Differential Entropy

* Extension of entropy for continuous 1.v.s.

* Let X be r.v. with pof f whose support is 7.

Differential entropy $h(x) = -\int f(n) \log f(n) dx$

Differential entropy
$$h(x) = -\int_{X} f(x) \log f(x) dx$$

$$f(7) = \frac{1}{b-a}$$

$$h(x) = -\int_{b-a}^{b} \frac{1}{b-a} \log \left(\frac{1}{b-a}\right) dx = -\left[\left(\frac{1}{b-a}\right) \log \frac{1}{b-a}\right]_{a}^{b}$$

$$= -\left[\left(\frac{1}{b-a}\right) \log \frac{1}{b-a}\right]$$

$$= -\left[\left(\frac{1}{b-a} \right) \frac{\log \frac{1}{b-a}}{b-a} \right]$$

$$= \log (b-a)$$

Differential entropy
$$h(x) = -\int f(x) \log f(x) dx$$

$$f(\pi) = \frac{1}{\sqrt{2\pi}6^2} \exp\left(-\frac{(\pi-\mu)^2}{26^2}\right) : \mathcal{T} = [-\infty, \infty]$$

(2) =
$$\frac{1}{2\pi p} \left(\frac{12-p}{2} \right)$$
 : $\pi = \left[-\infty, \infty \right]$

 $h(x) = -\left(\frac{1}{\sqrt{2\pi 6^2}} \exp\left(-\frac{(n-\mu)^2}{26^2}\right) dn\right)$

$$h(x) = -\int f(x) \log f(x) dx$$

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$f(\pi) = \frac{1}{\sqrt{276^2}}$	en (- (2-e)2	 N=	[-0,0	æ]	 	
J 27 62	7 (262)				 	

 $h(x) = -\int \frac{1}{\sqrt{2\pi6^2}} \exp\left(-\frac{(\pi-\mu)^2}{26^2}\right) \left[\log\left(\frac{1}{\sqrt{2\pi6^2}}\right) - \left(\frac{6-\mu}{26^2}\right)\right] dz$

 $= \log_e(6\sqrt{2\pi}e) = \frac{1}{2}\log(2\pi e^2)$

Differential entropy
$$h(x) = -\int_{x}^{x} f(x) \log f(x) dx$$

$$x \sim N_{\mu}(\mu, \Xi)$$

$$h(x) = \frac{1}{2} \log \left[(2\pi e)^{2} |\Xi| \right]$$

Differential entropy
$$h(x) = -\int_{x}^{x} f(x) \log f(x) dx$$

 $h(x) = \frac{1}{2} \log \left[(2\pi e)^{n} | \Xi | \right]$

e.g. $\times \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ $h_{1}(x) = \frac{1}{2} \log \left[\left(2\pi e\right)^{2}(1)\right]$

Differential entropy
$$h(x) = -\int f(x) \log f(x) dx$$

x~ N(1, E)

 $h(x) = \frac{1}{2} \log \left[(2\pi e)^{n} | \Xi | \right]$

eg. $\chi_{1}^{n} N \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \\ 0 & 1 \end{bmatrix}$ $h(\chi) = \frac{1}{2} log [(2\pi e)^{2}(1)] h(\chi_{2}) = \frac{1}{2} log [(2\pi e)^{2}(0.75)]$

Differential entropy
$$h(x) = -\int f(x) \log f(x) dx$$

$$X_{1} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$$

$$X_{2} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{2} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{3} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{4} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{5} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{6} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{1} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{2} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

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$$X_{5} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{1} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

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$$X_{2} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}1\\0.5\end{bmatrix}\right)$$

$$X_{1} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$h(x) = \frac{1}{2} \log \left[(2\pi e)^2 (1) \right] h(x_2) = \frac{1}{2} \log \left[(2\pi e)^2 0.75 \right]$$

$$2h(x_2) - 2h(x_1) = \log \left[\frac{(2\pi e)^2}{(2\pi e)^2} (0.75) \right]$$

$$2h(x_2) - 2h(x_1) = \log \left(\frac{(x_1 + 1)^2}{(x_1 + 1)^2} - \frac{(x_1 + 1)^2}{(x_1 + 1)^2}\right)$$

$$= \log (0.75) = -0.28$$

=) More endropy in X1

(ONDITIONAL DIFFERENTIAL ENTROPY

Let X and Y be continuous rus with joint paf

f(x,y)

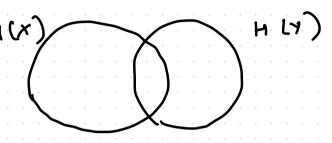
(on diffigural Entropy
$$h(X|Y) = -\int f(x,y) \log f(x|y) dxdy$$

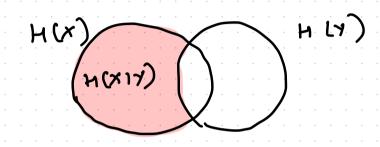
Conditional Entropy $h(X|Y) = -\int f(x,y) \log f(x|y) dxdy$

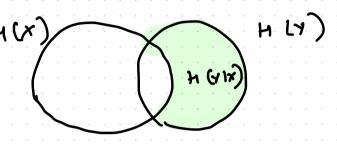
h(x|y) = h(x,y) - h(y)

