EMANUEL NICOLAS [illegible]

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Práctico 1

1 a) $AB = \{(1,2,3,4),(1,2,4,3),(3,2,1,4),(4,2,1,3)\} \Rightarrow \#AB = 4$

b) $\#(A \cup B) = \#A + \#B - \#AB = 3.3! + 3! - 4 = 20$

c) $\#ABC = 1$ ya que $ABC = \{(1,2,3,4)\}$

d) $\#(A \cup (BC)) = \#A + \#BC - \#ABC = 3.3! + 2! - 1 = 19$

2 a) $\boxed{A \cup B = A \cup (A^c B)}$

$$A \cup (A^c B) \overset{\text{distr.}}{=} (A \cup A^c)(A \cup B) \overset{\text{compl.}}{=} S(A \cup B) \overset{\text{intersec. univ.}}{=} A \cup B \blacksquare$$

$\boxed{A = (AB) \cup (AB^c)}$

$$(AB) \cup (AB^c) \overset{\text{distr.}}{=} A \cup (BB^c) \overset{\text{compl.}}{=} A \cup \varnothing \overset{\text{intersec. vacío}}{=} A \blacksquare$$

b) $P(A \cup B) = P(A) + P(B) - P(AB)$

* $A \cup B = A \cup (A^c B) \Rightarrow$ Como A, $A^c B$ son excluyentes, $P(A \cup B) = P(A) + P(A^c B)$ (1)

* $B = (AB) \cup (A^c B) \Rightarrow$ Como AB, $A^c B$ son excluyentes, $P(B) = P(AB) + P(A^c B)$. Luego, es lo mismo que $P(A^c B) = P(B) - P(AB)$

Por (1) y (2), $P(A \cup B) = P(A) + P(A^c B) = P(A) + P(B) - P(AB) \blacksquare$

3 a) $\dfrac{\binom{9}{2}}{\binom{16}{2}} = 0{,}3$ b) $\dfrac{9}{16} \cdot \dfrac{7}{15} = 0{,}2625$

④ a) $P(R_A) = \frac{6}{10} = 0{,}6$ $\qquad P(V_A) = \frac{4}{10} = 0{,}4$

b) $P(R_B|R_A) = \cancel{\quad} = \frac{8}{11} = 0{,}73$ $\qquad P(R_B|V_A) = \frac{7}{11} = 0{,}64$

c) $P(R_B R_A) = P(R_B|R_A)\,P(R_A) = \frac{8}{11} \cdot \frac{6}{10} = 0{,}44$

d) $P(R_B) = P(R_B R_A) + \underbrace{P(R_B V_A)} = 0{,}7$

$$P(R_B V_A) = P(R_B|V_A)\,P(V_A) = \frac{7}{11} \cdot \frac{4}{10} = 0{,}26$$

e) $P(R_B R_A) + \underbrace{P(V_B V_A)} = 0{,}59$

$$P(V_B V_A) = P(V_B|V_A)\,P(V_A) = \frac{4}{11} \cdot \frac{4}{10} = 0{,}15$$

⑤ a) Por prop., $1 = \sum_{i=1}^{4} P_i = \sum_{i=1}^{4} c \cdot i = c \cdot \sum_{i=1}^{4} i \overset{\text{Gauss}}{=} c \cdot \frac{4.5}{2} = 10c \implies c = \frac{1}{10}$

b) $P(2 \leq X \leq 3) = P_2 + P_3 = \frac{1}{10} \cdot 2 + \frac{1}{10} \cdot 3 = \frac{1}{2} = 0{,}5$

c) $E[X] \overset{\text{def.}}{=} \sum_{i=1}^{4} i \cdot P_i = \sum_{i=1}^{4} c \cdot i^2 = \frac{1}{10} \cdot \sum_{i=1}^{4} i^2 \overset{\text{prop.}}{=} \frac{1}{10} \cdot \frac{4.5.9}{6} = 3$

