Mechanism Design Project 2: Crank-Piston Mechanism

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Abstract

The Geometry of the Crank-Piston mechanism that creates maximum displacement (ΔS) was determined theoretically using MATLAB. The design was also analyzed using Solidworks Motion analysis and a physical model was created to validate the design. The overall dimensions for the Crank-Piston design is an L2 length of 6LU and L3 of 8LU. This will result in a displacement (ΔS) of 13.8LU or 110.8mm. The piston also has a maximum velocity of 6.05 m/s and maximum acceleration of 5.74 m/s^2. Future improvements of this experiment are discussed and possible sources of errors are examined.

Introduction

A Crank-Piston Mechanism is designed to convert rotary motion to a linear motion, a common system used in combustion engines and piston pumps. The mechanism is composed of a circular crank which produces the rotary motion, connecting rod, and a slider/ piston at the opposing end. In most cases the slider's motion is restricted by a cylinder or of a similar cross sectional guide to control the y-directional and z-directional movement. In this design case there will not be any restriction on the movement of the piston other than the connecting rod and the ground surface in which it slides on. This report is divided into the following sections: (1) A theoretical analysis and background work to determine the maximum displacement (ΔS) of the piston, and calculations to determine the angular velocity and acceleration of all angles, (2) an explanation of the experimental apparatus and procedure used to investigate the motion of the Crank-Piston Mechanism, (3) the results which were collected throughout the experiment which include the MATLAB calculations for the maximum displacement (ΔS), velocity and acceleration, (4) a comparison of results between the theoretical calculations and the experimental calculations, and (5) a conclusion which discusses possible areas of improvements on the experiment.

Background Work

To obtain the maximum overall movement of the piston $\Delta S = P_{max} - P_{min}$, we need to obtain maximum and minimum positions of the crankshaft based on the constraints given:

$$L_2 \leq 3H_1,$$
$$P_{all} = 18\ LU,$$
$$where\ H_1 = 2\ LU, 1\ LU = 8\ mm$$

From Appendix Figure 3 b, c we can derive P_{max} and P_{min}:

$$P_{max} = \sqrt{(L_2 + L_3)^2 - H_1^2},$$

$$P_{min} = \sqrt{(L_3 - L_2)^2 - H_1^2}$$

To calculate velocities and accelerations we need to analyze the geometry of the piston (Figure 1, Appendix Figure 3 a):

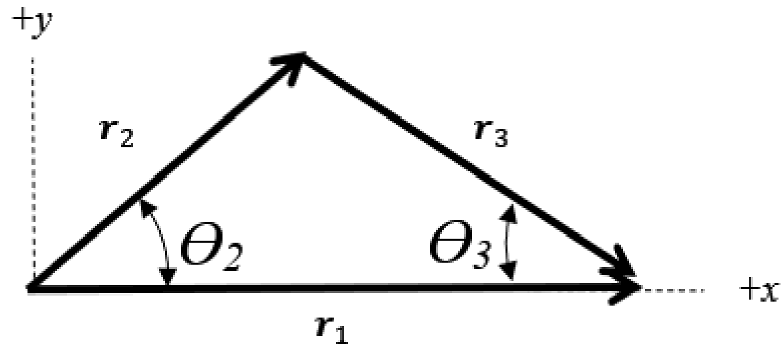

Figure 1. Vector Geometry Analysis.

$$\dot{\theta}_2 = \frac{d\theta_2}{dt} = \omega_2 = 1200\ RPM = \ 125.66\ \frac{rad}{sec} \rightarrow\ \alpha_2 = \frac{d\omega_2}{dt} = 0$$

$$\dot{\theta}_3 = \frac{d\theta_3}{dt} = \omega_3, \alpha_3 = \frac{d\omega_3}{dt}$$

$$\begin{Bmatrix} r_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} L_2 \cos\theta_2 + L_3 \cos\theta_3 \\ L_2 \sin\theta_2 + H_1\ - L_3 \cos\theta_3 \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \dot{r}_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -L_2\dot{\theta}_2 \sin\theta_2 - L_3\ \dot{\theta}_3 sin\ \theta_3 \\ L_2\dot{\theta}_2 \cos\theta_2 - L_3\ \dot{\theta}_3 cos\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} -L_2\omega_2 \sin\theta_2 - L_3\ \omega_3 sin\ \theta_3 \\ L_2\omega_2 \cos\theta_2 - L_3\ \omega_3 cos\ \theta_3 \end{Bmatrix} \tag{2}$$

