# One-Time Pad & Basics of Provable Security (I)

601.642/442: Modern Cryptography

Fall 2022

# Today's Agenda

- Private Communication via One-Time Pads
- Basics of Provable Security Approach

# Before we begin...

A few remarks

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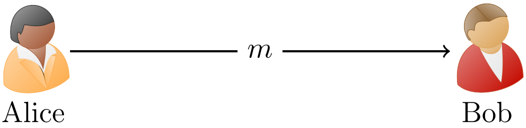
  - What matters is the underlying idea. Always ask "why?"
- View definitions as constructs. They are meant to capture the "rules" of the "game". Think about alternatives.
- Sometimes, the intuition may seem to not align with the proof. But eventually it will, once we make the intuition *robust*.

# Private Communication



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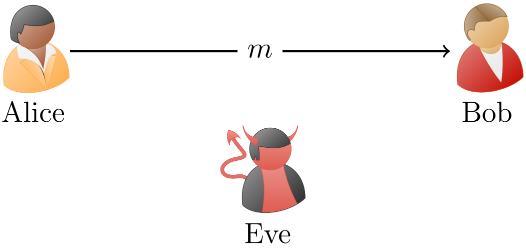

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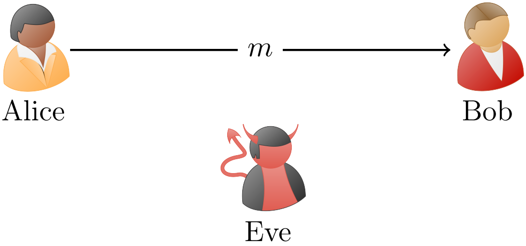


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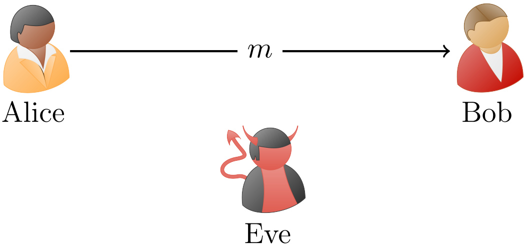
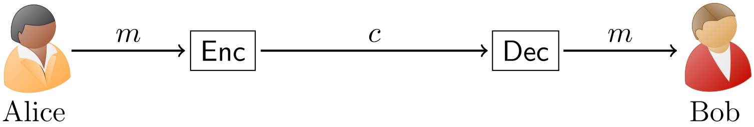



- Alice wants to send a message $m$ to Bob.
- How can Alice convey this message to Bob, while keeping it hidden from an eavesdropper Eve?

# Encryption




- **Encryption:** Alice transforms (encrypts) the message $m$ – also called the "plaintext" – into a "ciphertext" $c$.
- **Communication:** Alice sends $c$ to Bob.
- **Decryption:** Bob recovers (decrypts) the original $m$ from $c$.

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- Inventing good encryption algorithms is not an easy task.

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- The *length* of the secret key is called the **security parameter**.

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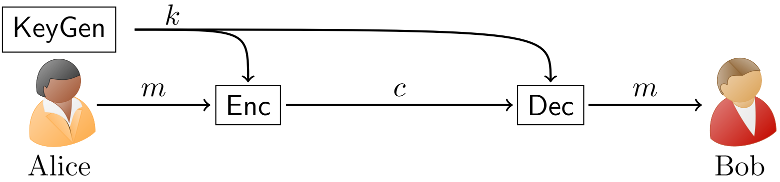

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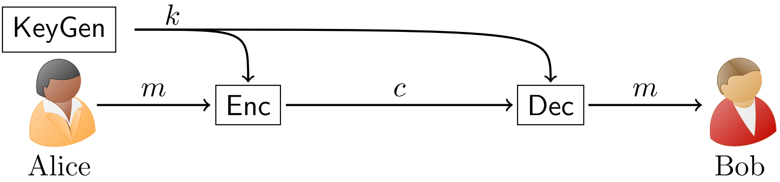


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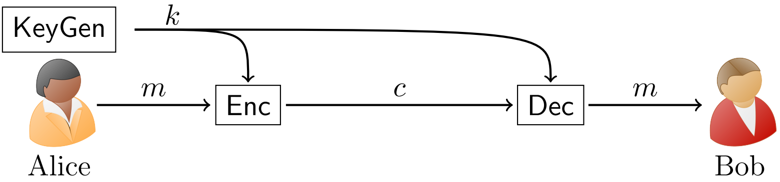


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**Encryption: Syntax**

An **encryption scheme** consists of the following three algorithms:

- $\mathsf{KeyGen}(1^n) \rightarrow k$
- $\mathsf{Enc}(k, m) \rightarrow c$
- $\mathsf{Dec}(k, c) \rightarrow m$

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- We are assuming that all users have the ability to generate random bits. Will discuss randomness generation and usage in more detail later in the course.

