Hazel PHI: 10-modules

July 2, 2021

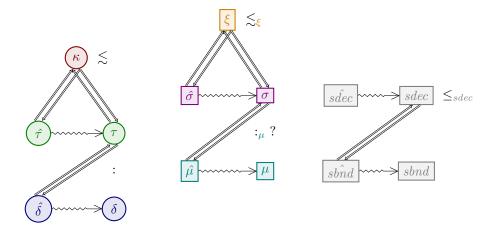
prerequisites

- Hazel PHI: 9-type-aliases-redux
 - github
 - $\ current \ commit: \ 4410cd565ce717707e580e44f64868d3175fe2a6$
- (optional) Hazel PHI: 1-labeled-tuples
 - github
 - $\ current \ commit: \ 0a7d0b53ee7286d03ea3be13a7ac91a86f1c90b1$

how to read

800000	kinds	D08000	signature kind
008000	types (constructors)	800080	signatures
000080	terms	008080	modules

notes



external typ/sig/mod syntax not written out yet (waiting for construction dust to settle); patterns not handled yet—will be left till end.

syntax



```
HTyp
                                                                                                                          type variable
                                               t
                                                bse
                                                                                                                              base type
                                                                                                                             type binop
                                               	au_1 \oplus 	au_2
                                               [	au]
                                                                                                                                list type
                                                                                                                         type function
                                               \lambda t :: \kappa.\tau
                                                                                                                      type application
                                                                                                   labelled product type (record)
                                               \{lab_1 \hookrightarrow \tau_1, \dots \, lab_n \hookrightarrow \tau_n\}
                                                                                                             module type projection
                                                                                                                      empty type hole
                                                (|\tau|)
                                                                                                                 nonempty type hole
                base type
                                bse
                                               Int
                                               Float
                                               Bool
           HTyp BinOp
                                 \oplus
   external expression
                                               signature s = \hat{\sigma} in \hat{\delta}
                                               module m=\hat{\mu} in \hat{\delta}
                                               module m{:}_{\mu}s=\hat{\mu} in \hat{\delta}
                                               functor something = something in \hat{\delta}
                                               \hat{\mu}.lab
                                                                                                            module term projection
    internal expression
                                 δ
                                         ::=
                                               signature s = \sigma in \delta
                                               module m:_{\mu} s = \mu in \delta
                                               functor something = something in \delta
                                               \mu.lab
                                                                                                            module term projection
         signature kind
                                               SSigKind(\sigma)
                                               SigKHole
     external signature
                                                                                                                    signature variable
                                                \{sdecs\}
                                                                                                                  structure signature
                                               \Pi_{m:_{\mu}\hat{\sigma_{1}}}.\hat{\sigma_{2}}
                                                                                                                     functor signature
                                                                                                               empty signature hole
                                                (s)^{\mathbf{u}}
                                                                                                           nonempty signature hole
                signature
                                                                                                                    signature variable
                                               \{sdecs\}
                                                                                                                  structure signature
                                               \Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2}
                                                                                                                     functor signature
                                                                                                               empty signature hole
                                                (s)^{\mathbf{u}}
                                                                                                           nonempty signature hole
                                                                                                        unbound signature variable
                                                \stackrel{s}{\sim}
                  module
                                                                                                                      module variable
                                               m
                                               \{sbnds\}
                                                                                                                               structure
                                               \lambda m:_{\mu} \sigma.\mu
                                                                                                                                 functor
                                                                                                                  functor application
                                               \mu_1 \mu_2
                                               \mu.lab
                                                                                                               submodule projection
                                                                                                                  empty module hole
                                               (\mu)
                                                                                                             nonempty module hole
signature declarations
                               sdecs
                                               sdec, sdecs
 signature declaration
                               sdec
                                               type lab::\kappa
                                               {\tt val}\ lab{:}\tau
                                               module lab:_{\mu}\sigma
                                               functor lab:_{\mu}\sigma
    structure bindings sbnds ::=
```

```
sbnd, sbnds
structure binding sbnd ::= type t = \tau
                                     | let x:\tau=\delta
                                         \texttt{module}\ m=\mu
                                         \text{module } m \text{:}_{\mu} s = \mu
```