$$\begin{aligned} \begin{Bmatrix} \ddot{r}_1 \\ 0 \end{Bmatrix} &= \begin{Bmatrix} \cancel{-L_2\ddot{\theta}_2 \sin\theta_2} - L_2\ \dot{\theta}_2 cos\ \theta_2 - L_3\ddot{\theta}_3 \sin\theta_3 - L_3\ \dot{\theta}_3 cos\ \theta_3 \\ \cancel{L_2\ddot{\theta}_2 \cos\theta_2} - L_2\ \dot{\theta}_2 sin\ \theta_2 - L_3\ddot{\theta}_3 \cos\theta_3 - L_3\ \dot{\theta}_3 sin\ \theta_3 \end{Bmatrix} \\ &= \begin{Bmatrix} -L_2\ \omega_2 cos\ \theta_2 - L_3\alpha_3 \sin\theta_3 - L_3\ \omega_3 cos\ \theta_3 \\ -L_2\ \omega_2 sin\ \theta_2 - L_3\alpha_3 \cos\theta_3 + L_3\ \omega_3 sin\ \theta_3 \end{Bmatrix} \end{aligned} \tag{3}$$

To obtain angular velocity and acceleration we need to have a relation between angles θ_2 and θ_3:

$$L_2 \sin\theta_2 + H_1\ - L_3 \cos\theta_3 = 0 \rightarrow\ \theta_3 = \arcsin\left(\frac{L_2 sin\theta_2 + H_1}{L_3}\right),$$

Then from (2), we can obtain angular velocity ω_3, and from (3) acceleration α_3:

$$L_2\omega_2 \cos\theta_2 - L_3\ \omega_3 cos\ \theta_3\ \rightarrow\ \omega_3 =\ \omega_2\ \frac{L_2 \cos\theta_2}{L_3\ cos\ \theta_3}$$

$$-L_2\ \omega_2 sin\ \theta_2 - L_3\alpha_3 \cos\theta_3 + L_3\ \omega_3 sin\ \theta_3 = 0 \rightarrow\ \alpha_3 = \frac{L_3\omega_3 \sin\theta_3 - L_2\ \omega_2 sin\ \theta_2}{L_3 \cos\theta_3}$$

Experimental Apparatus and Procedure

The experiment consisted of a 3D printed crank-piston setup that was dimensioned and designed on SOLIDWORKS. The setup consisted of a base plate with and elevated (H1) pin connection, a connecting rod L2 of length 42mm, a connecting rod of 64mm, a piston, and pins with radii of 2.5. All of these parts were printed separately and assembled to create the full functional model. The use of SOLIDWORKS assembly and animation helped in the design of the model because it allowed us to visualize a fully working model before constructing it. As shown below in Figure 2 the model with dimensions portray our final design .

