Cryptocurrency Market, A Bubble? - A Unit Root Approach

Bachelor's Thesis submitted

to

Examiners

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Abstract

The behaviour of the cryptocurrency market has often been characterized as a bubble in a vast number of publications. Due to the lack of the fundamental value, or rather the intrinsic value of cryptocurrencies, one can only apply several econometric methods to test for a bubble-like behaviour with accordance to the price development. This paper makes use of a recently developed econometric bubble method that relies on a unit root approach. Furthermore, the growing number of studies on whether a contagion effect by a specific cryptocurrency lays in another coin, is taken as a motivation in this study for the use of a bivariate Granger causality test. An empirical application uses six cryptocurrency coins in which a general evidence of a bubble is confirmed. It further concludes possible patterns to give a clue of such contagion effects. The findings of this paper could motivate further studies of testing bubbles in the cryptocurrency market with an augmentation of contagion market effects.

Keywords: cryptocurrencies, unit root tests, bubbles, Granger causality

The R code used in this paper can be found on Github [1]

[1] `https://github.com/heldaron/thesis18`

Contents

List of Abbreviations

crypto	Cryptocurrency
USD	US Dollar
BTC	Bitcoin
ETH	Ethereum
XRP	Ripple
LTC	Litecoin
DASH	Dashcoin
XLM	Stellar
IPO	initial public offering
ICO	initial coin offering
EMH	efficient market hypothesis
CV	critical value
MC	Monte Carlo
OLS	ordinary least squares
$log()$	natural logarithm
KPSS	Kwiatkowski Phillips Schmidt Shin
DF	Dickey-Fuller
ADF	augmented Dickey-Fuller
SADF	supremum augmented Dickey-Fuller
BADF	Backwards Augmented Dickey-Fuller
PWY	Phillips Wu Yu
AR	autoregressive
VAR	vector autoregression
ACF	autocorrelation function
iid	independent and identically distributed
$\mathcal{O}$	big O notation
$URSS$	residual sum of squares (unrestricted model)
$RRSS$	residual sum of squares (restricted model)

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1 Introduction

The attention by public media about cryptocurrencies has tremendously increased in recent time (Li et al., 2018). For instance, Bitcoin, as the biggest cryptocurrency and pioneer in crypto-coins, has experienced the recent major increase (by far the highest) of value from 917.17 USD in January 2017 to almost 20,000 USD (exact 19,205.11 USD) by December 18th 2017. This was an increase by over 2000%. However, shortly after the huge increase, Bitcoin experienced a drop again below 6,000 USD, or in other words around 70% loss of value. This drop happened in a time of only around a half year (peak December 18, 2017 to the low from June 26, 2018). This spectacular market behaviour is not rare, as in the year 2013 the market rose about 700%. This was the first big rise which was noted for Bitcoin.

After the recent explosive growth of the leading cryptocurrency, Bitcoin, the question between a currency and a financial product is still not clarified. (Jabotinsky, 2018)

Many debates about cryptocurrency‘s, in particular Bitcoins, intrinsic or fundamental value came up. The comparison by Gronwald (2014) between gold and Bitcoin, concludes some similarities with their complexity of mining them and the total supply (both limited). However, the problem still with cryptocurrencies is the lack of clearness by public media about its fundamental value (intrinsic value) by its lack of use for physical applications. Whereas, some people define the fundamental value of cryptocurrencies by their electricity costs to mine the cryptocurrencies, others by users of the particular crypto network.Cheah and Fry (2015) discussed in their paper that the fundamental price of Bitcoin is even by zero.

The question of whether cryptocurrencies are driven by speculative bubbles has been extensively debated in public media such as by Cheah and Fry (2015), Bianchetti et al. (2018), Urquhart (2016), and Hafner (2018).Urquhart (2016) devoted himself to Bitcoin and its inefficiency. He claims that it is used as an investment, or rather, as a speculative vehicle more than for its main purpose of being used as a currency, especially with the enormous increase of attention by public media and the increase of market capitalization within a short period of time (5.5 billion USD to 605.8 billion USD) of three years (2015-2017).

Our study examined the question of whether a bubble is present in the crypto market. Due to the lack of consensus about the fundamental value of cryptos, we have applied in our study several econometric tests to show the presence of a bubble within the crypto market. We have followed a unit root approach in accordance with Kwiatkowski et al. (1992), Diba and Grossman (1988) and Phillips et al. (2015). We showed the presence of unit roots within our set of cryptos and further examined the presence of a bubble we were able to date-stamp. Our study also provides an examination of whether a crypto in our set of cryptos show a contagion effect by a specific crypto. We found out that with reference to a bivariate Granger causality test, Ripple (XRP), gives a clue of such an effect on Bitcoin (BTC) as well as on Stellar (XLM) and Litecoin (LTC).

The paper is organized as follows. The next section provides some background information about cryptos and defines what a bubble is with reference to historical examples. Section 3 forms an overall terminology of time-series and explains what a unit root is. In addition, this section forms a hypothesis on how to define possible driver within our set of cryptos. The next section (4) describes the set of cryptos used in our study. Section 5 explains all our used methods and the intention based on our overall hypothesis. In Section 6 we will apply the tests on our set of cryptos and, finally, Section 7 concludes our findings and discusses them for further study.

2 Background

2.1 Cryptocurrency Market

The word ‘crypto’ is Greek and stands for “hidden, secret” (Liddell and Scott, 1996). According to the origin and meaning of the word, ‘cryptography’ is the practice of encrypting information. This practice is used within cryptocurrencies. In the following we will abbreviate the word “cryptocurrency” with “crypto”.

Cryptos are digital assets with the intention of being used as a medium of exchange (digital currency). They use cryptography to secure their transactions, control the creation of additional crypto-coins, and thus to secure the transfer of assets. They were founded on the incentive of being decentralized and global. This is a major difference with respect to the characteristics of classical monetary assets (fiat money). Whereas fiat money is always backed by a central government (e.g. the Federal Reserve or the European Central Bank), cryptos, by contrast, are founded on the principle of not being backed by any central government, institution or bank (GAO, 2013).

The architecture of the crypto-coins are provided by a blockchain. Nakamoto explains in his paper that Bitcoin (BTC) is used as a peer-to-peer technology for so-named “digital cash” (Nakamoto, 2009). A blockchain is more or less a digital book in which each transaction is secured from inception. Each transaction contains a time-stamp plus the transaction data in a “digital block”. This generated block is then linked to its previous block (previous transaction within the network). The special thing, compared to fiat money transactions through a bank account, is that the blockchain is open for the public containing all transactions. So, every transaction can be inspected. In this digital cash network, the address of the transaction data is linked to a unique wallet address. However, it does not necessarily have to be linkable to an individual person (containing their personal information). So, if person A pays person B an amount of ω then only the wallet address is stored in the block without knowing which individual person is behind it. Basically, the network communicates via address keys that are linked to a person’s own digital wallet that stores all the crypto-coins.

Nevertheless, every transaction needs to be validated by the network so that, for instance, sender A has enough coins in their wallet to send B the requested amount. This process is called “mining”. Every user can participate and receives for the effort of going through the validation process, a reward of new crypto-coins. This has another positive side effect as it reduces transaction fees by creating this kind of incentive for the users to contribute to the network.

Among many positive aspects with respect to the technology and the fact of the decentralization of cryptos, there arose many concerns by governments about whether they should be regulated, and if so, how. Some countries, for instance, Russia or China, have already prohibited the use of initial coin offering (ICO) (Jabotinsky, 2018). ICO is more or less a method to call for raising capital through cryptos in exchange for a "token" created by the fundraisers for their projects that is exclusively sold on the internet for which one can obtain their services and/or products. In addition, over time one can participate in the growth of the project by an increasing value of the "token" (Belleflamme et al., 2014). In short, ICO refers to the inception of a crypto and is comparable to an IPO in stocks. When a company is selling its stocks the first time publicly to interested investors, this is called an "initial public offering" (IPO) (inception of a company's stocks on a stock exchange).

In October 2008, the worlds leading first digital currency was launched with the name "Bitcoin". A pseudonymously named Satoshi Nakamoto introduced it. Beginning in 2009, the Bitcoin software was being released as an open source code to the public on Sourceforge (Cap, 2012).

From that point on, public attention grew and, thus, attracted a wave of people with a touch to technology to devote themselves to this new topic. They tried to create alternative cryptos based on the same fundamental technology, but with a specialized aim. (ElBahrawy et al., 2017)

With regard to this fact, currently there are more than 1700 cryptos. However, in accordance with the market cap, the top ten cryptos (BTC, ETH, XRP, Bitcoin Cash, EOS, XLM, LTC, Cardano, IOTA, Tether) control about 84% of the total crypto market cap. What is more, the top three cryptos (BTC, ETH and XRP) control 195.5 billion USD of market cap, which is around 70% (Coinmarketcap.com, 2018).

