  - github
  - current commit: 4410cd565ce717707e580e44f64868d3175fe2a6
- (optional) Hazel PHI: 1-labeled-tuples
  - github
  - current commit: 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1

how to read

800000	kinds	D08000	signature kind
008000	types (constructors)	800080	signatures
000080	terms	008080	modules

notes



external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet– will be left till end.

syntax

kind	κ	$::=$	$\texttt{Type}$	kind of types
		$\mid$	$\texttt{S}(\tau)$	singleton kind
		$\mid$	$\texttt{KHole}$	kind hole
		$\mid$	$\Pi_{t::\kappa_1}.\kappa_2$	dependent function kind

HTyp	τ	$::=$	t	type variable
		$\mid$	bse	base type
		$\mid$	$\tau_1 \oplus \tau_2$	type binop
		$\mid$	$[\tau]$	list type
		$\mid$	$\lambda t{::}\kappa.\tau$	type function
		$\mid$	$\tau_1\ \tau_2$	type application
		$\mid$	$\{lab_1 \hookrightarrow \tau_1, \ldots\ lab_n \hookrightarrow \tau_n\}$	labelled product type (record)
		$\mid$	$\mu.lab$	module type projection
		$\mid$	$\llparenthesis\rrparenthesis$	empty type hole
		$\mid$	$\llparenthesis\tau\rrparenthesis$	nonempty type hole
base type	bse	$::=$	$\texttt{Int}$	
		$\mid$	$\texttt{Float}$	
		$\mid$	$\texttt{Bool}$	
HTyp BinOp	$\oplus$	$::=$	$\times$	
		$\mid$	$+$	
		$\mid$	$\rightarrow$	
external expression	$\hat{\delta}$	$::=$	$\ldots$	
		$\mid$	x	
		$\mid$	$\texttt{signature}\ s = \hat{\sigma}\ \texttt{in}\ \hat{\delta}$	
		$\mid$	$\texttt{module}\ m = \hat{\mu}\ \texttt{in}\ \hat{\delta}$	
		$\mid$	$\texttt{module}\ m{:}_{\mu}s = \hat{\mu}\ \texttt{in}\ \hat{\delta}$	
		$\mid$	$\texttt{functor something} = \texttt{something in}\ \hat{\delta}$	
		$\mid$	$\hat{\mu}.lab$	module term projection
internal expression	δ	$::=$	$\ldots$	
		$\mid$	x	
		$\mid$	$\texttt{signature}\ s = \sigma\ \texttt{in}\ \delta$	
		$\mid$	$\texttt{module}\ m{:}_{\mu}s = \mu\ \texttt{in}\ \delta$	
		$\mid$	$\texttt{functor something} = \texttt{something in}\ \delta$	
		$\mid$	$\mu.lab$	module term projection
signature kind	ξ	$::=$	$\texttt{SSigKind}(\sigma)$	
		$\mid$	$\texttt{SigKHole}$	
external signature	$\hat{\sigma}$	$::=$	s	signature variable
		$\mid$	$\{\widehat{sdecs}\}$	structure signature
		$\mid$	$\Pi_{m:_{\mu}\hat{\sigma_1}}.\hat{\sigma_2}$	functor signature
		$\mid$	$\llparenthesis\rrparenthesis^{\texttt{u}}$	empty signature hole
		$\mid$	$\llparenthesis s\rrparenthesis^{\texttt{u}}$	nonempty signature hole
signature	σ	$::=$	s	signature variable
		$\mid$	$\{sdecs\}$	structure signature
		$\mid$	$\Pi_{m:_{\mu}\sigma_1}.\sigma_2$	functor signature
		$\mid$	$\llparenthesis\rrparenthesis^{\texttt{u}}$	empty signature hole
		$\mid$	$\llparenthesis s\rrparenthesis^{\texttt{u}}$	nonempty signature hole
		$\mid$	$\utilde{s}$	unbound signature variable
module	μ	$::=$	m	module variable
		$\mid$	$\{sbnds\}$	structure
		$\mid$	$\lambda m{:}_{\mu}\sigma.\mu$	functor
		$\mid$	$\mu_1\ \mu_2$	functor application
		$\mid$	$\mu.lab$	submodule projection
		$\mid$	$\llparenthesis\rrparenthesis$	empty module hole
		$\mid$	$\llparenthesis\mu\rrparenthesis$	nonempty module hole
signature declarations	$sdecs$	$::=$	$\cdot$	
		$\mid$	$sdec, sdecs$	
signature declaration	$sdec$	$::=$	$\texttt{type}\ lab{::}\kappa$	
		$\mid$	$\texttt{val}\ lab{:}\tau$	
		$\mid$	$\texttt{module}\ lab{:}_{\mu}\sigma$	
		$\mid$	$\texttt{functor}\ lab{:}_{\mu}\sigma$	
structure bindings	$sbnds$	$::=$	$\cdot$	

