Mechanism Design Project 3: Pendulum Mechanism

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Abstract

The dynamics and performance of a LEGO double pendulum has been calculated and verified using theoretical, analytical, ad experimental methods. The dynamics of the double pendulum was determined theoretically using MATLAB, computationally analyzed using solid works motion analysis, and experimentally using a 3D Printed model. Considering the results, we found out that the pins incur impact forces from the pendulums. We also realized that including damping is crucial to have better understanding of the underlying mechanism.

Introduction

A double Pendulum is a pendulum which is attached to another pendulum. This setup creates a chaotic system which is sensitive to the initial inputs and mass setup. The double pendulum found its way in the academic world as it exhibits non-linear behavior. The double pendulum is also involved applications such as robotics and human locomotion analysis. The dimensions and geometry of the double pendulum was provided and wan rendered using solid works which then was printed and assembled into the model which needed to be investigated. This report is separated into the following sections: (1) a theoretical derivation and analysis of the dynamics of the doble pendulum, (2) an explanation of the how the experimental setup was created which will be used to verify the motion, (3) the results and theoretical calculations which have been collection from Solid works motion analysis, MATLAB, and a visual tracker software (Tracker), (4) a comparison and discussion of theorical and experimental results, and (5) a conclusion of possible improvements that can be done to improve future experiments.

Background Work

The main motion of the mechanism can be found in Appendix Figure 1. From Figure 1 we can split the kinematics of 3 main points that describe the overall mechanism. The numerical iteration (Matlab Code Appendix) is based on timestep dt = 0.001s.

$$G_1: \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} = \begin{Bmatrix} \frac{L}{2} sin\varphi \\ -\frac{L}{2} cos\varphi \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{Bmatrix} = \begin{Bmatrix} \frac{L}{2} \omega_1 cos\varphi \\ -\frac{L}{2} \omega_1 sin\varphi \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{Bmatrix} = \begin{Bmatrix} \frac{L}{2} \alpha_1 cos\varphi - \frac{L}{2} \omega_1^2 sin\varphi \\ \frac{L}{2} \alpha_1 sin\varphi + \frac{L}{2} \omega_1^2 cos\varphi \end{Bmatrix},$$

$$where\ \omega_1 = \dot{\varphi}, \alpha_1 = \ddot{\varphi},$$
$$L = Beam\ Length$$

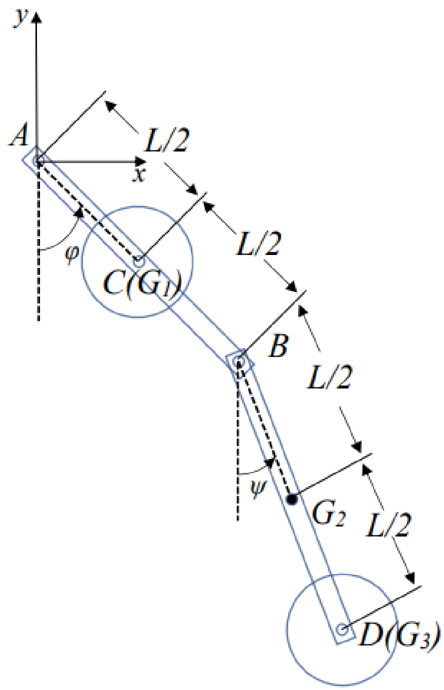


Figure 1. Pendulum Kinetics

$$G_2: \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} L sin\varphi + \frac{L}{2} sin\psi \\ -L cos\varphi - \frac{L}{2} cos\psi \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{Bmatrix} = \begin{Bmatrix} L\omega_1 cos\varphi + \frac{L}{2}\omega_2 cos\varphi \\ L\omega_1 sin\varphi + \frac{L}{2}\omega_2 sin\psi \end{Bmatrix}$$

