COSE474-2024F: Deep Learning HW1

2.1 Data manipulation

2.1.1. Getting Started

```
import torch
```

```
x = torch.arange(12, dtype=torch.float32)
x
```

```
tensor([ 0.,  1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.,  9., 10., 11.])
```

```
x.numel()
```

```
12
```

```
x.shape
```

```
torch.Size([12])
```

```
X = x.reshape(3, 4)
X
```

```
tensor([[ 0.,  1.,  2.,  3.],
        [ 4.,  5.,  6.,  7.],
        [ 8.,  9., 10., 11.]])
```

```
torch.zeros((2,3,4))
```

```
tensor([[[0., 0., 0., 0.],
         [0., 0., 0., 0.],
         [0., 0., 0., 0.]],

        [[0., 0., 0., 0.],
         [0., 0., 0., 0.],
         [0., 0., 0., 0.]]])
```

```
torch.ones((2,3,4))
```

```
tensor([[[1., 1., 1., 1.],
         [1., 1., 1., 1.],
         [1., 1., 1., 1.]],

        [[1., 1., 1., 1.],
         [1., 1., 1., 1.],
         [1., 1., 1., 1.]]])
```

```
torch.randn(3,4)
```

```
tensor([[ 1.0299,  1.7744,  0.2221,  1.5564],
        [-1.0925, -0.9889, -3.0579, -0.2205],
        [ 0.1886, -0.2370,  0.0943,  2.4571]])
```

```
torch.tensor([[2,1,4,3],[1,2,3,4],[4,3,2,1]])
```

```
tensor([[2, 1, 4, 3],
        [1, 2, 3, 4],
        [4, 3, 2, 1]])
```

2.1.2. Indexing and Slicing

```
X[-1], X[1:3]
```

```
(tensor([ 8.,  9., 10., 11.]),
 tensor([[ 4.,  5.,  6.,  7.],
         [ 8.,  9., 10., 11.]]))
```

```
X[1,2] = 17
X
```

```
tensor([[ 0.,  1.,  2.,  3.],
        [ 4.,  5., 17.,  7.],
        [ 8.,  9., 10., 11.]])
```

```
X[:2,:] = 12
X
```

```
tensor([[12., 12., 12., 12.],
        [12., 12., 12., 12.],
        [ 8.,  9., 10., 11.]])
```

2.1.3. Operations

```
torch.exp(x)
```

```
tensor([162754.7969, 162754.7969, 162754.7969, 162754.7969, 162754.7969,
        162754.7969, 162754.7969, 162754.7969,   2980.9580,   8103.0840,
         22026.4648,  59874.1406])
```

```
x = torch.tensor([1.0, 2, 4, 8])
y = torch.tensor([2, 2, 2, 2])
x+y, x-y, x*y, x/y, x**y
```

```
(tensor([ 3.,  4.,  6., 10.]),
 tensor([-1.,  0.,  2.,  6.]),
 tensor([ 2.,  4.,  8., 16.]),
 tensor([0.5000, 1.0000, 2.0000, 4.0000]),
 tensor([ 1.,  4., 16., 64.]))
```

```
X = torch. arange(12, dtype=torch.float32).reshape((3,4))
Y = torch. tensor([[2.0, 1, 4, 3], [1, 2, 3, 4], [4, 3, 2, 1]])
torch.cat((X,Y), dim=0), torch.cat((X, Y), dim=1)
```

```
(tensor([[ 0.,  1.,  2.,  3.],
         [ 4.,  5.,  6.,  7.],
         [ 8.,  9., 10., 11.],
         [ 2.,  1.,  4.,  3.],
         [ 1.,  2.,  3.,  4.],
         [ 4.,  3.,  2.,  1.]]),
 tensor([[ 0.,  1.,  2.,  3.,  2.,  1.,  4.,  3.],
         [ 4.,  5.,  6.,  7.,  1.,  2.,  3.,  4.],
         [ 8.,  9., 10., 11.,  4.,  3.,  2.,  1.]]))
```

```
X == Y
```

```
tensor([[False,  True, False,  True],
        [False, False, False, False],
        [False, False, False, False]])
```

```
X.sum()
```

```
tensor(66.)
```

2.1.4. Broadcasting

```
a = torch.arange(3).reshape((3,1))
b = torch.arange(2).reshape((1,2))
a, b
```

```
(tensor([[0],
         [1],
         [2]]),
 tensor([[0, 1]]))
```

```
a + b
```

```
tensor([[0, 1],
        [1, 2],
        [2, 3]])
```

2.1.5. Saving Memory

```
before = id(Y)
Y = Y + X
id(Y) == before
```

```
False
```

```
Z = torch.zeros_like(Y)
print('id(Z):', id(Z))
Z[:] = X + Y
print('id(Z):', id(Z))
```

```
id(Z): 137406168918240
id(Z): 137406168918240
```

```
before = id(X)
X += Y
id(X) == before
```

```
True
```

Conversion to Other Python objects

```
A = X.numpy()
B = torch.from_numpy(A)
type(A), type(B)
```

```
(numpy.ndarray, torch.Tensor)
```

```
a = torch.tensor([3.5])
a, a.item(), float(a), int(a)
```

```
(tensor([3.5000]), 3.5, 3.5, 3)
```

2.2 Data Preprocessing

2.2.1. Reading the Dataset

```
import os

os.makedirs(os.path.join('..', 'data'), exist_ok=True)
data_file = os.path.join('..', 'data', 'house_tiny.csv')
with open(data_file, 'w') as f:
    f.write('''NumRooms,RoofType,Price
NA,NA,127500
2,NA,106000
4,Slate,178100
NA,NA,140000''')
```

```
import pandas as pd

data = pd.read_csv(data_file)
print(data)
```

```
   NumRooms RoofType   Price
0       NaN      NaN  127500
1       2.0      NaN  106000
2       4.0    Slate  178100
3       NaN      NaN  140000
```

2.2.2. Data Preparation

```
inputs, targets = data.iloc[:, 0:2], data.iloc[:, 2]
inputs = pd.get_dummies(inputs, dummy_na=True)
print(inputs)
```

```
   NumRooms  RoofType_Slate  RoofType_nan
0       NaN           False          True
1       2.0           False          True
2       4.0            True         False
3       NaN           False          True
```

```
inputs = inputs.fillna(inputs.mean())
print(inputs)
```

```
   NumRooms  RoofType_Slate  RoofType_nan
0       3.0           False          True
1       2.0           False          True
2       4.0            True         False
3       3.0           False          True
```

2.2.3. Conversion to the Tensor Format

```
import torch

X = torch.tensor(inputs.to_numpy(dtype=float))
y = torch.tensor(targets.to_numpy(dtype=float))
X, y
```

```
(tensor([[3., 0., 1.],
         [2., 0., 1.],
         [4., 1., 0.],
         [3., 0., 1.]], dtype=torch.float64),
 tensor([127500., 106000., 178100., 140000.], dtype=torch.float64))
```

2.3 Linear Algebra

2.3.1. Scalars

```
x = torch.tensor(3.0)
y = torch.tensor(2.0)

x + y, x * y, x / y, x**y
```

```
(tensor(5.), tensor(6.), tensor(1.5000), tensor(9.))
```

2.3.2. Vectors

```
x = torch.arange(3)
x
```

```
tensor([0, 1, 2])
```

```
x[2]
```

```
tensor(2)
```

```
len(x)
```

```
3
```

```
x.shape
```

```
torch.Size([3])
```