$$\begin{array}{llcl} & & \mid & sbnd, sbnds \\ \text{structure binding} & sbnd & ::= & \mathtt{type}\ t = \tau \\ & & \mid & \mathtt{let}\ x{:}\tau = \delta \\ & & \mid & \mathtt{module}\ m = \mu \\ & & \mid & \mathtt{module}\ m{:}_{\mu} s = \mu \\ & & \mid & \mathtt{functor}\ m{:}_{\mu} s = \mu \end{array}$$

context definitions

$$\Delta; , ?; \Gamma; , x{:}\tau; \Phi; , t{::}\kappa; \Xi; , m{:}_{\mu}\sigma; \Psi; , s{::}_{\sigma}\xi$$

declarative statics

scratch

$\boxed{\Delta; ; \Phi; \vdash \kappa_1 \lesssim \kappa_2}$ κ_1 is a consistent subkind of κ_2

KCSubsumption

$$\frac{\mathit{test}}{\mathit{test}}$$

$\boxed{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \xi_1 \lesssim_{\xi} \xi_2}$ ξ_1 is a consistent sub signature kind of ξ_2

nameMe

$$\frac{\begin{array}{c}\exists sdec_x \in sdecs_1\ \ \mathit{st}\ \Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\{sdec_x\}) \lesssim_{\xi} \mathtt{SSigKind}(\{sdec_2\}) \\ \Delta; ; \Phi; , \mathsf{type}(\Delta; ; \Phi; ; \Xi; ; \Psi; , sdec_2); \Xi; , \mathsf{submodule}(sdec_2); \Psi; \vdash \{sdecs_1\} \lesssim_{\xi} \{sdecs_2\}\end{array}}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\{sdec_{11}, sdec_{12}, sdecs_{13}\ \mathtt{as}\ sdecs_1\}) \lesssim_{\xi} \mathtt{SSigKind}(\{sdec_2, sdecs_2\})}$$

single

$$\frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash sdec_1 \leq_{sdec} sdec_2}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\{sdec_1\}) \lesssim_{\xi} \mathtt{SSigKind}(\{sdec_2\})}$$

nil

$$\frac{}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\{sdecs\}) \lesssim_{\xi} \mathtt{SSigKind}(\{\cdot\})}$$

varprop

$$\frac{s{::}_{\sigma}\xi \in \Psi;}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(s) \lesssim_{\xi} \xi}$$

nameMe?delete?

$$\frac{\sigma_1 \neq s \qquad \Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \sigma_1\ \Leftarrow\ \mathtt{SSigKind}(\sigma_2)}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\sigma_1) \lesssim_{\xi} \mathtt{SSigKind}(\sigma_2)}$$

funct

$$\frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\sigma_{21}) \lesssim_{\xi} \mathtt{SSigKind}(\sigma_{11}) \qquad \Delta; ; \Phi; ; \Xi; , m{:}_{\mu}\sigma_{11}; \Psi; \vdash \mathtt{SSigKind}(\sigma_{12}) \lesssim_{\xi} \mathtt{SSigKind}(\sigma_{22})}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\Pi_{m{:}_{\mu}\sigma_{11}}.\sigma_{12}) \lesssim_{\xi} \mathtt{SSigKind}(\Pi_{m{:}_{\mu}\sigma_{21}}.\sigma_{22})}$$

holes

$$\frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \llparenthesis\rrparenthesis^{\mathtt{u}}\ \Leftarrow\ \xi}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\llparenthesis\rrparenthesis^{\mathtt{u}}) \lesssim_{\xi} \xi}$$

neholes

$$\frac{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \llparenthesis \mathsf{s} \rrparenthesis^{\mathtt{u}}\ \Leftarrow\ \xi}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SSigKind}(\llparenthesis \mathsf{s} \rrparenthesis^{\mathtt{u}}) \lesssim_{\xi} \xi}$$