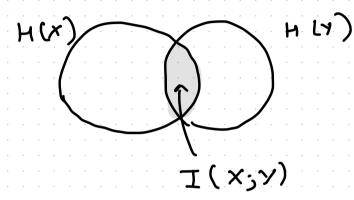


H- Discude h- Continuous

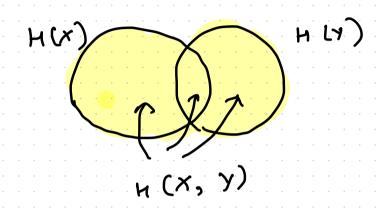








$$\frac{\left[T(x',y') - H(y') - H(y')x\right]}{\left[T(x',y') - h(y) - h(y')x\right]}$$
 Discrete
$$\frac{\left[T(x',y') - h(y')x\right]}{\left[T(x',y') - h(y')x\right]}$$
 Continuous



Let
$$X \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 3 & 1 \end{bmatrix})$$

 $h(x) = h(x_1, x_2) = \frac{1}{2} log [(2\pi e)^m | \Xi]$

= 1 log ((27-e) (1-p²))

Let
$$X \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 9 & 1 \end{bmatrix})$$

$$\begin{pmatrix} X_1 \\ m \end{pmatrix} \sim N \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix} \end{bmatrix}$$

Let
$$\times \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 3 & 1 \end{bmatrix})$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 9 \end{pmatrix} \end{bmatrix}$$

= 1 log ((27-e) (1-p²))

 $h(x_1) = \frac{1}{2} \log (2\pi e) = h(x_2)$

$$\begin{bmatrix} 1 & 3 & 5 \\ 5 & 1 & 1 \end{bmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

 $h(x) = h(x_1, x_2) = \frac{1}{2} log [(2\pi e)^m | \Xi |]$

Let
$$\times \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 3 & 1 \end{bmatrix})$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 9 \end{pmatrix} \end{bmatrix}$$

$$(x_1) \sim N(0), (19)$$

 $h(x) = \frac{1}{2} log((2\pi e)^2(1-p^2))$

 $h(x_1) = \frac{1}{2} log(2\pi e) = h(x_2)$

$$N \left(\left(\begin{array}{c} 0 \\ 0 \end{array} \right), \left(\begin{array}{c} 1 \\ P \end{array} \right) \right)$$

 $h(x_1|x_2) = h(x_1,x_2) - h(x_2) = h(x) - h(x_2)$ = $\frac{1}{2} log \frac{(2\pi e)^2 (1-g^2)}{(2\pi e)}$

Let
$$X \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 3 & 1 \end{bmatrix})$$

$$h(x) = \frac{1}{2} \log ((2\pi e)^2 (1-p^2))$$

 $h(x_1) = \frac{1}{2} log(2\pi e) = h(x_2)$

 $h(x_1|x_2) = h(x_1,x_2) - h(x_2) = h(x) - h(x_2)$ = $\frac{1}{2} log \frac{(2\pi e)^2(1-g^2)}{(2\pi e)}$

 $I(X_1;X_2) = h(X_1) - h(X_1|X_2)$ $= \frac{1}{2}\log 2\pi e - \frac{1}{2}\log 2\pi e \left(1-p^2\right) = \frac{-1}{2}\log \left(1-p^2\right)$

Let
$$X \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 9 & 1 \end{bmatrix})$$

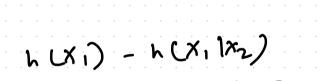
$$(x,x) = h(x_1) - h(x_1)x_2$$

$$I(X_1;X_2) = h(X_1) - h(X_1|X_2)$$

$$= \frac{1}{2} \log 2\pi e - \frac{1}{2} \log 2\pi e \left(1 - \frac{p^2}{2}\right) = \frac{1}{2} \log \left(1 - \frac{p^2}{2}\right)$$

 $p \rightarrow 1 = 1(x_1;x_2) \rightarrow \infty$

Perfect Cordat: M.I. -> 00





Let
$$\times \sim N_2(0, \begin{bmatrix} 1 & 9 \\ 9 & 1 \end{bmatrix})$$

$$h(x_1|x_2)$$

$$(x_2) = h(x_1) - h(x_1)x_2$$

$$=\frac{1}{3}\log 2\pi e - \frac{1}{2}\log 2\pi e$$

$$(x_1; x_2) = h(x_1) - x_1(x_2) = h(x_1) - x_2(x_2) = h(x_1) - x_1(x_2) = h(x_1) - x_2(x_2) = h(x_1) - x_$$

 $p \rightarrow \pm 1 \Rightarrow I(x_1; x_2) \rightarrow \infty$

Perfect Correlata: M.I. $\rightarrow \infty$ $g \rightarrow 0 \rightarrow I(X_1; X_2) \rightarrow 0$

$$I(X_1;X_2) = h(X_1) - h(X_1|X_2)$$

$$= \frac{1}{2}\log 2\pi e - \frac{1}{2}\log 2\pi e \left(1-p^2\right) = \frac{-1}{2}\log \left(1-p^2\right)$$

$$2\pi e (1-p^2) = -\frac{1}{2} \log (1-p^2)$$

No correlat : M. I. → O

