1 Theory

1.1 Distance

Hubbule parameter:

$$H^2 = H_0^2[\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} - \frac{Kc^2}{a^2H_0^2} + \Omega_\Lambda] \tag{1}$$

$$= H_0^2[\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{1-\Omega_0}{a^2} + \Omega_\Lambda] \tag{2}$$

Comving distance:

$$dt = \frac{da}{\dot{a}} \Rightarrow -dw = \frac{cdt}{a} = \frac{cda}{a\dot{a}} = \frac{cda}{a^2H}$$

$$\omega(z_1, z_2) = \frac{c}{H_0}\int_{a(z_2)}^{a(z_1)} \frac{da}{\sqrt{a\Omega_m + a^2(1-\Omega_m-\Omega_\Lambda) + a^4\Omega_\Lambda}}, z_1 < z_2 \tag{3}$$

$$= \frac{c}{H_0}\int_{z_1}^{z_2} \frac{dz}{\sqrt{(1+z)^3\Omega_m + (1+z)^2(1-\Omega_m-\Omega_\Lambda) + \Omega_\Lambda}}, z_1 < z_2 \tag{4}$$

$$= \frac{c}{H_0}\alpha(z_1, z_2) \tag{5}$$

The physical distance relates the angle to the size of a distant object is the angular diameter distance D_A.

$$D_A(z) = a(z)f_k(\omega(0, z))$$

The angular distance between z_1 and z_2 is

$$D_A(z_1, z_2) = a(z_2)f_k(\omega(z_1, z_2)) = \frac{c}{H_0}a(z_2)[\alpha(0, z_2) - \alpha(0, z_1)]$$

It is valid only for $\omega_k \geq 0$ (see astro-ph/9905116 or Principles of Physical Cosmology pp 336–337, Peebles)

1.2 Excess Surface Density

In comoving coordinate

$$\Sigma_{crit}(z_L, z_S) = \frac{c^2}{4\pi G}\frac{D_A(z_S)}{D_A(z_L)D_A(z_L, z_S)(1+z_L)^2} \tag{6}$$

$$\frac{c^2}{4\pi G} = \frac{c_0^2 \times 10^{16}\ \mathrm{m}^2 \cdot \mathrm{s}^{-2}}{4\pi G_0 \times 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{s}^{-2} \cdot \mathrm{Kg}^{-1}}$$

$$= \frac{c_0^2}{4\pi G_0} \times 10^{27}\ \mathrm{Kg} \cdot \mathrm{m}^{-1}$$

$$= \frac{c_0^2 l_{pc}}{4\pi G_0} \times 10^{43}\ \mathrm{Kg} \cdot \mathrm{pc}^{-1}$$

$$= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M}_\odot \cdot \mathrm{pc}^{-1} \tag{7}$$

$$
\begin{aligned}
&\frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)(1+z_L)^2} \\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)}\ \mathrm{h}\cdot\mathrm{Mpc}^{-1} && (8)\\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)} \times 10^{-6}\ \mathrm{h}\cdot\mathrm{pc}^{-1} && (9)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)}\frac{H_0}{c} && (10)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)}\frac{100\ \mathrm{h}\cdot\mathrm{Km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1}}{c_0\times 10^8\ \mathrm{m}\cdot\mathrm{s}^{-1}} && (11)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)}\frac{1}{c_0} \times 10^{-9}\ \mathrm{h}\cdot\mathrm{pc}^{-1} && (12)
\end{aligned}
$$

Using the **comoving distance** in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L,z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot}\times 10^{13}\ \mathrm{M}_\odot\cdot\mathrm{pc}^{-1}\frac{\omega(0,\mathrm{z_S})}{\omega(0,\mathrm{z_L})[\omega(0,\mathrm{z_S})-\omega(0,\mathrm{z_L})](1+\mathrm{z_L})}\times 10^{-6}\ \mathrm{h}\cdot\mathrm{pc}^{-1}\\
&= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot}\frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)}\times 10^{7}\ \mathrm{h}\cdot\mathrm{M}_\odot\cdot\mathrm{pc}^{-2}\\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)}\times 1662916.5401756007\ \mathrm{h}\cdot\mathrm{M}_\odot\cdot\mathrm{pc}^{-2}\\
&1662895.2081868195 \rightarrow 1662916.5401756007 && (13)
\end{aligned}
$$