CSubSigKindHoleL

$$\frac{}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \mathtt{SigKHole} \lesssim_{\xi} \xi}$$

CSubSigKindHoleR

$$\frac{}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \xi \lesssim_{\xi} \mathtt{SigKHole}}$$

$\boxed{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \sigma\ \Rightarrow\ \xi}$ σ synthesizes signature kind ξ

SynSigKndVar

$$\frac{s{::}_{\sigma}\xi \in \Psi;}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash s\ \Rightarrow\ \mathtt{SSigKind}(s)}$$

SynSigKndVarFail

$$\frac{s \notin \mathsf{dom}(\Psi;)}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash s\ \Rightarrow\ \mathtt{SigKHole}}$$

$$\frac{\{sdecs\}\mathit{wellformed?}}{\vdash \{sdecs\}\ \Rightarrow\ \mathtt{SSigKind}(\{sdecs\})}$$

$$\frac{\Delta; ; \Phi; ; \Xi; , m{:}_{\mu}\sigma_1; \Psi; \vdash \sigma_2\ \Rightarrow\ \xi}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \Pi_{m{:}_{\mu}\sigma_1}.\sigma_2\ \Rightarrow\ \mathtt{SSigKind}(\Pi_{m{:}_{\mu}\sigma_1}.\sigma_2)}$$

SynSigKndSigHole

$$\frac{u{::}_{\sigma}\xi \in \Delta;}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \llparenthesis\rrparenthesis^{\mathtt{u}}\ \Rightarrow\ \xi}$$

SynSigKndSigHole

$$\frac{u{::}_{\sigma}\xi \in \Delta; \qquad \Delta; ; \Phi; ; \Xi; ; \Psi; \vdash s\ \Rightarrow\ \xi_1}{\Delta; ; \Phi; ; \Xi; ; \Psi; \vdash \llparenthesis s \rrparenthesis^{\mathtt{u}}\ \Rightarrow\ \xi}$$

$\boxed{\Delta;;\Phi;;\Xi;;\Psi;\vdash \sigma \ \Leftarrow\ \xi}$ σ analyzes against signature kind ξ

Sub

$$\frac{\Delta;;\Phi;;\Xi;;\Psi;\vdash \sigma \ \Rightarrow\ \xi_1 \qquad \Delta;;\Phi;;\Xi;;\Psi;\vdash \xi_1 \lesssim_{\xi} \xi}{\Delta;;\Phi;;\Xi;;\Psi;\vdash \sigma \ \Leftarrow\ \xi}$$

$\boxed{\Delta;;\Phi;;\Xi;;\Psi;\vdash sdec_1 \leq_{sdec} sdec_2}$ $sdec_1$ is a subsdec of $sdec_2$

singleType

$$\frac{\vdash \kappa_1 \lesssim \kappa_2}{\Delta;;\Phi;;\Xi;;\Psi;\vdash \texttt{type}\ lab::\kappa_1 \leq_{sdec} \texttt{type}\ lab::\kappa_2}$$

singleVa

$$\frac{\Delta;;\Phi;;\Xi;;\Psi;\vdash \Delta;;\Phi;;\Xi;;\Psi;\equiv \tau_1\tau_2}{\Delta;;\Phi;;\Xi;;\Psi;\vdash \texttt{val}\ lab{:}\tau_1 \leq_{sdec} \texttt{val}\ lab{:}\tau_2}$$

singleMod

$$\frac{\Delta;;\Phi;;\Xi;;\Psi;\vdash \sigma_1 \ \Leftarrow\ \texttt{SSigKind}(\sigma_2)}{\Delta;;\Phi;;\Xi;;\Psi;\vdash \texttt{module}\ lab{:}_{\mu}\sigma_1 \leq_{sdec} \texttt{module}\ lab{:}_{\mu}\sigma_2}$$

elaboration

$\boxed{\Gamma;;\Phi;;\Xi;\vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta;}$ $\hat{\delta}$ synthesizes type τ and elaborates to δ with hole context $\Delta;$

