1 Theory

1.1 Distance

Hubbule parameter:

$$H^2 = H_0^2[\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} - \frac{Kc^2}{a^2H_0^2} + \Omega_\Lambda] \tag{1}$$

$$= H_0^2[\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{1-\Omega_0}{a^2} + \Omega_\Lambda] \tag{2}$$

Comving distance:

$$dt = \frac{da}{\dot{a}} \Rightarrow -dw = \frac{cdt}{a} = \frac{cda}{a\dot{a}} = \frac{cda}{a^2H}$$

$$\omega(z_1, z_2) = \frac{c}{H_0}\int_{a(z_2)}^{a(z_1)} \frac{da}{\sqrt{a\Omega_m + a^2(1-\Omega_m-\Omega_\Lambda) + a^4\Omega_\Lambda}}, z_1 < z_2 \tag{3}$$

$$= \frac{c}{H_0}\int_{z_1}^{z_2} \frac{dz}{\sqrt{(1+z)^3\Omega_m + (1+z)^2(1-\Omega_m-\Omega_\Lambda) + \Omega_\Lambda}}, z_1 < z_2 \tag{4}$$

$$= \frac{c}{H_0}\alpha(z_1, z_2) \tag{5}$$

The physical distance relates the angle to the size of a distant object is the angular diameter distance D_A.

$$D_A(z) = a(z)f_k(\omega(0, z))$$

The angular distance between z_1 and z_2 is

$$D_A(z_1, z_2) = a(z_2)f_k(\omega(z_1, z_2)) = \frac{c}{H_0}a(z_2)[\alpha(0, z_2) - \alpha(0, z_1)]$$

It is valid only for $\omega_k \geq 0$ (see astro-ph/9905116 or Principles of Physical Cosmology pp 336–337, Peebles)

1.2 Excess Surface Density

In comoving coordinate

$$\Sigma_{crit}(z_L, z_S) = \frac{c^2}{4\pi G}\frac{D_A(z_S)}{D_A(z_L)D_A(z_L, z_S)(1+z_L)^2} \tag{6}$$

$$\begin{aligned}\frac{c^2}{4\pi G} &= \frac{c_0^2 \times 10^{16}\ \mathrm{m}^2\cdot\mathrm{s}^{-2}}{4\pi G_0 \times 10^{-11}\ \mathrm{m}^3\cdot\mathrm{s}^{-2}\cdot\mathrm{Kg}^{-1}} \\ &= \frac{c_0^2}{4\pi G_0} \times 10^{27}\ \mathrm{Kg}\cdot\mathrm{m}^{-1} \\ &= \frac{c_0^2 l_{pc}}{4\pi G_0} \times 10^{43}\ \mathrm{Kg}\cdot\mathrm{pc}^{-1} \\ &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M}_\odot\cdot\mathrm{pc}^{-1}\end{aligned} \tag{7}$$

$$
\begin{aligned}
&\frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)(1+z_L)^2} \\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)} \ \mathrm{h\cdot Mpc^{-1}} && (8)\\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)} \times 10^{-6} \ \mathrm{h\cdot pc^{-1}} && (9)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)} \frac{H_0}{c} && (10)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)} \frac{100 \ \mathrm{h\cdot Km\cdot s^{-1}\cdot Mpc^{-1}}}{c_0\times 10^8 \ \mathrm{m\cdot s^{-1}}} && (11)\\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)} \frac{1}{c_0} \times 10^{-9} \ \mathrm{h\cdot pc^{-1}} && (12)
\end{aligned}
$$

Using the **comoving distance** in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L,z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13} \ \mathrm{M_\odot \cdot pc^{-1}} \frac{\omega(0,\mathrm{z_S})}{\omega(0,\mathrm{z_L})[\omega(0,\mathrm{z_S})-\omega(0,\mathrm{z_L})](1+\mathrm{z_L})} \times 10^{-6} \ \mathrm{h\cdot pc^{-1}} \\
&= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)} \times 10^{7} \ \mathrm{h\cdot M_\odot \cdot pc^{-2}} \\
&= \frac{\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)](1+z_L)} \times 1662895.2081868195 \ \mathrm{h\cdot M_\odot \cdot pc^{-2}} && (13)
\end{aligned}
$$