2.3.3. Matrices

```
A = torch.arange(6).reshape(3, 2)
A
```

```
tensor([[0, 1],
        [2, 3],
        [4, 5]])
```

```
A.T
```

```
tensor([[0, 2, 4],
        [1, 3, 5]])
```

```
A = torch.tensor([[1, 2, 3], [2, 0, 4], [3, 4, 5]])
A == A.T
```

```
tensor([[True, True, True],
        [True, True, True],
        [True, True, True]])
```

2.3.4. Tensors

```
torch.arange(24).reshape(2, 3, 4)
```

```
tensor([[[ 0,  1,  2,  3],
         [ 4,  5,  6,  7],
         [ 8,  9, 10, 11]],

        [[12, 13, 14, 15],
         [16, 17, 18, 19],
         [20, 21, 22, 23]]])
```

2.3.5. Basic Properties of Tensor Arithmetic

```
A = torch.arange(6, dtype=torch.float32).reshape(2, 3)
B = A.clone()
A, A + B
```

```
(tensor([[0., 1., 2.],
         [3., 4., 5.]]),
 tensor([[ 0.,  2.,  4.],
         [ 6.,  8., 10.]]))
```

```
A * B
```

```
tensor([[ 0.,  1.,  4.],
        [ 9., 16., 25.]])
```

```
a = 2
X = torch.arange(24).reshape(2, 3, 4)
a + X, (a * X).shape
```

```
(tensor([[[ 2,  3,  4,  5],
          [ 6,  7,  8,  9],
          [10, 11, 12, 13]],

         [[14, 15, 16, 17],
          [18, 19, 20, 21],
          [22, 23, 24, 25]]]),
 torch.Size([2, 3, 4]))
```

2.3.6. Reduction

```
x = torch.arange(3, dtype=torch.float32)
x, x.sum()
```

```
(tensor([0., 1., 2.]), tensor(3.))
```

```
A.shape, A.sum()
```

```
(torch.Size([2, 3]), tensor(15.))
```

```
A.shape, A.sum(axis=0).shape
```

```
(torch.Size([2, 3]), torch.Size([3]))
```

```
A.shape, A.sum(axis=1).shape
```

```
(torch.Size([2, 3]), torch.Size([2]))
```

```
A.sum(axis=[0, 1]) == A.sum()
```

```
tensor(True)
```

```
A.mean(), A.sum() / A.numel()
```

```
(tensor(2.5000), tensor(2.5000))
```

```
A.mean(axis=0), A.sum(axis=0) / A.shape[0]
```

```
(tensor([1.5000, 2.5000, 3.5000]), tensor([1.5000, 2.5000, 3.5000]))
```

2.3.7. Non-Reduction Sum

```
sum_A = A.sum(axis=1, keepdims=True)
sum_A, sum_A.shape
```

```
(tensor([[ 3.],
         [12.]]),
 torch.Size([2, 1]))
```

```
A / sum_A
```

```
tensor([[0.0000, 0.3333, 0.6667],
        [0.2500, 0.3333, 0.4167]])
```

```
A.cumsum(axis=0)
```

```
tensor([[0., 1., 2.],
        [3., 5., 7.]])
```

2.3.8. Dot Products

```
y = torch.ones(3, dtype = torch.float32)
x, y, torch.dot(x, y)
```

```
(tensor([0., 1., 2.]), tensor([1., 1., 1.]), tensor(3.))
```

```
torch.sum(x * y)
```

```
tensor(3.)
```

2.3.9. Matrix-Vector Products

```
A.shape, x.shape, torch.mv(A, x), A@x
```

```
(torch.Size([2, 3]), torch.Size([3]), tensor([ 5., 14.]), tensor([ 5., 14.]))
```

2.3.10. Matrix-Matrix Multiplication

```
B = torch.ones(3, 4)
torch.mm(A, B), A@B
```

```
(tensor([[ 3.,  3.,  3.,  3.],
         [12., 12., 12., 12.]]),
 tensor([[ 3.,  3.,  3.,  3.],
         [12., 12., 12., 12.]]))
```

2.3.11. Norms

```
u = torch.tensor([3.0, -4.0])
torch.norm(u)
```

```
tensor(5.)
```

```
torch.abs(u).sum()
```

```
tensor(7.)
```

```
torch.norm(torch.ones((4, 9)))
```

```
tensor(6.)
```

2.5 Automatic Differentitation

2.5.1. A Simple Function

```
x = torch.arange(4.0)
x
```

```
tensor([0., 1., 2., 3.])
```

```
x.requires_grad_(True)
x.grad  # The gradient is None by default
```

```
y = 2 * torch.dot(x, x)
y
```

```
tensor(28., grad_fn=<MulBackward0>)
```

```
y.backward()
x.grad
```

```
tensor([ 0.,  4.,  8., 12.])
```

```
x.grad == 4 * x
```

```
tensor([True, True, True, True])
```

```
x.grad.zero_()
y = x.sum()
y.backward()
x.grad
```

```
tensor([1., 1., 1., 1.])
```

2.5.2. Backward for Non-Scalar Variables

```
x.grad.zero_()
y = x * x
y.backward(gradient=torch.ones(len(y)))
x.grad
```

```
tensor([0., 2., 4., 6.])
```

2.5.3. Detaching Computation

```
x.grad.zero_()
y = x * x
u = y.detach()
z = u * x

z.sum().backward()
x.grad == u
```

```
tensor([True, True, True, True])
```

```
x.grad.zero_()
y.sum().backward()
x.grad == 2 * x
```

```
tensor([True, True, True, True])
```

2.5.4. Gradients and Python Control Flow

```
def f(a):
    b = a * 2
    while b.norm() < 1000:
        b = b * 2
    if b.sum() > 0:
        c = b
    else:
        c = 100 * b
    return c
```

```
a = torch.randn(size=(), requires_grad=True)
d = f(a)
d.backward()
```

```
a.grad == d / a
```

```
tensor(True)
```

