Untitled

: $X \sim B(n,p)$

- : $p(X=k) = C_n^k p^k (1-p)^{(n-k)}$
- $\mathrm{E}(X) = np; \mathrm{Var}(X) = np(1-p)$
- :

$$\mathrm{E}(X) = \sum_{k=0}^{n} kp(x=k) = \sum_{k=0}^{n} k\frac{n!}{k!(n-k)!}p^k(1-p)^{(n-k)}$$

$$= np\sum_{k=1}^{n}\frac{(n-1)!}{(k-1)!(n-k)!}p^{(k-1)}(1-p)^{(n-k)} = np$$

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}^2(X)$$

$$\mathrm{E}(X^2) = \sum_{k=0}^{n} k^2 p(x=k) = \sum_{k=2}^{n} k(k-1)p(x=k) + \mathrm{E}(X)$$

$$= p^2\sum_{k=2}^{n}\frac{n(n-1)(n-2)!}{(k-2)!(n-k)!}p^{(k-1)}(1-p)^{(n-k)} + np = n(n-1)p^2 + np$$

$$\Rightarrow \mathrm{Var}(X) = np(1-p)$$