An Introduction to Algorithms

By

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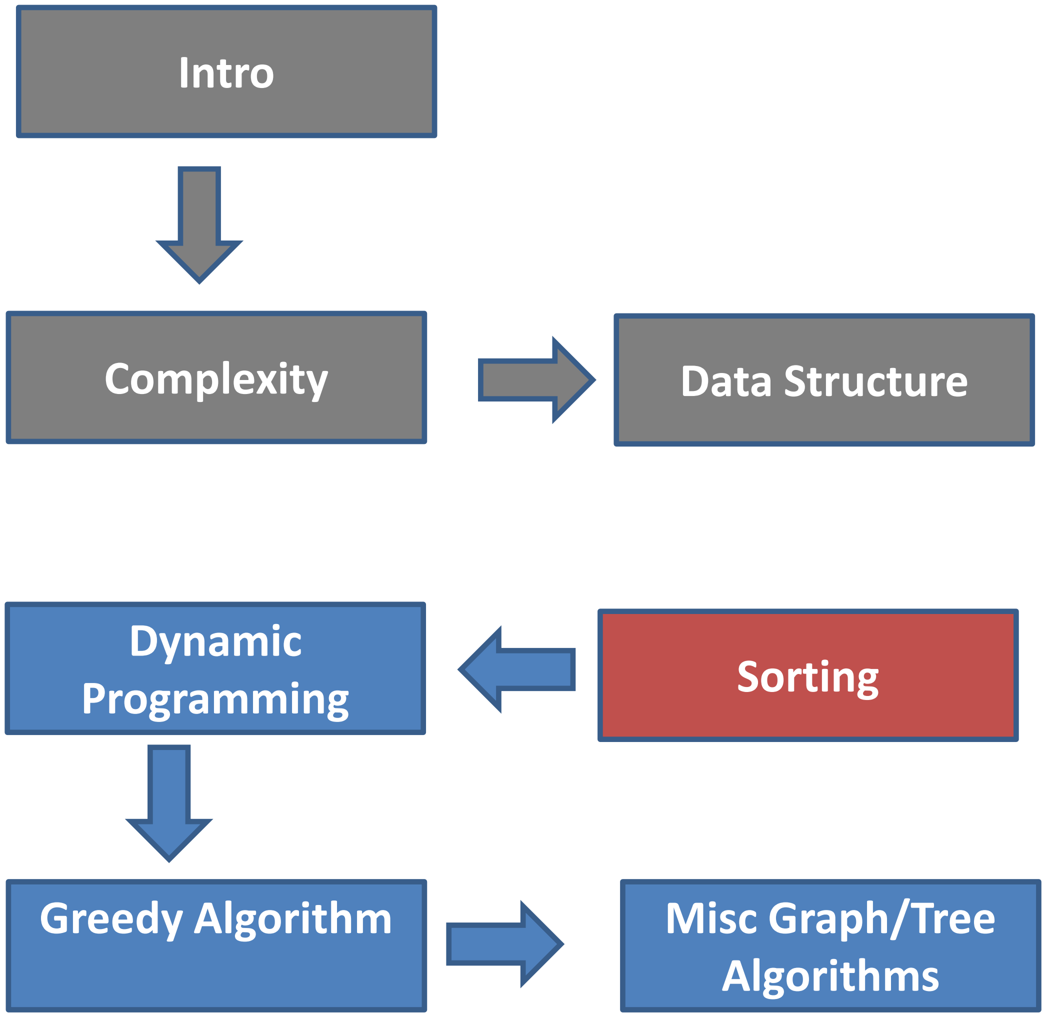

Intro
Complexity
Data Structure
Trees
Hash Functions
Sorting
Dynamic Programming
Greedy Algorithm
Misc Graph/Tree Algorithms

O(N log N) Sorting Algorithms

MergeSort

HeapSort

QuickSort

Merge Sort

```
MergeSort(A, left, right) {
   if (left < right) {
      mid = floor((left + right) / 2);
      MergeSort(A, left, mid);
      MergeSort(A, mid+1, right);
      Merge(A, left, mid, right);
   }
}

// Merge() takes two sorted subarrays of A and
// merges them into a single sorted subarray of A
//    (how long should this take?)
```