context definitions

 $\Delta, ?; \Gamma, x:\tau; \Phi, t::\kappa; \Xi, m:_{\mu}\sigma; \Psi, s::_{\sigma} \xi$

```
declarative statics
      scratch
        \Delta; \Phi \vdash \kappa_1 \lesssim \kappa_2 \kappa_1 is a consistent subkind of \kappa_2
                                                                                                                        KCSubsumption
                                                                                                                         test
                                                                                                                         test
 \Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim_{\xi} \xi_2 \mid \xi_1 \text{ is a consistent sub signature kind of } \xi_2
                                     nameMe
                                                     \exists sdec_x \in sdecs_1 \ st \ \Delta; \Phi; \Xi; \Psi \vdash \mathtt{SSigKind}(\{sdec_x\}) \lesssim_{\varepsilon} \mathtt{SSigKind}(\{sdec_2\})
                                                    \Delta; \Phi, type(\Delta; \Phi; \Xi; \Psi, sdec_2); \Xi, submodule(sdec_2); \Psi \vdash \{sdecs_1\} \lesssim_{\xi} \{sdecs_2\}
                                      \Delta; \Phi; \Xi; \Psi \vdash SSigKind(\{sdec_{11}, sdec_{12}, sdecs_{13} \text{ as } sdecs_1\}) \lesssim_{\varepsilon} SSigKind(\{sdec_2, sdecs_2\})
             single
             \frac{\Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_2}{\Delta; \Phi; \Xi; \Psi \vdash \mathsf{SSigKind}(\{sdec_1\}) \lesssim_{\xi} \mathsf{SSigKind}(\{sdec_2\})} \qquad \frac{\mathsf{nil}}{\Delta; \Phi; \Xi; \Psi \vdash \mathsf{SSigKind}(\{sdec_s\}) \lesssim_{\xi} \mathsf{SSigKind}(\{\cdot\})}
                                    \label{eq:sigma} \begin{array}{ll} \text{varprop} & \text{nameMe?delete?} \\ \frac{s::_{\sigma}\xi \in \Psi}{\Delta;\Phi;\Xi;\Psi \vdash \mathtt{SSigKind}(s) \lesssim_{\xi} \xi} & \frac{\sigma_{1} \neq s \quad \Delta;\Phi;\Xi;\Psi \vdash \sigma_{1} \iff \mathtt{SSigKind}(\sigma_{2})}{\Delta;\Phi;\Xi;\Psi \vdash \mathtt{SSigKind}(\sigma_{1}) \lesssim_{\xi} \mathtt{SSigKind}(\sigma_{2})} \end{array}
                    funct
                    \frac{\Delta; \Phi; \Xi; \Psi \vdash \mathsf{SSigKind}(\sigma_{21}) \lesssim_{\xi} \mathsf{SSigKind}(\sigma_{11})}{\Delta; \Phi; \Xi; \Psi \vdash \mathsf{SSigKind}(\Pi_{m:_{\mu}\sigma_{11}}, \sigma_{12})} \lesssim_{\xi} \mathsf{SSigKind}(\Pi_{m:_{\mu}\sigma_{21}}, \sigma_{22})}
                CSubSigKindHoleR
                                                                                                    \Delta; \Phi; \Xi; \Psi \vdash \xi \lesssim_{\varepsilon} SigKHole
 \Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi \mid \sigma \text{ synthesizes signature kind } \xi
               SynSigKndVar
       \frac{\Delta;\Phi;\Xi,m:_{\mu}\sigma_{1};\Psi\vdash\sigma_{2}\;\Rightarrow\;\xi}{\Delta;\Phi;\Xi;\Psi\vdash\Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2}\;\Rightarrow\;\mathsf{SSigKind}(\Pi_{m:_{\mu}\sigma_{1}}.\sigma_{2})} \qquad \frac{u::_{\sigma}\xi\in\Delta}{\Delta;\Phi;\Xi;\Psi\vdash())^{\mathsf{u}}\;\Rightarrow\;\xi} \qquad \frac{u::_{\sigma}\xi\in\Delta}{\Delta;\Phi;\Xi;\Psi\vdash())^{\mathsf{u}}\;\Rightarrow\;\xi} \qquad \frac{u::_{\sigma}\xi\in\Delta}{\Delta;\Phi;\Xi;\Psi\vdash(|s|)^{\mathsf{u}}\;\Rightarrow\;\xi}