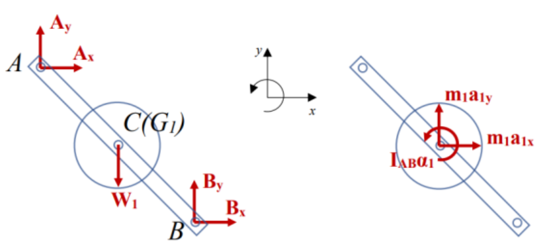
$$\begin{Bmatrix} \ddot{x}_2 \\ \ddot{y}_2 \end{Bmatrix} = \begin{Bmatrix} L\alpha_1 cos\varphi - L\omega_1^2 sin\varphi + \frac{L}{2}\alpha_2 cos\psi - \frac{L}{2}\omega_2^2 sin\psi \\ L\alpha_1 sin\varphi + L\omega_1^2 cos\varphi + \frac{L}{2}\alpha_2 sin\psi + \frac{L}{2}\omega_2^2 cos\psi \end{Bmatrix},$$

$$G_3: \begin{Bmatrix} x_3 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} L sin\varphi + L sin\psi \\ -L cos\varphi - L cos\psi \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{Bmatrix} = \begin{Bmatrix} L\omega_1 cos\varphi + L\omega_2 cos\varphi \\ L\omega_1 sin\varphi + L\omega_2 sin\psi \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{x}_2 \\ \ddot{y}_2 \end{Bmatrix} = \begin{Bmatrix} L\alpha_1 cos\varphi - L\omega_1^2 sin\varphi + L\alpha_2 cos\psi - L\omega_2^2 sin\psi \\ L\alpha_1 sin\varphi + L\omega_1^2 cos\varphi + L\alpha_2 sin\psi + L\omega_2^2 cos\psi \end{Bmatrix},$$

$$where\ \omega_2 = \dot{\psi}, \alpha_2 = \ddot{\psi}$$



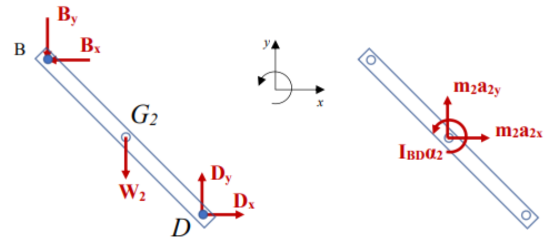
$$\sum F_{1x} = m_1 a_{1x} \rightarrow A_x + B_x - m_1 \frac{L}{2}\alpha_1 cos\varphi = -m_1 \frac{L}{2}\omega_1^2 sin\varphi$$

$$\sum F_{1y} = m_1 a_{1y} \rightarrow A_y + B_y - m_1 \frac{L}{2}\alpha_1 sin\varphi = m_1 \frac{L}{2}\omega_1^2 cos\varphi + W_1$$

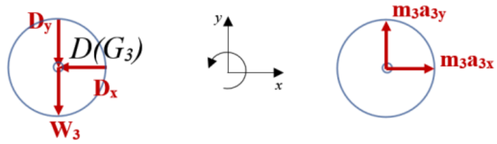
$$\sum M_{G_1} = I_{AB}\alpha_1 \rightarrow -A_x \frac{L}{2} cos\varphi - A_y \frac{L}{2} sin\varphi + B_x \frac{L}{2} cos\varphi + B_y \frac{L}{2} sin\varphi,$$

