# Scheduling of a Single Flow Shop for Minimal Energy Cost Under Real-Time Electricity Pricing

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A time-indexed integer programing approach is developed to optimize the manufacturing schedule of a factory for minimal energy cost under real-time pricing (RTP) of electricity. The approach is demonstrated using a flow shop operating during different time periods (i.e., day shift, swing shift, and night shift) in a microgrid, which also serves residential and commercial users. Results show that electricity cost can be reduced by 6.2%, 12.3%, and 21.5% for the three time periods considered, respectively. Additionally, a 6.3% cost reduction can be achieved by the residential and commercial buildings through adopting energy-conscious control strategies in this specific case study example.

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#### 1 Introduction

According to the International Energy Outlook 2013, world energy consumption will rise by 56% between 2010 and 2040 [1]. Industry is currently responsible for 37% of the total energy consumption; within the industrial sector, manufacturing is the largest user, accounting for 73% of the total [2]. Facing increasing energy prices and pressure from government entities and the general public to reduce their environmental footprint, manufacturing enterprises are moving toward energy-efficient manufacturing. One promising approach to achieve this is through manufacturing scheduling.

Electricity rate for manufacturing facilities in many countries is time-varying. Such pricing provides opportunities for manufacturing industries to reduce electricity cost by shifting their electricity usages from high-pricing periods to lower-pricing periods. For example, Nilsson and Söderström studied the impact of different electricity tariffs on industrial production planning and the potential of reducing electricity cost by shifting electricity usage from a high-rate period to a lower-rate period [3]. Two predetermined

electricity prices were used: a high price in the daytime and a lower price at night; such electricity pricing is somewhat simplistic. In reality, electricity pricing can be difficult to define and is time-varying. Generally, there are three different forms of time-varying electricity tariffs: time-of-use (TOU) rate, critical-peak pricing (CPP), and real-time pricing (RTP).

With TOU tariffs, the electricity price schedule is predefined, but it may vary by day, season, and weather [4]. Considering TOU electricity tariffs, Luo et al. presented a new ant colony optimization to optimize both makespan and cost in a hybrid flow shop which concerned both production and energy efficiency [5]. Fang et al. presented a mixed integer programing approach to find the global optimal solution for a flow shop scheduling problem with peak power consumption constraints [6]. Wang and Li researched on the manufacturing modeling under time-of-use electricity tariffs with considering both energy consumption and the demand [7]. However, if all the factories shift their electricity usage from the higher price period to the lower price period, the power demands during the lower price period will be dramatically increased and it may become a peak demand period. The RTP prevents this peak load issue by considering the dynamic interaction between the electricity supply and demand. Recently, Sun and Li used Markov decision process to identify machine control actions that could reduce power demand during peak hours [8]. However, to date no study has been done to access the possibility and potential of reducing energy cost via manufacturing scheduling in the scenario of RTP. This paper aims at filling this gap by presenting a time-index integer programing approach and demonstrating the approach using a simplified flow shop.

# 2 Model Description

In this paper, a manufacturing facility is assumed to consist of a single flow shop with a series of process steps. The facility is subject to real-time electricity pricing with the price determined by the balance between the supply and demand of the power grid serving the facility. For the sake of simplicity, a microgrid is considered in this paper, and there is only one manufacturing facility being served. In addition to the manufacturing facility, the microgrid also serves a number of residential and commercial buildings. The objective of scheduling is to minimize electricity cost while meeting a predetermined production throughput.

**2.1 Flow Shop Scheduling Formulation.** A time-indexed integer programing formulation (making decisions on assignment, sequencing, and timing of jobs using binary variables [9]) is employed to solve the flow shop energy-conscious scheduling problem. In a typical flow shop, there are several jobs and machines. Each job has to be processed in a defined order on a given set of machines, i.e., each job has to be processed on machine 1, then on machine 2, and so on.

