Authentication

CS 601.642/442 Modern Cryptography

Fall 2022

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- Adversary cannot *forge* a signature

1. Private Key: Message Authentication Codes

Two Types

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2. Public Key: Digital Signatures

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Security: An adversary can observe multiple (message,tag) pairs of its choice, but still cannot forge a tag on a new message

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- Security (UF-CMA): For all n.u. PPT adversary $\mathcal{A}$ there exists a negligible $\nu(\cdot)$ such that:

$$\Pr \left[\begin{array}{c} k \leftarrow \mathsf{Gen}(1^n) \\ (m,\sigma) \leftarrow \mathcal{A}^{\mathsf{Tag}_k(\cdot)}(1^n) \end{array} \colon \begin{array}{c} \mathcal{A} \text{ did not query } m \ \wedge \\ \mathsf{Ver}_k(m,\sigma) = 1 \end{array} \right] \leqslant \nu(n)$$

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- <u>Think</u>: Proof?

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- Think & Read

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<u>Think</u>: How to sign long messages?

Collision-resistant Hash Functions

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  - <u>Think</u>: Why?
- Need to consider a family of hash functions

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A family of functions $H = \{h_i : D_i \to R_i\}_{i \in I}$ is a collision-resistant hash function family (CRHF) if:

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  - Can be constructed from number-theoretic assumptions such as factoring, discrete log

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  - More efficient construction
    [Haitner-Holenstein-Reingold-Vadhan-Wee10]

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- <u>Read</u>: Tree-based signatures

Multi-message Signatures (via chain)

- $(sk_0, pk_0) \stackrel{\$}{\leftarrow} \mathsf{Gen}(1^n)$
- <u>Initialize</u>: $\tilde{\sigma}_i = \emptyset$, $i = 1$
- To sign m_i:
  - $(sk_i, pk_i) \stackrel{\$}{\leftarrow} \mathsf{Gen}(1^n)$
  - $\tilde{\sigma}_i \leftarrow \mathsf{Sign}_{sk_{i-1}}(m_i \| pk_i)$
  - Output: $\sigma_i = (i, \tilde{\sigma}_i, m_i, pk_i, \sigma_{i-1})$
  - Increment i
- <u>Think</u>: Proof?
- <u>Think</u>: How to reduce signature size?
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- <u>Read</u>: Full Domain Hash (RSA-based signatures)