Merge Sort Example

99	6	86	15	58	35	86	4	0

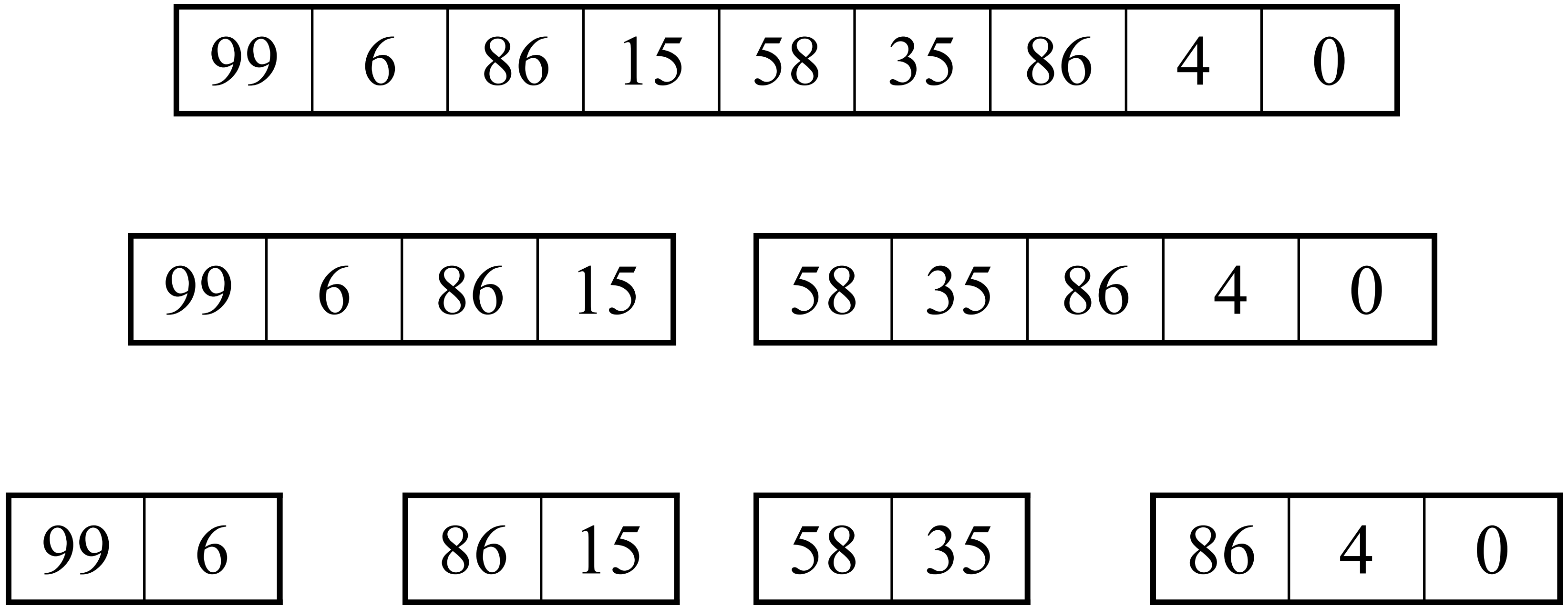
Merge Sort Example

99	6	86	15	58	35	86	4	0

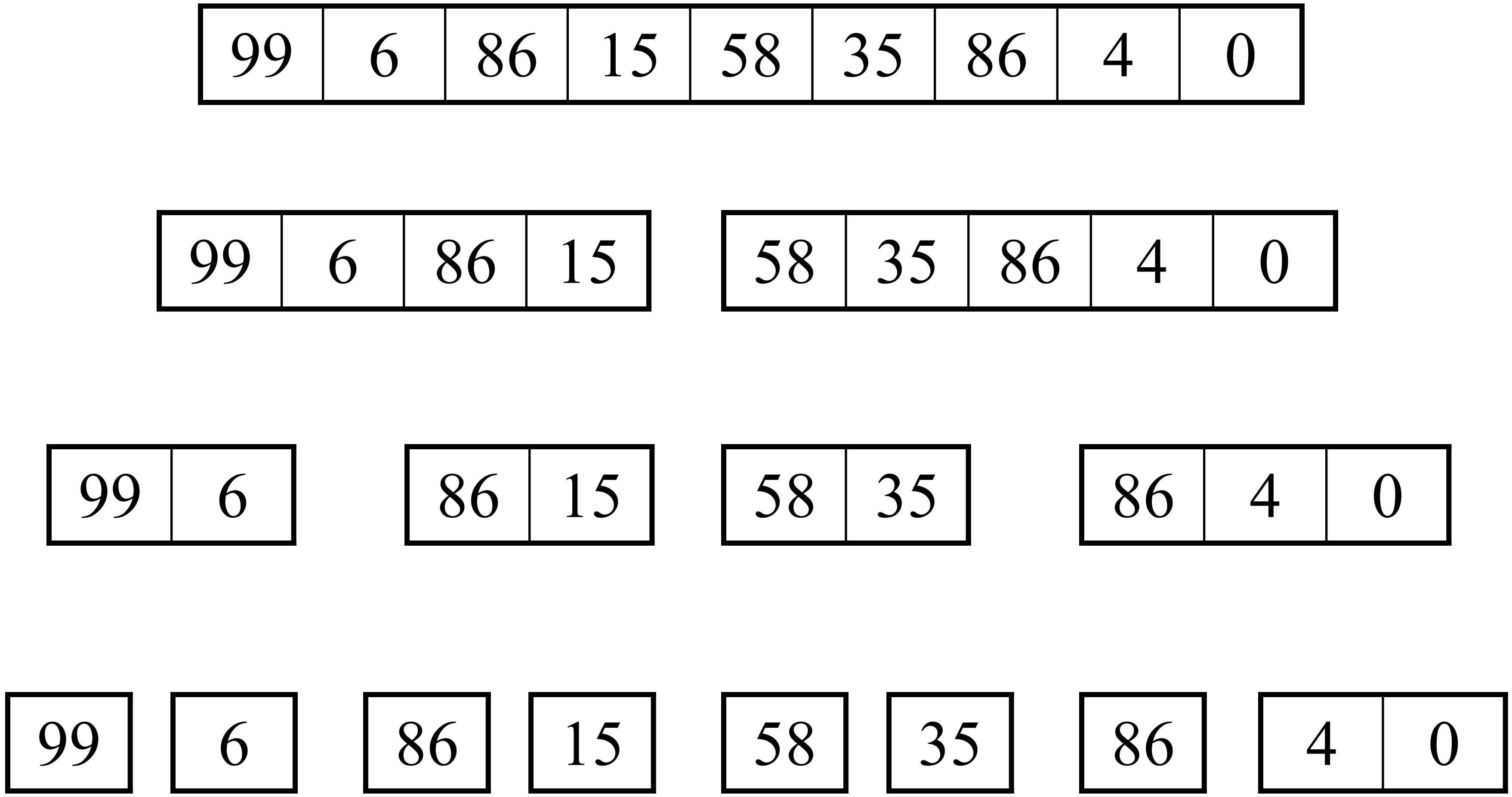
99	6	86	15

58	35	86	4	0

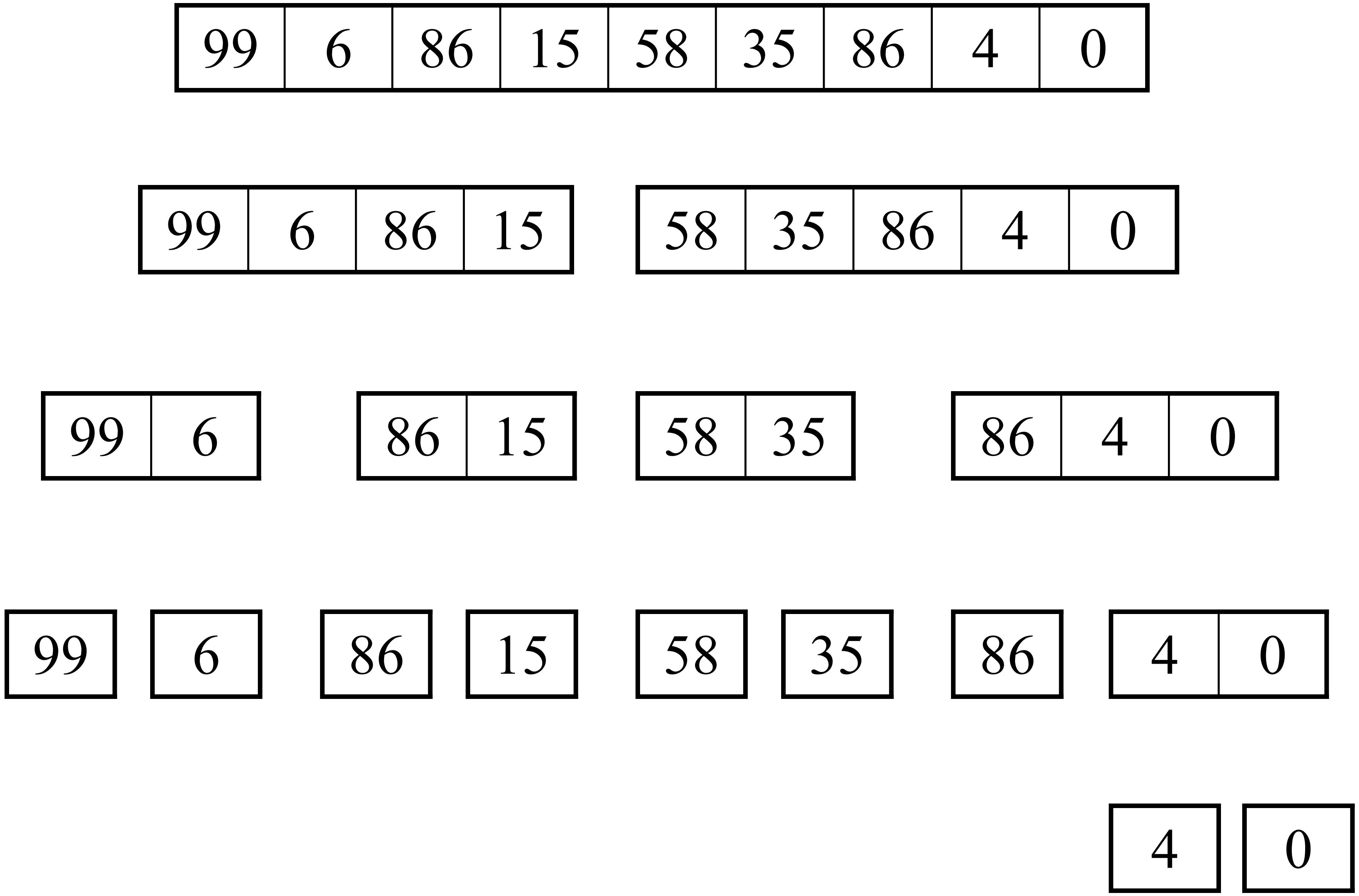
Merge Sort Example



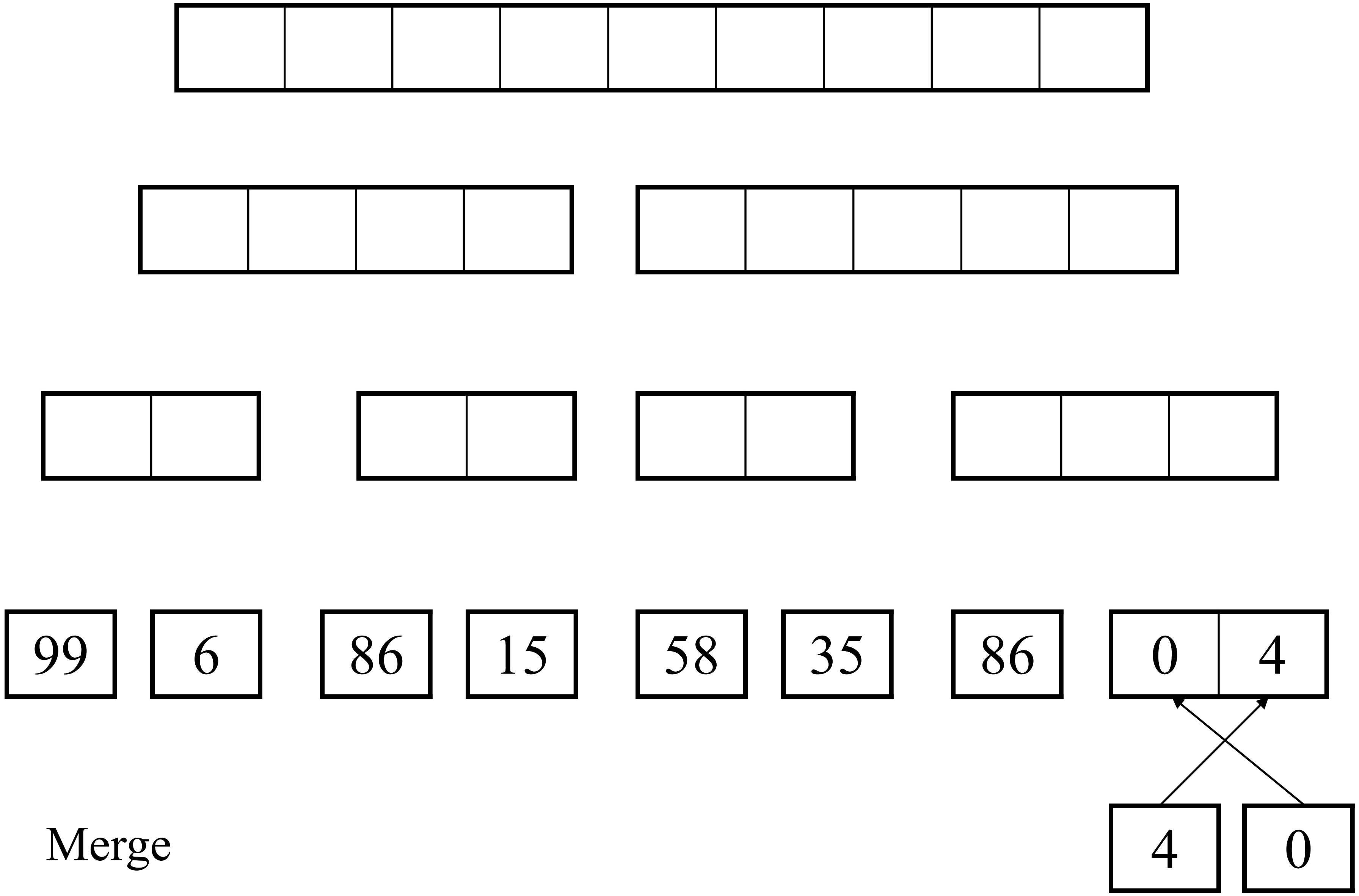
Merge Sort Example



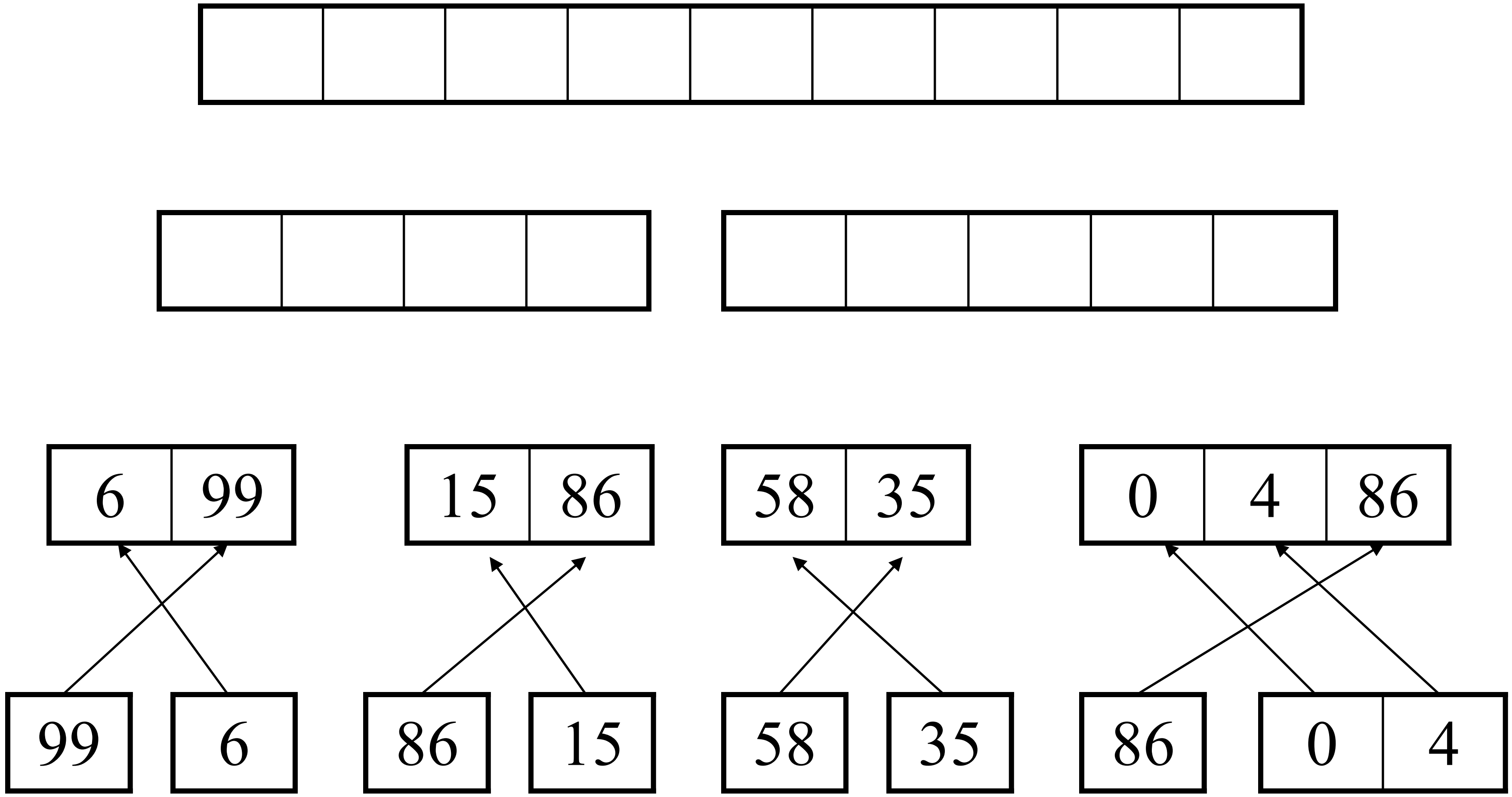
Merge Sort Example



Merge Sort Example

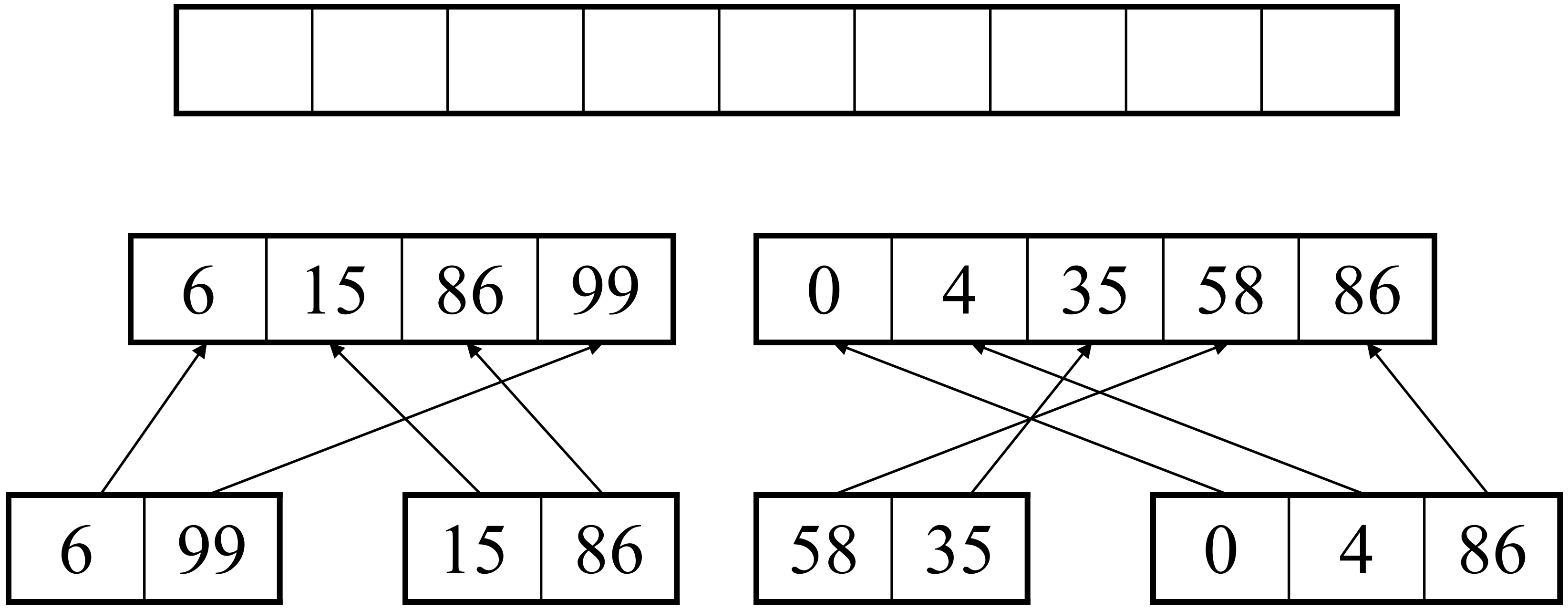


Merge Sort Example



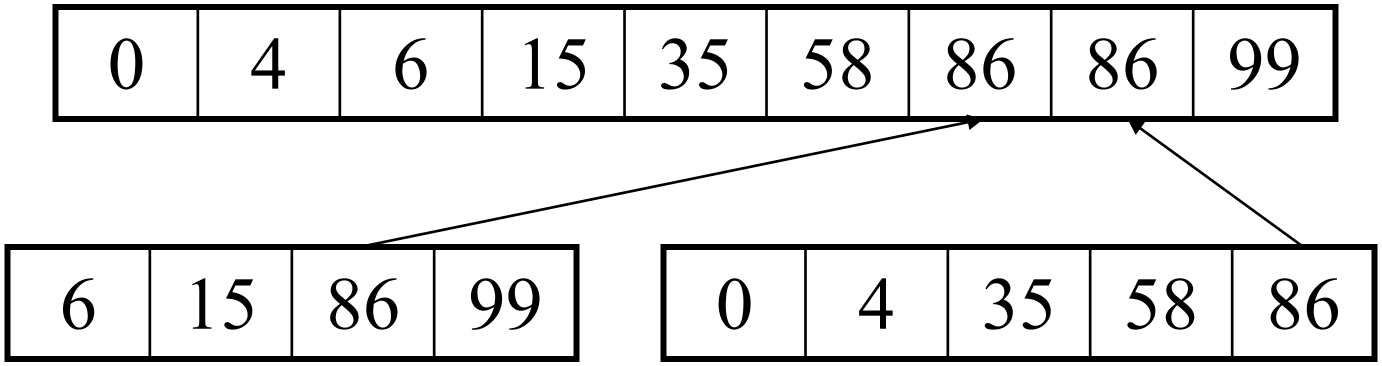