For this scheduling problem, the following assumptions/ simplifications are made: (a) all the machines have two modes, i.e., on-mode and off-mode, and the power demand in each mode is constant; (b) a machine cannot be switched from one mode to another while processing a job; (c) machines are automatically operated and labor is not included; (d) the operation speed of each machine is constant when it is on; (e) each machine is dedicated to only one of the process steps; and (f) there is only one type of job since all the products are the same.

Each flow shop has m machines and each product is processed by machine  $1 \to \text{machine } 2 \to, \ldots, \to \text{machine m}$ . The required production throughput is assumed to be  $N_0$ . When processing a product on machine i, its associated processing time is  $p_i$ , and its power demand is  $q_i$ . At time t, the electricity price is  $P(L_t)$ , where  $L_t$  is the corresponding total market electricity demand. The decision variables are (a)  $N_{it}$  is the number of products that have been completed on machine i by time t, (b)  $x_{it}$  is equal to 1 if machine i is processing products at time t, and 0 otherwise, (c)  $y_{it}$  is equal to 1 if machine i start processing new products at time t, and 0

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otherwise, and (d)  $B_{i_{-}i+1}$  is the buffer capacity between machines i and machine i+1.

The following time-indexed integer programing model seeks to find a schedule that minimizes the total electricity cost for this problem with the assumption that both t and  $p_i$  are integer values:

$$\min \sum_{i} \sum_{t} P(L_t) q_i x_{it} \tag{1}$$

Subject to

$$N_{it} = 0$$
,  $(t = 0, ..., p_i - 1; i = 1, ..., m)$  (2)

$$N_{it} = \sum_{k=0}^{t-p_i+1} y_{ik}, (t = p_i, ..., T, i = 1, ..., m)$$
 (3)

$$N_{it} \ge N_{i+1,t} + x_{i+1,t}, (i = 1, ..., m-1; t = 0, ..., T)$$
 (4)

$$N_{it} - N_{i+1,t} \le B_{i,+1}, (i = 1, ..., m-1; t = 0, ..., T)$$

$$N_{mT} \ge N_0 \tag{6}$$

(5)

(7)

$$N_{it} \in Z, (i = 1, ..., m; t = 0, ..., T)$$

$$x_{it}, y_{it} \in \{0, 1\}, (i = 1, ..., m; t = 0, ..., T)$$
 (8)

$$x_{it} = \sum_{k=0}^{t} y_{ik}, (t = 1, ..., p_i - 1; i = 1, ..., m)$$
(9)

$$x_{it} = \sum_{k=t-p_i+1}^{t} y_{ik}, (t = p_i, ..., T; i = 1, ..., m)$$
 (10)

$$\sum_{k=t}^{t+p_i-1} x_{ik} \ge p_i y_{it}, (t=1,...,T-p_i+1; i=1,...,m)$$
 (11)

Constraints (2) and (3) determine the number of products that have finished on machine group i by time t. Constraints (4) and (5) ensure that the products are produced in a flow shop with buffers. Constraints (6) ensure that the number of jobs produced by the end time T is at least  $N_0$ . Constraints (7)–(11) ensure that once a product begins processing on machine i, it cannot be interrupted until it is finished. It should be noted that the objective function sums the electricity cost across the duration time T. The electricity price  $P(L_1)$  is a function of the electricity demand of the entire grid and is determined by the model in Sec. 2.2. The optimization problem (1) is solved using mixed integer nonlinear programing (MINLP) solvers in general algebraic modeling system.

# 2.2 Real-Time Electricity Price Model. In this paper, the real-time electricity price model is based on two main

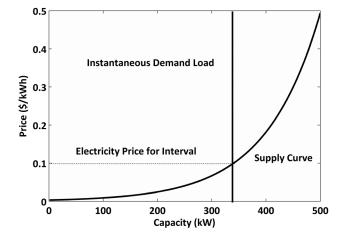


Fig. 1 Supply curve and instantaneous demand of a power grid

assumptions: (a) the supply curve is described with an exponential function, and (b) the electricity price is a function of load [10]. As is evident from Fig. 1, as the price of electricity increases, a greater supply of electricity can be provided.