⑥ $Var[aX+b] \overset{\text{def.}}{=} E[(aX+b)^2] - E[aX+b]^2 = E[a^2X^2 + 2abX + b^2] - E[aX+b]^2 \overset{\text{linealidad}}{=}$

$= a^2 E[X^2] + 2abE[X] + b^2 - (aE[X]+b)^2 = a^2E[X^2] + 2abE[X] + b^2 - a^2E[X]^2 - 2abE[X] - b^2 =$

$= a^2 E[X^2] - a^2 E[X] = a^2 (E[X^2] - E[X]) \overset{\text{def.}}{=} a^2 . Var[X]$ ■

⑦
$$\begin{cases} P_0 = e^{-\lambda} \\ P_{i+1} = \frac{\lambda}{i+1} P_i \quad \forall i \geq 0 \end{cases}$$

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⑧ Como X,Y son independientes, Z cumple que $p_Z(a)=p_X * p_Y(a)$. Luego, por def. de convolución y expandiendo, tenemos que:

$$p_Z(a)=p_X * p_Y(a) \overset{\text{def}}{=} \sum_x p_X(x)p_Y(a-x) \overset{\text{def. Poisson}}{=} \sum_{x=0}^{a} e^{-\lambda_1}\cdot\frac{\lambda_1^x}{x!}\cdot e^{-\lambda_2}\cdot\frac{\lambda_2^{a-x}}{(a-x)!} = \frac{e^{-(\lambda_1+\lambda_2)}}{a!}\cdot\sum_{x=0}^{a}\frac{\lambda_1^x\lambda_2^{a-x}\,a!}{x!(a-x)!}$$

$$\overset{\text{def. comb.}}{=} \frac{e^{-(\lambda_1+\lambda_2)}}{a!}\cdot\sum_{x=0}^{a}\binom{a}{x}\lambda_1^x\lambda_2^{a-x} \overset{\text{binom. Newton}}{=} e^{-(\lambda_1+\lambda_2)}\cdot\frac{(\lambda_1+\lambda_2)^a}{a!}$$

Luego, como se cumple $\forall a \geq 0$, tenemos que entonces $Z = X+Y \sim P(\lambda_1+\lambda_2)$ ■

⑨ Sea X la v.a. que lo representa, $X \sim H(15,10,12)$ por lo que

$$p(i) = \begin{cases} \dfrac{\binom{10}{i}\binom{12}{15-i}}{\binom{22}{15}} & \text{si } 3 \leq i \leq 15 \\ 0 & \text{c.c.} \end{cases}$$

⑩ Si $X \sim P(\lambda)$ entonces:

- $\boxed{E[X]=\lambda}$ (1)

$$E[X] \overset{\text{def.}}{=} \sum_{x=0}^{\infty} x\,p(x) \overset{\text{1er val. nulo}}{=} \sum_{x=1}^{\infty} x\,p(x) \overset{\text{def. prob. de Poisson}}{=} \sum_{x=1}^{\infty} x\cdot e^{-\lambda}\frac{\lambda^x}{x!} = \lambda\cdot\sum_{x=1}^{\infty} e^{-\lambda}\cdot\frac{\lambda^{x-1}}{(x-1)!} \overset{\text{Cambio var.}}{=} \lambda\cdot\sum_{y=0}^{\infty} e^{-\lambda}\frac{\lambda^y}{y!} \overset{\text{Por prop. probab.}}{=} \lambda \ \blacksquare$$

- $\boxed{Var[X]=\lambda}$

$$* \ Var[X] \overset{\text{def.}}{=} E[X^2]-E[X]^2 \overset{\text{ayuda}}{=} E[X(X-1)]+E[X]-E[X]^2 \overset{(1)}{=} E[X(X-1)]+\lambda-\lambda^2 \quad (2)$$

$$* \ E[X(X-1)] \overset{\text{def.}}{=} \sum_{x=0}^{\infty} x(x-1)\,e^{-\lambda}\frac{\lambda^x}{x!} \overset{\text{vals. nulos}}{=} \sum_{x=2}^{\infty} x(x-1)e^{-\lambda}\cdot\frac{\lambda^x}{x!} = \lambda^2\sum_{x=2}^{\infty} e^{-\lambda}\frac{\lambda^{x-2}}{(x-2)!} \overset{\text{Cambio var.}}{=}$$

$$= \lambda^2\cdot\sum_{y=0}^{\infty} e^{-\lambda}\frac{\lambda^y}{y!} \overset{\text{prop. probab.}}{=} \lambda^2 \quad (3)$$