$$where\ I_{AB} = \frac{m_{beam} L^2}{12} + \frac{m_{gear} r^2}{2},$$

$$r = gear\ radius$$



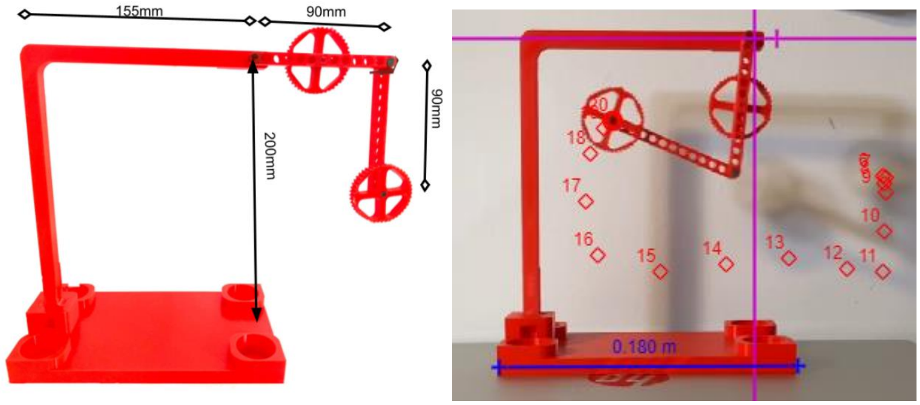
$$\sum F_{2x} = m_2 a_{2x} \rightarrow\ -B_x + D_x - m_2 L\alpha_1 cos\varphi$$
$$-m_2 \frac{L}{2}\alpha_2 cos\psi = -m_2 L\omega_1^2 sin\varphi - m_2 \frac{L}{2}\omega_2^2 sin\psi$$
$$\sum F_{2y} = m_2 a_{2y} \rightarrow\ -B_y + D_y - m_2 L\alpha_1 sin\varphi$$
$$-m_2 \frac{L}{2}\alpha_2 sin\psi = m_2 L\omega_1^2 cos\varphi + m_2 \frac{L}{2}\omega_2^2 cos\psi + W_2$$
$$\sum M_{G_2} = I_{BD}\alpha_2 \rightarrow\ B_x \frac{L}{2} cos\psi + B_y \frac{L}{2} sin\psi +$$
$$+D_x \frac{L}{2} cos\psi + D_y \frac{L}{2} sin\psi - I_{BD}\alpha_2,$$
$$where\ I_{BD} = \frac{(m_{beam} + 2m_{pin})L^2}{12}$$



$$\sum F_{3x} = m_3 a_{3x} \rightarrow\ -D_x - m_3 L\alpha_1 cos\varphi$$
$$-m_3 L\alpha_2 cos\psi = -m_3 L\omega_1^2 sin\varphi - m_2 L\omega_2^2 sin\psi$$
$$\sum F_{3y} = m_3 a_{3y} \rightarrow\ -D_y - m_3 L\alpha_1 sin\varphi$$
$$-m_3 L\alpha_2 sin\psi = m_3 L\omega_1^2 cos\varphi + m_3 L\omega_2^2 cos\psi + W_3$$

Experimental Apparatus and Procedure

The experimental apparatus which was described in the project has been modeled onto solid works keeping into consideration all the necessary dimensions and relationships described. The two technic beams are measured at 90mm length, and the two gears have a diameter of 35mm. Both the gear and the beam also have a mass of roughly 3.7g, this mass was given by the 3d-printing software. The total height at which the pendulum starts at is 200mm above the base plate. The solid works model was then analyzed on solid works motion analysis where it measured the forces acting on each pin for the 2 second duration. Another way for testing and recording the dynamics of the double pendulum is by using a tracker software that can calculate the position of a mass with the respective time, producing information such as angular displacement, velocity, and acceleration. For the software to produce accurate information it needed to have a reference length identified and an axis to calculate about shown in figure 2(b).



(a) (b)

Figure 2. (a) Dimensions of Physical Model. (b) TRACKER software

Results

From Appendix Figures 2, 3 we see that maximum angular velocities and accelerations are seen right before pendulum changes the direction. Interestingly the change in direction is accompanied with impulse-like forces in pins A, B, and D. Logically it makes sense because during the freefall the forces in the pins are almost 0, or negligible, and the change in direction is the same as abrupt hit of the object from the freefall into the solid surface. Also it is seen from Appendix Figure 1a and from video of the experiment that gear D and attached link starts rotating counter-clockwise, whilst link AB rotates clockwise, and after about 0.1s the link BD starts rotating clockwise, driven by link AB.

From Appendix Figures 4b (undamped), 4c (dampened) we see that in 2 seconds the theoretical motion and experimental motion differ. This is most likely due to the difference in inertia between the theoretical values and 3D printed parts, as well as the frictional forces. In 2 seconds theoretically we predicted ~2 full cycles, and practically obtained ~4.5 full cycles, with initial cycle being within reasonable margin of error. We also see that dampened results follow the heights of the peaks of the angle very closely with the experimental procedure.