The real-time electricity price is characterized as the intersection of the supply curve and instantaneous demand load. Thus, the real-time electricity price at a given time t is [11,12]

$$P(L_t) = \exp(aL_t + b) \tag{12}$$

where a and b characterize the electricity supply curve, and  $L_t$  is the instantaneous demand load at time t

$$L_t = R_{t,\text{load}} + C_{t,\text{load}} + I_{t,\text{load}}$$
 (13)

where  $R_{t,\mathrm{load}}$  represents the power demand of residential buildings,  $C_{t,\mathrm{load}}$  represents the power demand of commercial buildings, and  $I_{t,\mathrm{load}}$  represents the power demand of manufacturing facilities.

In Eq. (13), assuming that electricity demand of the manufacturing facility  $I_{t,load}$  only includes the electricity demand of processing machines, thus,  $I_{t,load}$  can be calculated using previously defined variables m,  $q_i$ , and  $x_{it}$  for a given manufacturing schedule

$$I_{t,\text{load}} = \sum_{i=1}^{m} q_i \cdot x_{it}$$
 (14)

In Eq. (13), the power demand of the residential and commercial buildings can be determined using the model presented in Sec. 2.3.

2.3 Residential and Commercial Buildings Model. The model used to simulate demands of the residential and commercial buildings is adopted from GRIDLAB-D. GRIDLAB-D is an open source software tool [13], which allows users to build a power distribution system with specific end-use equipment and devices to simulate real-time power demand. It also has a market auction module to simulate the time-varying electricity price based on total grid load. Although GRIDLAB-D does not have a module for manufacturing facilities, it does have modules for building envelope and energy-consuming devices, e.g., lights, water heaters, and HVAC (heating, ventilation, and air conditioning) systems, commonly used in the residential and commercial buildings. In brief, the model assumes that schedules of water heating and lighting are established in advance based on the consumers' requirements. For the operations of HVAC systems, the occupants could choose "business-as-usual." That is, they keep the temperature at a comfortable level and do not consider energy cost in their decisions of temperature setting. Alternatively, the occupants could choose to adopt some control strategies to reduce electricity cost while maintaining the temperature within an acceptable range

# 3 Case Study

To demonstrate the above scheduling approach, a microgrid serving one manufacturing facility consisting of a single flow shop, 200 residential homes, and 6 commercial buildings is considered. Scheduling is conducted for a typical summer day. In this case study, it is assumed that a = 0.0005 and b = -3.6052 in Eq. (12). In this case study, the time interval is selected to be 1 min.

For the residential and commercial buildings, schedules of HVAC systems are also slightly different from house to house. Tables 1 and 2 show the parameters for the residential and commercial buildings.

For the manufacturing facility, the flow shop has three process steps and each step is performed by ten parallel machines. The process order for each product is the same: process  $A \rightarrow \text{process } B \rightarrow \text{process } C$ . There are buffers after process A and process B, with the capacity of seven products. The processing time

Table 1 Average residential buildings characteristics

Residential house	Parameter	
Floor area (m <sup>2</sup> )	210	
Floor height (m)	3.35	
Ratio of window area to wall area	0.15	
Thermal resistance of the walls (W m <sup>2</sup> °C)	0.30	
Thermal resistance of the floor (W m <sup>2</sup> °C)	0.26	
Thermal resistance of the doors (W m <sup>2</sup> °C)	1.13	
Thermal resistance of the windows (W m <sup>2</sup> °C)	2.13	

Table 2 Average commercial buildings characteristics

Commercial building	Parameter
Office floor area (m <sup>2</sup> )	604
Office floor height (m)	5.33
Exterior thermal resistance (K/W)	0.94
Interior thermal resistance (K/W)	0.94
Windows facing south (m <sup>2</sup> )	3.39

(min/part) and power demand (kW) for each manufacturing process are listed in Table 3. The manufacturing facility is required to produce 400 products (or 40 batches as there are ten machines for each step) per shift.