To mention some specialized purposes, we now explain briefly ETH and XRP of the top three cryptos (in terms of their market cap). BTC was already explained earlier. ETH (Ethereum), as the second leading crypto, is a decentralized platform which uses smart contracts (Ethereum.org, 2018). The reason for being in the top three cryptos lays in the technology. With respect to the characteristics of the blockchain technology, smart contracts make it possible to close contracts independent of third parties, e.g. notaries, but with guaranteed legal certainty. Additionally, with the use of smart contracts, it is possible to take contracts automatically into effect under certain conditions (Bhargavan et al., 2016). The third biggest crypto XRP (Ripple) is, according to Ripple.com (2018) , specialized as a digital asset for payments all around the world. With their well known RippleNet, they are able to lower transaction costs and speed. Ripple works with an open-source technology, built on the principles of blockchain to allow the traceability of transactions. Currently, banks and payment providers are the target group, in contrast to ETH or BTC.

2.2 Financial Bubble

In bubble regimes assets experience explosive movements (Greenspan, 1996). Two types of bubbles can be differentiated: Rational and irrational (behavioral) bubbles. Rational bubbles are mainly driven by speculations that move the prices sharply without any proportional link to its intrinsic value. Irrational bubbles are, in most cases, linked to any behavioral aspect. Some of the behavioral aspects that move the asset sharply include herd instincts or any other psychological factors (Phillips et al., 2011).

Many researchers have already contributed to debates about the existence of financial bubbles and whether the bubbles were rational or irrational (behavioral). Some of the examples include Greenspan (1996), Shiller (2009), and so forth.

However, Greenspan (1996), with the phrase "irrational exuberance" on December 5, 1996, characterized a herding asset price behavior, and this explanation became the most-quoted one in terms of describing a bubble regime.

By far the most famous and first recorded irrational bubble in history is the Dutch Tulip Mania (1636-1637). Over a period of one year tulip prices skyrocketed in the Netherlands to a price where a tulip bulb was equivalent to that of a three-story town house (Singh and Zammit, 2010). Another example is the South Sea bubble in 1719-1721. Dale et al. (2005) explain the bubble as one of the largest stock scams in history by the plan to privatize the national debt of England. The shares of the company that was in charge of this scam appreciated enormously based on behavioral aspects such as rumors, speculations and false claims. Many people lost their life's fortune by participating in the speculations of this company. The more recent example of an irrational bubble is the dot-com bubble of 1997-2001. With the Taxpayer Relief Act of 1997 that provided a capital gain tax reduction from 28% to 20% (Dai et al., 2008), a major increase of capital flow towards major speculations and investments arose. Everything with a "dot-com" at the end of the name had been bought (Guttmann, 2009). The Federal Reserve reducing interest rates especially accelerated the burst of the dot-com bubble (Wollscheid, 2012). Shiller (2009) describes this phenomenon as a huge herding behavior that took place by retail investors as internet companies arose.

However, looking back, neither the Tulip Mania nor the South Sea bubble provided any positive side effect on the long-term to the economy, while the dot-com bubble, in contrast, gave an impressive impulse to the diffusion of broadband connection and the revolution of digitalization. The term “New Economy” (also known as Web 2.0) refers to the revolution of internet-based companies, which revolutionized the traditional economy (OReilly, 2007).

So, overall, we could characterize two types of bubbles (rational vs. irrational) with two different long-term side effects.

It is remarkable that all of our examples showed the same character of the course of a bubble. The character of an irrational bubble driven by behavioral aspects was identical throughout all three crises above. The evolution of the bubble and the final burst shows similarities to the crypto market prices over the period of 2013-2014, or more recently, of 2015 up until today. The enormous hype of cryptos in the public media, especially in recent times, has attracted more and more people to participate in the market. Whereas in the beginning of 2015, the total market cap was about 5.4 billion USD, it rose over a period of two years to 17.7 billion USD and exploded throughout 2017 to almost 606 billion USD in market cap, with a current all-time high marked on January 8, 2018 at 830.5 billion USD, and a sharp drop to 237.8 billion USD in less than six months (Coinmarketcap.com, 2018). Overall, the current price movements in the crypto market driven by psychological factors suggest there may as well be an irrational bubble.

However, as we summarized the general definition of a bubble with some examples in history and a classification of the movements in the crypto market, we now need to identify more precisely the presence of a bubble. Phillips et al. (2011) describe a bubble as when the price of an asset exceeds the fundamental value (intrinsic value).

The current problem with cryptos, which has been discussed in the previous chapter, is that there still is not any consensus about the fundamental (intrinsic) value as there is, for example, in other assets such as stocks (Phillips et al., 2011). So, as it seems like there is a consensus in academic literature about the possible presence of an irrational bubble in the crypto market, it is nevertheless useful to apply econometric tests or formulate a model that proves the bubble behavior while also taking into consideration the fact of the lack of definition about the fundamental value within cryptos.

3 Hypothesis

When analyzing time-series, we consider a sequence of measurements of our sample over a certain time. Taking this fact into consideration, we know that the arrangement of our sample plays a role. So, in time-series a model is used where a variable y from our sample (in our case the price of an asset) is measured in a period t (in our case day t for time). This fact leads us to an autoregressive model (AR model). This model regresses our value y_t from previous values in our time-series, e.g. y_t on y_{t-1}, where t-1 denotes the previous period. The number of values taken from the past to regress our model is called order. Thus, a model we use is an autoregressive model of order p, which looks as following:

$$AR(p) : y_t = c + \sum_{i=1}^{p} \rho_i y_{t-i} + \varepsilon_t = c + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \cdots + \rho_p y_{t-p} + \varepsilon_t \tag{1}$$

where y_t is the price of an asset on day t, c some constant, ρ some variable and ε_t an error term (white noise process $iid \sim \mathcal{N}(0, \sigma^2)$). Noticing, an AR process seems familiar with respect to a linear regression model. However, it auto-regresses our values $y_1, y_2, \ldots, y_n$ (n=amount of values in a data set) in which the arrangement of the y_t is taken into consideration. One can add to Equation 1 a time trend (βt). The characteristic of ρ, however, will play an important.

As mentioned above, the characteristics of an autoregressive model (AR(p)) manifest that it only depends on the value that the variable took in the previous periods (order p) plus an error term. The coefficient p implies the lag order.

For the explanation of ρ, we use the simplest form of an autoregressive process (Equation 1) of order 1, which looks as follows:

$$AR(1) : y_t = c + \rho y_{t-1} + \varepsilon_t \tag{2}$$

where y_t is the price of an asset on day t, c some constant, ρ a variable and ε_t an error term (white noise process $iid \sim \mathcal{N}(0, \sigma^2)$).
The characteristic of ρ is important. This coefficient is often called root.

Supposing $c = 0$ and $\rho = 0.4$: If $y_{t-1} = 50$, then we expect the value to be 20 in y_t plus some randomness expressed by ε_t with the respected property above. In period y_{t+1}, we expect the value to be 8 and so forth $(t + p)$. So, as the lag length increases, expressed by p, the time-series will converge back to the value c ($c = 0$). This is called a stationary time series, where the time-series has a long term mean (variable c) to which it will converge by time. On the other hand, when the given time-series has a root of an unit, i.e. $\rho = 1$, then the time-series will not converge to its origin (variable c). Using the Equation 2 and setting $\rho = 1$, one can see that by the same value as above (20 in y_t), the series will never converge back to c (=0). This leads to the characteristics and concept of unit roots, which is useful in interpreting time-series. Given the examples above, with this characteristics it provides insight whether a time-series will recover to its equilibrium price (variable c) as lag length increases. If one can reject this case, then the process will be highly persistent. Furthermore, then the process is hard to predict and control, in short it refers to a stochastic trend which is often named a random walk. Visually, one can imagine a drunken man walking on a street with moving to the right and left without any intention. The path of this person is unpredictable to which direction he may move next. This refers to a time-series with a unit root as well, in which e.g. prices will randomly move up and down without any stable path or rather equilibrium price. According to the definition of a unit root, another important motivation of it lays in an integrated process of order p. In other words, if a time-series has one unit root, one need to integrate the time-series (e.g. AR(1) process) once ($\Delta y_t : y_t - y_{t-1}$) in order to remove the non-stationary out of our time-series to obtain a stable, invertible autoregressive process, or rather an I(0) process. An integrated process is always denoted as I(p), where p is the order or differences.

Furthermore, another condition of Equation 1 is, if the root $\rho > 1$ for certain sub periods in a time-series. Then the process notes an explosive autoregressive behavior. In other words, the period y_t has an accelerated impact on the previous period y_{t-1} plus an error term.

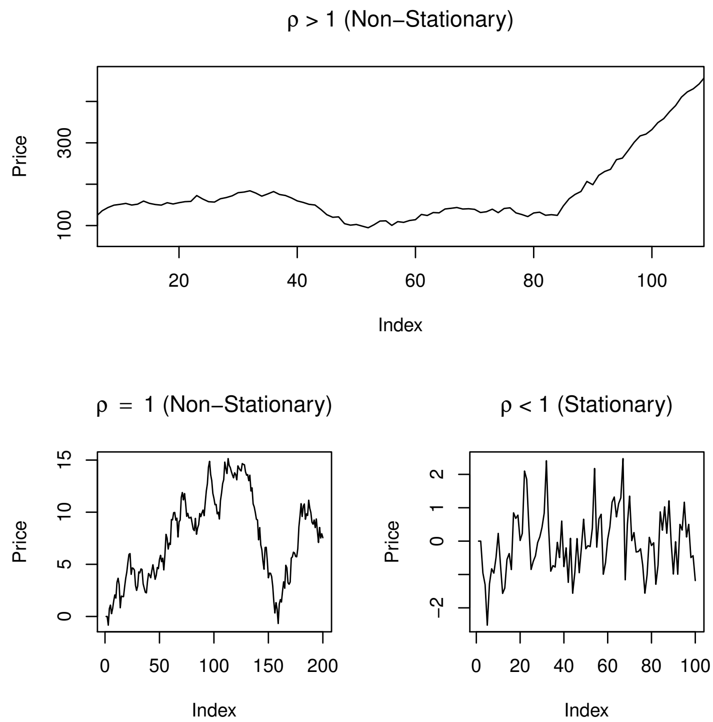

Figure 1: AR of different rhos (ρ).