$\cdots$

SynElabLetMod

$$\frac{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta;_1 \qquad \Gamma;;\Phi;;\Xi;,m{:}_{\mu}\sigma \vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta;_2}{\Gamma;;\Phi;;\Xi;\vdash \texttt{module}\ m = \hat{\mu}\ \texttt{in}\ \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \texttt{module}\ m = \mu\ \texttt{in}\ \delta \dashv \Delta;_1 \cup \Delta;_2}$$

SynElabLetModAnn

$$\frac{\Phi;;\Xi;\vdash \hat{\sigma} \ \Rightarrow\ \xi \rightsquigarrow \sigma \dashv \Delta;_1 \qquad \Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \ \Leftarrow\ \sigma \rightsquigarrow \mu \dashv \Delta;_2 \qquad \Gamma;;\Phi;;\Xi;,m{:}_{\mu}\sigma \vdash \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \delta \dashv \Delta;_3}{\Gamma;;\Phi;;\Xi;\vdash \texttt{module}\ m{:}_{\mu}\hat{\sigma} = \hat{\mu}\ \texttt{in}\ \hat{\delta} \ \Rightarrow\ \tau \rightsquigarrow \texttt{module}\ m{:}_{\mu}\sigma = \mu\ \texttt{in}\ \delta \dashv \Delta;_1 \cup \Delta;_2 \cup \Delta;_3}$$

SynElabModTermPrj

$$\frac{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta; \qquad \Phi;;\Xi;\vdash \sigma \ \Rightarrow\ \xi}{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu}.lab \ \Rightarrow\ \tau \rightsquigarrow \mu.lab \dashv \Delta;}$$

$\boxed{\Phi;;\Xi;\vdash \hat{\tau} \ \Rightarrow\ \kappa \rightsquigarrow \tau \dashv \Delta;}$ $\hat{\tau}$ synthesizes kind κ and elaborates to τ with hole context $\Delta;$

$\cdots$

SynElabModTypPrj

$$\frac{\Phi;;\Xi;\vdash m \ \Rightarrow\ \sigma \rightsquigarrow m \dashv \Delta; \qquad something\sigma\kappa}{\Phi;;\Xi;\vdash m.lab \ \Rightarrow\ \kappa \rightsquigarrow m.lab \dashv \Delta;}$$

$\boxed{\Phi;;\Xi;\vdash \hat{\tau} \ \Leftarrow\ \kappa \rightsquigarrow \tau \dashv \Delta;}$ $\hat{\tau}$ analyzes against kind κ and elaborates to τ with hole context $\Delta;$

$\boxed{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \ \Rightarrow\ \sigma \rightsquigarrow \mu \dashv \Delta;}$ $\hat{\mu}$ synthesizes signature σ and elaborates to μ with hole context $\Delta;$

SynElabModVar

$$\frac{m{:}_{\mu}\sigma \in \Xi;}{\Gamma;;\Phi;;\Xi;\vdash m \ \Rightarrow\ \sigma \rightsquigarrow m \dashv \cdot}$$

SynElabModVarFail

$$\frac{m \notin \mathsf{dom}(\Xi;)}{\Gamma;;\Phi;;\Xi;\vdash m \ \Rightarrow\ \llparenthesis\rrparenthesis \rightsquigarrow \llparenthesis m \rrparenthesis^{\mathtt{u}} \dashv u{:}_{\mu}\llparenthesis\rrparenthesis}$$

SynElabConsStruct

$$\frac{\begin{array}{c}\Gamma;;\Phi;;\Xi;\vdash \widehat{sbnd} \ \Rightarrow\ sdec \rightsquigarrow sbnd \dashv \Delta;_1 \\ \Gamma;,\mathsf{val}(sdec);\Phi;,\mathsf{type}(\Delta;_1;\Phi;;\Xi;;\Psi;,sdec);\Xi;,\mathsf{submodule}(sdec) \vdash \{\widehat{sbnds}\} \ \Rightarrow\ \{sdecs\} \rightsquigarrow \{sbnds\} \dashv \Delta;_2\end{array}}{\Gamma;;\Phi;;\Xi;\vdash \{\widehat{sbnd}, \widehat{sbnds}\} \ \Rightarrow\ \{sdec, sdecs\} \rightsquigarrow \{sbnd, sbnds\} \dashv \Delta;_1 \cup \Delta;_2}$$

SynElabNilStruct

$$\frac{}{\Gamma;;\Phi;;\Xi;\vdash \{\cdot\} \ \Rightarrow\ \{\cdot\} \rightsquigarrow \{\cdot\} \dashv \cdot}$$