Merge

Merge Sort Example



Merge

Merge Sort Example



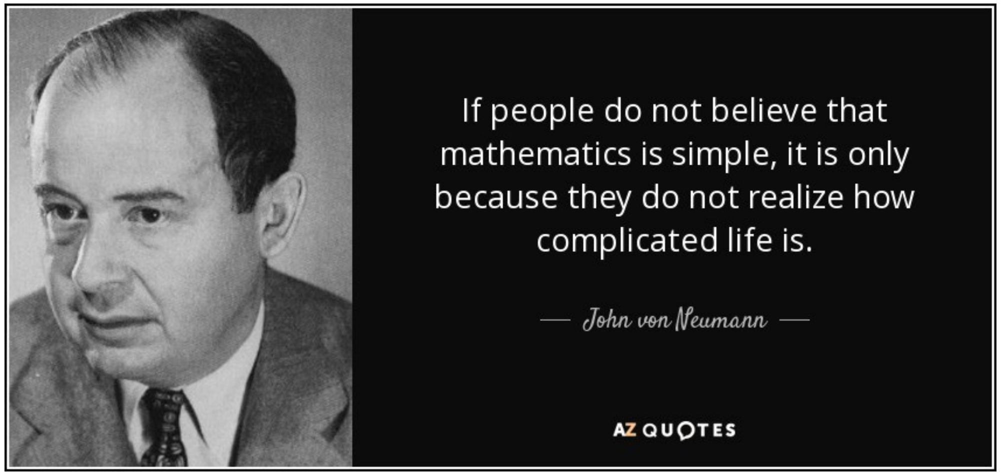
Merge

Merge Sort Example

0	4	6	15	35	58	86	86	99

Merge Sort

Don Knuth cites John von Neumann as the creator of this algorithm


If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

— John von Neumann —

AZ QUOTES

The famous mathematician John von Neumann (1903-1957) was born in a Jewish family in Budapest, Austro-Hungarian Empire, as János Lajos von Neumann. A child-prodigy, János