

Secure Computation - I

CS 601.642/442 Modern Cryptography

Fall 2022

Motivating Example

Consider two billionaires Alice and Bob with net worths x and y , respectively:

- They want to find out who is richer by computing the following function

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

Motivating Example

Consider two billionaires Alice and Bob with net worths x and y , respectively:

- They want to find out who is richer by computing the following function

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

- Potential Solution: Alice sends x to Bob, who sends y to Alice. They each compute f on their own.

Motivating Example

Consider two billionaires Alice and Bob with net worths x and y , respectively:

- They want to find out who is richer by computing the following function

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

- Potential Solution: Alice sends x to Bob, who sends y to Alice. They each compute f on their own.
- Problem: Alice learns Bob's net worth (and vice-versa). No privacy!

Motivating Example

Consider two billionaires Alice and Bob with net worths x and y , respectively:

- They want to find out who is richer by computing the following function

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

- Potential Solution: Alice sends x to Bob, who sends y to Alice. They each compute f on their own.
- Problem: Alice learns Bob's net worth (and vice-versa). No privacy!
- Main Question: Can Alice and Bob compute f in a “secure manner” s.t. they only learn the output of f , and *nothing more*?

General Setting

Two parties A and B , with private inputs x and y , respectively:

- They want to “securely” compute a function f

General Setting

Two parties A and B , with private inputs x and y , respectively:

- They want to “securely” compute a function f
- If both A and B are honest, then they should learn the output $f(x, y)$

General Setting

Two parties A and B , with private inputs x and y , respectively:

- They want to “securely” compute a function f
- If both A and B are honest, then they should learn the output $f(x, y)$
- Even if one party is adversarial, it should not learn anything beyond the output (and its own input)

General Setting

Two parties A and B , with private inputs x and y , respectively:

- They want to “securely” compute a function f
- If both A and B are honest, then they should learn the output $f(x, y)$
- Even if one party is adversarial, it should not learn anything beyond the output (and its own input)
- Think: How to formalize this security requirement?

Types of Adversaries

Two types of adversaries:

Types of Adversaries

Two types of adversaries:

- **Honest but curious (a.k.a. semi-honest):** Such an adversary follows the instructions of the protocol, but will later analyze the protocol transcript to learn any “extra information” about the input of the other party

Types of Adversaries

Two types of adversaries:

- **Honest but curious (a.k.a. semi-honest):** Such an adversary follows the instructions of the protocol, but will later analyze the protocol transcript to learn any “extra information” about the input of the other party
- **Malicious:** Such an adversary can deviate from the protocol instructions and follow an arbitrary strategy

Types of Adversaries

Two types of adversaries:

- **Honest but curious (a.k.a. semi-honest):** Such an adversary follows the instructions of the protocol, but will later analyze the protocol transcript to learn any “extra information” about the input of the other party
- **Malicious:** Such an adversary can deviate from the protocol instructions and follow an arbitrary strategy

Note: We will only consider *semi-honest* adversaries

Secure Computation: Intuition

- Want to formalize that no semi-honest adversary learns anything from the protocol execution beyond its input and the (correct) output

Secure Computation: Intuition

- Want to formalize that no semi-honest adversary learns anything from the protocol execution beyond its input and the (correct) output
- Idea: Use simulation paradigm, as in zero-knowledge proofs

Secure Computation: Intuition

- Want to formalize that no semi-honest adversary learns anything from the protocol execution beyond its input and the (correct) output
- Idea: Use simulation paradigm, as in zero-knowledge proofs
- View of adversary in the protocol execution can be efficiently simulated given only its input and output, and without the input of the honest party

Secure Computation: Definition

Definition (Semi-honest Secure Computation)

A protocol $\pi = (A, B)$ securely computes a function f in the semi-honest model if there exists a pair of non-uniform PPT simulator algorithms $\mathcal{S}_A, \mathcal{S}_B$ such that for every security parameter n , and all inputs $x, y \in \{0, 1\}^n$, it holds that:

$$\left\{ \mathcal{S}_A(x, f(x, y)), f(x, y) \right\} \approx \left\{ e \leftarrow [A(x) \leftrightarrow B(y)] : \text{View}_A(e) \right\},$$

$$\left\{ \mathcal{S}_B(y, f(x, y)), f(x, y) \right\} \approx \left\{ e \leftarrow [A(x) \leftrightarrow B(y)] : \text{View}_B(e) \right\}.$$

Remarks on Definition

- In addition to the above security requirement, we also need protocol correctness property, namely, that the outputs of the parties corresponds to the correct function evaluation on the inputs when all the parties are honest

Remarks on Definition

- In addition to the above security requirement, we also need protocol correctness property, namely, that the outputs of the parties corresponds to the correct function evaluation on the inputs when all the parties are honest
- Since semi-honest adversary behaves honestly during the protocol, the above property implies that outputs of honest parties are correct even when some parties are semi-honest