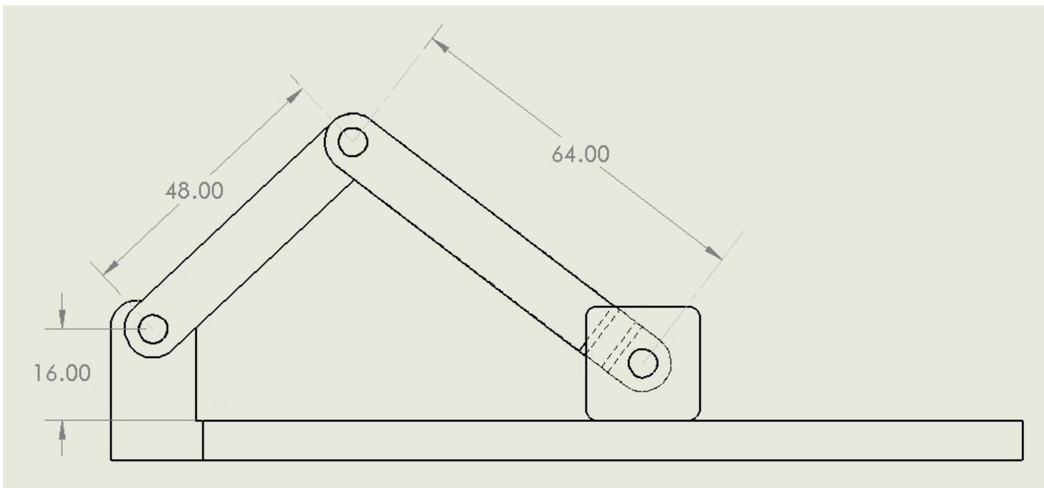

Figure 2. Mechanism Main Dimensions.

In order to validate our results we measured the displacement of the constructed model with a caliper shown in Figure 3 both in displacement max and displacement min position.

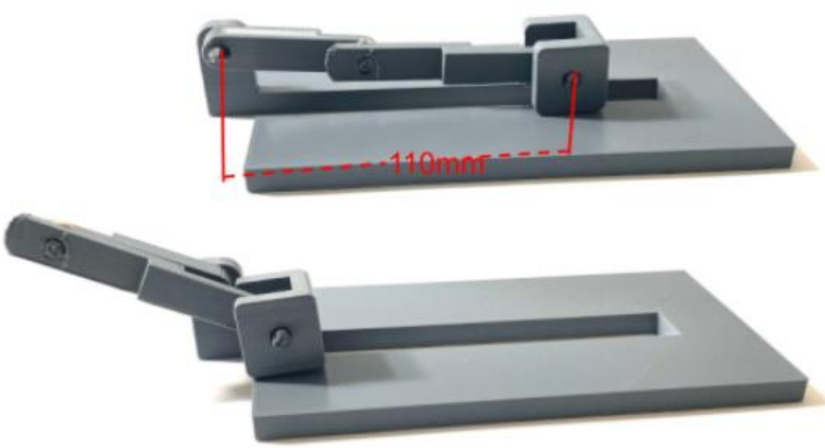

Figure 3. Mechanism Physical Assembly

Results

We use the iterative method (Appendix Matlab Code 1) to find the crank arm length and maximum movement of the piston:

$$L_2 = 6\ LU, L_3 = 8\ LU, \Delta\mathrm{S} = 13.86\ \mathrm{LU}, P_{max} = 13.86\ LU, P_{min} = 0\ LU,$$

Angular velocity and acceleration:

$$\omega_3 = 0.51\frac{rad}{sec} = 4.91\ RPM, \qquad \alpha_3 = 21.895\frac{rad}{sec}$$

Maximum Piston Velocity and Acceleration:

$$\dot{r}_{1\ max} = 6.05\frac{m}{s}, \ddot{r}_{1\ max} = 5.74\ \frac{m}{s^2}$$

Plots of Piston velocity and Piston acceleration vs crankshaft angle can be observed in Figure 4, and Figure 5 (code – Appendix Matlab Code 2)