SynElabEmptyModHole

$$\frac{}{\Gamma;;\Phi;;\Xi;\vdash \llparenthesis\rrparenthesis^{\mathtt{u}} \ \Rightarrow\ \llparenthesis\rrparenthesis \rightsquigarrow \llparenthesis\rrparenthesis^{\mathtt{u}} \dashv u{:}_{\mu}\llparenthesis\rrparenthesis}$$

SynElabNonemptyModHole

$$\frac{}{\Gamma;;\Phi;;\Xi;\vdash \llparenthesis m \rrparenthesis^{\mathtt{u}} \ \Rightarrow\ \llparenthesis\rrparenthesis \rightsquigarrow \llparenthesis m \rrparenthesis^{\mathtt{u}} \dashv u{:}_{\mu}\llparenthesis\rrparenthesis}$$

functor stuff

$\boxed{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \;\Leftarrow\; \sigma \leadsto \mu \dashv \Delta;}$ $\hat{\mu}$ analyzes against signature σ and elaborates to μ with hole context $\Delta;$

`AnaElabModSubsumption`

$$\frac{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \;\Rightarrow\; \sigma \leadsto \mu \dashv \Delta;}{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \;\Leftarrow\; \sigma \leadsto \mu \dashv \Delta;}$$

$\boxed{\Gamma;;\Phi;;\Xi;\vdash \widehat{sbnd} \;\Rightarrow\; sdec \leadsto sbnd \dashv \Delta;}$ $\widehat{sbnd}$ synthesizes declaration $sdec$ and elaborates to $sbnd$ with hole context $\Delta;$

`SynElabTypeSbnd`

$$\frac{\Phi;;\Xi;\vdash \hat{\tau} \;\Rightarrow\; \kappa \leadsto \tau \dashv \Delta;}{\Gamma;;\Phi;;\Xi;\vdash \texttt{type}\ t = \hat{\tau} \;\Rightarrow\; \texttt{type}\ t{::}\kappa \leadsto \texttt{type}\ t = \tau \dashv \Delta;}$$

`SynElabValSbnd`

$$\frac{\Phi;;\Xi;\vdash \hat{\tau} \;\Rightarrow\; \kappa \leadsto \tau \dashv \Delta;_1 \qquad \Gamma;;\Phi;;\Xi;\vdash \hat{\delta} \;\Leftarrow\; \tau \leadsto \delta \dashv \Delta;_2}{\Gamma;;\Phi;;\Xi;\vdash \texttt{let}\ x{:}\hat{\tau} = \hat{\delta} \;\Rightarrow\; \texttt{val}\ x{:}\tau \leadsto \texttt{let}\ x{:}\tau = \delta \dashv \Delta;_1 \cup \Delta;_2}$$

`SynElabModSbnd`

$$\frac{\Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \;\Rightarrow\; \sigma \leadsto \mu \dashv \Delta;}{\Gamma;;\Phi;;\Xi;\vdash \texttt{module}\ m = \hat{\mu} \;\Rightarrow\; \texttt{module}\ m{:}_{\mu}\sigma \leadsto \texttt{module}\ m{:}_{\mu}\sigma = \mu \dashv \Delta;}$$

`SynElabModAnnSbnd`

$$\frac{\Phi;;\Xi;\vdash \hat{\sigma} \;\Rightarrow\; \xi \leadsto \sigma_1 \dashv \Delta;_1 \qquad \Gamma;;\Phi;;\Xi;\vdash \hat{\mu} \;\Rightarrow\; \sigma_2 \leadsto \mu \dashv \Delta;_2 \qquad \Phi;;\Xi;;\Psi;\vdash \sigma_2 \;\Leftarrow\; \xi}{\Gamma;;\Phi;;\Xi;\vdash \texttt{module}\ m{:}_{\mu}\hat{\sigma} = \hat{\mu} \;\Rightarrow\; \texttt{module}\ m{:}_{\mu}\sigma_1 \leadsto \texttt{module}\ m{:}_{\mu}\sigma_1 = \mu \dashv \Delta;_1 \cup \Delta;_2}$$