Note. The motion tracking software uses angle between point A and point D – ϴ (Appendix Figure 4a)

Interpretation

Experimental and theoretical calculations were conducted to analyze the performance of the double pendulum. As seen in the results (*Appendix Figure 1)*, the dynamics of the double pendulum portrays a random and chaotic movement. The calculated movement recorded theoretically and experimentally does not match directly which can be attributed to friction of the real model and inertia differences. Although there are differences present in the graphs the overall motion, depicted by the peaks and directional changes remain the same, validifying the results. The forces recorded during the 2 second period of motion can be seen in figure 1. These reaction forces can be correlated to the dynamics of the pendulum as we can see that peaks in the reaction force figures occur coinciding with sudden changes in direction and velocity occur. When examining the dynamics of the model in solid works animation and in the physical, we can also conclude that the dynamics of the model depends greatly on the starting position and external inputs that act on the model. For example, a double pendulum starting with a little back and forth movement on the second leg will drastically change the output dynamics of the model.

Conclusion

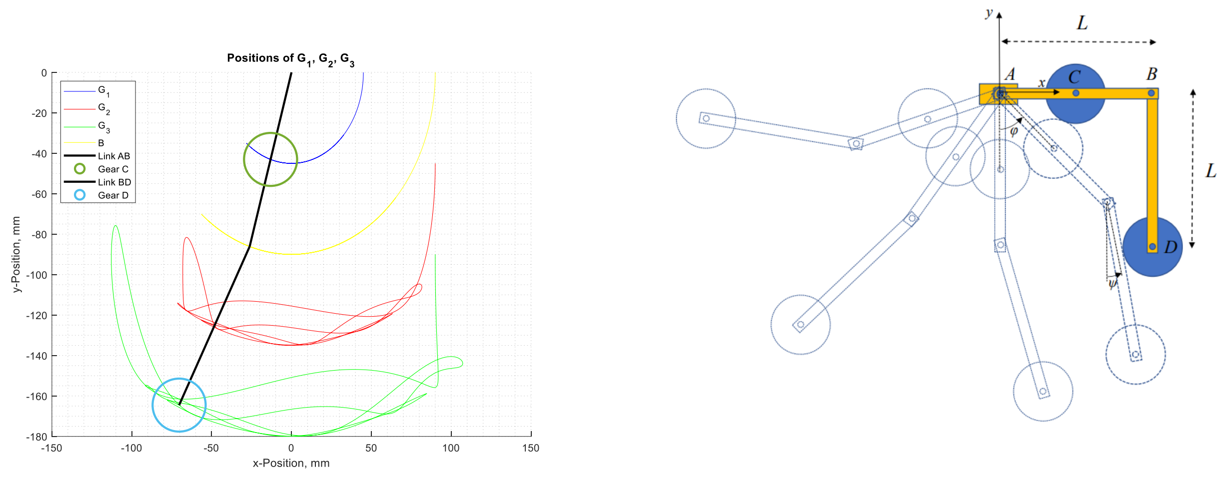
A double pendulum was designed and tested to verify the performance of its movement. Calculations were also conducted to compute the dynamics of the pendulum during the first 2 seconds. We were also able to determine for the reaction forces in the pins and relate it to the dynamics of the pendulum. We also have described the motion of the double pendulum as being nonlinear. A non-linear motion is defined when the direction of the net forces is not parallel to the motion and when the motion is constantly accelerating. The experimental and theoretical results show correlation within some marginal error which can be brought on due to the real-world frictional forces that occur within the pins and the inertial differences due to the slight difference in mass. This system is also highly sensitive to initial conditions which could have also created differences in the experimental procedure to portray in the results. Some ways that this experiment can be improved is to measure the masses of the physical model to increase theoretical accuracy and to use a higher quality camera, so the frames are clearer without motion blur, creating an accurate frame by frame depiction of the movement.