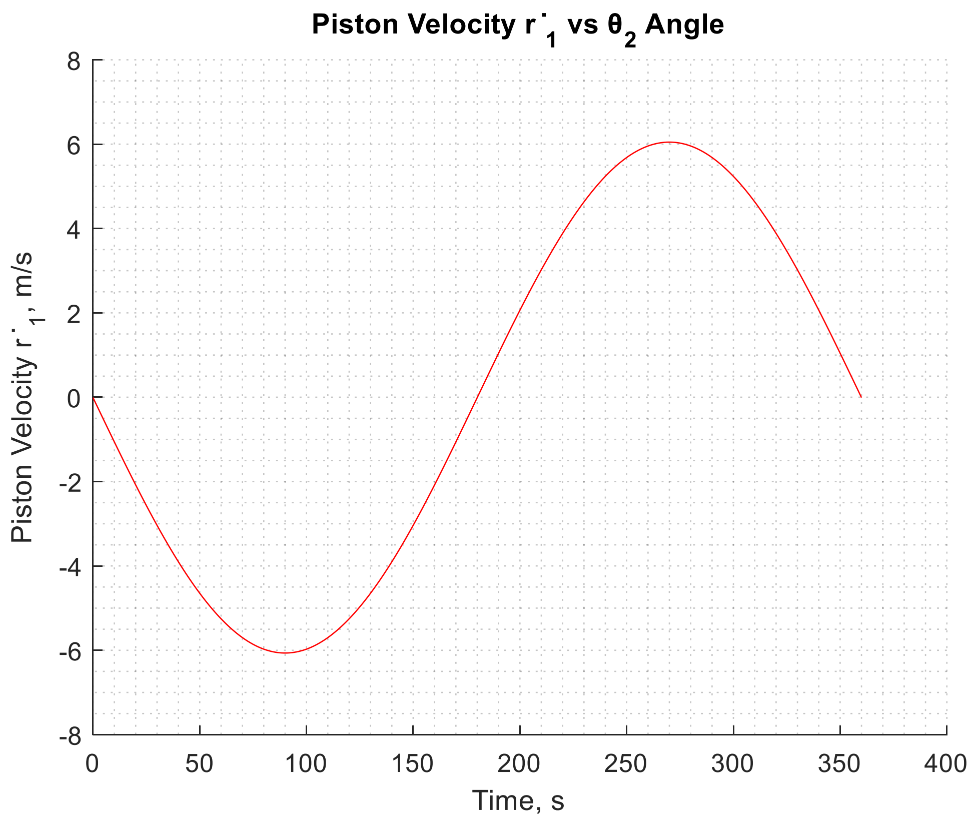

Figure 4. Plot Piston Velocity vs Angle of the Crank Shaft (Motor)

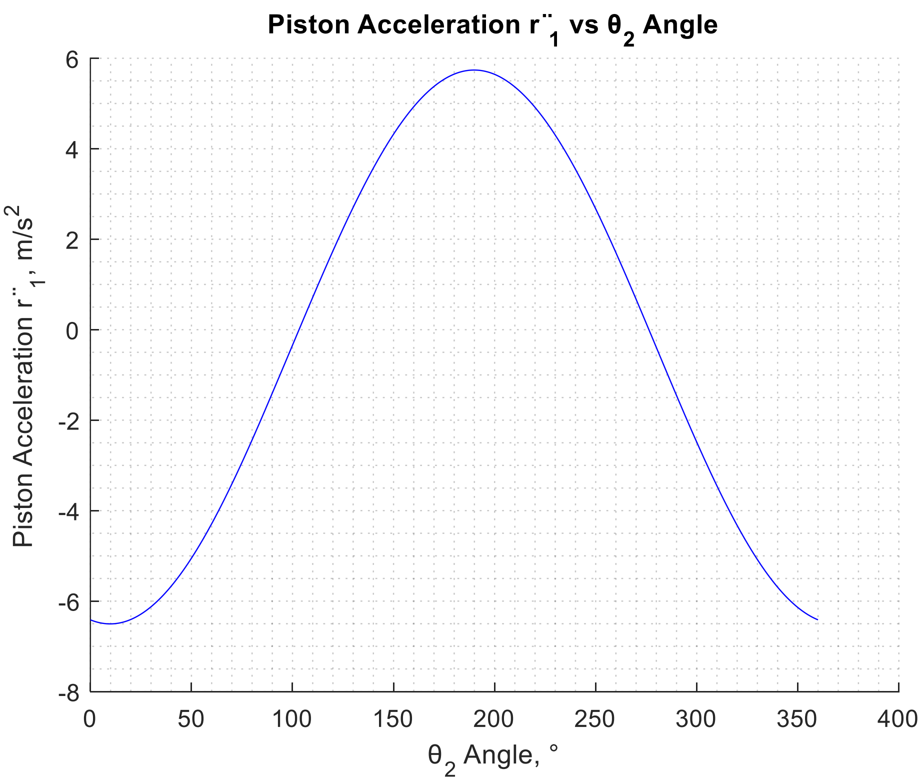

Figure 5. Plot Piston Acceleration vs Angle of the Crank Shaft (Motor)

Interpretation

In this configuration where the motor is located on the height H_1 from the piston height (ground), the acceleration is smoothed and no backlash is observed (Figure 5 vs Appendix Figure 2). We can see from the results that the dimensions of the mechanism is 6LU for L2 and 8LU for L3 connecting rod with a displacement of 13.86LU or 110.8mm . The analytical solution matched that of the physical model as shown in Figure 3. The piston also has a maximum velocity of 6.04m/s and acceleration of 5.74m/s^2. The mechanism's velocity and acceleration could not be validated experimentally due to the extent of the requirements. Some discrepancies arose between the analytical results and the physical dimensions because the design requirements did not specify dimensioning of piston P. There are also discrepancies in the physical model due to the final cutting and trimming in assembly which may be portrayed in Figure 5. Future analysis of this experiment should consider the exact design requirements and geometries of the piston and whether the two bases which hold both the pin A and piston P are connected or not.