```

```
\Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi \mid \sigma \text{ analyzes against signature kind } \xi
                                                                                           \frac{\Delta; \Phi; \Xi; \Psi \vdash \sigma \Rightarrow \xi_1 \qquad \Delta; \Phi; \Xi; \Psi \vdash \xi_1 \lesssim_{\xi} \xi}{\Delta; \Phi; \Xi; \Psi \vdash \sigma \Leftarrow \xi}
  \Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_2
                                                                    sdec_1 is a subsdec of sdec_2
                                  singleType
                                  \frac{-\kappa_1 \lesssim \kappa_2}{\Delta; \Phi; \Xi; \Psi \vdash \mathsf{type} \; lab :: \kappa_1 \leq_{sdec} \mathsf{type} \; lab :: \kappa_2}
                                                                                                                                        \Delta; \Phi; \Xi; \Psi \vdash \Delta; \Phi; \Xi; \Psi \equiv \tau_1 \tau_2
\Delta; \Phi; \Xi; \Psi \vdash \text{val } lab: \tau_1 \leq_{sdec} \text{val } lab: \tau_2
                                                                                      singleMod
                                                                                                         \Delta; \Phi; \Xi; \Psi \vdash \sigma_1 \Leftarrow SSigKind(\sigma_2)
                                                                                      \frac{1}{\Delta;\Phi;\Xi;\Psi\vdash \text{module } lab:_{\mu}\sigma_{1}\leq_{sdec}\text{module } lab:_{\mu}\sigma_{2}}
elaboration
                                                                     \hat{\delta} synthesizes type \tau and elaborates to \delta with hole context \Delta
                                                                                            SynElabLetMod
                                                                                            SynElabLetModAnn
                                           \Phi;\Xi\vdash\hat{\sigma}\ \Rightarrow\ \underline{\xi}\leadsto\sigma\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Leftarrow\ \sigma\leadsto\mu\dashv\Delta_2\qquad \Gamma;\Phi;\Xi,m:_{\mu}\sigma\vdash\hat{\delta}\ \Rightarrow\ \tau\leadsto\delta\dashv\Delta_3
                                                           \Gamma; \Phi; \Xi \vdash \text{module } m:_{\mu} \hat{\sigma} = \hat{\mu} \text{ in } \hat{\delta} \Rightarrow \tau \leadsto \text{module } m:_{\mu} \sigma = \mu \text{ in } \delta \dashv \Delta_1 \cup \Delta_2 \cup \Delta_3
                                                                                                   SynElabModTermPrj
                                                                                                   \frac{\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta \qquad \Phi; \Xi \vdash \sigma \Rightarrow \xi}{\Gamma; \Phi; \Xi \vdash \hat{\mu}.lab \Rightarrow \tau \leadsto \mu.lab \dashv \Delta}
 \Phi;\Xi\vdash\hat{\tau}\Rightarrow\kappa\leadsto\tau\dashv\Delta \hat{\tau} synthesizes kind \kappa and elaborates to \tau with hole context \Delta
                                                                                                                       SynElabModTypPrj
