lensing potential

$$\Psi(\boldsymbol{\theta},\chi)=\frac{2}{c^2}\int_0^{\chi}d\chi'\,\frac{f_K(\chi-\chi')}{f_K(\chi)f_K(\chi')}\Phi(f_K(\chi')\boldsymbol{\theta},\chi') \tag{1}$$

Lensing matrix:

$$A=\delta_{ij}-\partial_i\partial_j\Psi=\begin{pmatrix}1-\kappa-\gamma_1 & -\gamma_2\\ -\gamma_2 & 1-\kappa+\gamma_1\end{pmatrix} \tag{2}$$

where

$$\begin{aligned}\gamma_1&=\frac{1}{2}(\partial_1\partial_1-\partial_2\partial_2)\Psi;\quad \gamma_2=\partial_1\partial_2\Psi,\\ \kappa&=\frac{1}{2}(\partial_1\partial_1+\partial_2\partial_2)\Psi.\end{aligned} \tag{3}$$

$$\begin{aligned}\kappa&=\frac{1}{2}(\partial_1\partial_1+\partial_2\partial_2)\int\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}d^2l\\
&=-\frac{1}{2}\int(l_1^2+l_2^2)\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}d^2l\\
\gamma&=\gamma_1+i\gamma_2\\
&=\frac{1}{2}(\partial_1\partial_1-\partial_2\partial_2+2i\partial_1\partial_2)\int\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}d^2l\\
&=-\frac{1}{2}\int(l_1^2-l_2^2+2il_1l_2)\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}d^2l\\
\tilde{\kappa}(\boldsymbol{l'})&=-\frac{1}{2}\int(l_1^2+l_2^2)\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}e^{-i\boldsymbol{l'}\cdot\boldsymbol{\theta}}d^2l\,d^2\theta\\
&=-\frac{1}{2}\int(l_1^2+l_2^2)\tilde{\Psi}(\boldsymbol{l})e^{i(\boldsymbol{l}-\boldsymbol{l'})\cdot\boldsymbol{\theta}}d^2l\,d^2\theta\\
&=-\frac{(2\pi)^2}{2}\int(l_1^2+l_2^2)\tilde{\Psi}(\boldsymbol{l})\delta^2(\boldsymbol{l}-\boldsymbol{l'})d^2l\\
&=-\frac{(2\pi)^2}{2}({l'_1}^2+{l'_2}^2)\tilde{\Psi}(\boldsymbol{l'})\\
\tilde{\gamma}(\boldsymbol{l'})&=-\frac{1}{2}\int(l_1^2-l_2^2+2il_1l_2)\tilde{\Psi}(\boldsymbol{l})e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}e^{-i\boldsymbol{l'}\cdot\boldsymbol{\theta}}d^2l\,d^2\theta\\
&=-\frac{(2\pi)^2}{2}({l'_1}^2-{l'_2}^2+2il'_1l'_2)\tilde{\Psi}(\boldsymbol{l'})\\
&=-\frac{(2\pi)^2}{2}(l'_1+il'_2)^2\tilde{\Psi}(\boldsymbol{l'})\end{aligned} \tag{4}$$

Therefore, we have

$$\tilde{\gamma}(\boldsymbol{l})=\frac{(l_1+il_2)^2}{l_1^2+l_2^2}\tilde{\kappa}(\boldsymbol{l})=[\cos^2(\beta)-\sin^2(\beta)+2i\cos(\beta)\sin(\beta)]\tilde{\kappa}(\boldsymbol{l})=e^{2i\beta}\tilde{\kappa}(\boldsymbol{l}). \tag{5}$$

The power spectrum of γ equals that of κ, *i.e.* $P_\gamma(\boldsymbol{l})=P_\kappa(\boldsymbol{l})$.

Relate $\kappa(\boldsymbol{\theta},\chi)$ to the overdensity $\delta(f_K(\chi)\boldsymbol{\theta},\chi)$.

$$\begin{aligned}
\kappa(\boldsymbol{\theta},\chi) &= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)f_K(\chi')}\left(\frac{\partial^2}{\partial\theta_1^2}+\frac{\partial^2}{\partial\theta_2^2}\right)\Phi(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)f_K(\chi')} f_K(\chi')^2\left(\frac{\partial^2}{\partial(f_K(\chi')\theta_1)^2}+\frac{\partial^2}{\partial(f_K(\chi')\theta_2)^2}\right)\Phi(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')\left(\frac{\partial^2}{\partial(f_K(\chi')\theta_1)^2}+\frac{\partial^2}{\partial(f_K(\chi')\theta_2)^2}+\frac{\partial^2}{\partial\chi'^2}\right)\Phi(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')\nabla^2\Phi(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')4\pi G a(\chi')^2\bar{\rho}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')4\pi G a(\chi')^2\bar{\rho}_0 a(\chi')^{-3}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} d\chi' \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')4\pi G a(\chi')^2\rho_{crit}\Omega_{m0} a(\chi')^{-3}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{1}{c^2}\int_0^{\chi} \frac{d\chi'}{a(\chi')} \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)} 4\pi G\frac{3H_0^2}{8\pi G}\Omega_{m0}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi} \frac{d\chi'}{a(\chi')} \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)}\delta(f_K(\chi')\boldsymbol{\theta},\chi')
\end{aligned} \tag{6}$$

The mean convergence

$$\begin{aligned}
\kappa(\boldsymbol{\theta}) &= \int_0^{\chi_{lim}} d\chi n(\chi)\kappa(\boldsymbol{\theta},\chi) \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi_{lim}} d\chi n(\chi)\int_0^{\chi} \frac{d\chi'}{a(\chi')} \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi_{lim}} d\chi'\int_{\chi'}^{\chi_{lim}} d\chi\frac{n(\chi)}{a(\chi')} \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)}\delta(f_K(\chi')\boldsymbol{\theta},\chi') \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi_{lim}} d\chi\int_{\chi}^{\chi_{lim}} d\chi'\frac{n(\chi')}{a(\chi)} \frac{f_K(\chi'-\chi)f_K(\chi)}{f_K(\chi')}\delta(f_K(\chi)\boldsymbol{\theta},\chi) \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi_{lim}} \frac{d\chi}{a(\chi)} f_K(\chi)\delta(f_K(\chi)\boldsymbol{\theta},\chi)\int_{\chi}^{\chi_{lim}} d\chi' n(\chi')\frac{f_K(\chi'-\chi)}{f_K(\chi')} \\
&= \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\chi_{lim}} \frac{d\chi}{a(\chi)} q(\chi)f_K(\chi)\delta(f_K(\chi)\boldsymbol{\theta},\chi); \quad q(\chi)=\int_{\chi}^{\chi_{lim}} d\chi' n(\chi')\frac{f_K(\chi'-\chi)}{f_K(\chi')}
\end{aligned} \tag{7}$$