3.1 Linear Regression

```
%matplotlib inline
import math
import time
import numpy as np
import torch
!pip install d2l
from d2l import torch as d2l
```

```
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-packages (from python-dateutil>=2.7->matplotli
Requirement already satisfied: ipython-genutils in /usr/local/lib/python3.10/dist-packages (from ipykernel->jupyter==1.0
Requirement already satisfied: ipython>=5.0.0 in /usr/local/lib/python3.10/dist-packages (from ipykernel->jupyter==1.0.0
Requirement already satisfied: jupyter-client in /usr/local/lib/python3.10/dist-packages (from ipykernel->jupyter==1.0.0
Requirement already satisfied: tornado>=4.2 in /usr/local/lib/python3.10/dist-packages (from ipykernel->jupyter==1.0.0->
Requirement already satisfied: widgetsnbextension~=3.6.0 in /usr/local/lib/python3.10/dist-packages (from ipywidgets->ju
Requirement already satisfied: jupyterlab-widgets>=1.0.0 in /usr/local/lib/python3.10/dist-packages (from ipywidgets->ju
Requirement already satisfied: prompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0 in /usr/local/lib/python3.10/dist-packages (
Requirement already satisfied: pygments in /usr/local/lib/python3.10/dist-packages (from jupyter-console->jupyter==1.0.0
Requirement already satisfied: lxml in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0->d2l) (4.
Requirement already satisfied: beautifulsoup4 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0
Requirement already satisfied: bleach in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0->d2l) (
Requirement already satisfied: defusedxml in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0->d2
Requirement already satisfied: entrypoints>=0.2.2 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1
Requirement already satisfied: jinja2>=3.0 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0->d
Requirement already satisfied: jupyter-core>=4.7 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.
Requirement already satisfied: jupyterlab-pygments in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==
Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.
Requirement already satisfied: mistune<2,>=0.8.1 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.
Requirement already satisfied: nbclient>=0.5.0 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.
Requirement already satisfied: nbformat>=5.1 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0-
Requirement already satisfied: pandocfilters>=1.4.1 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter=
Requirement already satisfied: tinycss2 in /usr/local/lib/python3.10/dist-packages (from nbconvert->jupyter==1.0.0->d2l)
Requirement already satisfied: pyzmq<25,>=17 in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0.0->
Requirement already satisfied: argon2-cffi in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0.0->d2
Requirement already satisfied: nest-asyncio>=1.5 in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0
Requirement already satisfied: Send2Trash>=1.8.0 in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0
Requirement already satisfied: terminado>=0.8.3 in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0.
Requirement already satisfied: prometheus-client in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0
Requirement already satisfied: nbclassic>=0.4.7 in /usr/local/lib/python3.10/dist-packages (from notebook->jupyter==1.0.
Requirement already satisfied: qtpy>=2.4.0 in /usr/local/lib/python3.10/dist-packages (from qtconsole->jupyter==1.0.0->d
Requirement already satisfied: setuptools>=18.5 in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykern
Requirement already satisfied: jedi>=0.16 in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykernel->ju
Requirement already satisfied: decorator in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykernel->jup
Requirement already satisfied: pickleshare in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykernel->j
Requirement already satisfied: backcall in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykernel->jupy
Requirement already satisfied: pexpect>4.3 in /usr/local/lib/python3.10/dist-packages (from ipython>=5.0.0->ipykernel->j
Requirement already satisfied: platformdirs>=2.5 in /usr/local/lib/python3.10/dist-packages (from jupyter-core>=4.7->nbc
Requirement already satisfied: notebook-shim>=0.2.3 in /usr/local/lib/python3.10/dist-packages (from nbclassic>=0.4.7->n
Requirement already satisfied: fastjsonschema>=2.15 in /usr/local/lib/python3.10/dist-packages (from nbformat>=5.1->nbco
Requirement already satisfied: jsonschema>=2.6 in /usr/local/lib/python3.10/dist-packages (from nbformat>=5.1->nbconvert
Requirement already satisfied: wcwidth in /usr/local/lib/python3.10/dist-packages (from prompt-toolkit!=3.0.0,!=3.0.1,<3
Requirement already satisfied: ptyprocess in /usr/local/lib/python3.10/dist-packages (from terminado>=0.8.3->notebook->j
Requirement already satisfied: argon2-cffi-bindings in /usr/local/lib/python3.10/dist-packages (from argon2-cffi->notebo
Requirement already satisfied: soupsieve>1.2 in /usr/local/lib/python3.10/dist-packages (from beautifulsoup4->nbconvert-
Requirement already satisfied: webencodings in /usr/local/lib/python3.10/dist-packages (from bleach->nbconvert->jupyter=
Requirement already satisfied: parso<0.9.0,>=0.8.3 in /usr/local/lib/python3.10/dist-packages (from jedi>=0.16->ipython>
Requirement already satisfied: attrs>=22.2.0 in /usr/local/lib/python3.10/dist-packages (from jsonschema>=2.6->nbformat>
Requirement already satisfied: jsonschema-specifications>=2023.03.6 in /usr/local/lib/python3.10/dist-packages (from jso
Requirement already satisfied: referencing>=0.28.4 in /usr/local/lib/python3.10/dist-packages (from jsonschema>=2.6->nbf
Requirement already satisfied: rpds-py>=0.7.1 in /usr/local/lib/python3.10/dist-packages (from jsonschema>=2.6->nbformat
Requirement already satisfied: jupyter-server<3,>=1.8 in /usr/local/lib/python3.10/dist-packages (from notebook-shim>=0.
Requirement already satisfied: cffi>=1.0.1 in /usr/local/lib/python3.10/dist-packages (from argon2-cffi-bindings->argon2
Requirement already satisfied: pycparser in /usr/local/lib/python3.10/dist-packages (from cffi>=1.0.1->argon2-cffi-bindi
Requirement already satisfied: anyio<4,>=3.1.0 in /usr/local/lib/python3.10/dist-packages (from jupyter-server<3,>=1.8->
Requirement already satisfied: websocket-client in /usr/local/lib/python3.10/dist-packages (from jupyter-server<3,>=1.8-
Requirement already satisfied: sniffio>=1.1 in /usr/local/lib/python3.10/dist-packages (from anyio<4,>=3.1.0->jupyter-se
Requirement already satisfied: exceptiongroup in /usr/local/lib/python3.10/dist-packages (from anyio<4,>=3.1.0->jupyter-
```

3.1.2. Vectorization for Speed

```
n = 10000
a = torch.ones(n)
b = torch.ones(n)
```

```
c = torch.zeros(n)
t = time.time()
for i in range(n):
    c[i] = a[i] + b[i]
f'{time.time() - t:.5f} sec'
```

```
'0.12797 sec'
```

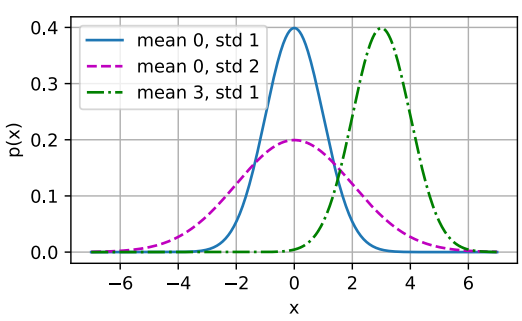
```
t = time.time()
d = a + b
f'{time.time() - t:.5f} sec'
```

```
'0.00091 sec'
```

3.1.3. The Normal Distribution and Squared Loss

```
def normal(x, mu, sigma):
    p = 1 / math.sqrt(2 * math.pi * sigma**2)
    return p * np.exp(-0.5 * (x - mu)**2 / sigma**2)
```

```
x = np.arange(-7, 7, 0.01)

params = [(0, 1), (0, 2), (3, 1)]
d2l.plot(x, [normal(x, mu, sigma) for mu, sigma in params], xlabel='x',
         ylabel='p(x)', figsize=(4.5, 2.5),
         legend=[f'mean {mu}, std {sigma}' for mu, sigma in params])
```



3.2 Object-Oriented Design for Implementation

```
import time
import numpy as np
import torch
from torch import nn
from d2l import torch as d2l
```

3.2.1. Utilities

```
def add_to_class(Class):
    def wrapper(obj):
        setattr(Class, obj.__name__, obj)
    return wrapper
```

```
class A:
    def __init__(self):
        self.b = 1

a = A()
```