Using the **integrate part of comoving distance** instead of distance itself in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L,z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot}\times 10^{13}\ \mathrm{M}_\odot\cdot\mathrm{pc}^{-1}\frac{\alpha(0,\mathrm{z_S})}{\alpha(0,\mathrm{z_L})[\alpha(0,\mathrm{z_S})-\alpha(0,\mathrm{z_L})](1+\mathrm{z_L})}\frac{1}{\mathrm{c_0}}\times 10^{-9}\ \mathrm{h}\cdot\mathrm{pc}^{-1}\\
&= \frac{c_0 l_{pc}}{4\pi G_0 m_\odot}\frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)}\times 10^{4}\ \mathrm{h}\cdot\mathrm{M}_\odot\cdot\mathrm{pc}^{-2}\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)}\times 554.6821355281792\ \mathrm{h}\cdot\mathrm{M}_\odot\cdot\mathrm{pc}^{-2} && (14)
\end{aligned}
$$

In physical coordinate

$$
\Sigma_{crit}(z_L,z_S) = \frac{c^2}{4\pi G}\frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)} \tag{15}
$$

$$
\begin{aligned}
&\frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)} \\
&= \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)]}\times 10^{-6}\ \mathrm{h}\cdot\mathrm{pc}^{-1} && (16)\\
&= \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)]}\frac{1}{c_0}\times 10^{-9}\ \mathrm{h}\cdot\mathrm{pc}^{-1} && (17)
\end{aligned}
$$

Using the **comoving distance** in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L, z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M_\odot \cdot pc^{-1}} \frac{(1+\mathrm{z_L})\omega(0,\mathrm{z_S})}{\omega(0,\mathrm{z_L})[\omega(0,\mathrm{z_S}) - \omega(0,\mathrm{z_L})]} \times 10^{-6}\ \mathrm{h \cdot pc^{-1}} \\
&= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S) - \omega(0,z_L)]} \times 10^{7}\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \\
&= \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S) - \omega(0,z_L)]} \times 1662895.2081868195\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \qquad (18)
\end{aligned}
$$

Using the **integrate part of comoving distance** instead of distance itself in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L, z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M_\odot \cdot pc^{-1}} \frac{(1+\mathrm{z_L})\alpha(0,\mathrm{z_S})}{\alpha(0,\mathrm{z_L})[\alpha(0,\mathrm{z_S}) - \alpha(0,\mathrm{z_L})]} \frac{1}{\mathrm{c_0}} \times 10^{-9}\ \mathrm{h \cdot pc^{-1}} \\
&= \frac{c_0 l_{pc}}{4\pi G_0 m_\odot} \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S) - \alpha(0,z_L)]} \times 10^{4}\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \\
&= \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S) - \alpha(0,z_L)]} \times 554.6821355281792\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \qquad (19)
\end{aligned}
$$

$c_0 = 2.99\,792\,458$; $\quad l_{pc} = 3.085\,677\,581$; $\quad m_\odot = 1.9885,$ $\quad M_\odot = 1.9885{\times}10^{30}$ Kg; $\quad \mathrm{G_0} = 6.67\,408\,313$

1.3 Rotation

To calculate the tangential shear or Excess Surface Density, one must rotate the ellipticity according the position angle definition in Figure **??**. The position angle and the direction of rotation will be different in many papers.

1.4 sphere

1.4.1 shear estimators rotation

Rotation of shear:

$$
\begin{aligned}
e_+ &= e_1 \cos 2\theta - e_2 \sin 2\theta \\
e_\times &= e_1 \sin 2\theta + e_2 \cos 2\theta
\end{aligned}
\qquad (20)
$$

The rotation of the shear estimator : G_1,G_2,N,U,V.