As shown in Figure 1, a $\rho > 1$ in Equation 1 will have a significant impact on the overall course of the time-series. Before the time-series shows this massive increase of price (with respect to $\rho > 1$), we have more or less a random walk behavior with $\rho = 1$ (also shown more detailed in Figure 1 in the bottom left). In other words, we have a unit root in which the given time-series follows a stochastic trend. So, once the ρ in the AR process (Equation 1) turns to be greater than 1, the characteristic that y_t only depends on the value that the variable took in the previous period (y_{t-1}),tempts now to an accelerating price move. Prices are hard to predict and control. Furthermore, with $\rho > 1$ the shock of accelerating prices will be highly persistent.

On the other hand, if the root is smaller than 1, i.e. $\rho < 1$, the time-series may look like in the bottom right of Figure 1 in which the price always converges to its long-term mean. In other words, the price fluctuates around the constant c out of Equation 1.

The property of Equation 1 that y_t depends on y_{t-1} (plus the randomness) is often shown by an autocorrelation function (ACF). This function shows how the ongoing values are correlated to y_t. In a stationary time-series the AFC shows always the same autocorrelation in particular lags, which can be interesting for forecasting matters. In contrast, in time-series of non-stationary, the ACF is only gradually decaying or not at all due to the characteristics of AR process, and in particular the property of ρ, which has been explained earlier.

So, supposing it is known that an asset will increase the invested capital by buying in the asset on day t and selling the asset for an expected profit the very next day (t+1), then efficient speculation will drive up the current price of the asset. Without any doubt, no one is going to hold an asset which increased in price and is then expected to depreciate over a certain period of time. Therefore, a path of an asset can't follow stationary. Therefore, Fama (1970) explains with the efficient-market hypothesis (EMH) that the behavior of asset prices should follow a random walk. The same holds true for cryptos with accordance to (Caporale et al., 2018). As explained, in a random walk model (with $\rho = 1$ in AR(1)) prices will move now randomly up and down, which makes it impossible to forecast its path. In a process where a time-series characterizes an explosive autoregressive behavior, the ρ is greater than 1. Without any doubt, such a process in which prices move randomly and experience, in some cases, an explosive autoregressive behavior (price shock) can only lay in a non-stationary time-series in which a time-series has a time-dependent variance without any long-term mean.

These characteristics of a time-series explained above motivated Diba and Grossman (1988) and Phillips et al. (2011), to examine the presence of a unit root in a time-series to conclude whether price shocks could have a permanent effect on the series (non-stationary) or a temporary effect (stationary). The presence of a unit root suggests that after a price shock the series does not revert to its equilibrium price (long-term mean), or in other words its stable path, which means that a shock is highly persistent. For that, they applied unit root tests. Further, Phillips et al. (2011) uses a method for examining whether a bubble is present in a time-series. In case of detecting a bubble, they are also able to date-stamp them.

We will test our set of cryptos for the presence of a unit root by applying the augmented Dickey-Fuller (ADF) and KPSS test to conclude whether we have a bubble behavior (unit root behavior) in the crypto market or not. Further, if we come to the conclusion that there is a unit root, we will further test our set of cryptos for the presence of a bubble and then date-stamp them.

Therefore, our hypothesis is as follows:

- H_0 = Our set of cryptos show no presence of a bubble with regard to a unit root approach

- H_1 = Our set of cryptos show the presence of a bubble with regard to a unit root approach

Furthermore, Huynh et al. (2018) study tail dependence between BTC and XRP, BTC and LTC, and XRP and LTC to test for contagion effect. In other words, they have found that their tested cryptos depend on left-tail, which means that a decrease in price of BTC, for instance, might lead to a loss for XRP and LTC. Ergo, the price effect on BTC causes all their other tested cryptos (XRP, LTC) to follow. White (2015) describes BTC as the lion in the crypto market. These studies motivate us to find out if a crypto coin in our set of cryptos shows a contagion effect by a specific crypto coin. If so, this could be an important fact to further examine bubbles in the crypto market. Further, a bubble examination in terms of a crypto could then be traced back to a very few crypto coins.

Therefore, in our study we will try to find out whether there are any possible drivers within our set of cryptos. For that we will apply a bivariate Granger causality test to test whether any coin Granger-causes another. If so, we may be able to gain important information as to the presence of a bubble, thus opening ways for further research. We will test our set of cryptos on the following hypothesis:

- H_0 = Certain cryptos within our set of cryptos show no clue of possible drivers with regard to a bivariate Granger causality test

- H_1 = Certain cryptos within our set of cryptos show clues of possible drivers with regard to a bivariate Granger causality test

4 Data

In the following study we will use six different cryptos, including the top three in terms of market capitalization. We have chosen the other three cryptos for their amount of data size. Each of these six markets are according to the data provider Coinmarketcap.com. Each data point in our sample refers to the corresponding daily closing price (in USD per coin) for our chosen time frame. Our data points for each crypto are structured in a chronological order and, thus, form a time-series.

	Mean	St. Dev.	Min	Max	Skew	Kurt	Start Date	Numb. Obs.
BTC	2033.04	3382.46	68.43	19497.40	2.36	8.35	04/28/2013	1921
LTC	29.64	55.51	1.16	358.34	2.74	10.73	04/28/2013	1921
ETH	211.16	292.17	0.43	1396.42	1.46	4.46	08/07/2015	1090
DASH	120.57	230.71	0.31	1550.85	2.65	10.80	02/14/2014	1629
XRP	0.15	0.35	0.00	3.38	4.20	26.95	08/04/2013	1823
XLM	50.18	92.29	0.22	469.20	2.17	7.06	05/21/2014	1532

Table 1: Descriptive statistics of our set of cryptos.

Table 1 gives an overview of the data of our study. All of the cryptos, excluding BTC, have their start date corresponding to the date of the ICO. BTC, however, already launched on January 3, 2009. The last day of the sample is July 31, 2018. In short, the cryptos from our data set vary from 1090 to 1921 number of observations (days). Furthermore, all of our set of cryptos use the blockchain technology.

A first look at the table already demonstrates the tremendous upside moves with respect to an overall low amount of data points. BTC, for instance, moves last from 68.43 USD per BTC (minimum price) up to 19,497.40 USD (maximum price). With respect to its standard deviation of around 3,382.46 USD, it shows that high price fluctuations around the mean price of 2,033.03 USD are possible. Also, the other cryptos in our set show characteristics of high fluctuation prices around the given mean price. The lowest maximum price measured in USD has XRP with 3.38 USD per XRP, whereas BTC notes, without any doubt, the maximum price. A further description of our set of cryptos include skewness and kurtosis. They provide details about the location and the variability of our given data set. With regard to the kurtosis coefficients for our set of cryptos, they tend to have heavy tails. In other words, the coefficients, which are all positive, refer to a so-called “leptokurtic distribution with fat tails”. XRP has the highest coefficient with around 26.95. This observation already gives a clue that XRP has the greatest outliers (price fluctuation) compared to its mean price. The lowest coefficient is with ETH, which has around 4.46.

Furthermore, taking a look at the measure of symmetry with accordance to the skewness coefficient we already find an interesting observation. All of our set of cryptos are positively skewed again, with XRP at the highest of around 4.20, and ETH at the lowest of around 1.46. Therefore, our set of cryptos have overall no symmetry around the mean whereas the left and right sides look the same. The fact of the positive skewness of all our samples shown in Table 1 already supports the fact that our set of cryptos, with respect to their daily closing price (in USD), cant be normally distributed. Further study supported the fact, in which classical normality tests such as the Shapiro-Wilk or Jarque-Bera reject normality at all conventional significant levels (10%, 5%, 1%). For a visual comparison, it is appropriate to plot our set of cryptos with regard to log prices, due to the fact that they vary massively in USD per coin (from a few USD up to a couple thousand USD).