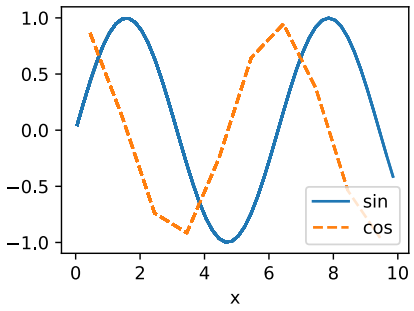
```
@add_to_class(A)
def do(self):
    print('Class attribute "b" is', self.b)

a.do()
```

```
Class attribute "b" is 1
```

```
class HyperParameters:
    def save_hyperparameters(self, ignore=[]):
        raise NotImplemented



class B(d2l.HyperParameters):
    def __init__(self, a, b, c):
        self.save_hyperparameters(ignore=['c'])
        print('self.a =', self.a, 'self.b =', self.b)
        print('There is no self.c =', not hasattr(self, 'c'))

b = B(a=1, b=2, c=3)
```

```
self.a = 1 self.b = 2
There is no self.c = True
```

```
class ProgressBoard(d2l.HyperParameters):
    def __init__(self, xlabel=None, ylabel=None, xlim=None,
                 ylim=None, xscale='linear', yscale='linear',
                 ls=['-', '--', '-.', ':'], colors=['C0', 'C1', 'C2', 'C3'],
                 fig=None, axes=None, figsize=(3.5, 2.5), display=True):
        self.save_hyperparameters()

    def draw(self, x, y, label, every_n=1):
        raise NotImplemented


board = d2l.ProgressBoard('x')
for x in np.arange(0, 10, 0.1):
    board.draw(x, np.sin(x), 'sin', every_n=2)
    board.draw(x, np.cos(x), 'cos', every_n=10)
```



3.2.2. Models

```
class Module(nn.Module, d2l.HyperParameters):
    def __init__(self, plot_train_per_epoch=2, plot_valid_per_epoch=1):
        super().__init__()
        self.save_hyperparameters()
        self.board = ProgressBoard()

    def loss(self, y_hat, y):
        raise NotImplementedError

    def forward(self, X):
        assert hasattr(self, 'net'), 'Neural network is defined'
        return self.net(X)

    def plot(self, key, value, train):
        """Plot a point in animation."""
        assert hasattr(self, 'trainer'), 'Trainer is not inited'
        self.board.xlabel = 'epoch'
        if train:
            x = self.trainer.train_batch_idx / \
                self.trainer.num_train_batches
            n = self.trainer.num_train_batches / \
                self.plot_train_per_epoch
        else:
            x = self.trainer.epoch + 1
            n = self.trainer.num_val_batches / \
                self.plot_valid_per_epoch
        self.board.draw(x, value.to(d2l.cpu()).detach().numpy(),
                        ('train_' if train else 'val_') + key,
                        every_n=int(n))

    def training_step(self, batch):
```

```
        l = self.loss(self(*batch[:-1]), batch[-1])
        self.plot('loss', l, train=True)
        return l

    def validation_step(self, batch):
        l = self.loss(self(*batch[:-1]), batch[-1])
        self.plot('loss', l, train=False)

    def configure_optimizers(self):
        raise NotImplementedError
```

3.2.3. Data

```
class DataModule(d2l.HyperParameters):
    def __init__(self, root='../data', num_workers=4):
        self.save_hyperparameters()

    def get_dataloader(self, train):
        raise NotImplementedError

    def train_dataloader(self):
        return self.get_dataloader(train=True)

    def val_dataloader(self):
        return self.get_dataloader(train=False)
```

3.2.4. Training

```
class Trainer(d2l.HyperParameters):
    def __init__(self, max_epochs, num_gpus=0, gradient_clip_val=0):
        self.save_hyperparameters()
        assert num_gpus == 0, 'No GPU support yet'

    def prepare_data(self, data):
        self.train_dataloader = data.train_dataloader()
        self.val_dataloader = data.val_dataloader()
        self.num_train_batches = len(self.train_dataloader)
        self.num_val_batches = (len(self.val_dataloader)
                                if self.val_dataloader is not None else 0)

    def prepare_model(self, model):
        model.trainer = self
        model.board.xlim = [0, self.max_epochs]
        self.model = model

    def fit(self, model, data):
        self.prepare_data(data)
        self.prepare_model(model)
        self.optim = model.configure_optimizers()
        self.epoch = 0
        self.train_batch_idx = 0
        self.val_batch_idx = 0
        for self.epoch in range(self.max_epochs):
            self.fit_epoch()

    def fit_epoch(self):
        raise NotImplementedError
```

3.4. Linear Regression Implementation from Scratch

```
%matplotlib inline
import torch
from d2l import torch as d2l
```

3.4.1. Defining the Model

```
class LinearRegressionScratch(d2l.Module):
    def __init__(self, num_inputs, lr, sigma=0.01):
        super().__init__()
        self.save_hyperparameters()
        self.w = torch.normal(0, sigma, (num_inputs, 1), requires_grad=True)
        self.b = torch.zeros(1, requires_grad=True)
```

```
@d2l.add_to_class(LinearRegressionScratch)
def forward(self, X):
    return torch.matmul(X, self.w) + self.b
```

3.4.2. Defining the Loss Function

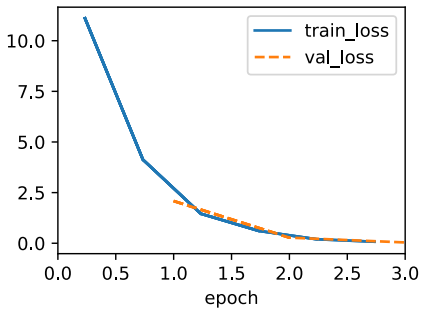
```
@d2l.add_to_class(LinearRegressionScratch)
def loss(self, y_hat, y):
    l = (y_hat - y) ** 2 / 2
    return l.mean()
```

3.4.3. Defining the Optimization Algorithm

```
class SGD(d2l.HyperParameters):
    """Minibatch stochastic gradient descent."""
    def __init__(self, params, lr):
        self.save_hyperparameters()

    def step(self):
        for param in self.params:
            param -= self.lr * param.grad

    def zero_grad(self):
        for param in self.params:
            if param.grad is not None:
                param.grad.zero_()

@d2l.add_to_class(LinearRegressionScratch)
def configure_optimizers(self):
    return SGD([self.w, self.b], self.lr)
```

3.4.4. Training

```
@d2l.add_to_class(d2l.Trainer)
def prepare_batch(self, batch):
    return batch

@d2l.add_to_class(d2l.Trainer)
def fit_epoch(self):
    self.model.train()
    for batch in self.train_dataloader:
        loss = self.model.training_step(self.prepare_batch(batch))
        self.optim.zero_grad()
        with torch.no_grad():
            loss.backward()
            if self.gradient_clip_val > 0:
                self.clip_gradients(self.gradient_clip_val, self.model)
            self.optim.step()
        self.train_batch_idx += 1
    if self.val_dataloader is None:
        return
    self.model.eval()
    for batch in self.val_dataloader:
        with torch.no_grad():
            self.model.validation_step(self.prepare_batch(batch))
        self.val_batch_idx += 1

model = LinearRegressionScratch(2, lr=0.03)
data = d2l.SyntheticRegressionData(w=torch.tensor([2, -3.4]), b=4.2)
trainer = d2l.Trainer(max_epochs=3)
trainer.fit(model, data)
```




```
with torch.no_grad():
    print(f'error in estimating w: {data.w - model.w.reshape(data.w.shape)}')
    print(f'error in estimating b: {data.b - model.b}')
```

```
error in estimating w: tensor([ 0.1050, -0.1723])
error in estimating b: tensor([0.2082])
```