Using the **integrate part of comoving distance** instead of distance itself in calculation,

$$
\begin{aligned}
\Sigma_{crit}(z_L,z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13} \ \mathrm{M_\odot \cdot pc^{-1}} \frac{\alpha(0,\mathrm{z_S})}{\alpha(0,\mathrm{z_L})[\alpha(0,\mathrm{z_S})-\alpha(0,\mathrm{z_L})](1+\mathrm{z_L})} \frac{1}{\mathrm{c_0}} \times 10^{-9} \ \mathrm{h\cdot pc^{-1}} \\
&= \frac{c_0 l_{pc}}{4\pi G_0 m_\odot} \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)} \times 10^{4} \ \mathrm{h\cdot M_\odot \cdot pc^{-2}} \\
&= \frac{\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)](1+z_L)} \times 554.6821355281792 \ \mathrm{h\cdot M_\odot \cdot pc^{-2}} && (14)
\end{aligned}
$$

In physical coordinate

$$
\Sigma_{crit}(z_L,z_S) = \frac{c^2}{4\pi G} \frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)} \tag{15}
$$

$$
\begin{aligned}
&\frac{D_A(z_S)}{D_A(z_L)D_A(z_L,z_S)} \\
&= \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S)-\omega(0,z_L)]} \times 10^{-6} \ \mathrm{h\cdot pc^{-1}} && (16)\\
&= \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S)-\alpha(0,z_L)]} \frac{1}{c_0} \times 10^{-9} \ \mathrm{h\cdot pc^{-1}} && (17)
\end{aligned}
$$

Using the **comoving distance** in calculation,

$$\begin{aligned}
\Sigma_{crit}(z_L, z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M_\odot \cdot pc^{-1}} \frac{(1+\mathrm{z_L})\omega(0,\mathrm{z_S})}{\omega(0,\mathrm{z_L})[\omega(0,\mathrm{z_S}) - \omega(0,\mathrm{z_L})]} \times 10^{-6}\ \mathrm{h \cdot pc^{-1}} \\
&= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S) - \omega(0,z_L)]} \times 10^{7}\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \\
&= \frac{(1+z_L)\omega(0,z_S)}{\omega(0,z_L)[\omega(0,z_S) - \omega(0,z_L)]} \times 1662895.2081868195\ \mathrm{h \cdot M_\odot \cdot pc^{-2}}
\end{aligned} \tag{18}$$

Using the **integrate part of comoving distance** instead of distance itself in calculation,

$$\begin{aligned}
\Sigma_{crit}(z_L, z_S) &= \frac{c_0^2 l_{pc}}{4\pi G_0 m_\odot} \times 10^{13}\ \mathrm{M_\odot \cdot pc^{-1}} \frac{(1+\mathrm{z_L})\alpha(0,\mathrm{z_S})}{\alpha(0,\mathrm{z_L})[\alpha(0,\mathrm{z_S}) - \alpha(0,\mathrm{z_L})]} \frac{1}{\mathrm{c_0}} \times 10^{-9}\ \mathrm{h \cdot pc^{-1}} \\
&= \frac{c_0 l_{pc}}{4\pi G_0 m_\odot} \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S) - \alpha(0,z_L)]} \times 10^{4}\ \mathrm{h \cdot M_\odot \cdot pc^{-2}} \\
&= \frac{(1+z_L)\alpha(0,z_S)}{\alpha(0,z_L)[\alpha(0,z_S) - \alpha(0,z_L)]} \times 554.6821355281792\ \mathrm{h \cdot M_\odot \cdot pc^{-2}}
\end{aligned} \tag{19}$$

$c_0 = 2.99\,792\,458$; $l_{pc} = 3.085\,677\,581$; $m_\odot = 1.9885$, $M_\odot = 1.9885{\times}10^{30}$ Kg; $\mathrm{G_0} = 6.67\,408\,313$

1.3 Rotation

To calculate the tangential shear or Excess Surface Density, one must rotate the ellipticity according the position angle definition in Figure 1. The position angle and the direction of rotation will be different in many papers.