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**One-Time Pad: Construction**

- $\mathsf{KeyGen}(1^n) := k \xleftarrow{\$} \{0,1\}^n$
- $\mathsf{Enc}(k, m) := c = k \oplus m$
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- Recall: $k \xleftarrow{\$} \{0,1\}^n$ refers to sampling $k$ uniformly at random from the set of all $n$-bit strings.

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*Proof.* For all $k, m \in \{0,1\}^n$, we have:

$$\begin{aligned} \mathsf{Dec}(k, \mathsf{Enc}(k, m)) &= \mathsf{Dec}(k, (k \oplus m)) \\ &= k \oplus (k \oplus m) \\ &= 0^n \oplus m \\ &= m \end{aligned}$$

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- From the viewpoint of Eve, the following happens: Alice has an $n$-bit message message $m$. She samples some key $k$ *uniformly at random* from the space of $n$-bit strings and then outputs $c = k \oplus m$. Let's analyze this *distribution*.

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**Example**

Let $n = 3$, $m = 010$. What is the value of $c$ for every possible value of $k$?

# One-Time Pad: Security

| Pr | $k$ | $c = k \oplus 010$ |
|---|---|---|
| 1/8 | 000 | 010 |
| 1/8 | 001 | 011 |
| 1/8 | 010 | 000 |
| 1/8 | 011 | 001 |
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- This holds for each $m \in \{0,1\}^3$, and not just $m = 010$.

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  1. Doesn't the above claim conflict with the claim on correctness?
  2. No, because Eve's view does NOT include the key.

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  2. Ones that specify what can happen to a system in the presence of an attacker. E.g., security.
- **Eventual Goal:** How to formally define any *secure* system, in a way that captures all the properties that we need from that system.

# Encryption: Correctness

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Does $\mathsf{Enc}(k, m) = 0^n$ satisfy correctness? Is it a useful encryption scheme?

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An encryption scheme is a good one if its ciphertexts look like random values to Eve, when each key is secret and used to encrypt only one plaintext, even when Eve chooses the plaintexts.

# Encryption: One-Time Uniform Ciphertext Security

## One-Time Uniform Ciphertext Security

We say that an encryption scheme is one-time uniform ciphertext secure if $\forall m \in \mathcal{M}$ chosen by Eve, the ciphertext is uniformly distributed (over the ciphertext space $\mathcal{C}$), i.e., the following distributions are identical:

1. $\mathcal{D}_1 := \left\{ c := \mathsf{Enc}(k, m); k \leftarrow \mathsf{KeyGen}(1^\lambda) \right\}$
2. $\mathcal{D}_2 := \left\{ c \xleftarrow{\$} \mathcal{C} \right\}$

# Insecure Encryption

## Insecure Encryption Scheme

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## Example

Is the following encryption scheme secure?

- $\mathsf{KeyGen}(1^n) := k \xleftarrow{\$} \{0,1\}^n$
- $\mathsf{Enc}(k, m) := c = k \wedge m$ (here $\wedge$ denotes bit-wise AND)

# Insecure Encryption

## Insecure Encryption Scheme

An encryption scheme does not satisfy uniform ciphertext security, if $\exists m \in \mathcal{M}$, such that the following distributions are not identical:

1. $\mathcal{D}_1 := \{c := \mathsf{Enc}(k, m); k \leftarrow \mathsf{KeyGen}(1^n)\}$
2. $\mathcal{D}_2 := \left\{c \stackrel{\$}{\leftarrow} \mathcal{C}\right\}$

## Example

Is the following encryption scheme secure?

- $\mathsf{KeyGen}(1^n) := k \stackrel{\$}{\leftarrow} \{0,1\}^n$
- $\mathsf{Enc}(k, m) := c = k \wedge m$ (here $\wedge$ denotes bit-wise AND)

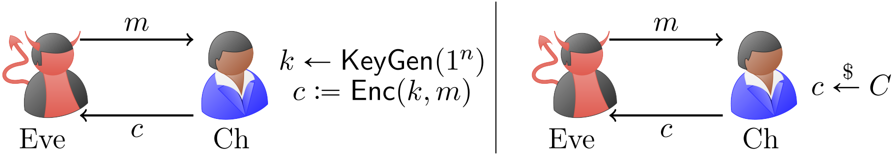
For $m = 0^n$,

$$\Pr[c = 0^n | \mathcal{D}_1] = 1$$
$$\Pr[c = 0^n | \mathcal{D}_2] = 1/2^n$$

Clearly the two distributions are not identical in this case.

# Encryption: One-Time Uniform Ciphertext Security

Consider the following two interactions between Eve and a challenger.




- Interaction with a *challenger* helps us model what Eve can see during encryption, and what remains hidden.
- We say that an encryption scheme is secure if for any $m$ chosen by Eve, the above two scenarios seem identical to Eve.