4.1. Softmax Regression

The Softmax

softmax function:

$$\hat{\mathbf{y}} = \mathrm{softmax}(\mathbf{o}) \quad \text{where} \quad \hat{y}_i = \frac{\exp(o_i)}{\sum_j \exp(o_j)}.$$

the largest coordinate of $\mathbf{o}$ corresponds to the most likely class according to $\hat{\mathbf{y}}$. Moreover, because the softmax operation preserves the ordering among its arguments, we do not need to compute the softmax to determine which class has been assigned the highest probability. Thus,

$$\operatorname*{argmax}_j \hat{y}_j = \operatorname*{argmax}_j o_j.$$

Log-likelihood

The softmax function gives us a vector $\hat{\mathbf{y}}$, which we can interpret as the (estimated) conditional probabilities of each class, given any input $\mathbf{x}$, such as $\hat{y}_1 = P(y = \text{cat} \mid \mathbf{x})$. In the following we assume that for a dataset with features $\mathbf{X}$ the labels $\mathbf{Y}$ are represented using a one-hot encoding label vector. We can compare the estimates with reality by checking how probable the actual classes are according to our model, given the features:

$$P(\mathbf{Y} \mid \mathbf{X}) = \prod_{i=1}^{n} P(\mathbf{y}^{(i)} \mid \mathbf{x}^{(i)}).$$

We are allowed to use the factorization since we assume that each label is drawn independently from its respective distribution $P(\mathbf{y} \mid \mathbf{x}^{(i)})$. Since maximizing the product of terms is awkward, we take the negative logarithm to obtain the equivalent problem of minimizing the negative log-likelihood:

$$-\log P(\mathbf{Y} \mid \mathbf{X}) = \sum_{i=1}^{n} -\log P(\mathbf{y}^{(i)} \mid \mathbf{x}^{(i)}) = \sum_{i=1}^{n} l(\mathbf{y}^{(i)}, \hat{\mathbf{y}}^{(i)}),$$

where for any pair of label $\mathbf{y}$ and model prediction $\hat{\mathbf{y}}$ over q classes, the loss function l is

$$l(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{j=1}^{q} y_j \log \hat{y}_j.$$

Softmax and Cross-Entropy Loss

Since the softmax function and the corresponding cross-entropy loss are so common, it is worth understanding a bit better how they are computed. Plugging :eqref: `eq_softmax_y_and_o` into the definition of the loss in :eqref: `eq_l_cross_entropy` and using the definition of the softmax we obtain

$$l(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{j=1}^{q} y_j \log \frac{\exp(o_j)}{\sum_{k=1}^{q} \exp(o_k)}$$
$$= \sum_{j=1}^{q} y_j \log \sum_{k=1}^{q} \exp(o_k) - \sum_{j=1}^{q} y_j o_j$$
$$= \log \sum_{k=1}^{q} \exp(o_k) - \sum_{j=1}^{q} y_j o_j.$$

To understand a bit better what is going on, consider the derivative with respect to any logit o_j. We get

$$\partial_{o_j} l(\mathbf{y}, \hat{\mathbf{y}}) = \frac{\exp(o_j)}{\sum_{k=1}^{q} \exp(o_k)} - y_j = \text{softmax}(\mathbf{o})_j - y_j.$$

4.2. THe Image Classification Dataset

```
%matplotlib inline
import time
import torch
import torchvision
from torchvision import transforms
from d2l import torch as d2l

d2l.use_svg_display()
```

4.2.1. Loading the Dataset

```
class FashionMNIST(d2l.DataModule):
    """The Fashion-MNIST dataset."""
    def __init__(self, batch_size=64, resize=(28, 28)):
        super().__init__()
        self.save_hyperparameters()
        trans = transforms.Compose([transforms.Resize(resize),
                                    transforms.ToTensor()])
        self.train = torchvision.datasets.FashionMNIST(
            root=self.root, train=True, transform=trans, download=True)
        self.val = torchvision.datasets.FashionMNIST(
            root=self.root, train=False, transform=trans, download=True)
```

```
data = FashionMNIST(resize=(32, 32))
len(data.train), len(data.val)
```

```
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-images-idx3-ubyte.gz to ../data/FashionMNIS
100%|██████████| 26421880/26421880 [00:03<00:00, 7736396.53it/s]
Extracting ../data/FashionMNIST/raw/train-images-idx3-ubyte.gz to ../data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/train-labels-idx1-ubyte.gz to ../data/FashionMNIS
100%|██████████| 29515/29515 [00:00<00:00, 136357.65it/s]
Extracting ../data/FashionMNIST/raw/train-labels-idx1-ubyte.gz to ../data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-images-idx3-ubyte.gz to ../data/FashionMNIST
100%|██████████| 4422102/4422102 [00:08<00:00, 508420.52it/s]
Extracting ../data/FashionMNIST/raw/t10k-images-idx3-ubyte.gz to ../data/FashionMNIST/raw

Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz
Downloading http://fashion-mnist.s3-website.eu-central-1.amazonaws.com/t10k-labels-idx1-ubyte.gz to ../data/FashionMNIST
100%|██████████| 5148/5148 [00:00<00:00, 9520404.32it/s]Extracting ../data/FashionMNIST/raw/t10k-labels-idx1-ubyte.gz to


(60000, 10000)
```

```
data.train[0][0].shape
```

```
torch.Size([1, 32, 32])
```

```
@d2l.add_to_class(FashionMNIST)
def text_labels(self, indices):
    labels = ['t-shirt', 'trouser', 'pullover', 'dress', 'coat',
              'sandal', 'shirt', 'sneaker', 'bag', 'ankle boot']
    return [labels[int(i)] for i in indices]
```

4.2.2. Reading a Minibatch

```
@d2l.add_to_class(FashionMNIST)
def get_dataloader(self, train):
    data = self.train if train else self.val
    return torch.utils.data.DataLoader(data, self.batch_size, shuffle=train,
                                       num_workers=self.num_workers)
```

```
X, y = next(iter(data.train_dataloader()))
print(X.shape, X.dtype, y.shape, y.dtype)
```

```
/usr/local/lib/python3.10/dist-packages/torch/utils/data/dataloader.py:557: UserWarning: This DataLoader will create 4 w
  warnings.warn(_create_warning_msg(
torch.Size([64, 1, 32, 32]) torch.float32 torch.Size([64]) torch.int64
```

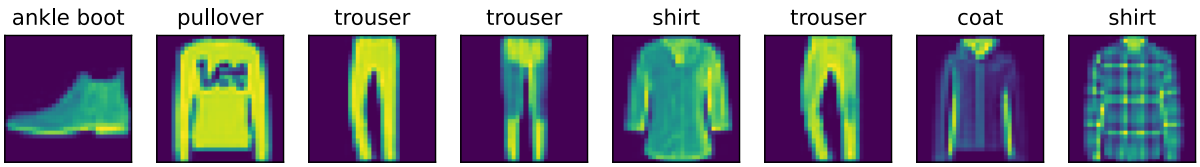
```
tic = time.time()
for X, y in data.train_dataloader():
    continue
f'{time.time() - tic:.2f} sec'
```

```
'13.76 sec'
```

4.2.3. Visualization

```
def show_images(imgs, num_rows, num_cols, titles=None, scale=1.5):
    raise NotImplementedError
```

```
@d2l.add_to_class(FashionMNIST)
def visualize(self, batch, nrows=1, ncols=8, labels=[]):
    X, y = batch
    if not labels:
        labels = self.text_labels(y)
    d2l.show_images(X.squeeze(1), nrows, ncols, titles=labels)
batch = next(iter(data.val_dataloader()))
data.visualize(batch)
```