References

[1] Neumann, F. (2004). *Double pendulum*. myPhysicsLab Double Pendulum. Retrieved April 11, 2022, from https://www.myphysicslab.com/pendulum/double-pendulum-en.html

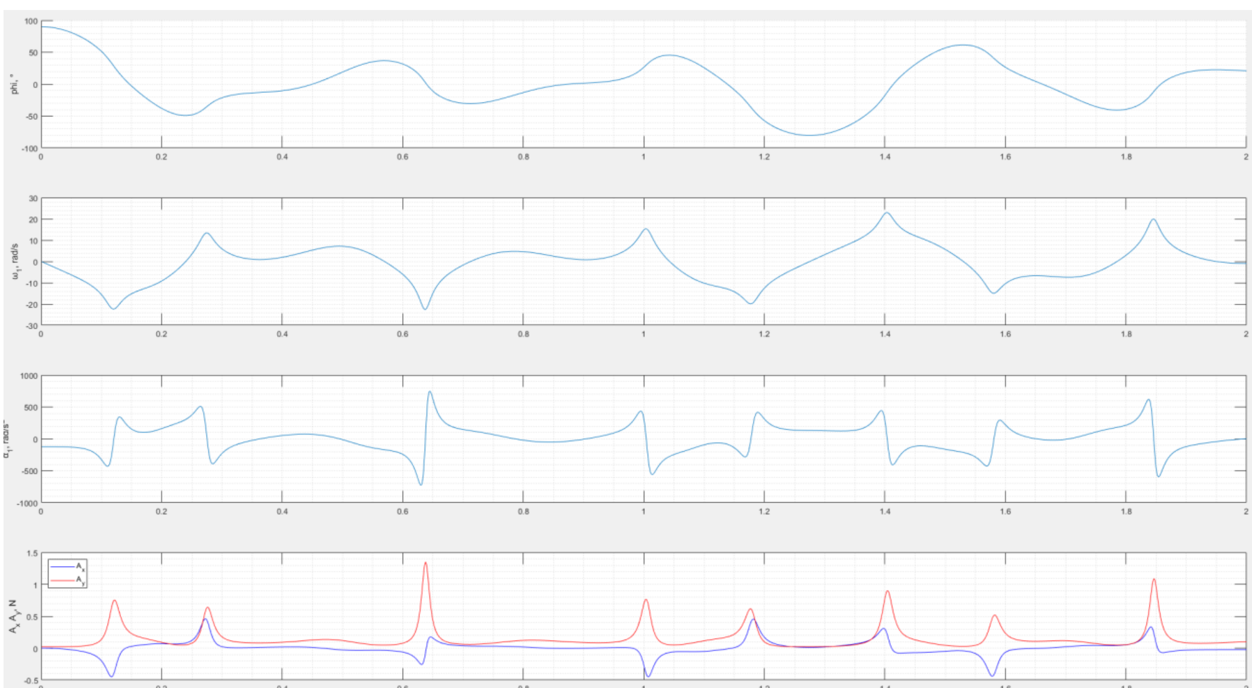
[2] Weisstein, E. W. (1996). *Double pendulum -- from Eric Weisstein's world of physics*. scienceworld.wolfram.com. Retrieved April 11, 2022, from https://scienceworld.wolfram.com/physics/DoublePendulum.html