Remarks on Definition

- In addition to the above security requirement, we also need protocol correctness property, namely, that the outputs of the parties corresponds to the correct function evaluation on the inputs when all the parties are honest
- Since semi-honest adversary behaves honestly during the protocol, the above property implies that outputs of honest parties are correct even when some parties are semi-honest
- More tricky in the case of malicious adversaries (security definition much more non-trivial)

Oblivious Transfer

Consider the following functionality, called, 1-out-of-2 oblivious transfer (OT):

Oblivious Transfer

Consider the following functionality, called, 1-out-of-2 oblivious transfer (OT):

- Two parties: Sender A , and Receiver B

Oblivious Transfer

Consider the following functionality, called, 1-out-of-2 oblivious transfer (OT):

- Two parties: Sender A , and Receiver B
- Inputs: A 's input is a pair of bits (a_0, a_1) , and B 's input is a bit b

Oblivious Transfer

Consider the following functionality, called, 1-out-of-2 oblivious transfer (OT):

- Two parties: Sender A , and Receiver B
- Inputs: A 's input is a pair of bits (a_0, a_1) , and B 's input is a bit b
- Outputs: B 's output is a_b , and A receives no output

Oblivious Transfer

Consider the following functionality, called, 1-out-of-2 oblivious transfer (OT):

- Two parties: Sender A , and Receiver B
- Inputs: A 's input is a pair of bits (a_0, a_1) , and B 's input is a bit b
- Outputs: B 's output is a_b , and A receives no output

Note: Definition of secure computation promises that in a secure OT protocol, A does not learn b and B does not learn a_{1-b}

Importance of Oblivious Transfer

- Can be realized from physical channels [Wiener,Rabin]

Importance of Oblivious Transfer

- Can be realized from physical channels [Wiener,Rabin]
- **OT is complete:** given a secure protocol for OT, any function can be securely computed

Importance of Oblivious Transfer

- Can be realized from physical channels [Wiener,Rabin]
- **OT is complete:** given a secure protocol for OT, any function can be securely computed
- **OT is necessary:** OT is the minimal assumption for secure computation

Oblivious Transfer: Construction

Let $\{f_i\}_{i \in \mathcal{I}}$ be a family of trapdoor permutations with sampling algorithm Gen . Let h be a hardcore predicate for any f_i .

Sender's input: (a_0, a_1) where $a_i \in \{0, 1\}$

Receiver's input: $b \in \{0, 1\}$

Protocol OT = (A, B) :

$A \rightarrow B$: A samples $(f_i, f_i^{-1}) \leftarrow \text{Gen}(1^n)$ and sends f_i to B

$B \rightarrow A$: B samples $x \xleftarrow{\$} \{0, 1\}^n$ and computes $y_b = f_i(x)$. It also samples $y_{1-b} \xleftarrow{\$} \{0, 1\}^n$. B sends (y_0, y_1) to A

$A \rightarrow B$: A computes the inverse of each value y_j and XORs the hard-core bit of the result with a_j :

$$z_j = h(f_i^{-1}(y_j)) \oplus a_j$$

A sends (z_0, z_1) to B

$B(x, b, z_0, z_1)$: B outputs $h(x) \oplus z_b$

OT = (A, B) is Semi-honest Secure : Intuition

- Security against A : Both y_0 and y_1 are uniformly distributed and therefore independent of b . Thus, b is hidden from A

OT = (A, B) is Semi-honest Secure : Intuition

- Security against A : Both y_0 and y_1 are uniformly distributed and therefore independent of b . Thus, b is hidden from A
- Security against B : If B could learn a_{1-b} , then it would be able to predict the hardcore predicate

OT = (A, B) is Semi-honest Secure : Intuition

- Security against A : Both y_0 and y_1 are uniformly distributed and therefore independent of b . Thus, b is hidden from A
- Security against B : If B could learn a_{1-b} , then it would be able to predict the hardcore predicate

OT = (A, B) is Semi-honest Secure : Intuition

- Security against A : Both y_0 and y_1 are uniformly distributed and therefore independent of b . Thus, b is hidden from A
- Security against B : If B could learn a_{1-b} , then it would be able to predict the hardcore predicate

Note: A *malicious* B can easily learn a_{1-b} by deviating from the protocol strategy

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_A

Simulator $\mathcal{S}_A((a_0, a_1), \perp)$:

- ① Fix a random tape r_A for A . Run honest emulation of A using (a_0, a_1) and r_A to obtain the first message f_i
- ② Choose two random strings $y_0, y_1 \in \{0, 1\}^n$ as B 's message
- ③ Run honest emulation of A using (y_0, y_1) to obtain the third message (z_0, z_1)
- ④ Stop and output \perp

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_A

Simulator $\mathcal{S}_A((a_0, a_1), \perp)$:

- ① Fix a random tape r_A for A . Run honest emulation of A using (a_0, a_1) and r_A to obtain the first message f_i
- ② Choose two random strings $y_0, y_1 \in \{0, 1\}^n$ as B 's message
- ③ Run honest emulation of A using (y_0, y_1) to obtain the third message (z_0, z_1)
- ④ Stop and output \perp