4.3. The Base Classification Model

```
import torch
from d2l import torch as d2l
```

4.3.1. The Classifier Class

```
class Classifier(d2l.Module):
    def validation_step(self, batch):
        Y_hat = self(*batch[:-1])
        self.plot('loss', self.loss(Y_hat, batch[-1]), train=False)
        self.plot('acc', self.accuracy(Y_hat, batch[-1]), train=False)
```

```
@d2l.add_to_class(d2l.Module)
def configure_optimizers(self):
    return torch.optim.SGD(self.parameters(), lr=self.lr)
```

4.3.2. Accuracy

```
@d2l.add_to_class(Classifier)
def accuracy(self, Y_hat, Y, averaged=True):
    Y_hat = Y_hat.reshape((-1, Y_hat.shape[-1]))
    preds = Y_hat.argmax(axis=1).type(Y.dtype)
```

```
    compare = (preds == Y.reshape(-1)).type(torch.float32)
    return compare.mean() if averaged else compare
```

4.4. Softmax Regression Implementation from Scratch

```
import torch
from d2l import torch as d2l
```

4.4.1. The Softmax

```
X = torch.tensor([[1.0, 2.0, 3.0], [4.0, 5.0, 6.0]])
X.sum(0, keepdims=True), X.sum(1, keepdims=True)
```

```
(tensor([[5., 7., 9.]]),
 tensor([[ 6.],
         [15.]]))
```

```
def softmax(X):
    X_exp = torch.exp(X)
    partition = X_exp.sum(1, keepdims=True)
    return X_exp / partition
```

```
X = torch.rand((2, 5))
X_prob = softmax(X)
X_prob, X_prob.sum(1)
```

```
(tensor([[0.2160, 0.2001, 0.1335, 0.2468, 0.2036],
         [0.2039, 0.1533, 0.2870, 0.2354, 0.1204]]),
 tensor([1.0000, 1.0000]))
```

4.4.2. The Model

```
class SoftmaxRegressionScratch(d2l.Classifier):
    def __init__(self, num_inputs, num_outputs, lr, sigma=0.01):
        super().__init__()
        self.save_hyperparameters()
        self.W = torch.normal(0, sigma, size=(num_inputs, num_outputs),
                              requires_grad=True)
        self.b = torch.zeros(num_outputs, requires_grad=True)

    def parameters(self):
        return [self.W, self.b]
```

```
@d2l.add_to_class(SoftmaxRegressionScratch)
def forward(self, X):
    X = X.reshape((-1, self.W.shape[0]))
    return softmax(torch.matmul(X, self.W) + self.b)
```

4.4.3. The Cross-Entropy Loss

```
y = torch.tensor([0, 2])
y_hat = torch.tensor([[0.1, 0.3, 0.6], [0.3, 0.2, 0.5]])
y_hat[[0, 1], y]
```

```
tensor([0.1000, 0.5000])
```

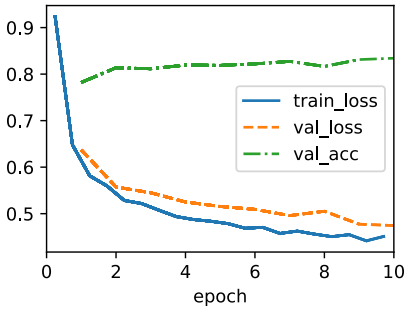
```
def cross_entropy(y_hat, y):
    return -torch.log(y_hat[list(range(len(y_hat))), y]).mean()

cross_entropy(y_hat, y)
```

```
tensor(1.4979)
```

```
@d2l.add_to_class(SoftmaxRegressionScratch)
def loss(self, y_hat, y):
    return cross_entropy(y_hat, y)
```

4.4.4. Training

```
data = d2l.FashionMNIST(batch_size=256)
model = SoftmaxRegressionScratch(num_inputs=784, num_outputs=10, lr=0.1)
trainer = d2l.Trainer(max_epochs=10)
trainer.fit(model, data)
```

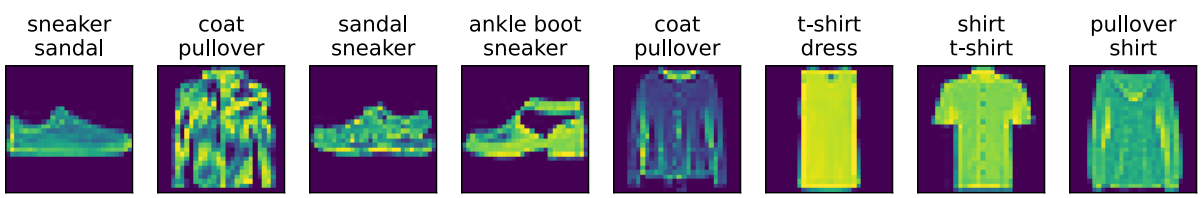


4.4.5. Prediction

```
X, y = next(iter(data.val_dataloader()))
preds = model(X).argmax(axis=1)
preds.shape
```

```
torch.Size([256])
```

```
wrong = preds.type(y.dtype) != y
X, y, preds = X[wrong], y[wrong], preds[wrong]
labels = [a+'\n'+b for a, b in zip(
    data.text_labels(y), data.text_labels(preds))]
data.visualize([X, y], labels=labels)
```



5.1 Multilayer Perceptrons

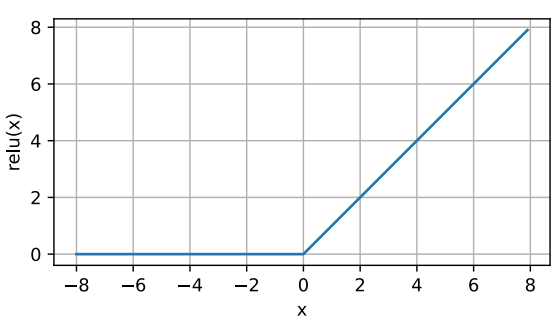
```
%matplotlib inline
import torch
from d2l import torch as d2l
```

5.1.1. Hidden Layers

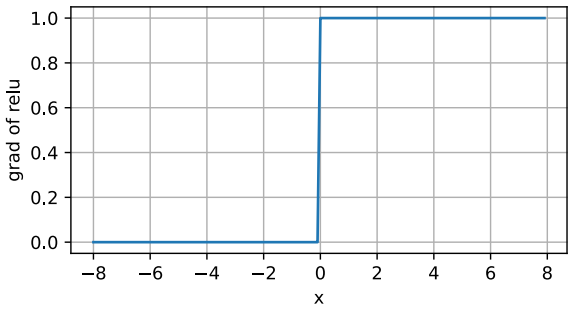
5.1.2. Activation Functions

5.1.2.1. ReLU Function

```
x = torch.arange(-8.0, 8.0, 0.1, requires_grad=True)
y = torch.relu(x)
d2l.plot(x.detach(), y.detach(), 'x', 'relu(x)', figsize=(5, 2.5))
```

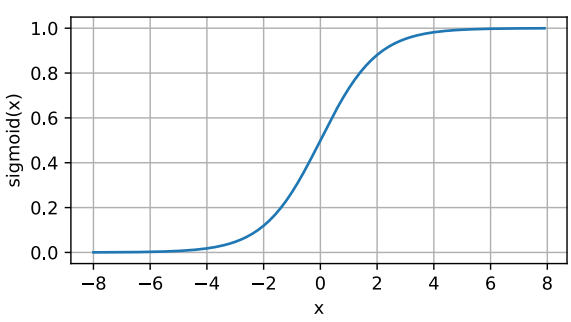


```
y.backward(torch.ones_like(x), retain_graph=True)
d2l.plot(x.detach(), x.grad, 'x', 'grad of relu', figsize=(5, 2.5))
```