Conclusion

A Crank piston design was created with dimensions L2, L3, and H1 to optimize the displacement (ΔS) of the piston. The dimensions of the mechanism is 6LU for L2 and 8LU for L3 connecting rod with a displacement of 13.86LU or 110.8mm. The design dimensions were validated by a physical model and with a CAD model. Both Piston velocity and acceleration were calculated and shown in graph format. Small discrepancies arose between the analytical solution and the physical model which has not affected the analytical solution of the design but rather the physical Contruction of it. A more accurate design can be created to provide space for the piston to reach minimum displacement without interfering with the pin at location A.

References

[1] Britannica, The Editors of Encyclopaedia. "slider-crank mechanism". Encyclopedia Britannica, 11 Apr. 2016, https://www.britannica.com/technology/slider-crank-mechanism. Accessed 14 March 2022.

Appendix

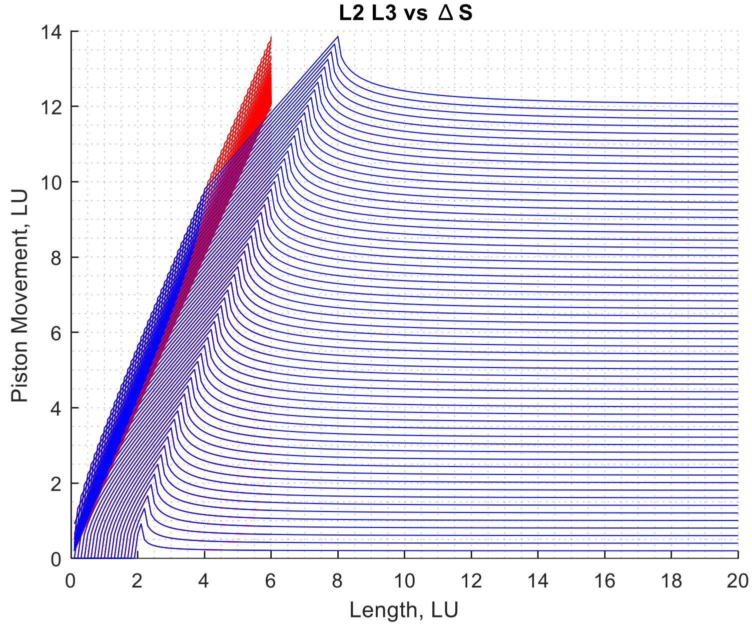

Figure 1. L_2 vs ΔS (red), L_3 vs ΔS (blue)