Claim: The following two distributions are identical:

$$\left\{ \mathcal{S}_A((a_0, a_1), \perp), a_b \right\} \text{ and } \left\{ e \leftarrow [A(a_0, a_1) \leftrightarrow B(b)] : \text{View}_A(e), \text{Out}_B(e) \right\}$$

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_A

Simulator $\mathcal{S}_A((a_0, a_1), \perp)$:

- 1 Fix a random tape r_A for A . Run honest emulation of A using (a_0, a_1) and r_A to obtain the first message f_i
- 2 Choose two random strings $y_0, y_1 \in \{0, 1\}^n$ as B 's message
- 3 Run honest emulation of A using (y_0, y_1) to obtain the third message (z_0, z_1)
- 4 Stop and output \perp

Claim: The following two distributions are identical:

$$\left\{ \mathcal{S}_A((a_0, a_1), \perp), a_b \right\} \text{ and } \left\{ e \leftarrow [A(a_0, a_1) \leftrightarrow B(b)] : \text{View}_A(e), \text{Out}_B(e) \right\}$$

Proof: The only difference between \mathcal{S}_A and real execution is in step 2. However, since f is a permutation, y_0, y_1 are identically distributed in both cases.

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_B

Simulator $\mathcal{S}_B(b, a_b)$:

- ① Sample f_i
- ② Choose random tape r_B for B . Run honest emulation of B using (b, r_B, f_i) to produce (x, y_0, y_1) s.t. $y_b = f_i(x)$ and $y_{1-b} \xleftarrow{\$} \{0, 1\}^n$
- ③ Compute $z_b = h(x) \oplus a_b$ and $z_{1-b} \xleftarrow{\$} \{0, 1\}$
- ④ Output (z_0, z_1) as third message and stop

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_B

Simulator $\mathcal{S}_B(b, a_b)$:

- ① Sample f_i
- ② Choose random tape r_B for B . Run honest emulation of B using (b, r_B, f_i) to produce (x, y_0, y_1) s.t. $y_b = f_i(x)$ and $y_{1-b} \xleftarrow{\$} \{0, 1\}^n$
- ③ Compute $z_b = h(x) \oplus a_b$ and $z_{1-b} \xleftarrow{\$} \{0, 1\}$
- ④ Output (z_0, z_1) as third message and stop

Claim: The following two distributions are indistinguishable:

$$\left\{ \mathcal{S}_B(b, a_b), \perp \right\} \text{ and } \left\{ e \leftarrow [A(a_0, a_1) \leftrightarrow B(b)] : \text{View}_B(e), \text{Out}_A(e) \right\}$$

OT = (A, B) is Semi-honest Secure : Simulator \mathcal{S}_B

Simulator $\mathcal{S}_B(b, a_b)$:

- 1 Sample f_i
- 2 Choose random tape r_B for B . Run honest emulation of B using (b, r_B, f_i) to produce (x, y_0, y_1) s.t. $y_b = f_i(x)$ and $y_{1-b} \xleftarrow{\$} \{0, 1\}^n$
- 3 Compute $z_b = h(x) \oplus a_b$ and $z_{1-b} \xleftarrow{\$} \{0, 1\}$
- 4 Output (z_0, z_1) as third message and stop

Claim: The following two distributions are indistinguishable:

$$\left\{ \mathcal{S}_B(b, a_b), \perp \right\} \text{ and } \left\{ e \leftarrow [A(a_0, a_1) \leftrightarrow B(b)] : \text{View}_B(e), \text{Out}_A(e) \right\}$$

Proof: The only difference is in step 3, where \mathcal{S}_B computes z_{1-b} as a random bit. However, since $h(f_i^{-1}(y_{1-b}))$ is indistinguishable from random (even given y_{1-b}), this change is indistinguishable

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

Semi-honest vs Malicious:

- In reality, adversary may be malicious and not semi-honest

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

Semi-honest vs Malicious:

- In reality, adversary may be malicious and not semi-honest
- Goldreich-Micali-Wigderson [GMW] gave a compiler to transform *any* protocol secure against semi-honest adversary into one secure against malicious adversary

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

Semi-honest vs Malicious:

- In reality, adversary may be malicious and not semi-honest
- Goldreich-Micali-Wigderson [GMW] gave a compiler to transform *any* protocol secure against semi-honest adversary into one secure against malicious adversary
- The transformation uses coin-flipping (to make sure that adversary's random tape is truly random) and zero-knowledge proofs (to make sure that adversary is following the protocol instructions)

1-out-of- k OT:

- The previous protocol can be easily generalized to construct 1-out-of- k OT for $k > 2$

Semi-honest vs Malicious:

- In reality, adversary may be malicious and not semi-honest
- Goldreich-Micali-Wigderson [GMW] gave a compiler to transform *any* protocol secure against semi-honest adversary into one secure against malicious adversary
- The transformation uses coin-flipping (to make sure that adversary's random tape is truly random) and zero-knowledge proofs (to make sure that adversary is following the protocol instructions)
- Details outside the scope of this class