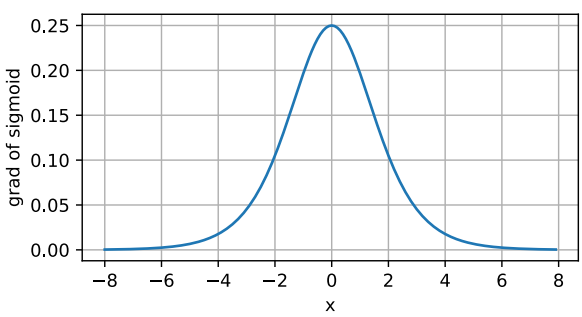


5.1.2.2. Sigmoid Function

```
y = torch.sigmoid(x)
d2l.plot(x.detach(), y.detach(), 'x', 'sigmoid(x)', figsize=(5, 2.5))
```

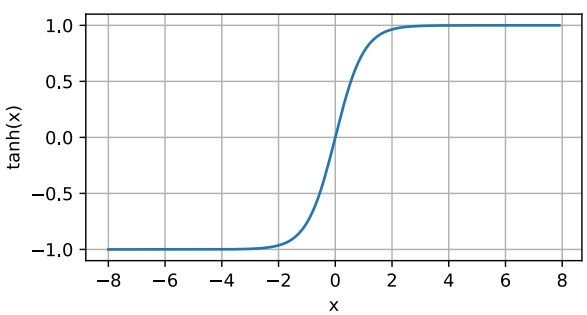



```
# Clear out previous gradients
x.grad.data.zero_()
y.backward(torch.ones_like(x),retain_graph=True)
d2l.plot(x.detach(), x.grad, 'x', 'grad of sigmoid', figsize=(5, 2.5))
```



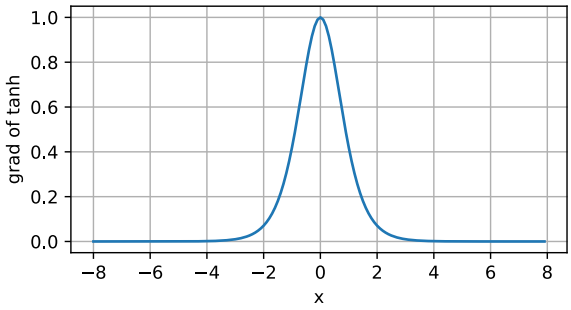
5.1.2.3. Tanh Function

```
y = torch.tanh(x)
d2l.plot(x.detach(), y.detach(), 'x', 'tanh(x)', figsize=(5, 2.5))
```




```
x.grad.data.zero_()
y.backward(torch.ones_like(x),retain_graph=True)
```

```
d2l.plot(x.detach(), x.grad, 'x', 'grad of tanh', figsize=(5, 2.5))
```



5.2. Implementation of Multilayer Perceptrons

5.2.1. Implementation from Scratch

5.2.1.1. Intializing Model Parameters

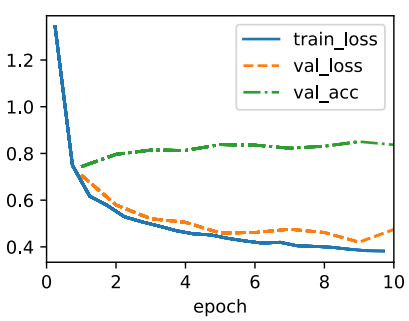
```
class MLPScratch(d2l.Classifier):
    def __init__(self, num_inputs, num_outputs, num_hiddens, lr, sigma=0.01):
        super().__init__()
        self.save_hyperparameters()
        self.W1 = nn.Parameter(torch.randn(num_inputs, num_hiddens) * sigma)
        self.b1 = nn.Parameter(torch.zeros(num_hiddens))
        self.W2 = nn.Parameter(torch.randn(num_hiddens, num_outputs) * sigma)
        self.b2 = nn.Parameter(torch.zeros(num_outputs))
```

5.2.1.2. Model

```
def relu(X):
    a = torch.zeros_like(X)
    return torch.max(X, a)

@d2l.add_to_class(MLPScratch)
def forward(self, X):
    X = X.reshape((-1, self.num_inputs))
    H = relu(torch.matmul(X, self.W1) + self.b1)
    return torch.matmul(H, self.W2) + self.b2
```

5.2.1.3. Training

```
model = MLPScratch(num_inputs=784, num_outputs=10, num_hiddens=256, lr=0.1)
data = d2l.FashionMNIST(batch_size=256)
trainer = d2l.Trainer(max_epochs=10)
trainer.fit(model, data)
```



5.2.2. Consice Implementation

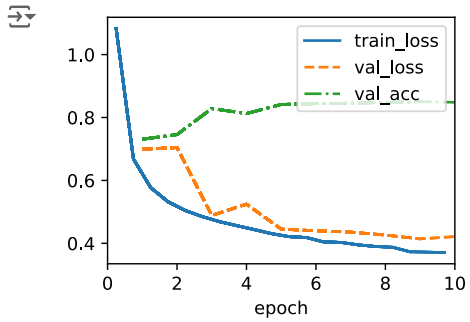
5.2.2.1. Model

```
class MLP(d2l.Classifier):
    def __init__(self, num_outputs, num_hiddens, lr):
```

```
        super().__init__()
        self.save_hyperparameters()
        self.net = nn.Sequential(nn.Flatten(), nn.LazyLinear(num_hiddens),
                                 nn.ReLU(), nn.LazyLinear(num_outputs))
```

5.2.2.2. Training

```
model = MLP(num_outputs=10, num_hiddens=256, lr=0.1)
trainer.fit(model, data)
```



5.3. Forward Propagation, Backward Propogation, and Computational Graphs

5.3.1. Forward Propagation

the intermediate variable is:

$$\mathbf{z} = \mathbf{W}^{(1)}\mathbf{x},$$

where $\mathbf{W}^{(1)} \in \mathbb{R}^{h \times d}$ is the weight parameter of the hidden layer. After running the intermediate variable $\mathbf{z} \in \mathbb{R}^h$ through the activation function ϕ we obtain our hidden activation vector of length h:

$$\mathbf{h} = \phi(\mathbf{z}).$$

The hidden layer output $\mathbf{h}$ is also an intermediate variable. Assuming that the parameters of the output layer possess only a weight of $\mathbf{W}^{(2)} \in \mathbb{R}^{q \times h}$, we can obtain an output layer variable with a vector of length q:

$$\mathbf{o} = \mathbf{W}^{(2)}\mathbf{h}.$$

Assuming that the loss function is l and the example label is y, we can then calculate the loss term for a single data example,

$$L = l(\mathbf{o}, y).$$

As we will see the definition of ℓ_2 regularization to be introduced later, given the hyperparameter λ, the regularization term is

$$s = \frac{\lambda}{2}\left(\|\mathbf{W}^{(1)}\|_F^2 + \|\mathbf{W}^{(2)}\|_F^2\right),$$

:eqlabel: `eq_forward-s`

where the Frobenius norm of the matrix is simply the ℓ_2 norm applied after flattening the matrix into a vector. Finally, the model's regularized loss on a given data example is:

$$J = L + s.$$

5.3.3. Backpropagation

he first step is to calculate the gradients of the objective function $J = L + s$ with respect to the loss term L and the regularization term s:

$$\frac{\partial J}{\partial L} = 1 \text{ and } \frac{\partial J}{\partial s} = 1.$$