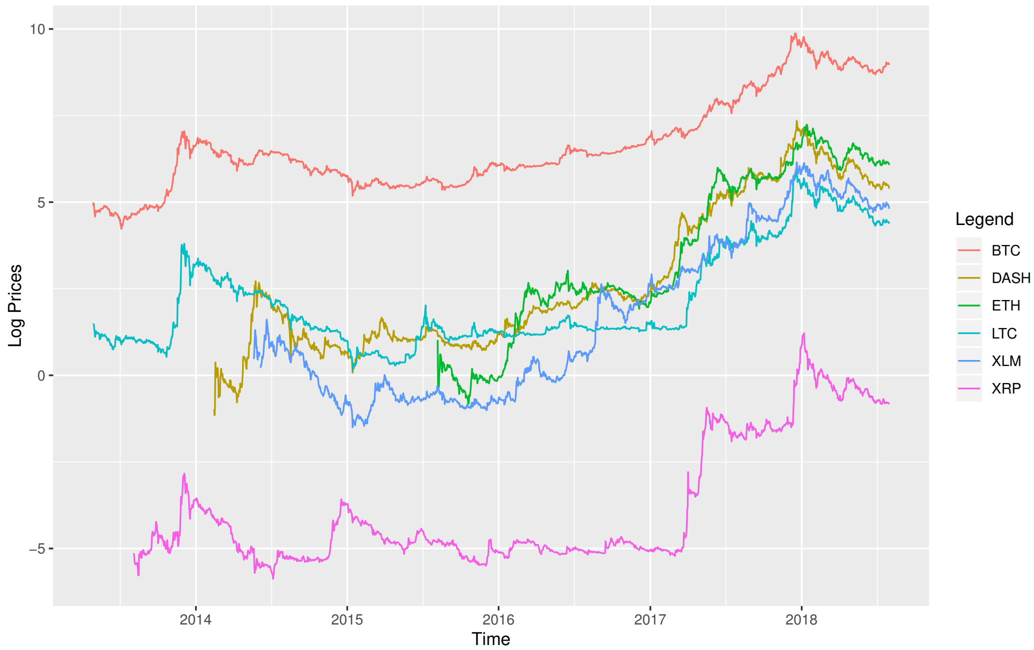

Figure 2: Plot of our set of cryptos in log prices.

Figure 2 shows the log (natural logarithm) prices of all corresponding cryptos used for our study.

With respect to Figure 2 we can visually see an overall upwards trend in our sample of crypto coins. What stands out is the fact that DASH, ETH, LTC, and XLM show visually an equal path of log prices overall in our sample size. BTC shows, besides the higher log prices, an overall equal path compared to the other coins, excluding XRP. XRP, on the other side provides an interesting case. Whereas, the market more or less fluctuated around a visual mean (log) price of around -5 over the path of three years, it then suddenly exploded by the beginning of 2017 to around 0. Visually, XRP has the greatest sudden upward shift. Visual inspection of Figure 2, supports the fact that XRP can obviously only be the market with the highest skewness coefficient.

Figure 5 in the Appendix shows the ACF for all of our set of crypto for first visual evaluation. The autocorrelation between the variable t and $t - p$ (p=lag) only gradually decays for all our cryptos. This gives a first signal of non-stationary for the time-series (set of cryptos).

5 Methods

In the following part, we will explain all statistical tests in detail with respect to their use and interpretation for our study.

5.1 Bubble Testing

In accordance to Phillips et al. (2015), for the test for unit roots, we transform our set of cryptos first into natural log data.

5.1.1 Testing for a unit root using the ADF and the KPSS test

Following the approach by Kwiatkowski et al. (1992) and Diba and Grossman (1988) we start our study by testing our time-series for the presence of a unit root.

The ADF Test:

The intuition behind the augmented Dickey-Fuller test (the ADF test) is the same as for the Dickey-Fuller test, but with added lags to the model. Let us recapitulate once again the Dickey-Fuller (DF) test and its intention for continuing with the augmented version of the DF test, taking into consideration the DF test proposed by Hill et al. (2012).
If we subtract from an AR(1) process, referring to Equation (2), y_{t-1} in order to integrate it in first order, we obtain the following:

$$\Delta y_t = c + \delta y_{t-1} + \varepsilon_t \tag{3}$$

where Δ denotes the difference operator $(y_t - y_{t-1})$ and $\delta = (\rho - 1)$.
According to the DF test, which is a left-tailed test, δ can't be positive where:

- $H_0 : \delta = 0$ (equivalent to $\rho = 1$)
- $H_1 : \delta < 0$ (equivalent to $\rho < 1$)

For the estimation of δ , the ordinary least squares (OLS) method is applied.

If the test statistic from the DF test is smaller than its critical value (CV) (needs to be smaller due to a left-tailed test), given from the DF table, we can reject the H_0, that there is a unit root in our time-series on the given significant level. Otherwise we cannot reject the H_0, that the time-series has a unit root.

Note that there are three different versions of the DF test:

- $\Delta y_t = \delta y_{t-1} + \varepsilon_t$ (no constant, no trend)
- $\Delta y_t = c + \delta y_{t-1} + \varepsilon_t$ (constant, no trend)
- $\Delta y_t = c + \beta t + \delta y_{t-1} + \varepsilon_t$ (constant, trend)

Testing Equation (3) with regard to the DF test, we consequently refer to the second version.

The augmented Dickey-Fuller test (ADF) works the same as the DF test but it adds lags to the model to make it more robust (Hill et al. 2012) (Diba and Grossman, 1988). Compared to the DF test where autocorrelation is present, the augmented version of the DF test tries to reduce autocorrelation out of the model by using lag orders in order to test for a unit root. Note that the same CVs are derived for the sample of the model as in the Dickey-Fuller test. Now we augment Equation (3) by adding $p - 1$ lagged differences to our model:

$$\Delta y_t = c + \delta y_{t-1} + \rho_1 \Delta y_{t-1} + \cdots + \rho_{p-1} \Delta y_{t-p+1} + \varepsilon_t \tag{4}$$

where p denotes the lag order or the autoregressive process.

If the series is integrated in I(1) and we have a unit root in the time-series, then only the lagged changes (y_{t-p}, p : lag) contain relevant information in predicting the change in y_t ($\delta y_{t-1} = 0$). Again, the H_0 in this case is rejected if $\delta < 0$. If $\delta = 0$, we cannot reject that we have a differences stationary $AR(p-1)$ process. In other words, we cannot reject the null hypothesis of having a unit root in our AR(p) process and, thus, can conclude based on Diba and Grossman (1988) that we have a bubble behavior in our series.

In case of the presence of a unit root derived by the ADF test, we want to verify it by applying the KPSS test.

The KPSS Test:

The benefit of additionally using the KPSS (Kwiatkowski et al., 1992) for detecting unit roots is that its setup is interestingly different. Whereas the approach by Diba and Grossman (1988) follows stationary under the H_0, Kwiatkowski et al. (1992) follows stationary under the H_0. In this case, we are testing our time-series from both sides for the presence of a unit root. However, we need to remember that in only using the KPSS test alone, there is a high rate of type I errors. This is a major disadvantage of the test. However, once combined with the ADF test, the overall statement about a unit root can be more powerful (Wang, 2006). Therefore, we will combine the ADF test with the KPSS test in our study for an increase of the statistical power of testing whether our set of cryptos has a unit root.

The setup of the KPSS works by regressing y_t on a constant and time trend. Taking into consideration the KPSS test, it is also based on a linear regression model, but consists out of three components: a random walk (r_t), a deterministic time trend (βt, where β is an intercept from the linear regression and t denotes the time index) and a stationary error u_t.

In this setup our regression denotes:

$$y_t = c + \beta t + (c + r_t) + u_t \tag{5}$$

where $r_t = (r_{t-1} + \varepsilon_t)$ with $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ and the c serves as intercept for the initial value r_0.

Taking into consideration Kwiatkowski et al. (1992) and Shin and Schmidt (1992), the KPSS test examines whether the variance of t is equal to zero (H_0) or unequal to zero (H_1). If the variance of t is unequal to zero, then there is a random walk in y_t. In that case we can reject H_0 that the series follows trend stationary. Note that the residuals of r_t (t) are estimated by the OLS method from Kwiatkowski et al. (1992). Once we detect a unit root in our set of cryptos we, nevertheless, cannot say if we have a bubble in our set of cryptos. The only thing we can conclude is, if we detect a unit root process in our set of cryptos and ρ will turn to be greater than 1, then we have a bubble. Therefore, Phillips et al. (2015) have created a supremum ADF (SADF) test. With the use of the SADF test, we are able to test whether our set of cryptos has a bubble.

5.1.2 Sup Augmented Dickey-Fuller test (SADF)

We follow the approach from Phillips et al. (2011). For the following we note the SADF test as PWY test. The advantage of the PWY is the detection of a bubble in a time-series based on a sequential right-tailed ADF test in which the diagnostic extends the sample sequence to a more flexible range. The PWY tests the H_0 of a unit root ($\delta = 1$) against the alternative H_1 of an explosive root ($\delta > 1$). Phillips et al. (2011) describe the presence of an explosive root within a time-series as a bubble.

