some notes of mine

# hejohns' notes

hejohns

December 21, 2022

made with kaobook
set in EB Garamond

# Contents

# List of Figures

# List of Tables

# Papers/Articles

## 2012 - Soare, *Formalism and intuition in computability*

**Tags:** computability,history

**Date:** 2022-12-17

up to p.9

f

## 1981 - Kleene, *Origins of recursive function theory*

**Tags:** logic,computability,history

**Date:** December 21, 2022

need to read last 10 pages

nice review by Steward Shapiro, 1990

$\lambda$-defineable = Church, recursive = Gödel, Herbrand, computable = Turing,

(although until 193?, recursive $\mapsto$ primitive recursive, for Gödel, now recursive $\mapsto$ Herbrand-Gödel general recursive)

## 1995 - Makiko Nakano, *Makiko's Diary*

**Tags:**

**Date:** December 21, 2022

Translated by Kazuko Smith

sticky rice (desserts) = ½sticky rice + ½normal

# Presentations/Lectures

## 2007 - Bryan Cantrill, *Dtrace*

**Tags:**

**Date:** December 21, 2022

**Link:** recording

A

# Dictionary

## AFAI*

**Tags:** acronyms

**Date:** 2022-12-21

As Far As I (Can Tell / Know / ...)

## Craig's Trick

**Tags:** logic,computability

**Date:** 2022-12-18

**Theorem** (Craig's Trick)
From Mathew (MATH 684): $S$ ce set of sentences $\implies \exists S^*.S^*$ computable $\wedge$ they have the same theory.

So ce theories are just as effective as computable ones, which is good news for axiom schemas. A priori, it's not clear that theories w/ axiom schemas are as effective as finitely axiomatizable ones, but intuitively, axiom schemas are of the same character, are "easily checkable". Craig's trick formally grounds this.
eg $PA$ is as effective as $PA^-$. (although $PA^-$ has nice utility for the Entscheidungsproblem.)

(my terminology) a theory is computable $\iff$ it is ce [1]

1: ...sentences up to logical equivalence

ie a theory is computably axiomatized $\iff$ it is computably enumerably axiomatized

Wikipedia has a similar sketch.

*Proof.* MATH 684: $S$ ce, so you only have a listing of the sentences. We can make it strictly monotonic by relisting, but by adding a bunch of tautological noise or padding to each sentence (say, by conjuncting tautologies, assuming eg Gödel's prime factorization encoding) st the Gödel number is much larger. (Each sentence is relisted logically equivalently.) $\square$

Somehow the proof itself doesn't feel intuitive, but the "intended use" of the theorem *is*.

**See:** Theory

## Complete Lattice

**Tags:** order

**Date:** 2022-12-21

**Definition** (Complete Lattice)
TFAE

- lattice w/ all joins and meets $\implies$
- lattice w/ all joins, meets, top, bottom
- lattice w/ all joins [2]

2: People tend to say that the reals are "*the* complete ordered field", *in the sense that every bounded (above) set has a supremum. Which implies that every bounded (below) set has a infimum. (NOTE: "has" has a very specific meaning that everyone learns to live with.)

**See:** Thin Yoneda Embedding

* TODO: apparently the ambiguity of "complete" makes a difference, but afaik the following is itself correct.

## Effective

**Tags:** logic,computability

**Date:** December 21, 2022

An informal notion.

I typically use "effective" to mean alt.

- "morally computable"
- "probably computable, but then I'd have to actually check"

## Emotion

**Tags:** personal terminology

**Date:** December 21, 2022

I use "emotionally" similarly to "intuitively" or "morally", but "emotionally" has more to do with the gut feeling. Something that is pre-, or maybe post-, or maybe anti-, rational.

**See:** Informal Notion

## Enumeration Operator

**Tags:** logic,computability

**Date:** 2022-12-18

From Rogers' 1967 *Theory of Recursive Functions*.

An archetypical example: In Gödel's incompleteness theorems, each computable enumeration of a theory (my sense) gives rise to an enumeration of the theory (the deductive closure). In Miller's terms, there is the "deducibility operator" $D$ that gives for each axiom set $B$, $D(B)$, the set of consequences.

I'm thinking of the deducibility operator the whole time.

**Definition** (Enumeration Operator)
Each enumeration reduction witness $z$ and $B$ determine the $A$, so each $z$ determines a enumeration operator $\Phi_z : 2^\omega \to 2^\omega$.

ie $\Phi_z(B) = A \iff A \leq_e X$ witnessed by $z$.

$A \equiv_e B \iff A \leq_e B \wedge B \leq_e A$

**Theorem**

- enumeration operators compose, by inspection
- $A \subseteq B \implies \Phi(A) \subseteq \Phi(B)$ (monotonicity)
- $x \in \Phi(B) \implies \exists C.C$ finite $\wedge\, C \subseteq B \wedge x \in \Phi(C)$ (continuity) [3]

3: which I'd call compactness

**See:** Dana Scott's graph model of λ-calculus

## Enumeration Reducibility

**Tags:** logic,computability

**Date:** 2022-12-18

From Rogers' 1967 *Theory of Recursive Functions.*

**Definition** (Enumeration Reduc(tion/ible))
$A \leq_e B \iff \exists z.\forall x.x \in A \leftrightarrow \exists u.\langle x, y\rangle \in W_z \wedge D_u \subseteq B$

This definition is not as nice as (many-)one or Turing reduction, but the idea is that we want to "effectively list $A$ using any listing (computable or not) of $B$". Note that enumeration reductions "only use positive information about $B$, and produce only positive information about $A$; whereas Turing reductions use and produce both positive and negative information." (paraphrased from the introduction to Russell Miller's *Non-coding Enumeration Operators.*)

where $z$ is the Gödel code of the reduction witness, and $D_u$ is the finite set associated with $u$ as a canonical index (ie a tuple). [4]

4: The idea w/ $u$ is that to list $A$ while watching elements enter $B$, you should only need (to see) a finite amount of $B$ to list a particular element $x \in A$.