$*$ Por (2) y (3), $Var[X] = \lambda^2+\lambda-\lambda^2 = \lambda$ ■

Escaneado con CamScanner

(11) (a) $F_{X|Y}(x|y) \overset{def.}{=} \dfrac{F_{X,Y}(x,y)}{F_Y(y)} \overset{X,Y\ ind.}{=} \dfrac{F_X(x)\,F_Y(y)}{F_Y(y)} = F_X(x) \overset{def}{=} \lambda e^{-\lambda x}\ ,\ x>0$

(b) $P(X<Y) \overset{def.}{=} \displaystyle\int_0^{\infty}\int_0^{y} F_{X,Y}(x,y)\,dx\,dy \overset{X,Y\ ind.}{=} \int_0^{\infty}\int_0^{y} F_X(x)F_Y(y)\,dx\,dy = \int_0^{\infty} F_Y(y)\int_0^{y} F_X(x)\,dx\,dy \overset{def.}{=}$

$$= \int_0^{\infty} \mu e^{-\mu y}\int_0^{y} \lambda e^{-\lambda x}\,dx\,dy = \int_0^{\infty} -\mu e^{-\mu y}\int_0^{y} -\lambda e^{-\lambda x}\,dx\,dy = \int_0^{\infty} -\mu e^{-\mu y}\cdot\left[e^{-\lambda x}\right]_0^{y} =$$

$$= \int_0^{\infty} -\mu e^{-\mu y}\left(e^{-\lambda y}-1\right)dy = \int_0 \mu e^{-\mu y}\,dy - \int_0^{\infty} \mu e^{-(\mu+\lambda)y}\,dy = -\int_0 -\mu e^{-\mu y}\,dy +$$

$$+ \frac{\mu}{\mu+\lambda}\cdot\int_0^{\infty} -(\mu+\lambda)e^{-(\mu+\lambda)y}\,dy = -\left[e^{-\mu y}\right]_0^{\infty} + \frac{\mu}{\mu+\lambda}\cdot\left[e^{-(\mu+\lambda)y}\right]_0^{\infty} = -(-1) + \frac{\mu}{\mu+\lambda}\cdot(-1) =$$

$$= 1 - \frac{\mu}{\mu+\lambda}.$$

Luego, entonces, $P(X<Y) = 1 - \dfrac{\mu}{\mu+\lambda}$.

(12) $* \ F_{X|X+Y}(x|t) \overset{def.}{=} \dfrac{F_{X,X+Y}(x,t)}{F_{X+Y}(t)}$ (1)

$* \ F_{X,X+Y}(x,t) = F_{X,Y}(x,t-x) \overset{X,Y\ ind.}{=} F_X(x)F_Y(t-x) \overset{def.}{=} \lambda e^{-\lambda x}\,\mathbb{I}_{(0,\infty)}(x)\,\lambda e^{-\lambda(t-x)}\,\mathbb{I}_{(0,\infty)}(t-x) =$

$= \lambda^2 e^{-\lambda t}\cdot\mathbb{I}_{(0,t)}(x)$ (2)

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$$* f_{X+Y}(t) \overset{X,Y \text{ ind.}}{=} f_X * f_Y(t) \overset{\text{def. conv.}}{=} \int_0^t f_X(r) f_Y(t-r)\, dr \overset{\text{def.}}{=} \int_0^t \lambda e^{-\lambda r} \lambda e^{-\lambda(t-r)}\, dr =$$

$$= \int_0^t \lambda^2 e^{-\lambda t}\, dr = \lambda^2 e^{-\lambda t} \int_0^t dr = \lambda^2 e^{-\lambda t} \cdot [r]_0^t = \lambda^2 \cdot e^{-\lambda t} \cdot t \cdot \mathbb{I}_{(0,\infty)}(t) \quad (3)$$