$\boxed{\Gamma;;\Phi;;\Xi;\vdash \widehat{sbnd} \;\Leftarrow\; sdec \leadsto sbnd \dashv \Delta;}$ $\widehat{sbnd}$ analyzes against declaration $sdec$ and elaborates to $sbnd$ with hole context $\Delta;$

`subsump`

$$\frac{\Gamma;;\Phi;;\Xi;;l\Psi;\vdash \widehat{sbnd} \;\Rightarrow\; sdec_1 \leadsto sbnd \dashv \Delta; \qquad \Delta;;\Phi;;\Xi;;\Psi;\vdash sdec_1 \leq_{sdec} sdec}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \widehat{sbnd} \;\Leftarrow\; sdec \leadsto sbnd \dashv \Delta;}$$

$\boxed{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \widehat{sdec} \leadsto sdec \dashv \Delta;}$ $\widehat{sdec}$ elaborates to $sdec$ with hole context $\Delta;$

`opq`

$$\frac{}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \texttt{type}\ lab \leadsto \texttt{type}\ lab{::}\texttt{Type} \dashv \cdot}$$

`trn`

$$\frac{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \hat{\tau} \;\Rightarrow\; \kappa \leadsto \tau \dashv \Delta;}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \texttt{type}\ lab{::}\hat{\tau} \leadsto \texttt{type}\ lab{::}\kappa \dashv \Delta;}$$

`val`

$$\frac{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \hat{\tau} \;\Rightarrow\; \kappa \leadsto \tau \dashv \Delta;}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \texttt{val}\ lab{:}\hat{\tau} \leadsto \texttt{val}\ lab{:}\tau \dashv \Delta;}$$

`mod`

$$\frac{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \hat{\sigma} \;\Rightarrow\; \xi \leadsto \sigma \dashv \Delta;}{\Gamma;;\Phi;;\Xi;;\Psi;\vdash \texttt{module}\ lab{:}_{\mu}\hat{\sigma} \leadsto \texttt{module}\ lab{:}_{\mu}\sigma \dashv \Delta;}$$

`we're going to need HOFunctors so we don't need to preclude users from typing a functor into a module and vice versa`

$\boxed{\Phi;;\Xi;;\Psi;\vdash \hat{\sigma} \;\Rightarrow\; \xi \leadsto \sigma \dashv \Delta;}$ $\hat{\sigma}$ synthesizes signature kind ξ and elaborates to σ with hole context $\Delta;$

`SynSigEmptyHole`

$$\frac{}{\Phi;;\Xi;;\Psi;\vdash (\!|\,|\!)^{u} \;\Rightarrow\; \texttt{SigKHole} \leadsto (\!|\,|\!)^{u} \dashv u{::}_{\sigma}\texttt{SigKHole}}$$

`SynSigNonEmptyHole`

$\boxed{\Phi;;\Xi;\vdash \hat{\sigma} \;\Leftarrow\; \xi \leadsto \sigma \dashv \Delta;}$ $\hat{\sigma}$ analyzes against signature kind ξ and elaborates to σ with hole context $\Delta;$

misc functions

$$\mathsf{val}(sdec) = \begin{cases} lab{:}\tau & sdec = \texttt{val}\ lab{:}\tau \\ \cdot & \text{otherwise} \end{cases}$$

$$\mathsf{type}(cntxts, sdec) = \begin{cases} lab{::}\kappa & sdec = \texttt{type}\ lab{::}\kappa \\ \cdot & \text{otherwise} \end{cases}$$

$$\mathsf{submodule}(sdec) = \begin{cases} lab{:}_{\mu}\sigma & sdec = \texttt{module}\ lab{:}_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases}$$

$$\begin{aligned}
\mathtt{S}_{\mathtt{Type}}(\tau) &:= \mathtt{S}(\tau)\\
\mathtt{S}_{\mathtt{S}(\tau_1)}(\tau) &:= \mathtt{S}(\tau)\\
\mathtt{S}_{\mathtt{KHole}}(\tau) &:= \mathtt{KHole}
\end{aligned}$$

theorems

Kind Synthesis Precision

If $\mathrm{cntxts} \vdash \tau \;\Rightarrow\; \kappa$ then $\forall \kappa_1 . \Delta;;\Phi;;\Xi;;\Psi; \vdash \mathrm{cntxts}{::}\tau\kappa_1 \implies \mathrm{cntxts} \vdash \kappa \lesssim \kappa_1$