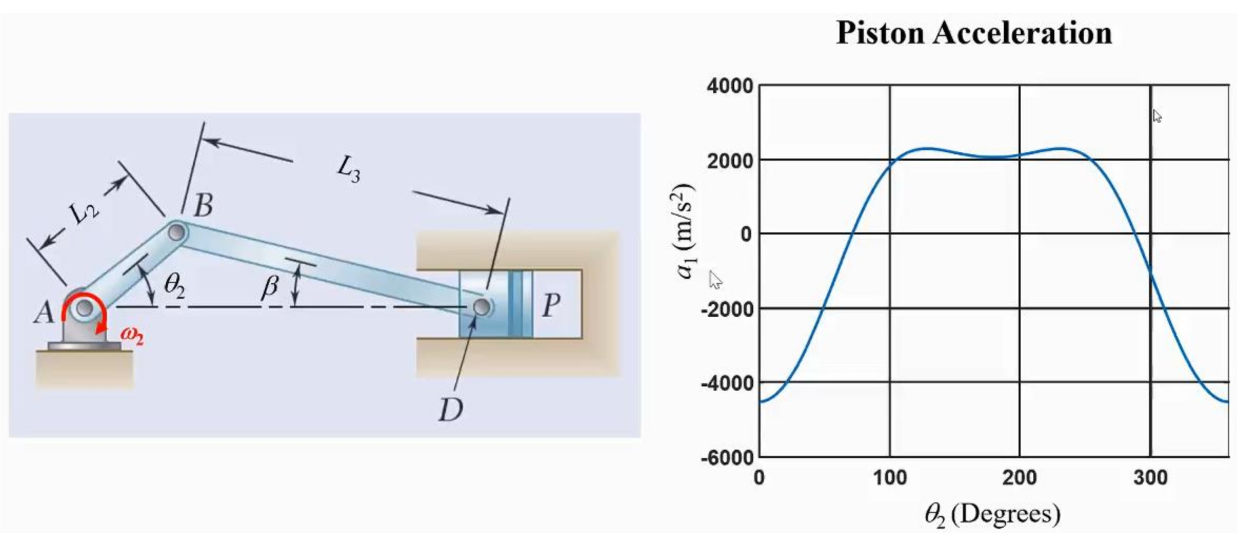

Figure 2. Same Level Piston Configuration

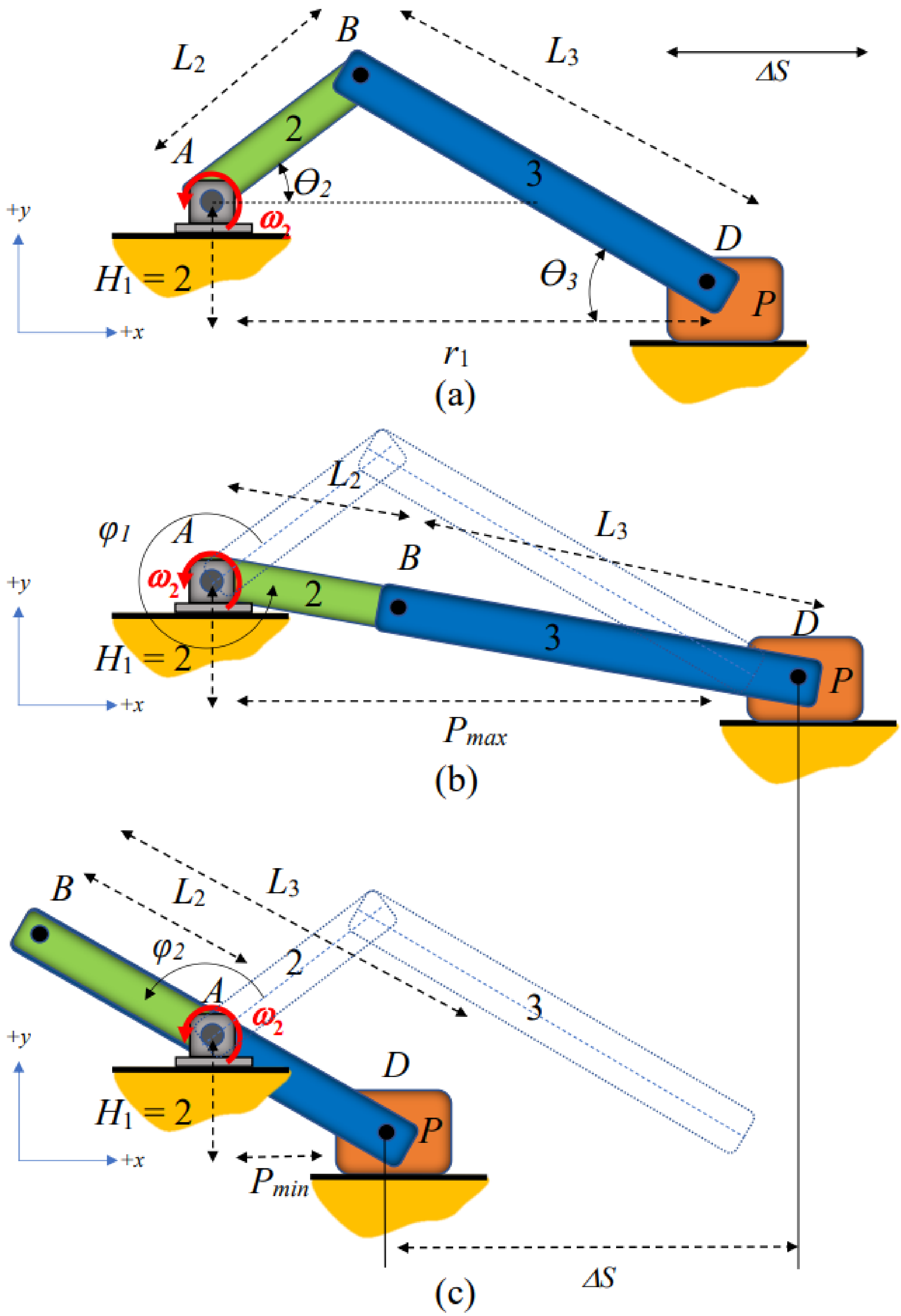

Figure 3. Original Problem (a); Maximum and Minimum Piston Position (b, c)

Matlab Code 1:

```
close all
L2=0.1:0.1:6;
L3=0.1:0.1:20;
Pmax=0;
delS=0;
i=1;
maxS=0;
indexi=0;
indexj=0;
for i=1:1:60
    for j = 1:1:200
        Pmax = sqrt((L2(i)+L3(j))^2-4);
        Pmin = sqrt((L3(j)-L3(i))^2-4);
        delS(i,j) = Pmax - Pmin;
        if delS(i,j)>maxS
            maxS = delS(i,j);
            indexi=i;
            indexj=j;
        end
    end

end
maxS
Pmax = sqrt((L2(indexi)+L3(indexj))^2-4)
Pmin = sqrt((-L2(indexi)+L3(indexj))^2-4)
L2(indexi)
L3(indexj)

%L2 vs delS L3 vs delS
figure
title('L2 L3 vs ?S')
xlabel('Length, LU')
ylabel('Piston Movement, LU')
grid minor
hold on
plot (L2, delS, "color", "red",'DisplayName', 'L2')
plot (L3, delS, "color", "blue",'DisplayName', 'L3')
%legend%('Location','northwest')
```

Matlab Code 2:

```
close all
L2=6;
L3=8;
H1=2;
Omega2=1200*0.10472
Theta2=(linspace(0,pi*2,360));

sinTh3 = (L2.*sin(Theta2)+H1)/L3