$$G_+ + iG_\times = (G_1 + iG_2)\exp(2i\phi) \qquad (21)$$

$$G_+ = G_1 \cos(2\phi) - G_2 \sin(2\phi) \qquad (22)$$

$$G_\times = G_1 \sin(2\phi) + G_2 \cos(2\phi) \qquad (23)$$

$$N \to N \qquad (24)$$

$$U' + iV' = (U + iV)\exp(4i\phi) \qquad (25)$$

$$U' = U \cos(4\phi) - V \sin(4\phi) \qquad (26)$$

$$V' = U \sin(4\phi) + V \cos(4\phi) \qquad (27)$$

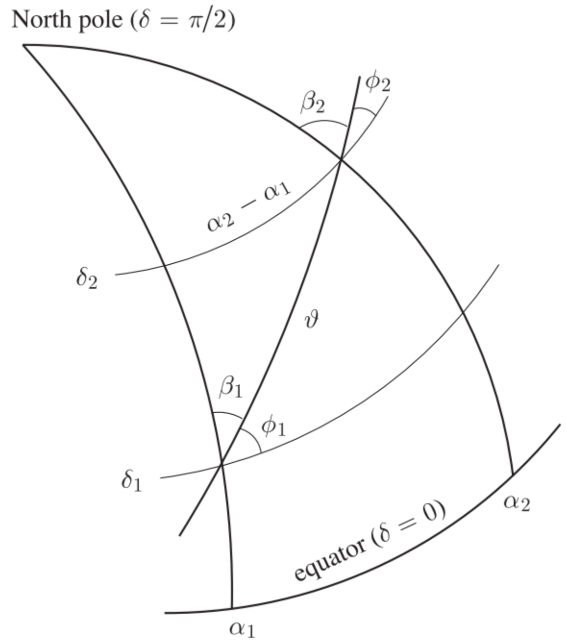

Figure 1: Sphere

1.5 Parameters

Mass of sun:

$$M_\odot = 1.9885 \times 10^{30}\text{Kg}, \quad \text{m}_\odot = 1.9885 \tag{28}$$

Gravitational constant:

$$G = 6.67\ 408(31) \times 10^{-11}\ \text{m}^3 \cdot \text{Kg}^{-1} \cdot \text{s}^{-2} \tag{29}$$

Speed of light:

$$c = 299\ 792\ 458\ \text{m} \cdot \text{s}^{-1} = 2.99\ 792\ 458 \times 10^8\ \text{m} \cdot \text{s}^{-1} \tag{30}$$

$$1\ \text{AU} = 149\ 597\ 870\ 700\ \text{m} = 1.49\ 597\ 870\ 700 \times 10^{11}\ \text{m} \tag{31}$$

$$1\ \text{Persec} = 3.085\ 677\ 581 \times 10^{16}\ \text{m} \tag{32}$$

1.6 NFW profile

$$\rho(r) = \frac{\rho_s}{[\frac{r}{r_s}][1 + \frac{r}{r_s}]^2} \tag{33}$$

2 How to...

There two similar pipelines for **CFHT** and **Fourier_Quad**.

Firstly, the distance should be calculated. "**calculate_co-distance.py**" calculates the comoving distance (Mpc/h) and the integrate part in the distance calculate for the final GGL calculation. The