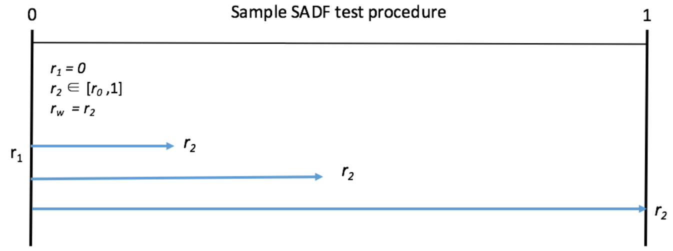

Figure 3: Sample PWY test procedure

We will set our sample interval $[0, 1]$. As seen in Figure 3, the PWY sets r_1 to 0 and r_2 $[r_0, 1]$. It will use the initial window $[0, r_2]$ and varies r_2. Further, the window width is $r_w = r_2$.
Basically, the PWY starts in $r_1 = 0$ with the ADF test for the first sample, which is $r_1 + r_w = r_2$ and then runs the ADF statistic on a forward expanding sample sequence. The SADF statistic is written as:

$$SADF(r_0) = \sup_{r_2 \in [r_0,1]} \{ADF_0^{r_2}\} \tag{6}$$

where $ADF_0^{r_2}$ is the t-ratio for the OLS estimation of δ in the regression:

$$\Delta y_t = c + \delta y_{t-1} + \sum_{i=1}^{p} \rho_p \Delta y_{t-p+1} + \varepsilon_t \tag{7}$$

The PWY test is a supremum statistic based on this forward recursive regression. In other words, for the outcome of the SADF (r_0) it will return the supremum of the δs estimated by the OLS for our regression Equation 7 in a forward sequential method.
For the recursive unit root test (SADF) to detect a bubble, we calculate the CV by applying T times the Monte Carlo (MC) simulation for M samples generated from a normal distribution. For the CVs we will use the 10%, 5% and 1% significance levels. Then we compare the t-statistic out of the PWY with the respective CVs. If the t-statistic is above the CV, then

we can reject the H_0 of the PWY test in favor of an explosive root laying in our time-series. In other words, we could speak of a bubble laying in our time-series (Phillips et al., 2011).

5.1.3 Backward test for date-stamping a bubble episode

Suppose our system detects a bubble, then we want to know when the bubble started and when it collapsed. Phillips et al. (2011) feature that one advantage of the recursive types of tests (eg. SADF) is that they allow us to pin-point the start and collapse of a bubble. For that Phillips et al. (2011) and Phillips and Yu (2011) propose a backwards regression technique, which identifies the points of start and collapse of a bubble.

Detection of a bubble:

The technique compares each element of the estimated ADF_{r2} sequence to the corresponding right-tailed CVs of the standard ADF statistic received, out of the MC simulation, for the identification of a bubble initiation at time Tr_2.

For that, the estimated starting point of the bubble is the first chronological observation ($\hat{\tau_e}$), the ADF_{r2} crosses the corresponding CV from below. The estimated termination point is the first chronological observation after $\hat{\tau_e}$. It is denoted by $\hat{\tau_f}$. The ADF_{r2} now crosses the CV from above. In short, the starting and ending of the explosive episode (bubble) is noted as $\hat{\tau_{e/f}} = [n\hat{r_{e/f}}]$, where n is the total sample size with respect to the cryptos.

Formally, the dating of the start and collapse of the bubble is written as:

$$
\begin{aligned}
\hat{r_e} &= \inf_{r_2 \in [r_0, 1]} \{r_2 : ADF_{r_2} > CV^{adf}(r_2)\} \\
\hat{r_f} &= \inf_{r_2 \in [\hat{r_e}, 1]} \{r_2 : ADF_{r_2} < CV^{adf}(r_2)\}
\end{aligned}
\tag{8}
$$

where $CV^{adf}(r_2)$ is the CV of the ADF_{r_2} statistic based on $[nr_2]$ observations (Phillips and Yu, 2011). With the choice of Phillips et al. (2015), we use a minimum rolling window of $log(n)$, in order to remove small periods of explosiveness and date-stamp a bubble.

5.1.4 Time-series behavior under the hypothesis of a bubble - a summing up

To understand this process, we explain how the time-series behaves before the bubble origination, the actual bubble period, and after the bubble termination. We use the approach by Phillips et al. (2011) and Phillips and Yu (2011).

For detecting an explosive bubble behavior in our-time series, we will look at the following equation:

$$
\begin{aligned}
y_t = y_{t-1}\,\mathrm{II}\left\{t < \tau_e\right\} + \delta_n y_{t-1}\,\mathrm{II}\left\{\tau_e \le t \le \tau_f\right\} + \\
+ \left(\sum_{i=\tau_f+1}^{t} \varepsilon_i + y^*_{\tau_f}\right)\mathrm{II}\left\{t > \tau_f\right\} + \varepsilon_t\,\mathrm{II}\left\{t \le \tau_f\right\}
\end{aligned}
\tag{9}
$$

with $\delta_n = 1 + \frac{v}{n^{\alpha}}$, $v > 0$, $\alpha \in (0,1) \implies \delta_n > 1$
where $\varepsilon_t \sim \mathcal{N}(0,\sigma^2)$, τ_e denotes the date of bubble starting and τ_f is the collapse date of the explosive episode (bubble). II denotes an indicator function. Intuitively with accordance to Phillips et al. (2011), if there is no explosive episode (bubble) in the time-series, then $v = 0$ and, therefore, $\delta_n = 1$.

With regard to Equation (9), the time-series behaves as a random walk (unit root behavior) during the process before the bubble period starts ($t < \tau_e$), then between $[\tau_e, \tau_f]$ it switches to an $AR(1)$ process with the coefficient δ_n. After τ_f, the process falls back to $y^*_{\tau_f}$ plus the sum of the error terms (ε_i) that occurred during the bubble period. According to Phillips and Yu (2009), $y^*_{\tau_f}$ is expressed by $y_{\tau_e} + y^*$, with $y^* = \mathcal{O}_p(1)$. In short, the process in ($t > \tau_f$) may relate to the earlier unit root process ($t < \tau_e$), however, with some random deviation.

5.2 Drivers Test Granger Causality Test

For the second approach in our study to examine whether a crypto in our set is having an information gain in forecasting another crypto, we explain the Granger causality test with respect to Granger (1969) and Mahdavi and Sohrabian (1991). However, first things first, we need to explain the vector autoregressive (VAR) model, which is a prerequisite for applying an Granger causality test.

Before we start with the VAR model we have to transform our set of cryptos into log returns to be stationary.

$$
Log - returns = \frac{y_t}{y_{t-1}} = \log(y_t) - \log(y_{t-1}) \tag{10}
$$

where we use again the natural logarithm ($\log(y) = \ln(y)$). As we can see, $log - returns$ are simply the integration of log data in the first order ($y_t - y_{t-1}$).

5.2.1 The VAR model

The VAR model, is one of the most used models in multivariate time-series analysis. It is more or less a natural extension of the univariate AR(p) model to a dynamic multivariate time-series. It is used for describing dynamic behavior of time-series, for forecasting, but also for structural analysis.

Each considered equation for the VAR model is estimated using the OLS method.

The basic p-lag VAR(p) model consists of the following form:

$$VAR(p) = y_t = c + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \varepsilon_t \tag{11}$$

where y_t is a $(n \times 1)$ vector, A_i are $(n \times n)$ coefficient matrices and ε_t is a vector with white noise ($iid \sim \mathcal{N}(0, \sigma^2)$) and the same dimension as y_t.

We only use a bivariate model, consisting, thus, out of two equations. Each equation represents a crypto. For instance, a bivariate VAR(2) model equation by equation, which intuitively consist out of two lags, has the following vector form:

$$\begin{aligned} VAR(2) = \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \\ + \begin{pmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \end{aligned} \tag{12}$$

or in equation notation:

$$\begin{aligned} y_{1t} &= c_1 + a_{11}^1 y_{1t-1} + a_{12}^1 y_{2t-1} + a_{11}^2 y_{1t-2} + a_{12}^2 y_{2t-2} + \varepsilon_{1t} \\ y_{2t} &= c_2 + a_{21}^1 y_{1t-1} + a_{22}^1 y_{2t-1} + a_{21}^2 y_{1t-2} + a_{22}^2 y_{2t-2} + \varepsilon_{2t} \end{aligned} \tag{13}$$

with $E(\varepsilon_{1t}, \varepsilon_{2t}) = 0$ and $E(\varepsilon_{1t} * \varepsilon_{2t}) = 0$, ergo expected residuals of zero and the error terms are not auto correlated.

With this model equation, it allows us to make dynamic relationship interpretations between the indicated variables, and thus, we can make use of the Granger causality test.

5.2.2 Bivariate Granger causality test

With the VAR(p) model of lag p, the Granger causality test is performed.

Note, we need to clarify that Granger causality is not the result of a statistically causality statement in its power (Maziarz, 2015). The argument is much weaker. Nevertheless, we can test the bivariate time-series, like whether y_2 has any influence on y_1 ($a_{12} \neq 0$). If so, we can say y_2 Granger-causes y_1, vice versa y_1 Granger-causes y_2, if y_1 has any influence on y_2 ($a_{21} \neq 0$).

For our sample we have used the f-type Granger causality test. In this case, the vector of the endogenous variables y_t is split into two sub-vectors y_{1t} and y_{2t} with dimensions $(n \times 1)$. To decide whether the additional information in the other time-series Granger-causes the other (increases the R^2), an f-test must be used to test the H_0 of non-causality.