**3.1 Scenarios Considered.** For comparison purposes, three scenarios are considered. For each scenario, the manufacturing facility operates three shifts, i.e., day shift (8:00-16:00), swing shift (16:00-24:00), and night shift (0:00-8:00). In scenario 1, the flow shop operates under business-as-usual conditions, as so the residential and commercial buildings. This scenario serves as the benchmark for evaluating cost-saving potentials of adopting energy-conscious schedules for manufacturing facility or control strategies for the residential and commercial buildings. In scenario 2, the flow shop operates on schedules that minimize electricity cost while maintaining a predetermined production quota. The cost-saving potential is evaluated for the day shift, swing shift, and night shift, respectively. For all the shifts in this scenario, the residential and commercial buildings served by the same microgrid operate with business-as-usual. In scenario 3, the flow shop operates with business-as-usual, and the residential and commercial buildings adopt strategies that aim at reducing electricity costs.

**3.2 Scenario 1.** In scenario 1, the manufacturing factory, residential buildings, and commercial buildings operate as business-as-usual, i.e., they operate without considering electricity costs. Figure 2 shows the power demand for the residential and commercial buildings over a 24-h period.

The schedules for the three machines in a flow shop for the 24 hrs period are shown in Fig. 3. As mentioned previously, there are ten identical machines operating in parallel for each process step.

Thus, the total electricity cost of the manufacturing factory is \$2287 for the 24-h period, while that of the residential and commercial buildings is \$2042.

**3.3 Scenario 2.** In scenario 2, the schedules for the manufacturing facility are selected to minimize the electricity cost for each

Table 3 Flow shop parameters

	Processing time (min/part)	Power demand (kW)
Process A	5	80
Process B	8	60
Process C	6	40

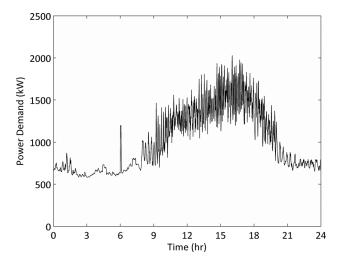


Fig. 2 Power demand of the business-as-usual residential and commercial buildings

shift. The residential and commercial buildings are operated with business-as-usual conditions over the 24-h period. To find the manufacturing schedule, the time-indexed integer programing model presented in Sec. 2.1 was solved. The optimized manufacturing schedules and the associated electricity cost for each shift are shown in Fig. 4. It can be seen that all the machines switch between on-mode and off-mode frequently.

Again the price is determined by the total power demand of the residential buildings, commercial buildings, and the manufacturing factory at each time interval. For the day shift, when compared with the business as usual case, the total power demand  $L_t$  is reduced overall, which leads to lower electricity price  $P(L_t)$ . The total electricity cost of the manufacturing factory for the day shift is \$741 in scenario 1, while it drops to \$695 in scenario 2, which represents a 6.2% reduction in the electricity cost of the manufacturing factory for the day shift. For the swing shift, the total electricity cost of manufacturing factory in scenario 1 is \$921, while it is \$723 for scenario 2 (a reduction of 21.5%). For the night shift, the total electricity cost for the manufacturing facility is \$625 in scenario 1, while it is \$548 for scenario 2 (a reduction of 12.3%). Thus, the total electricity cost of the manufacturing factory over a 24 h period is \$2287 in scenario 1, while it is \$1966 in scenario 2 with a reduction of 14.0%. The above results are achieved, when the buffer capacity is equal to 7. Repeating the optimization with