                                                                                                                        \Phi; \Xi \vdash m \Rightarrow \sigma \leadsto m \dashv \Delta something \sigma \kappa
                                                                                                                                    \Phi:\Xi \vdash m.lab \Rightarrow \kappa \leadsto m.lab \dashv \Delta
 \Phi;\Xi\vdash\hat{\tau}\iff\kappa\leadsto\tau\dashv\Delta | \hat{\tau} analyzes against kind \kappa and elaborates to \tau with hole context \Delta
                                                             \hat{\mu} synthesizes signature \sigma and elaborates to \mu with hole context \Delta
 \Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta
                                                       SynElabModVar
                                                                                                                                                         SynElabModVarFail
                                                                                                                                                          \frac{m \notin \mathsf{dom}(\Xi)}{\Gamma; \Phi; \Xi \vdash m \ \Rightarrow \ (\!\!\!\! \| \leadsto (\!\!\! \| m)^{\mathtt{u}} \dashv u {:}_{\mu} (\!\!\!\! \| )}
                                                       \frac{m:_{\mu}\sigma\in\Xi}{\Gamma;\Phi;\Xi\vdash m\ \Rightarrow\ \sigma\leadsto m\dashv\cdot}
                                  SynElabConsStruct
                                                                                                     \Gamma; \Phi; \Xi \vdash s\hat{bnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta_1
                                  \Gamma, \mathsf{val}(\mathit{sdec}); \Phi, \mathsf{type}(\Delta_1; \Phi; \Xi; \Psi, \mathit{sdec}); \Xi, \mathsf{sub} \underline{\mathsf{module}(\mathit{sdec})} \vdash \{\mathit{sbnds}\} \ \Rightarrow \ \{\mathit{sdecs}\} \leadsto \{\mathit{sbnds}\} \dashv \Delta_2
                                                                 \Gamma; \Phi; \Xi \vdash \{\hat{sbnd}, \hat{sbnds}\} \Rightarrow \{\hat{sdec}, \hat{sdecs}\} \rightsquigarrow \{\hat{sbnd}, \hat{sbnds}\} \dashv \Delta_1 \cup \Delta_2
SynElabNilStruct
                                                                             SynElabEmptyModHole
                                                                                                                                                                 SynElabNonemptyModHole
                                                                                                                                                                                                                                                               functor stuff
\overline{\Gamma;\Phi;\Xi\vdash\{\cdot\}\ \Rightarrow\ \{\cdot\}\rightsquigarrow\{\cdot\}\dashv\cdot}\qquad \overline{\Gamma;\Phi;\Xi\vdash(\mathbb{J}^{\mathbf{u}}\ \Rightarrow\ (\mathbb{J})\rightsquigarrow(\mathbb{J}^{\mathbf{u}}\dashv u:_{\iota\iota}(\mathbb{J})}\qquad \overline{\Gamma;\Phi;\Xi\vdash(\mathbb{J}^{\mathbf{u}})^{\mathbf{u}}\ \Rightarrow\ (\mathbb{J})\rightsquigarrow(\mathbb{J}^{\mathbf{u}}\dashv u:_{\iota\iota}(\mathbb{J})}
```