We will illustrate below how the Granger causality works on an example of a two-lagged bivariate VAR(2) model. To test whether we can reject our null hypothesis, that y_2 does not Granger-causes y_1, we will consider our Equation (13) in a rewritten way as the unrestricted model:

$$y_{1tu} = c_t + \sum_{j=1}^{2} a_{11}^{j} y_{1t-j} + \sum_{m=1}^{2} a_{12}^{m} y_{2t-m} + \varepsilon_t \tag{14}$$

Out of our bivariate VAR(2) model (Equation 13), we receive all coefficients for Equation (14).

In terms to assume that the coefficients for the lagged values of y_2 are equal to zero (for testing the H_0), we additionally need to form the restricted model from our bivariate VAR(2) model. It will have the following form:

$$y_{1tr} = c_t + \sum_{j=1}^{2} a_{11}^{j} y_{1t-j} + \varepsilon_t \tag{15}$$

Now the method will calculate the residual sum of squares for the restricted model (Equation 15) as well for the unrestricted model (Equation 14) by using the OLS method.

The next step of the method is the calculation of the f-statistic, which is done as follows:

$$F = \frac{RRSS - URSS}{URSS}\left(\frac{T-k}{q}\right) \tag{16}$$

where $RRSS$ is the residual sum of squares out of the restricted model, $URSS$ is the residual sum of squares for the unrestricted model, T is the sample size, k the number of regressors of the model and q is the number of restrictions to test (in our case of Equation (14), it's two).

Once it has obtained the f-statistic from the Equation (16), it compares the value with the f-critical value at the 5% significance level (5% is used our study). If the f-statistic is higher than the f-critical value, we will reject the H_0 and then we can say that the y_2 Granger-causes y_1. In other words, the fit with the additional information out of y_2 in y_1 will be improved (increase of the R^2).

So, for instance, using $crypto_1$ and $crypto_2$. If $crypto_2$ rejects the H_0 of the bivariate Granger causality test with regard to $crypto_1$, we have a clue that $crypto_2$ provides additional information in forecasting $crypto_1$.

6 Data Analysis – Empirical Part

All of our set of cryptos are transformed first into natural log data to continue with the unit root tests. We apply the KPSS together with the ADF test to our set of cryptos. Tables 6 to 10 in the Appendix report the main results. The KPSS states that on all possible significant levels (10%, 5%, 1%), the H_0 is being rejected and thus favors following non-stationary on all our tested lags (up to lag 5). The ADF results on all possible significant levels (10%, 5%, 1%) with the use of all three types of ADF tests, that the H_0 cannot be rejected of the time-series (cryptos) follow non-stationary (on all tested lags). In other words, the first two applied tests show strong evidence of a unit root laying in our set of cryptos. Continuing with our study, we apply the PWY test to our set of cryptos (log prices) with one lag. We implemented the test using the starting point r_0, to the choice of Phillips et al. (2015) based on a lower bound of 1% of the full interval of our data ($r_0 = 0.01 + 1.8/\sqrt{n}$, where n is the total sample size). For the calculation of the CVs, we applied 1000 times the MC simulation for 1000 samples generated from a normal distribution. Table 2 reports the main results.

	Test Stat	10%	5%	1%
BTC	4.35	1.14	1.43	2.10
LTC	4.62	1.24	1.45	2.00
ETH	1.21	1.25	1.55	2.21
DASH	0.41	1.28	1.57	2.02
XRP	1.97	1.22	1.46	2.11
XLM	1.59	1.24	1.50	1.94

Table 2: The SADF p-values with regard to the CVs.

The first column contains the obtained t-statistic from the PWY test. The next columns give the obtained CVs. For each crypto we applied the MC simulations, with their respective settings from above, separately. The CVs closely correspond to the values reported in Table 1 of Phillips et al. (2011). Given our results from the Table 2, we cannot reject the H_0 of a unit root in favor of having a explosive root (H_1) in some cryptos out of our set. Since the PWY test is a right-tailed test, the t-statistic needs to be bigger than the critical values obtained from the MC simulations in order to reject the H_0. Let us first compare the t-statistics from the PWY test and the corresponding statement by Phillips et al. (2015) of having a bubble or not.

BTC and LTC show strong evidence of an explosive root, or rather a bubble in accordance to Phillips et al. (2015), as they reject the null hypothesis on all given significance levels (10%, 5%, 1%). XRP and XRM reject the null hypothesis up to the 5% significance level. However, on a 1% significance level they do not. ETH and DASH cannot reject the null hypothesis in favor of having a bubble (explosive root).

The effect of having a bubble seems, therefore, strongest in BTC and LTC. For the visual examination and a possible date-stamping of a bubble episode, we turn to the analysis of our set of cryptos of the time-series of recursive ADF statistics to pin-point the start and collapse of a bubble, as used in Phillips et al. (2011). The date-stamping algorithm was applied as it was in Phillips et al. (2011), using a backwards regression technique. In short, the algorithm is called BADF.

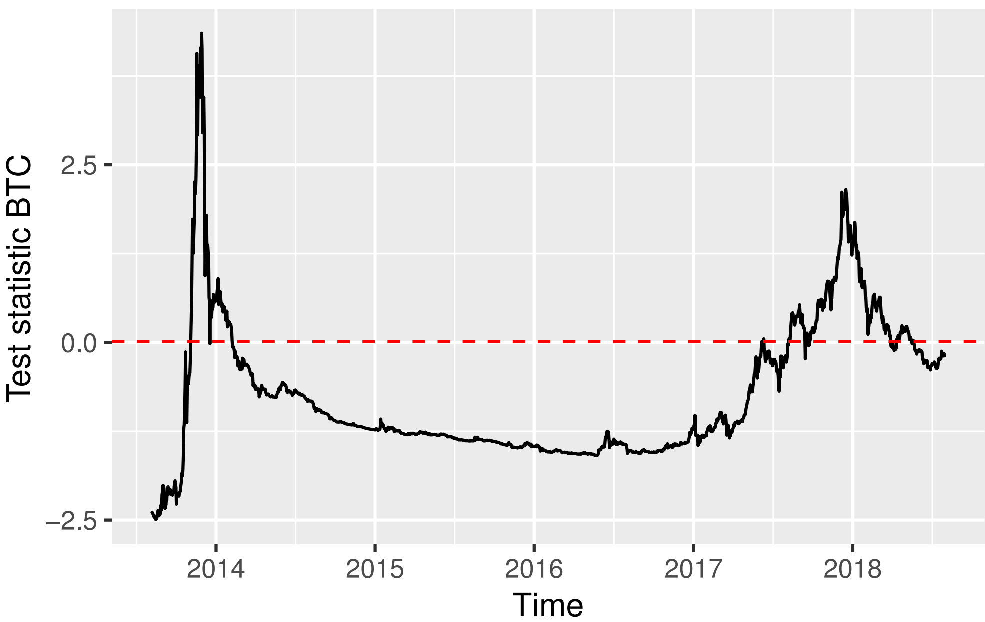

Figure 4: Series of backwards *ADF* test statistics for BTC together with its respected CV on a 5% significant level.

Figure 4 shows the series of ADF statistics for BTC together with the respected CV. Note that we have used the CVs on a 5% significance level obtained by the ADF test for our BADF algorithm (Phillips et al., 2011). Combined with a very short crossing beginning in 2014, there are two bubble periods observed. The first bubble starting on November 4, 2013, and collapsing on February 15, 2014, whereas the second bubbles origination is on August 14, 2017 and has shortly collapsed (May 30, 2018). In Figure 6 - 10, the series of the ADF statistics are shown for the other cryptos in our set of data.

Overall, what stands out is that for almost all the sets of cryptos we can clearly conclude explosive roots. For LTC the first explosive root (bubble) also notes beginning around 2014 and the second is observed recently with its origin around the end of 2017. XRP shows an impressive explosive root (bubble) which already started in the first quarter of 2017. XRM on the other hand shows the presence of an explosive root in mid-2016. Again, for ETH and especially for DASH it is not clear whether we have an explosive root (bubble) with respect to our econometric test. As visually seen for DASH in Figure 8, the series of ADF statistics only vaguely exceeds the CV. Regarding Figure 7 for ETH, the series of ADF statistics tends to fluctuate around the CV, which makes it difficult to give a clear statement. Nevertheless, we can state that ETH shows as well evidence of several explosive behaviors within the market. Further, our set of cryptos show a lowering ADF statistic in between their respected explosive processes.

For our study to find potential interdependence, or rather drivers, within the set of cryptos, we transformed our log data into log returns in order to get the unit root out of our data. In other words we integrated the data once (I(1)) and formed a VAR(p) model of lag p. Our findings for the bivariate Granger causality tests for the VAR(p) models of lag $p = [1, 5]$ are reported in Table 11 − 15. We firstly can conclude that possible patterns are only to report up to lag 3. On lag 4 we see some shifts in these patterns. However, we included lag 4 and even 5 as well in Table 14 and 15 to visually compare our findings.

We have to mention that we only seek the result in which the H_0 (=Granger does not cause) is present for one side, but for the other side we can reject the H_0. With this result we can better conclude in which direction an information gain can be reported. When we have on both sides a rejection of the H_0, we are not able to make any statement.

When taking into consideration these prerequisites, we can conclude some possible patterns.