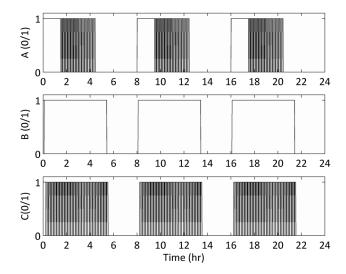


Fig. 3 Schedule of the business-as-usual manufacturing factory

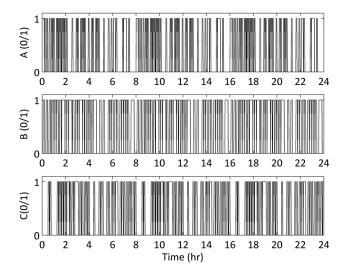


Fig. 4 "Minimal electricity cost" schedules of the manufacturing factory

different buffer capacity reveals that if the size of the buffer is larger or equal to 6 the optimal manufacturing schedules will stay the same. With the size of buffer smaller than 6, energy cost in both scenarios 1 and 2 will increase, and the energy cost saving varies. For example, the minimized total electricity cost of manufacturing factory is \$2080 in scenario 2, which represents a reduction of 9.6% when compared with \$2301 in scenario 1.

**3.4 Scenario 3.** In scenario 3, the manufacturing factory operates under business-as-usual conditions, while the residential and commercial buildings adopt control strategies to reduce their electricity cost. A control scheme is applied to the operation of the HVAC system by adjusting the set point within the occupants' comfort range. With this control scheme, the occupants can adopt a cost-saving operation strategy.

Since in this scenario, the manufacturing factory operates under business-as-usual, the schedules and power demand are the same as in scenario 1. The power demand of the residential and commercial buildings with cost reduction strategies is shown in Fig. 5. Again the electricity price is determined by the time-varying total power demand of residential buildings, commercial buildings, and manufacturing factory.

The electricity cost of the residential and commercial buildings using cost reduction strategies is \$1914, and a 6.3% of electricity

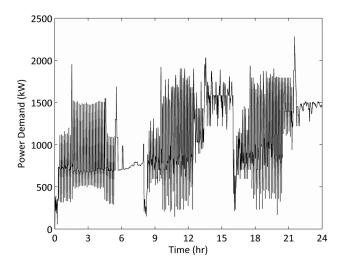


Fig. 5 Power demand of the residential and commercial buildings with cost-saving operation strategies

cost reduction can be achieved compared with that in scenario 1 (\$2042).

#### 4 Conclusions

In this paper, electricity cost of a manufacturing facility operating a single flow shop under real-time electricity price is minimized while maintaining the production throughput. The timevarying electricity price is determined by the total power demand from the manufacturing facilities as well as the residential and commercial users served by the same grid. Electricity demand from the residential and commercial buildings is simulated using GRIDLAB-D. The energy-conscious scheduling problem is formulated as a time-indexed integer program. To demonstrate the approach, a microgrid serving 200 homes, 6 commercial buildings, and a manufacturing facility operating a flow shops is considered. Schedules for three shifts (day shift, swing shift, and night shift) of the manufacturing factory are identified, and the total electricity cost is determined. The results show that electricity cost can be reduced by 6.2%, 21.5%, and 12.3% for each of the three shifts, respectively, and the average saving over the 24-h time period is 14.0%. Additionally, the residential and commercial buildings with adopting control strategies have a 6.3% cost reduction, which is lower than the cost saving achieved by the manufacturing factory through manufacturing scheduling in this specific example. The result also shows that electricity cost can be reduced through optimizing manufacturing schedules in the manufacturing facility or adopting control strategies in the residential and commercial buildings. This indicates that the time-indexed integer program can be used to develop new schedules for flow shop subject to real-time electricity pricing to minimize energy cost without sacrificing production throughput and could get a significant reduction on electricity cost at least comparable with the residential and commercial energy-saving case.

Time-indexed integer programing can identify the global optimum but requires significant computational efforts. The flow shop considered in this study is a much simplified one. For more realistic systems, it may not be the best option. There is an urgent need to develop more efficient formulations and algorithms (e.g., metaheuristics) suitable for large-size problems (flow shop and other types of configurations). In addition, under a fully implemented smart grid, a large number of users exchange information with the central controller and develop their control strategies and schedules accordingly. It may require a different framework to solve the scheduling problem in which many manufacturing facilities work collaboratively along with the residential and commercial users to reduce their electricity cost.

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