Luego, por (1), (2) y (3) llegamos a que:

$$f_{X|X+Y}(x|t) = \frac{\lambda^2 e^{-\lambda t} \cdot \mathbb{I}_{(0,t)}(x)}{\lambda^2 e^{-\lambda t} \cdot t \cdot \mathbb{I}_{(0,\infty)}(t)} = \frac{1}{t} \cdot \mathbb{I}_{(0,t)}(x)$$

(13) Sea $X \sim N(4{,}8\ ,\ 1{,}4^2)$ tenemos:

(a) $P(X \leq 2) \overset{\text{prop.}}{=} \Phi\left(\frac{2-4{,}8}{1{,}4}\right) = \Phi(-2) \overset{\text{prop.}}{=} 1 - \Phi(2) \overset{\text{tabla}}{\approx} 1 - 0{,}9772 = 0{,}0228$

(b) Buscamos k tq. $0{,}05 = P(X \leq k) = \Phi\left(\frac{k-4{,}8}{1{,}4}\right) \Rightarrow \frac{k-4{,}8}{1{,}4} \overset{\text{def.}}{=} z_{0,95} \overset{\text{prop.}}{=} -z_{0,05} \overset{\text{tabla}}{\approx}$

$\approx -1{,}645 \Rightarrow k \approx 2{,}497.$

(14) Sea $X \sim B(12000, 1/6)$, buscamos $\sum_{x=1900}^{2150} p(x)$. Como el n es grande, por TCL

$X \sim N\left(\frac{12000}{6}, 12000 \cdot \frac{5}{6^2}\right) \sim N(2000, (40{,}82)^2)$. Luego:

$$P(1900 \leq X \leq 2150) \overset{\text{prop.}}{=} \Phi\left(\frac{2150-2000}{40{,}82}\right) - \Phi\left(\frac{1900-2000}{40{,}82}\right) \approx \Phi(3{,}67) - \Phi(-2{,}45) \overset{\text{prop.}}{=} \Phi(3{,}67) - 1 + \Phi(2{,}45) \approx$$

$\overset{\text{tabla}}{\approx} 1 - 1 + 0{,}9929 = 0{,}9929.$

(15) Tenemos:

$$p_{G(c)}(x) = \begin{cases} 1/100 & \text{si } x = 70 - c \\ 99/100 & \text{si } x = -c \\ 0 & \text{c.c.} \end{cases}$$

Luego:

Escaneado con CamScanner

ⓐ $E[G(1)] \overset{\text{def.}}{=} 69 \cdot \frac{1}{100} + (-1) \cdot \frac{99}{100} = -0{,}3$

ⓒ $E[G(c)] = 0 \iff 0 = E[G(c)] \overset{\text{def.}}{=} (70-c) \cdot \frac{1}{100} + (-c) \cdot \frac{99}{100} = \frac{70}{100} - c \iff c = 0{,}7.$

ⓑ Sea X la v.a. que representa la ganancia en los 60 días con $c=1$, como $E[G(1)] = -0{,}3$ y $Var[G(1)] \overset{\text{def.}}{=} E[G(1)^2] - E[G(1)] \overset{\text{def.}}{=} 69^2 \cdot \frac{1}{100} + (-1)^2 \cdot \frac{99}{100} - (-0{,}3)^2 = 48{,}51$, por el TCL tenemos que $X \sim N(60 \cdot (-0{,}3), 60 \cdot 48{,}51) \sim N(-18, 54^2)$.

Luego, [redacted] $= P(X \leq -15) \overset{\text{prop.}}{=} \Phi\left(\frac{-15+18}{54}\right) \approx \Phi(0{,}06) \underset{\text{tabla}}{\approx} 0{,}5239.$