Definition of position angle and rotation of shear

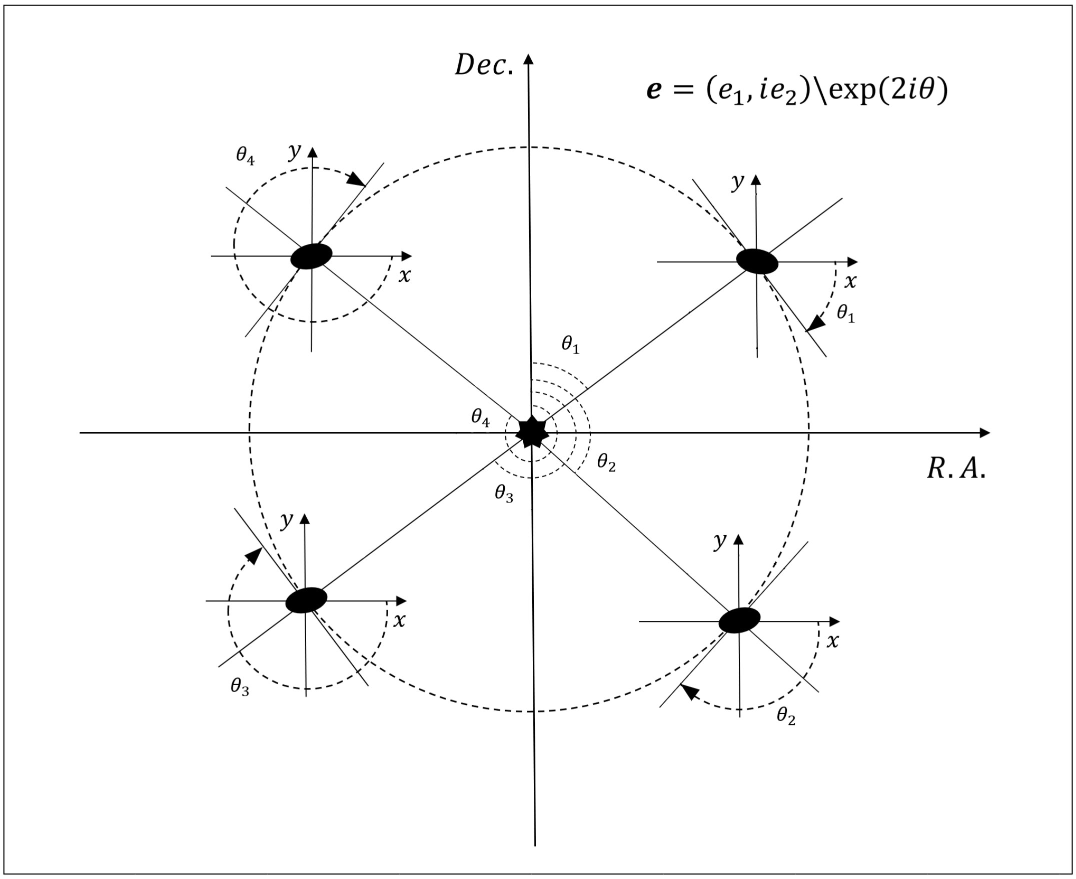

Figure 2: Rotation

parameters should be specified in code. The data will be saved in a hdf5 file. The distances will be signed to the source catalog in "**prepare_background_cata.py**".
"/**OM0_H0_C**" contains a array of Ω_{m0}, H_0, and $C_0(\sim 2.9)$.
"/**Z**" contains the redshifts ($0 \sim Z_{max}$).
"/**DISTANCE**" contains the distances (Mpc/h).
"/**DISTANCE_INTEG**" the integrate part of the distance.

2.1 CFHT catalog

2.1.1 Prepare data

1. "**add_ODD_Z_B.py**" adds **Z_MIN**, **Z_MAX**, and **ODDS** (from the .csv files) to the **CFHT** catalog for source selection. It will create two new files (.hdf5 & _new.dat) that contains the added parameters.

The hdf5 file contains 3 arrays:
"/**data**": the catalog with the 3 added parameters. The column: *"RA DEC Flag FLUX_RADIUS e1 e2 weight fitclass SNratio MASK Z_B m c2 LP_Mi star_flag MAG_i **Z_B_MIN Z_B_MAX ODDS**"*. The last three are added.
"/**mask**": it should be 1 for each source
"/**dRA_dDEC**": delta RA and delta DEC, they should be very small for each source ($< 10^{-5}$)

2. Run "**prepare_background_cata.py**" in "**collect**" mode with MPI to stack the data from each field. It creates the "**cfht_cata.hdf5**" in the parent directory of the one contain the field catalog. The data in i-th area will be in "/w_i" in the .hdf5 file. **If the catalog file (cfht_cata.hdf5) doesn't exist, run it firstly!** Before this step, **CFHT** catalog contains 19 ($0 \sim 18$) columns. After this the 19'th & 20'th column are the PZ data from Dong FY.

3. Run "**prepare_background_cata.py**" in "**select**" mode with CPU's as the same number as the area. The result will be in **cfht_cata_cut.hdf5**. The cutoff should be specified in the code. The program will create a few additional data for GGL calculation (see the code). At the end, the first thread will call "add_com_dist (add_com_dist.cpp)" to sign distance to the source.