Appendix

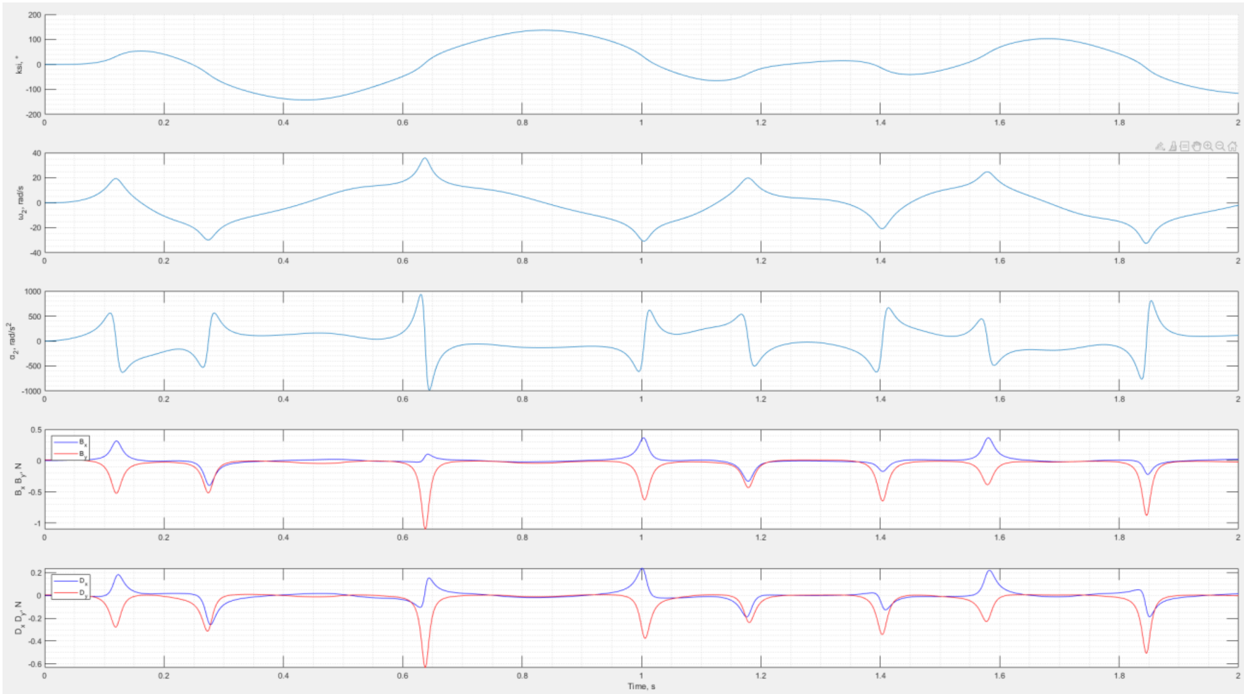


(a) (b)

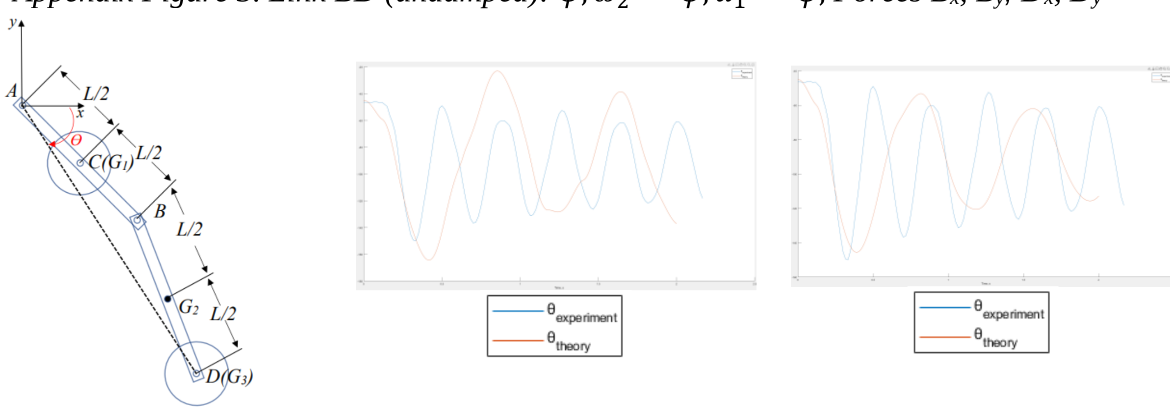
Appendix Figure 1. Mechanism Motion. (a) Matlab. (b) Graphical Interpretation



Appendix Figure 2. Link AB (undamped) vs Time: $\varphi, \omega_1 = \dot{\varphi}, \alpha_1 = \ddot{\varphi}$, Forces A_x and A_y



Appendix Figure 3. Link BD (undamped): $\psi, \omega_2 = \dot{\psi}, \alpha_1 = \ddot{\psi}$, Forces B_x, B_y, D_x, D_y



(a) *(b)* *(c)*

Appendix Figure 4. Angle Study. a – setup, b – undamped, c – dampened

MATLAB Code.

[Link to the Matlab Code.] Note: only CCNY accounts can access the file.