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مهندسی کامپیوتر

1) $T(n) = 3T(\frac{n}{2}) + n^2$ $\quad a=3, b=2, f(n)=n^2$

$n^{\log_b^a} = n^{\log_2^3} \longrightarrow \Omega(n^{\log_2^3 + \varepsilon}) = n^2 \quad 0<\varepsilon<1 \longrightarrow ✓$

$af(\frac{n}{b}) < cf(n) \quad \frac{3}{4}n^2 < cn^2 \Rightarrow c = \frac{3}{4}$

$\longrightarrow$ case 3 $\longrightarrow \theta(n^2) = T(n)$

2. $T(n) = 4T(\frac{n}{2}) + n^2$ $\quad a=4 \quad b=2 \quad f(n)=n^2$

$n^{\log_b^a} = n^{\log_2^4} = n^2 = n^2 \longrightarrow$ case 2

$T(n) = \theta(n^2 \log^n)$

3) $T(n) = T(\frac{n}{2}) + 2^n$ $\quad a=1 \quad b=2 \quad f(n)=2^n$

$n^{\log_b^a} = n^{\log_2^1} \longrightarrow \Omega(n^{\log_2^1 + \varepsilon}) = 2^n$ ✓

$af(\frac{n}{b}) < cf(n) = 2^{\frac{n}{2}} < c2^n \Rightarrow c=1$ ✓

$\longrightarrow$ case 3 $\longrightarrow T(n) = \theta(2^n)$

4. $T(n) = 2^n(T(\frac{n}{2}) + n^n)$

در این سوال شرط اولیه مستر تئوریم $a \geq 1$ برقرار است و 2^n تغییر می‌کند

(از نتیجه نمی‌توانیم برای حل از مستر تئوریم استفاده کنیم)

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(5). $16T(\frac{n}{4}) + n$ $a = 16$ $b = 4$ $f(n) = n$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$

$f(n) = \theta(n^{2-\epsilon}) \longrightarrow 0 < \epsilon < 1$ ✓

case 1 $\longrightarrow T(n) = \theta(n^2)$

(6). $T(n) = 2T(\frac{n}{2}) + n^{\log^n}$ $a = 2$ $b = 2$ $f(n) = n\log^n$

~~$n^{\log_b^a}$~~ $n^{\log_b^a} = n^{\log_2^2} = n \longrightarrow \Omega(n) = n\log^n$

$af(\frac{n}{b}) \leq cf(n) \longrightarrow \frac{2n}{2}\log^{n/2} \leq cf(n) = cn\log^n$

$n\log\frac{n}{2} \leq cn\log^n \longrightarrow \log^{n/2} \leq c\log^n \longrightarrow c = 1$

case 2 $\longrightarrow T(n) = n\log_n^2$

(7). $T(n) = 2T(\frac{n}{2}) + n\log^n$ این صورت سوال

متاسفانه حل نشد [illegible]

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⑧ $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$ $\qquad a=2,\ b=4,\ f(n) = n^{0.51}$

$n^{\log_b^a} = n^{\log_4^2} = n^{\frac{1}{2}} \longrightarrow \Omega\left(n^{\frac{1}{2}+\varepsilon}\right) = n^{0.51}$

$0<\varepsilon<1 \Rightarrow \Omega\left(n^{\frac{1}{2}}\right) = n^{0.51}$

$af\left(\frac{n}{b}\right) \leq cf(n) = 2\left(\frac{n}{4}\right)^{0.51} \leq c(n)^{0.51} \longrightarrow c=1$ ✓

case3 $\rightarrow T(n) = \theta\left(n^{0.51}\right)$

⑨. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

چون یکی از شروط مستر قضیه $a \geq 1$ بوده است پس الان نمی‌توانیم این
سوال را با مستر حل کنیم

⑩. $16T\left(\frac{n}{4}\right) + n!$ $\qquad a=16 \quad b=4 \quad f(n) = n!$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$

$\Omega\left(n^{2^n+\varepsilon}\right) = n! \xrightarrow{0<\varepsilon<1}$ ✓

$af\left(\frac{n}{b}\right) \leq f(n) \longrightarrow 16\left(\frac{n}{4}\right)! \leq cn!$ $\qquad c = 0.9$ ✓

case3 $\longrightarrow T(n) = \theta(n!)$

(11). $T(n) = \sqrt{2}\, T\left(\frac{n}{2}\right) + \log^n$ $a = \sqrt{2}$ $b = 2$

$f(n) = \log^n$

$n^{\log_b^a} = n^{\log_2^{\sqrt{2}}} = n^{\frac{1}{2}}$

$O\left(n^{\frac{1}{2} - \varepsilon}\right) = \log^n \xrightarrow{0 < \varepsilon < 1}$ ✓

Case 1 $\Rightarrow T(n) = \theta(\sqrt{n})$

(12). $T(n) = 3T\left(\frac{n}{2}\right) + n$ $a = 3$ $b = 2$ $f(n) = n$

$n^{\log_b^a} = n^{\log_2^3}$

$\theta\left(n^{\log_2^3 - \varepsilon}\right) = n \xrightarrow{0 < \varepsilon < 1}$ ✓

Case 1 $\longrightarrow T(n) = \theta\left(n^{\log_2^3}\right)$