Granger causality	1 lag	2 lags	3 lags	4 lags
BTC-XRP	0.2934	0.3360	0.1785	0.0000
XRP-BTC	0.0001	0.0000	0.0000	0.0000

Table 3: p-values out of the Granger causality test between BTC and XRP on the tested lags

For the bivariate Granger causality test between BTC and XRP, reported in the table above (Table 3), we can find a pattern in which we can reject the H_0, for XRP does not Granger-cause BTC up to lag 3 (included). On the other side we cannot reject the H_0 for BTC Granger-cause XRP up to lag 3 (included).

Granger causality	1 lag	2 lags	3 lags	4 lags
XRP-XLM	0.0001	0.0000	0.0000	0.0000
XLM-XRP	0.9509	0.1595	0.3922	0.0101

Table 4: p-values out of the Granger causality test between XRP and XLM on the tested lags

Furthermore, we noticed another possible pattern between XRP and XLM, reported in the table above (Table 4). For the bivariate Granger causality test, whether XRP does not Granger-causes XLM, we can reject the H_0 up to lag 3 (included) as well. On the other side, with the bivariate Granger causality test, whether XLM does not Granger-causes XRP, we cannot reject the H_0 up to lag 3 (included).

Granger causality	1 lag	2 lags	3 lags	4 lags
LTC-XRP	0.4124	0.4146	0.5188	0.0006
XRP-LTC	0.0000	0.0000	0.0000	0.0000

Table 5: p-values out of the Granger causality test between LTC and XRP on the tested lags

Last but not least, we can conclude another possible pattern between LTC and XRP, reported in the table above (Table 4). For the bivariate Granger causality test, whether XRP does not Granger-causes LTC, we can reject the H_0 up to lag 3 (included) as well. On the other side, with the bivariate Granger causality test, whether LTC does not Granger-causes XRP, we cannot reject the H_0 up to lag 3 (included).

In other words, we can conclude from the bivariate Granger causality test, that we found three interesting patterns. For predicting BTC, the additional information from XRP decreases the OLS estimates for the VAR model up to lag 3 and, thus, increases the R^2 for the VAR. Same for XLM and LTC, the additional information from XRP decreases the OLS estimates and therefore increases the fit of the VARs (R^2) up to lag 3, as well. However, note that our findings do not give a clue about the percentage increase. The findings from the bivariate Granger causality test literally report that the information from the additional coin increase the fit (R^2) and give us a clue as to an additional information gain from a crypto.

To summarize our findings in this study, the first results from our study show that our chosen cryptos have an I(1) process. In other words, all our cryptos show strong evidence of a unit root with accordance to the ADF and KPSS Test on all our tested lags and ADF-types (no constant, no time trend; constant, no time trend; and constant, time trend). Even our preferred ADF test with the inclusion of a constant and a time trend (referring to visual inspection of Figure 2), cannot reject the H_0 of having a unit root. Further study by applying the SADF test (with regard to lag 1) concludes that, at a 10% significance level there is strong evidence of the presence of a speculative bubble in most of our cryptos. The strongest evidence of a speculative bubble, even at the 1% significance level, with accordance to Phillips et al. (2015) show for BTC and LTC. The date-stamping procedure allowed us to identify a clear bubble in most of our cryptos. Interestingly, we were able to detect a bubble in XLM at the end of the sample on July 31, 2018, which has not collapsed yet. Furthermore, our findings for Granger causality state that XRP Granger-causes BTC, XLM and LTC (only into these directions; not vice versa). The recently collapsed bubble in XRP for our sample and the still not collapsed bubble in XLM are interesting. The recent media hype within XRP and the henceforth bubble in this market show a potential information gain for the presence of the bubble in XLM. Additionally, our study gives a possible clue that XRP provides an information gain for BTC due to our Granger causality findings.

7 Conclusions

We can conclude that the crypto market in general shows evidence of a bubble with accordance to our applied econometric tests. Thus, we can reject our H_0 of no bubble within our set of cryptos.

Furthermore, the steps in our study show that the application of a size correction and improvement of the power of test with the use of the PWY test gives a more detailed picture of the crypto market. The majority of our tested cryptos show strong evidence of a speculative bubble. Moreover, in the Bitcoin (BTC) and Litecoin (LTC) cases, our date-stamping procedure in accordance with Phillips et al. (2011) and Phillips and Yu (2011) suggests strong evidence of the presence of two bubbles. Ethereum (ETH), as the second biggest crypto coin (in terms of market capitalization), suggests no presence of a speculative bubble on our conventional significant levels. However, by applying the date-stamping procedure, it shows as well evidence of several explosive behaviors within the market. With respect to this, one can extend the PWY test with more lags. One can also apply information criterion to automatically compute the lag size, e.g. the Akaike or Bayesian (Schwarz) information criterion. Due to the lack of the fundamental value of cryptos we can only result our findings on one side with accordance to the price development without taking into consideration the fundamental value.

Another important part of our study was the examination of whether our chosen coins have any interdependence. With the application of the Granger causality test we came across potential interesting patterns. With regard to our H_0, we were able to reject the hypothesis that our set of cryptos shows no clue of possible drivers. These patterns (of possible drivers), especially with regard to bubble development could be an important consideration for further study, in particular for bubble definition, timing and explanation within the crypto market. More in-depth, we recommend examining these patterns in further studies with the application of impulse response functions by shocking independent variables of the VAR model to analyze the time response. In other words, to analyze the time response when shocking the independent variables (e.g. of XRP) in the VAR (i.e. XLM). This could lead to interesting findings. With respect to this, one can extend the VAR model for further input. We want to mention that we found some interesting conclusions, which should not be neglected by analyzing for potential bubbles in the crypto market as a whole.

Further, as Hafner (2018) has already studied and stated, the volatility of cryptos is time-varying, which points to another important consideration of examining bubbles within the crypto market. In further examination, the consideration of time-varying volatility by Hafner (2018) can also be implemented for our course of study.

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A Figures

ACF for our set of cryptos:

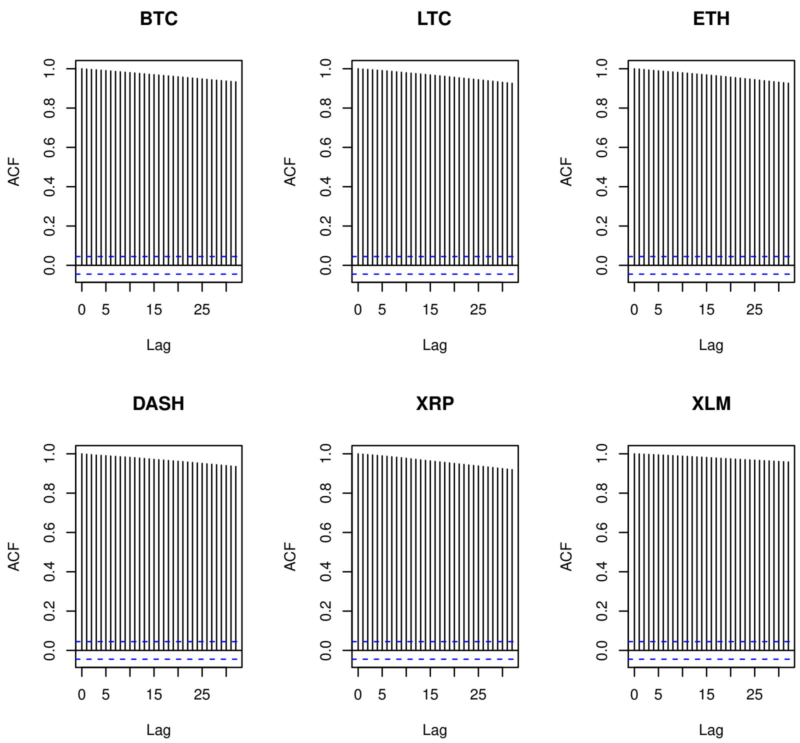

Figure 5: ACF for all our set of cryptos.

ADF LTC:

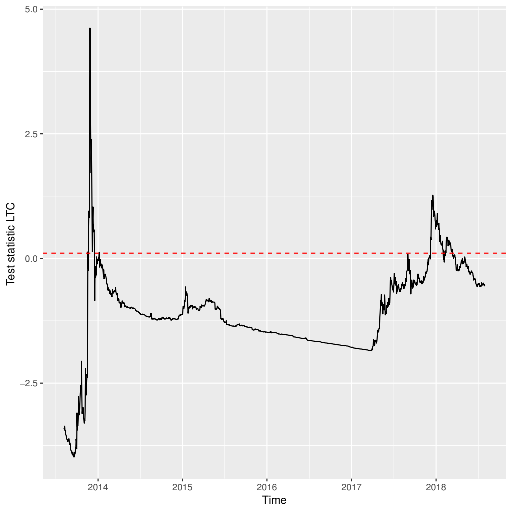

Figure 6: Series of backwards *ADF* statistics for LTC together with its respected CV on a 5% significant level.