cosTh3 = sqrt(1-((L2.*sin(Theta2)+H1)/L3).^2);
Omega3 = (L2*Omega2.*cos(Theta2))/(L2.*cosTh3)
Omega3/0.10472
alpha3 = (L3*Omega3.*sinTh3-L2*Omega2.*sin(Theta2))/(L3*cosTh3)
```

```
%distance
r = L2.*cos(Theta2)+L3.*cosTh3;
max(r)

%Velocity
r_dot=-L2*Omega2.*sin(Theta2)-L3*Omega3.*sinTh3;
max(r_dot)*8/1000

%Acceleration
r_d_dot = -L2*Omega2.*cos(Theta2)-L3*alpha3.*sinTh3-L3*Omega3.*cosTh3;
max(r_d_dot)*8/1000

% Distance vs angle
figure
title('Piston Distance r_1 vs ?_2 Angle')
xlabel('Time, s')
ylabel('Piston Distance r_1, m')
grid minor
hold on
plot (Theta2*180/pi,r*8/1000, "color", "green",'DisplayName', 'm/sec')

% Vel vs angle
figure
title('Piston Velocity r ?_1 vs ?_2 Angle')
xlabel('Time, s')
ylabel('Piston Velocity r ?_1, m/s')
grid minor
hold on
plot (Theta2*180/pi,r_dot*8/1000, "color", "red",'DisplayName', 'm/sec')

%Accel vs angle
figure
title('Piston Acceleration r ?_1 vs ?_2 Angle')
xlabel('?_2 Angle, °')
ylabel('Piston Acceleration r ?_1, m/s^2')
grid minor
hold on
plot (Theta2*180/pi,r_d_dot*8/1000, "color", "blue",'DisplayName',
'mm/sec^2')

%legend%('Location','northwest')
```