Next, we compute the gradient of the objective function with respect to variable of the output layer $\mathbf{o}$ according to the chain rule:

$$\frac{\partial J}{\partial \mathbf{o}} = \text{prod}\left(\frac{\partial J}{\partial L}, \frac{\partial L}{\partial \mathbf{o}}\right) = \frac{\partial L}{\partial \mathbf{o}} \in \mathbb{R}^q.$$

Next, we calculate the gradients of the regularization term with respect to both parameters:

$$\frac{\partial s}{\partial \mathbf{W}^{(1)}} = \lambda \mathbf{W}^{(1)} \text{ and } \frac{\partial s}{\partial \mathbf{W}^{(2)}} = \lambda \mathbf{W}^{(2)}.$$

Now we are able to calculate the gradient $\partial J / \partial \mathbf{W}^{(2)} \in \mathbb{R}^{q \times h}$ of the model parameters closest to the output layer. Using the chain rule yields:

$$\frac{\partial J}{\partial \mathbf{W}^{(2)}} = \text{prod}\left(\frac{\partial J}{\partial \mathbf{o}}, \frac{\partial \mathbf{o}}{\partial \mathbf{W}^{(2)}}\right) + \text{prod}\left(\frac{\partial J}{\partial s}, \frac{\partial s}{\partial \mathbf{W}^{(2)}}\right) = \frac{\partial J}{\partial \mathbf{o}}\mathbf{h}^\top + \lambda \mathbf{W}^{(2)}.$$

To obtain the gradient with respect to $\mathbf{W}^{(1)}$ we need to continue backpropagation along the output layer to the hidden layer. The gradient with respect to the hidden layer output $\partial J/\partial \mathbf{h} \in \mathbb{R}^h$ is given by

$$\frac{\partial J}{\partial \mathbf{h}} = \text{prod}\left(\frac{\partial J}{\partial \mathbf{o}}, \frac{\partial \mathbf{o}}{\partial \mathbf{h}}\right) = \mathbf{W}^{(2)\top}\frac{\partial J}{\partial \mathbf{o}}.$$

Since the activation function ϕ applies elementwise, calculating the gradient $\partial J/\partial \mathbf{z} \in \mathbb{R}^h$ of the intermediate variable $\mathbf{z}$ requires that we use the elementwise multiplication operator, which we denote by $\odot$:

$$\frac{\partial J}{\partial \mathbf{z}} = \text{prod}\left(\frac{\partial J}{\partial \mathbf{h}}, \frac{\partial \mathbf{h}}{\partial \mathbf{z}}\right) = \frac{\partial J}{\partial \mathbf{h}} \odot \phi'(\mathbf{z}).$$

Finally, we can obtain the gradient $\partial J/\partial \mathbf{W}^{(1)} \in \mathbb{R}^{h \times d}$ of the model parameters closest to the input layer. According to the chain rule, we get

$$\frac{\partial J}{\partial \mathbf{W}^{(1)}} = \text{prod}\left(\frac{\partial J}{\partial \mathbf{z}}, \frac{\partial \mathbf{z}}{\partial \mathbf{W}^{(1)}}\right) + \text{prod}\left(\frac{\partial J}{\partial s}, \frac{\partial s}{\partial \mathbf{W}^{(1)}}\right) = \frac{\partial J}{\partial \mathbf{z}}\mathbf{x}^\top + \lambda \mathbf{W}^{(1)}.$$

Discussion & Excercises

Memo

2.2. Data Preprocessing

- we must pay attention to data quality. Real-world datasets are often plagued by outliers, faulty measurements from sensors, and recording errors, which must be addressed before feeding the data into any model.

2.5. Automatic Differentiation

- basic steps

(i) attach gradients to those variables with respect to which we desire derivatives;

(ii) record the computation of the target value;

(iii) execute the backpropagation function; and

(iv) access the resulting gradient.

3.2. Object-Oriented Design for Implementation

- we wish to have three classes:

(i) Module contains models, losses, and optimization methods;

(ii) DataModule provides data loaders for training and validation;

(iii) both classes are combined using the Trainer class, which allows us to train models on a variety of hardware platforms.

4.1. Softmax Regression

- derivative of the cross-entropy loss when combined with softmax behaves very similarly to the derivative of squared error.
- Deep Fried Convnets (Yang et al., 2015) uses a combination of permutations, Fourier transforms, and scaling to reduce the cost from quadratic to log-linear. Similar techniques work for more advanced structural matrix approximations (Sindhwani et al., 2015).

4.2. The Image Classification Dataset

- data iterators are a key component for efficient performance. For instance, we might use GPUs for efficient image decompression, video transcoding, or other preprocessing.
- Whenever possible, you should rely on well-implemented data iterators that exploit high-performance computing to avoid slowing down your training loop.

5.2. Multilayer Perceptrons

Problem of nonlinearity ex:

- decision trees in their most basic form use a sequence of binary decisions to decide upon class membership (Quinlan, 1993).
- kernel methods have been used for many decades to model nonlinear dependencies (Aronszajn, 1950). This has found its way into nonparametric spline models (Wahba, 1990) and kernel methods (Schölkopf and Smola, 2002).

Universal Approximators:

- Because a single-hidden-layer network can learn any function does not mean that you should try to solve all of your problems with one. In fact, in this case kernel methods are way more effective, since they are capable of solving the problem exactly even in infinite dimensional spaces (Kimeldorf and Wahba, 1971, Schölkopf et al., 2001)
- we can approximate many functions much more compactly by using deeper (rather than wider) networks (Simonyan and Zisserman, 2014).

- A secondary benefit is that ReLU is significantly more amenable to optimization than the sigmoid or the tanh function.
- the GELU (Gaussian error linear unit) activation function xΦ(x) by Hendrycks and Gimpel (2016) (Φ(x) is the standard Gaussian cumulative distribution function) and the Swish activation function σ(x)=xsigmoid(βx) as proposed in Ramachandran et al. (2017) can yield better accuracy in many cases.

Exercise

2.5.6.1.

- It involves computing n^2 partial derivatives compared to n for the first derivative, leading to quadratic growth in computational cost.
- It requires calculating cross-terms that describe interactions between all pairs of variables.
- The computational graph becomes significantly more complex when calculating second derivatives.

3.1.6.4.

1. when XTX is rank-deficient, the problem has no unique solution, and there may be an infinite number of solutions or no solution at all. This typically happens if there is multicollinearity in the features, meaning some features are linear combinations of others.
2. perturb the matrix X by adding small amounts of noise to its entries. Specifically, you can add coordinate-wise independent Gaussian noise $N(0,\sigma 2$) to all entries of X. This can "break" linear dependencies between columns and make XTX full rank, though it does slightly modify the data.
3. Stochastic gradient descent can still work, but it may suffer from slow convergence or oscillation. Regularization can help SGD perform better when $XTX$is not full rank.

4.1.5.2.

Problem: Binary encoding for three equally probable classes leads to inefficiency because we need 2 bits to encode 3 outcomes, wasting one bit.

By encoding multiple observations jointly, the code becomes more efficient. For two observations, we use 4 bits instead of 2 bits per observation. As n increases, the efficiency approaches the optimal entropy of $\log_2 3 \approx 1.5.85$ bits per observation.

5.3.6.1.

the gradient $\nabla x f$ will also be an $n \times m$ matrix. Each element of this matrix corresponds to the partial derivative of f with respect to the corresponding element of f