received his Ph.D. in mathematics from Pázmány Péter University in Budapest at the age of 22, simultaneously earning a diploma in chemical engineering from the ETH Zurich in Switzerland. Between 1926 and 1930, he taught as a Privatdozent at the University of Berlin, the youngest in its history. By age 25, he had already published a dozen of major papers.

John von Neumann emigrated to the United States just in time—in 1930, where he was invited to Princeton University, and, subsequently, was one of the first four people selected for the faculty of the Institute for Advanced Study (two of the others being Albert Einstein and Kurt Gödel!), where he remained a mathematics professor from its formation in 1933 until his death.

Von Neumann was an important figure in computer science.

Heap Sort

Heap data structure

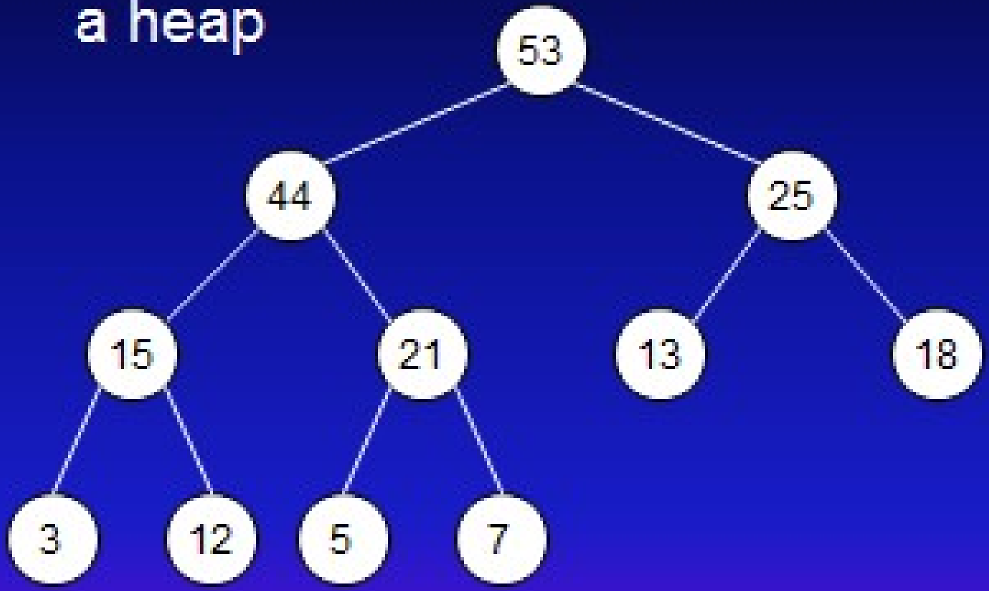
- Binary tree
- Balanced
- Left-justified or Complete
- (Max) Heap property: no node has a value greater than the value in its parent