Rogers' (really simple) examples:

- $\{2n|n \in \omega\} \leq_e \omega$
- $A$ ce $\implies \forall B.A \leq_e B$

**See:** Enumeration Operator

## Dana Scott's Graph Model

**Tags:** logic,λ-calculus

**Date:** 2022-12-18

**Definition**

$$[\![\lambda x.]\!] :=$$
$$[\![e_1 e_2]\!] :=$$

**See:** Enumeration Operator

## Herbrand's Theorem

**Tags:** logic

**Date:** 2022-12-17

TODO: convert notes from Prof. Blass' November seminar Herbrand's Theorem

## Informal Notion

**Tags:**

**Date:** December 21, 2022

TODO:

- a formal notion probably applies, but then I'd have to actually check
- the formal notion doesn't quite emotionally capture the concept

# Knaster-Tarski

**Tags:** logic

**Date:** 2022-12-19

**Theorem** (Knaster-Tarski Fixpoint Theorem)
Every monotone function on a complete lattice has a complete lattice of fixpoints.

This theorem has many statements, and this is the easiest for me to remember. The complete lattice is often a powerset lattice.

*Proof.* Widely available. □

**Example**

- The deducibility operator is a monotone function on sets of sentences. [5]
  The bottom (least) fixpoint is the (deductively closed) empty theory. The top (greatest) fixpoint is the inconsistent theory, ie the set of all sentences. Consistency of the empty theory (by Gentzen's original cut elimination, or by existence of a model) says this complete lattice is nontrivial.
  Any consistent, computably axiomatizable (deductively closed) theory that proves more than the empty theory is an intermediate fixpoint– eg PA. Incompleteness says there is no intermediate fixpoint above PA that is complete, but there are at least $2^{\aleph_0}$ intermediate fixpoints above PA where we keep adding $Con(T)$ or $\neg Con(T)$. [6]

5: For simplicity, assume everything is about– and still true about– a fixed language of arithmetic.

6: Are there complete intermediate fixpoints?

**See:** Enumeration Operator, Dana Scott's Graph Model

# Locus Solum

**Tags:**

**Date:** December 21, 2022

This is my version of Girard's dictionary.

Also afaik the most Girard paper there is

# PA

**Tags:**

**Date:** December 21, 2022

Peano Arithmetic w/ induction. $PA^-$ := PA \ induction

# Realizability

**Tags:**

**Date:** December 21, 2022

This is how we can attach beamer presentations

## Theory

**Tags:** logic

**Date:** 2022-12-18

An unfortunately ambiguous term, but you can usually figure it out from context, if it really matters.

I tend to use "Theory" to just mean a set of sentences, as in the $\Gamma$ in the sequent $\Gamma \vdash$. So I see a finite set for "the theory of groups", and a finite set unioned w/ a schema for "Peano Arithmetic". (and the empty set for the Entscheidungsproblem.) Sometimes, people mean a deductively closed set of sentences.

## TFAE

**Tags:** acronyms

**Date:** 2022-12-21

The Following Are Equivalent

When there are multiple characterizations of the same thing. Typically followed by a bullet list.

## Universal Property

**Tags:** category theory

**Date:** December 21, 2022

NOTE: this is going to be in constant flux for a while...

TODO: semiformal definition goes here

afaik this is an informal notion. Things defined by universal properties should automatically be respected by lower level mappings on the base structure, that respect the lower level base structure. [†]
TODO: how does Yoneda tie in?

## Thin Yoneda Embedding

**Tags:** order

**Date:** 2022-12-21

NOTE: re name: iirc this is mentioned in Awodey in the Yoneda chapter

**Theorem** (see Stoy,*Denotational Semantics* 6.29. Theorem.)
Every partial order can be embedded in a complete lattice.

The partial order is often a lattice already. afaik, the lattice structure is also respected *because* join, meet, ..., are defined by universal properties.

*Proof.* alt.

- The presheaf category in the Yoneda embedding is morally a complete lattice.
- Dedekind cuts.
  ie map each object to the downards closure, and utilize the lattice structure of *Set*
  ie $\cup, \cap$ and reasonable $\top, \bot$

A nice example from Stoy (starting ~p.89): (everything standard $\le$ ordering) $\mathbb{Q}$ is a partial order. [‡]$\mathbb{Q}$ can be completed by adding $\top = +\infty, \bot = -\infty$ and taking Dedekind cuts.

[†] not at all how I want to phrase this, but I'm still trying to figure out what I mean
[‡] actually $\mathbb{Q}$ is a non-empty total order, thus a lattice, but for the theorem statement, we'll describe it this way.

$\square$

**See:** Complete Lattice, Universal Property

# Alphabetical Index