ADF ETH:

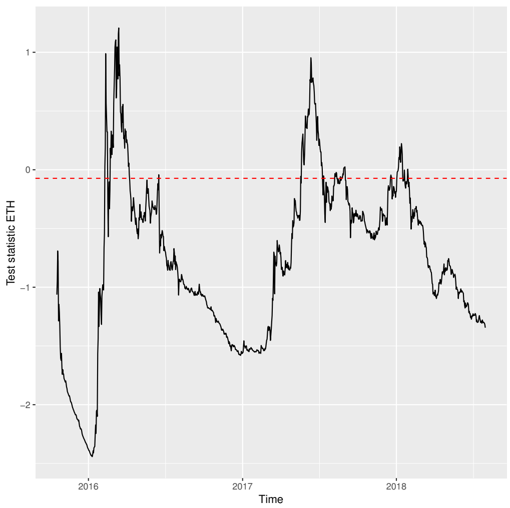

Figure 7: Series of backwards *ADF* statistics for ETH together with its respected CV on a 5% significant level.

ADF DASH:

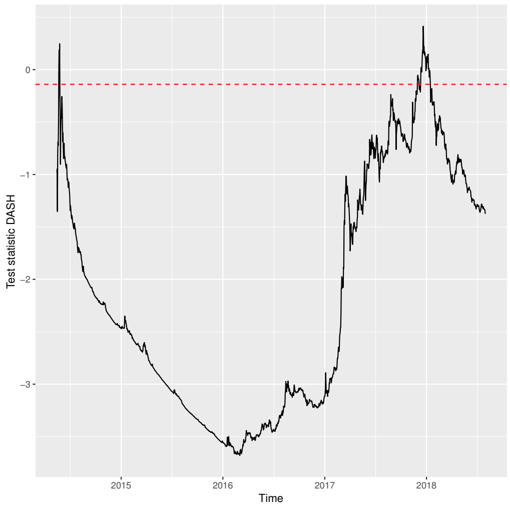

Figure 8: Series of backwards *ADF* statistics for DASH together with its respected CV on a 5% significant level.

ADF XRP:

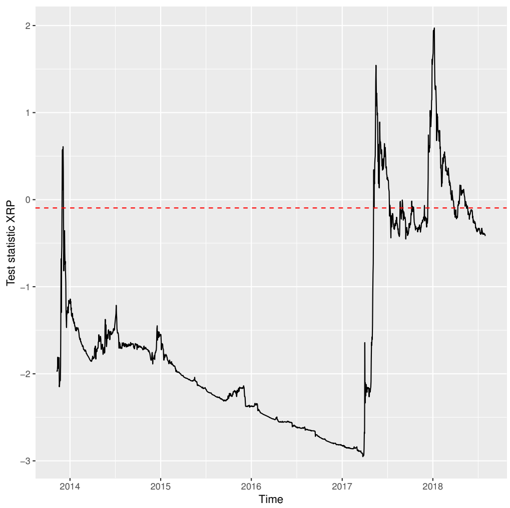

Figure 9: Series of backwards *ADF* statistics for XRP together with its respected CV on a 5% significant level.

ADF XLM:

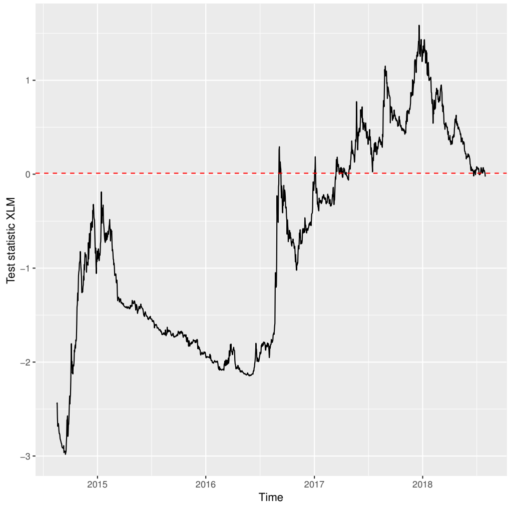

Figure 10: Series of backwards *ADF* statistics for XLM together with its respected CV on a 5% significant level.

B Tables

KPSS test results:

	LAG 1	LAG 2	LAG 3	LAG 4	LAG 5
BTC	15.85	10.58	7.94	6.36	5.30
LTC	17.12	11.42	8.57	6.86	5.72
ETH	2.60	1.74	1.31	1.05	0.88
DASH	12.78	8.53	6.41	5.13	4.28
XRP	16.42	10.96	8.22	6.58	5.49
XLM	13.58	9.07	6.81	5.46	4.56

Table 6: The KPSS test results up to lag 5 with regard to its CVs (below):

KPSS CV:

	10pct	5pct	1pct
critical values	0.12	0.15	0.22

Table 7: The KPSS critical values.

ADF with constant:

	LAG 1	LAG 2	LAG 3	LAG 4	LAG 5
BTC	0.93	0.93	0.92	0.90	0.86
LTC	0.85	0.87	0.86	0.83	0.81
ETH	0.56	0.56	0.58	0.68	0.72
DASH	0.55	0.60	0.83	0.78	0.77
XRP	0.90	0.87	0.85	0.83	0.80
XLM	0.95	0.96	0.96	0.96	0.96

Table 8: The ADF p-values with constant.

ADF without constant:

	LAG 1	LAG 2	LAG 3	LAG 4	LAG 5
BTC	0.99	0.99	0.99	0.99	0.99
LTC	0.77	0.79	0.79	0.78	0.77
ETH	0.96	0.96	0.96	0.96	0.97
DASH	0.85	0.89	0.92	0.91	0.90
XRP	0.21	0.19	0.18	0.18	0.19
XLM	0.86	0.86	0.86	0.88	0.84

Table 9: The ADF p-values without constant.

ADF constant and trend:

	LAG 1	LAG 2	LAG 3	LAG 4	LAG 5
BTC	0.88	0.90	0.90	0.90	0.87
LTC	0.84	0.86	0.87	0.86	0.85
ETH	0.90	0.89	0.91	0.88	0.92
DASH	0.68	0.75	0.76	0.76	0.76
XRP	0.84	0.83	0.82	0.81	0.77
XLM	0.42	0.30	0.21	0.28	0.27

Table 10: The ADF p-values with constant and trend.

Bivariate Granger causality test on 1 lag:

	BTC	LTC	ETH	DASH	XRP	XLM
BTC		0.0000	0.2514	0.2948	0.2934	0.0000
LTC	0.0000		0.2192	0.0479	0.4124	0.0004
ETH	0.0000	0.0001		0.9890	0.0018	0.0295
DASH	0.0000	0.5278	0.1938		0.0000	0.1439
XRP	0.0001	0.0000	0.0000	0.0000		0.0001
XRM	0.0008	0.0365	0.2654	0.5133	0.9509	

Table 11: p-values of the bivariate Granger causality test on lag 1

Bivariate Granger causality test on 2 lags:

	BTC	LTC	ETH	DASH	XRP	XLM
BTC		0.0000	0.0001	0.0000	0.3360	0.0000
LTC	0.0000		0.0011	0.0000	0.4146	0.0000
ETH	0.0000	0.0001		0.6349	0.0025	0.0291
DASH	0.0000	0.0000	0.3831		0.0000	0.2533
XRP	0.0000	0.0000	0.0000	0.0000		0.0000
XRM	0.0030	0.0000	0.2750	0.0144	0.1595	

Table 12: p-values of the bivariate Granger causality test on lag 2

Bivariate Granger causality test on 3 lags:

	BTC	LTC	ETH	DASH	XRP	XLM
BTC		0.0000	0.0001	0.0000	0.1785	0.0000
LTC	0.0000		0.0006	0.0000	0.5188	0.0000
ETH	0.0000	0.0000		0.0342	0.0027	0.0073
DASH	0.0000	0.0000	0.0659		0.0000	0.0001
XRP	0.0000	0.0000	0.0000	0.0000		0.0000
XRM	0.0104	0.0000	0.4608	0.0039	0.3922	

Table 13: p-values of the bivariate Granger causality test on lag 3

Bivariate Granger causality test on 4 lags:

	BTC	LTC	ETH	DASH	XRP	XLM
BTC		0.0000	0.0001	0.0000	0.0000	0.0000
LTC	0.0000		0.0003	0.0000	0.0006	0.0000
ETH	0.0000	0.0000		0.0411	0.0032	0.0155
DASH	0.0000	0.0000	0.0000		0.0000	0.0000
XRP	0.0000	0.0000	0.0000	0.0000		0.0000
XRM	0.0000	0.0000	0.0015	0.0023	0.0101	

Table 14: p-values of the bivariate Granger causality test on lag 4

Bivariate Granger causality test on 5 lags:

	BTC	LTC	ETH	DASH	XRP	XLM
BTC		0.0000	0.0001	0.0000	0.0000	0.0000
LTC	0.0000		0.0000	0.0000	0.0001	0.0000
ETH	0.0000	0.0001		0.0338	0.0076	0.0207
DASH	0.0000	0.0000	0.0001		0.0000	0.0001
XRP	0.0000	0.0000	0.0000	0.0000		0.0000
XRM	0.0000	0.0000	0.0058	0.0013	0.0059	

Table 15: p-values of the bivariate Granger causality test on lag 5

Declaration of Authorship

I hereby confirm that I have authored this Bachelor's thesis independently and without use of others than the indicated sources. All passages which are literally or in general matter taken out of publications or other sources are marked as such.

Berlin, September 1, 2018

Aron Felix Held