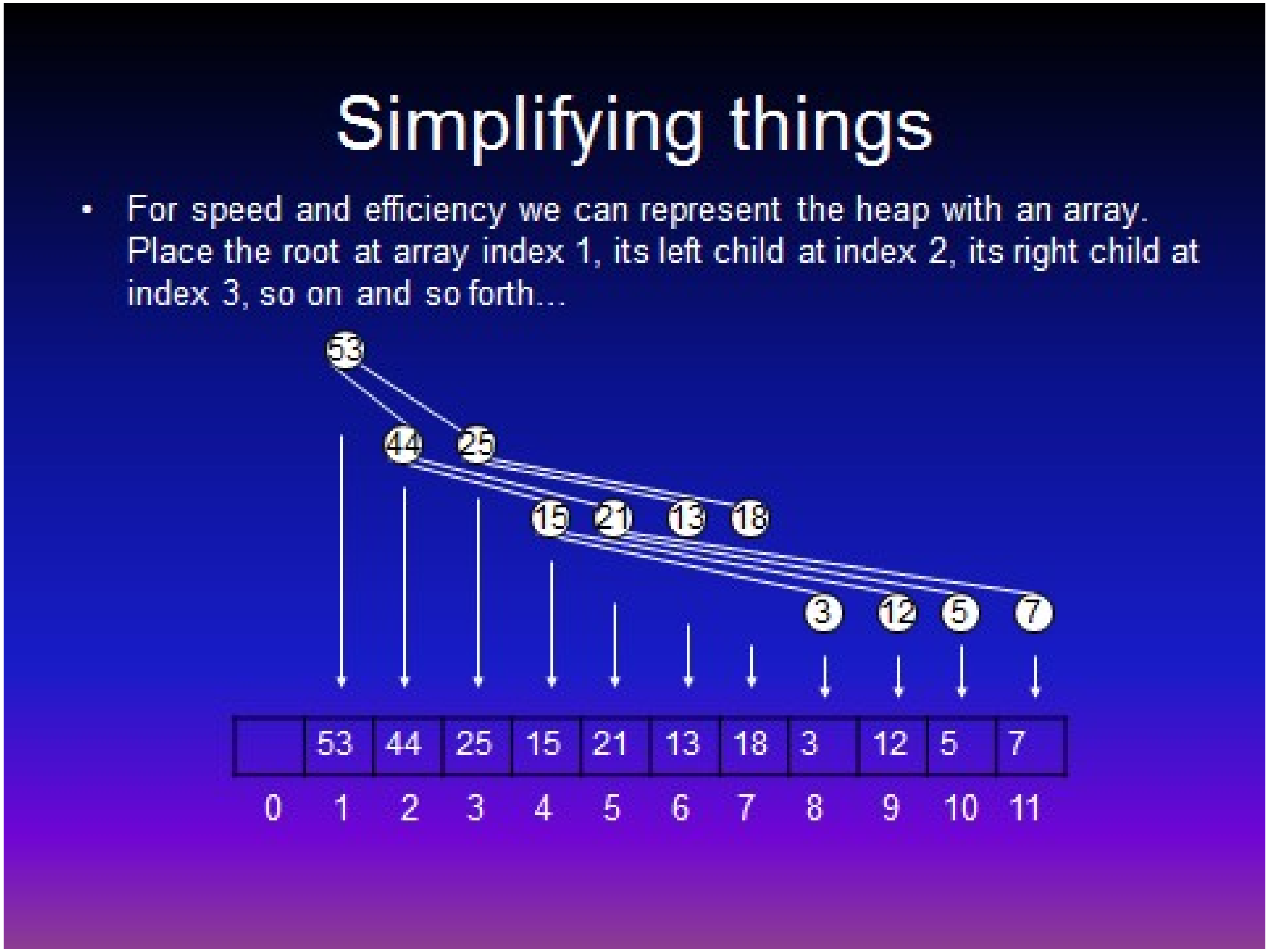
Analysis

- Construct the heap O(n log n)
- Remove and re-heap O(log n)
  - Do this n times O(n log n)
- Total time O(n log n) + O(n log n)

Example:

a heap

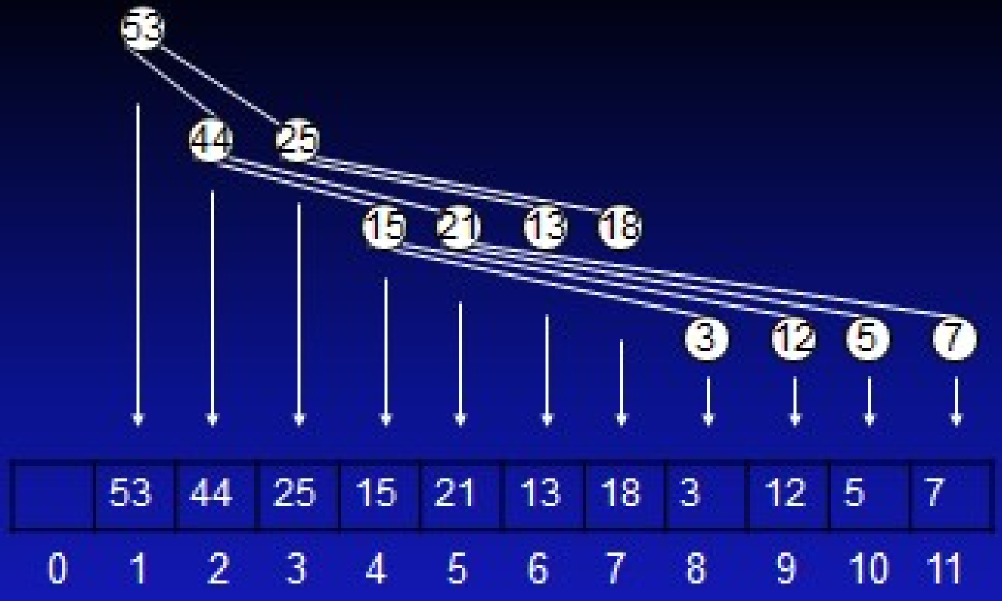




Simplifying things

- For speed and efficiency we can represent the heap with an array. Place the root at array index 1, its left child at index 2, its right child at index 3, so on and so forth…

0	1	2	3	4	5	6	7	8	9	10	11
	53	44	25	15	21	13	18	3	12	5	7



	53	44	25	15	21	13	18	3	12	5	7
0	1	2	3	4	5	6	7	8	9	10	11

For any node i, the following formulas apply:

The index of its parent = i / 2

Index of left child = 2 * i

Index of right child = 2 * i + 1

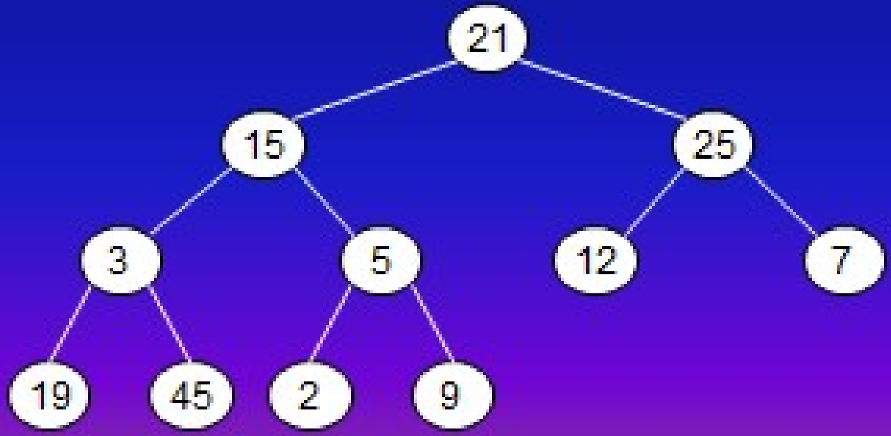
Sample Run

- Start with unordered array of data

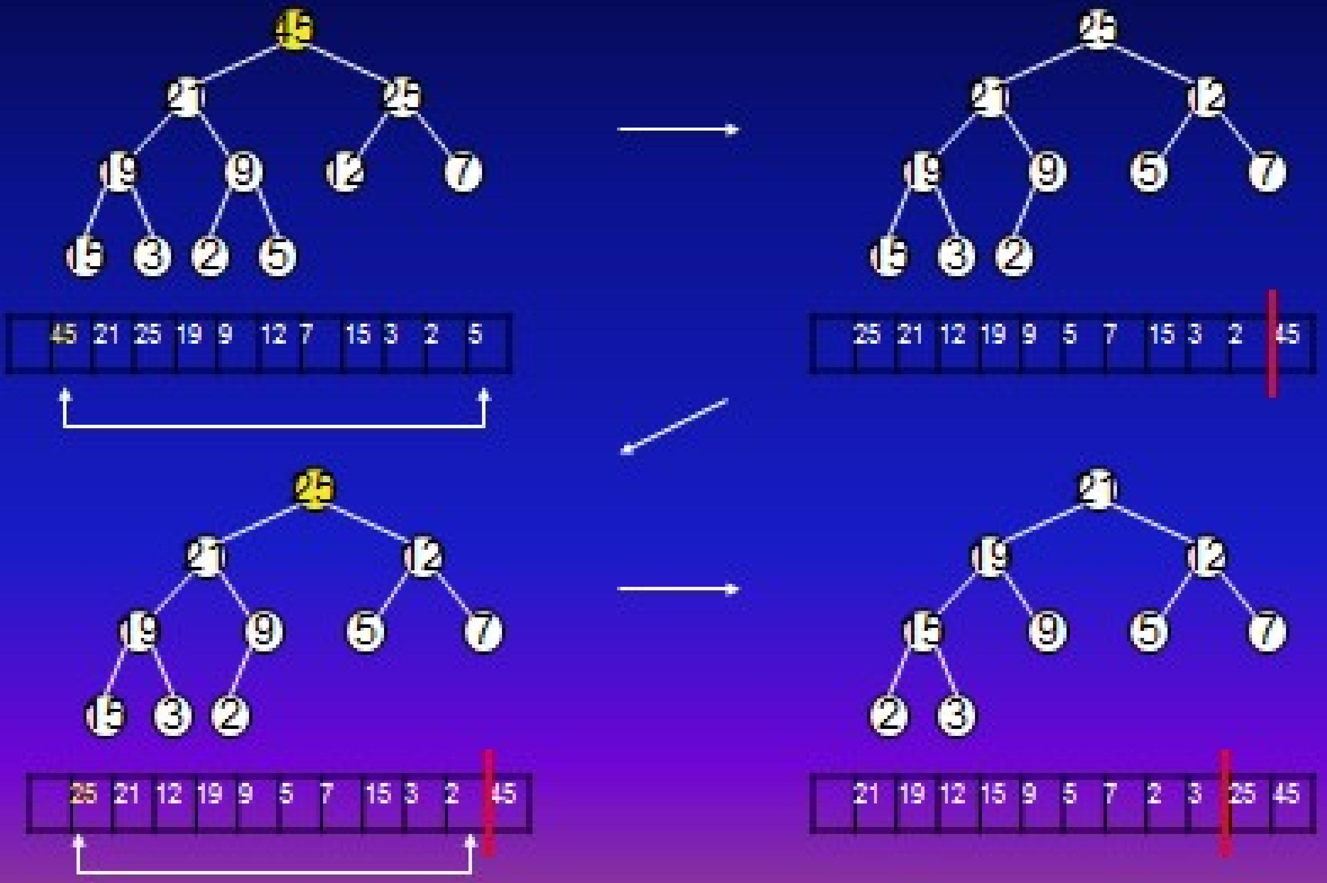
Array representation:

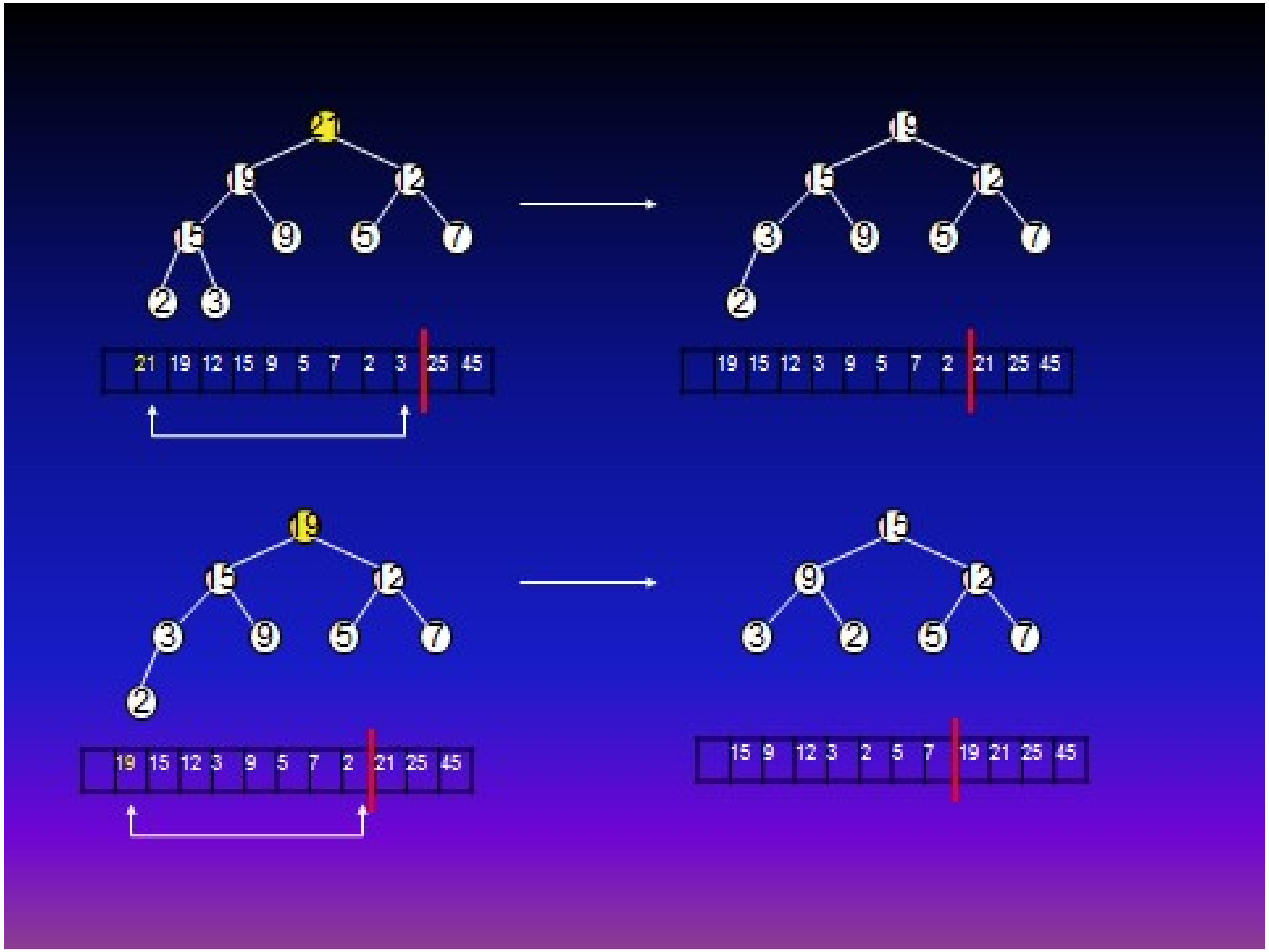
	21	15	25	3	5	12	7	19	45	2	9

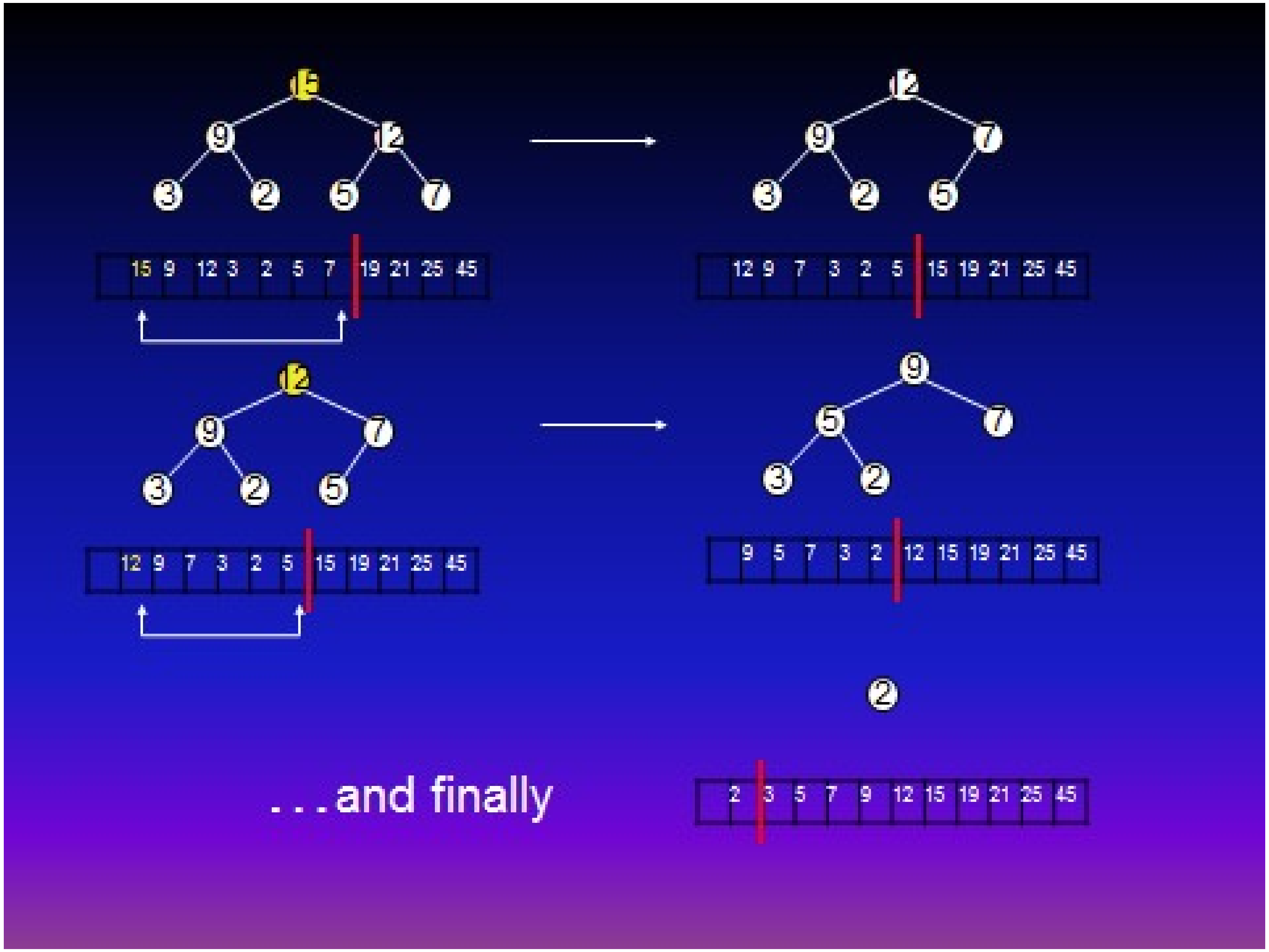
Binary tree representation:



Perform n – 1 deleteMax(es), and replace last element in heap with first, then re-heapify. Place deleted element in the last nodes position.




21 19 12 15 9 5 7 2 3 25 45
19 15 12 3 9 5 7 2 21 25 45
19 15 12 3 9 5 7 2 21 25 45
15 9 12 3 2 5 7 19 21 25 45



15 9 12 3 2 5 7 19 21 25 45
12 9 7 3 2 5 15 19 21 25 45
12 9 7 3 2 5 15 19 21 25 45
9 5 7 3 2 12 15 19 21 25 45
. . . and finally
2 3 5 7 9 12 15 19 21 25 45

Quiz 1

Which sorting algorithm will take least time when <u>all</u> elements of input array are <u>identical</u>? Consider typical implementations of sorting algorithms.

A	Insertion Sort
B	Heap Sort
C	Merge Sort
D	Selection Sort

Quiz 2

Which of the following sorting algorithms has the lowest worst-case complexity?

A Merge Sort
B Bubble Sort
C Quick Sort
D Selection Sort

Quiz 3

What are the best, average, and worst case asymptotic costs for Mergesort?

Quiz 4

In a modified merge sort, the input array is splitted at a position one-third of the length(N) of the array. What is the worst case time complexity of this merge sort?

A	N(logN base 3)
B	N(logN base 2/3)
C	N(logN base 1/3)
D	N(logN base 3/2)

Quiz 5

Suppose we are sorting an array of eight integers using heapsort, and we have just finished some heapify (either maxheapify or minheapify) operations. The array now looks like this: 16 14 15 10 12 27 28 How many heapify operations have been performed on root of heap?