Rotation of shear:

$$\begin{aligned}
e_+ &= e_1 \cos 2\theta - e_2 \sin 2\theta \\
e_\times &= e_1 \sin 2\theta + e_2 \cos 2\theta
\end{aligned} \tag{20}$$

The rotation of the shear estimator : G_1,G_2,N,U,V.

$$G_+ + iG_\times = (G_1 + iG_2)\exp(2i\phi) \tag{21}$$
$$G_+ = G_1 \cos(2\phi) - G_2 \sin(2\phi) \tag{22}$$
$$G_\times = G_1 \sin(2\phi) + G_2 \cos(2\phi) \tag{23}$$
$$N \to N \tag{24}$$
$$U' + iV' = (U + iV)\exp(4i\phi) \tag{25}$$
$$U' = U \cos(4\phi) - V \sin(4\phi) \tag{26}$$
$$V' = U \sin(4\phi) + V \cos(4\phi) \tag{27}$$

Definition of position angle and rotation of shear

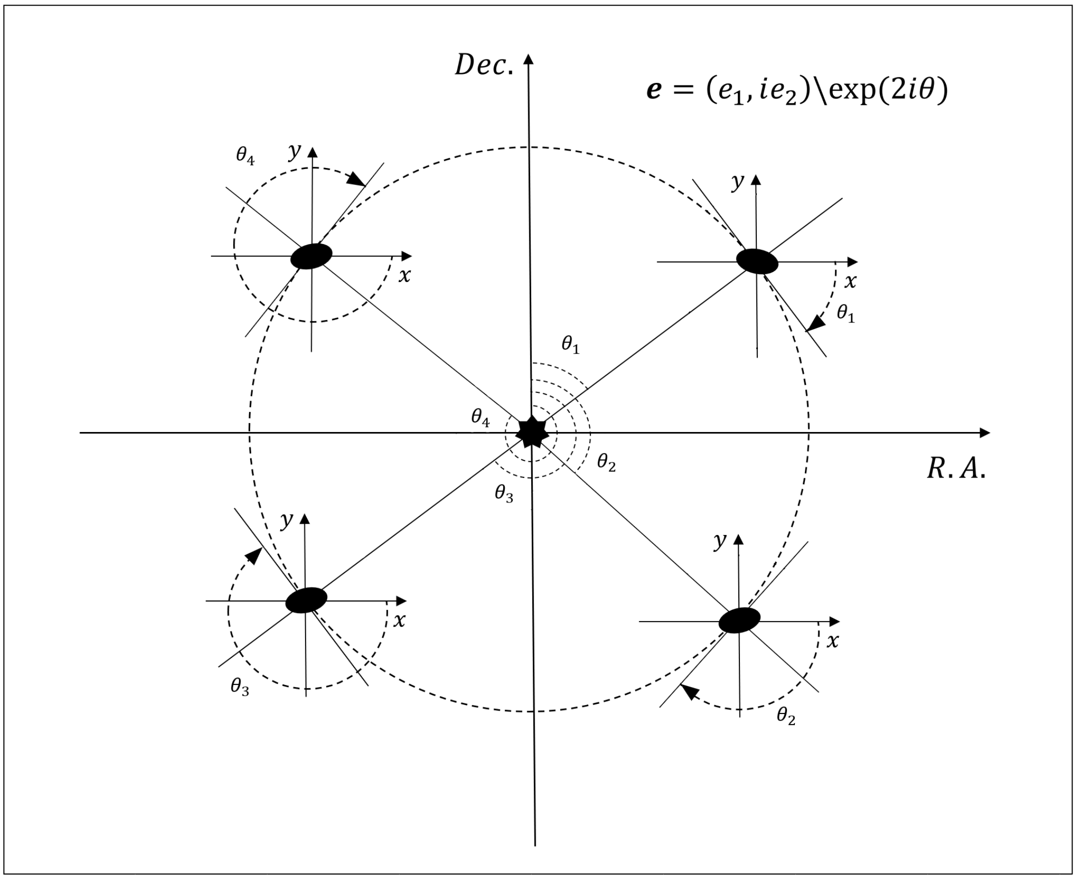

Figure 1: Rotation

1.4 Parameters

Mass of sun:

$$M_{\odot} = 1.9885 \times 10^{30}\text{Kg}, \quad \text{m}_{\odot} = 1.9885 \tag{28}$$

Gravitational constant:

$$G = 6.67\ 408(31) \times 10^{-11}\ \text{m}^3 \cdot \text{Kg}^{-1} \cdot \text{s}^{-2} \tag{29}$$

Speed of light:

$$c = 299\ 792\ 458\ \text{m} \cdot \text{s}^{-1} = 2.99\ 792\ 458 \times 10^8\ \text{m} \cdot \text{s}^{-1} \tag{30}$$

$$1\ \text{AU} = 149\ 597\ 870\ 700\ \text{m} = 1.49\ 597\ 870\ 700 \times 10^{11}\ \text{m} \tag{31}$$

$$1\ \text{Persec} = 3.085\ 677\ 581 \times 10^{16}\ \text{m} \tag{32}$$

1.5 NFW profile

$$\rho(r) = \frac{\rho_s}{[\frac{r}{r_s}][1 + \frac{r}{r_s}]^2} \tag{33}$$

2 How to...

There two similar pipelines for **CFHT** and **Fourier_Quad**.

Firstly, the distance should be calculated. "**calculate_co-distance.py**" calculates the comoving distance (Mpc/h) and the integrate part in the distance calculate for the final GGL calculation. The parameters should be specified in code. The data will be saved in a hdf5 file. The distances will be signed to the source catalog in "**prepare_background_cata.py**".
"**/OM0_H0_C**" contains a array of Ω_{m0}, H_0, and $C_0(\sim 2.9)$.
"**/Z**" contains the redshifts ($0 \sim Z_{max}$).
"**/DISTANCE**" contains the distances (Mpc/h).
"**/DISTANCE_INTEG**" the integrate part of the distance.

2.1 CFHT catalog

2.1.1 Prepare data

1. "**add_ODD_Z_B.py**" adds **Z_MIN**, **Z_MAX**, and **ODDS** (from the .csv files) to the **CFHT** catalog for source selection. It will create two new files (.hdf5 & _new.dat) that contains the added parameters.

The hdf5 file contains 3 arrays:
"**/data**": the catalog with the 3 added parameters. The column: *"RA DEC Flag FLUX_RADIUS e1 e2 weight fitclass SNratio MASK Z_B m c2 LP_Mi star_flag MAG_i* ***Z_B_MIN Z_B_MAX ODDS***". The last three are added.
"**/mask**": it should be 1 for each source
"**/dRA_dDEC**": delta RA and delta DEC, they should be very small for each source ($< 10^{-5}$)

2. Run "**prepare_background_cata.py**" in "**collect**" mode with MPI to stack the data from each field. It creates the "**cfht_cata.hdf5**" in the parent directory of the one contain the field catalog. The data in i-th area will be in "/w_i" in the .hdf5 file. **If the catalog file (cfht_cata.hdf5) doesn't exist, run it firstly!** Before this step, **CFHT** catalog contains 19 ($0 \sim 18$) columns. After this the 19'th & 20'th column are the PZ data from Dong FY.

3. Run "**prepare_background_cata.py**" in "**select**" mode with CPU's as the same number as the area. The result will be in **cfht_cata_cut.hdf5**. The cutoff should be specified in the code. The program will create a few additional data for GGL calculation (see the code). At the end, the first thread will call "add_com_dist (add_com_dist.cpp)" to sign distance to the source.