 $\Gamma; \Phi; \Xi \vdash \hat{\mu} \Leftarrow \sigma \leadsto \mu \dashv \Delta$ $\hat{\mu}$ analyzes against signature σ and elaborates to μ with hole context Δ

$$\frac{\Gamma; \Phi; \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta}{\Gamma \vdash \Phi \vdash \Gamma \vdash \Phi}$$

$$\overline{\Gamma; \Phi; \Xi \vdash \hat{\mu} \iff \sigma \leadsto \mu \dashv \Delta}$$

 $\Gamma; \Phi; \Xi \vdash \hat{sbnd} \Rightarrow sdec \leadsto sbnd \dashv \Delta$ $s\hat{bnd}$ synthesizes declaration sdec and elaborates to sbnd with hole context Δ

 ${\tt SynElabTypeSbnd}$

$$\Phi:\Xi\vdash\hat{\tau} \Rightarrow \kappa \leadsto \tau\dashv \Lambda$$

$$\Gamma: \Phi: \Xi \vdash \mathsf{type}\ t = \hat{\tau} \Rightarrow \mathsf{type}\ t :: \kappa \leadsto \mathsf{type}\ t = \tau \dashv \Delta$$

SynElabValSbnd

$$\frac{\Phi;\Xi\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta}{\Gamma;\Phi;\Xi\vdash\mathsf{type}\ t=\hat{\tau}\ \Rightarrow\ \mathsf{type}\ t::\kappa\leadsto\mathsf{type}\ t=\tau\dashv\Delta} \qquad \frac{\Phi;\Xi\vdash\hat{\tau}\ \Rightarrow\ \kappa\leadsto\tau\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\delta}\ \Leftarrow\ \tau\leadsto\delta\dashv\Delta_2}{\Gamma;\Phi;\Xi\vdash\mathsf{let}\ x:\hat{\tau}=\hat{\delta}\ \Rightarrow\ \mathsf{val}\ x:\tau\leadsto\mathsf{let}\ x:\tau=\delta\dashv\Delta_1\cup\Delta_2}$$

$$\Gamma: \Phi: \Xi \vdash \hat{\mu} \Rightarrow \sigma \leadsto \mu \dashv \Delta$$

$$\frac{\Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Rightarrow\ \sigma\leadsto\mu\dashv\Delta}{\Gamma;\Phi;\Xi\vdash \mathrm{module}\ m=\hat{\mu}\ \Rightarrow\ \mathrm{module}\ m{:}_{\mu}\sigma\leadsto\mathrm{module}\ m{:}_{\mu}\sigma=\mu\dashv\Delta}$$

SynElabModAnnSbnd

$$\Phi:\Xi\vdash\hat{\sigma}\Rightarrow \xi\leadsto\sigma_1\dashv\Delta$$

$$\Gamma: \Phi: \Xi \vdash \hat{\mu} \Rightarrow \sigma_2 \rightsquigarrow \mu \dashv \Delta_2$$

$$\Phi: \Xi: \Psi \vdash \sigma_{\varrho} \Leftarrow$$

$$\frac{\Phi;\Xi\vdash\hat{\sigma}\ \Rightarrow\ \xi\leadsto\sigma_1\dashv\Delta_1\qquad \Gamma;\Phi;\Xi\vdash\hat{\mu}\ \Rightarrow\ \sigma_2\leadsto\mu\dashv\Delta_2\qquad \Phi;\Xi;\Psi\vdash\sigma_2\ \Leftarrow\ \xi}{\Gamma;\Phi;\Xi\vdash \mathsf{module}\ m:_{\mu}\hat{\sigma}=\hat{\mu}\ \Rightarrow\ \mathsf{module}\ m:_{\mu}\sigma_1\leadsto\mathsf{module}\ m:_{\mu}\sigma_1=\mu\dashv\Delta_1\cup\Delta_2}$$

 $\Gamma; \Phi; \Xi \vdash \hat{sbnd} \Leftarrow sdec \leadsto sbnd \dashv \Delta$ sbnd analyzes against declaration sdec and elaborates to sbnd with hole context Δ

$$\underline{\Gamma; \Phi; \Xi; l\Psi \vdash s\^{bnd}} \; \Rightarrow \; sdec_1 \leadsto sbnd \dashv \Delta \qquad \Delta; \Phi; \Xi; \Psi \vdash sdec_1 \leq_{sdec} sdec_1$$

$$\Gamma; \Phi; \Xi; \Psi \vdash \hat{sbnd} \Leftarrow sdec \leadsto sbnd \dashv \Delta$$

 $\Gamma; \Phi; \Xi; \Psi \vdash s \hat{dec} \leadsto s dec \dashv \Delta$ s \hat{dec} elaborates to s dec with hole context Δ

$$\Gamma$$

 Γ ; Φ ; Ξ ; Ψ \vdash type $lab \rightsquigarrow$ type lab ::Type \dashv ·

$$\begin{array}{c} \operatorname{val} & \operatorname{mod} \\ \Gamma; \Phi; \Xi; \Psi \vdash \hat{\tau} \ \Rightarrow \ \kappa \leadsto \tau \dashv \Delta \\ \hline \Gamma; \Phi; \Xi; \Psi \vdash \operatorname{val} \ \mathit{lab} : \hat{\tau} \leadsto \operatorname{val} \ \mathit{lab} : \tau \dashv \Delta \\ \end{array} \qquad \begin{array}{c} \operatorname{mod} \\ \Gamma; \Phi; \Xi; \Psi \vdash \hat{\sigma} \ \Rightarrow \ \xi \leadsto \sigma \dashv \Delta \\ \hline \Gamma; \Phi; \Xi; \Psi \vdash \operatorname{module} \ \mathit{lab} :_{\mu} \hat{\sigma} \dashv \Delta \end{array}$$

$$\frac{\Gamma; \Phi; \Xi; \Psi \vdash \hat{\tau} \implies \kappa \leadsto \tau \dashv \Delta}{\Gamma; \Phi; \Xi; \Psi \vdash \mathsf{type} \ lab :: \hat{\tau} \leadsto \mathsf{type} \ lab :: \kappa \dashv \Delta}$$

$$\Gamma; \Phi; \Xi; \Psi \vdash \mathsf{type} \ lab :: \hat{\tau} \leadsto \mathsf{type} \ lab :: \kappa \dashv \Delta$$

$$\overline{\Gamma;\Phi;\Xi;\Psi\vdash \mathtt{val}\ \mathit{lab}{:}\hat{\tau}\leadsto\mathtt{val}\ \mathit{lab}{:}\tau\dashv\Delta}$$

$$\Gamma \cdot \Phi \cdot \Xi \cdot \Psi \vdash \hat{\sigma} \implies \mathcal{E} \leadsto \sigma \dashv \Lambda$$

$$\Gamma: \Phi: \Xi: \Psi \vdash \text{module } lab: _u \hat{\sigma} \leadsto \text{module } lab: _u \sigma \dashv \Delta$$

we're going to need HOFunctors so we don't need to preclude users from typing a functor into a module and vice versa

 $\Phi;\Xi;\Psi\vdash\hat{\sigma}\Rightarrow \xi\leadsto\sigma\dashv\Delta$ $\hat{\sigma}$ synthesizes signature kind ξ and elaborates to σ with hole context Δ

SynSigEmptyHole

SynSigNonEmptyHole

$$\overline{\Phi;\Xi;\Psi\vdash (\!(\!)^\mathtt{u}\ \Rightarrow\ \mathtt{SigKHole}\leadsto (\!(\!)^\mathtt{u}\dashv u{::}_\sigma\mathtt{SigKHole}}$$

 $\Phi;\Xi\vdash\hat{\sigma}\leftarrow \xi\leadsto\sigma\dashv\Delta$ $\hat{\sigma}$ analyzes against signature kind ξ and elaborates to σ with hole context Δ

misc functions

$$\begin{aligned} \operatorname{val}(sdec) &= \begin{cases} lab{:}\tau & sdec = \operatorname{val}\ lab{:}\tau \\ \cdot & \text{otherwise} \end{cases} \\ \operatorname{type}(cntxts, sdec) &= \begin{cases} lab{:}:\kappa & sdec = \operatorname{type}\ lab{:}:\kappa \\ \cdot & \text{otherwise} \end{cases} \\ \operatorname{submodule}(sdec) &= \begin{cases} lab{:}_{\mu}\sigma & sdec = \operatorname{module}\ lab{:}_{\mu}\sigma \\ \cdot & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{split} \mathbf{S}_{\mathtt{Type}}(\tau) &:= \mathbf{S}(\tau) \\ \mathbf{S}_{\mathbf{S}(\tau_I)}(\tau) &:= \mathbf{S}(\tau) \\ \mathbf{S}_{\mathtt{KHole}}(\tau) &:= \mathtt{KHole} \end{split}$$

theorems

Kind Synthesis Precision

```
If cntxts \vdash \tau \implies \kappa then \forall \kappa_1.cntxts \vdash \tau :: \kappa_1 \implies \text{cntxts} \vdash \kappa \lesssim \kappa_1
```