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CIS 262

Problem Set 3

Problem 1

a) $a^* b^*$

In: ab, aabb

Out: aba, baba

b) $a(ba)^* b$

In: ab, abab

Out: a, aba

c) $a^* \cup b^*$

In: a, b

Out: ab, ba

d) $(aaa)^*$

In: aaa, aaaaaa

Out: a, aaaa

e) $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$

In: aaabaaa, babbbab

Out: a, b

Problem 2

Prove that if L is regular, then $h(L) = \{h(w) \mid w \in L\}$ also regular

Want to prove for any regular expression R,

$$L(h(R)) = h(L(R))$$

Base Cases

$R = \varepsilon$ or ϕ

$h(R) = R$

$h(L(R)) = L(R)$

$R = a$

$L(R) = \{a\}$

$h(L(R)) = h\{a\} = L(h(a)) = L(h(R))$

Induction Step

For $R = R_1 \cup R_2$, $h(R) = h(R_1) \cup h(R_2)$

$h(L(R)) = h(L(R_1) \cup L(R_2)) = h(L(R_1)) \cup h(L(R_2))$

By Induction Hypothesis, $h(L(R_i)) = L(h(R_i))$

It follows that $h(L(R)) = L(h(R_1) \cup h(R_2))$

For $R = R_1 R_2$, $h(R) = h(R_1) h(R_2)$

$h(L(R)) = h(L(R_1) L(R_2)) = h(L(R_1)) h(L(R_2))$

By IH, it follows that $h(L(R)) = L(h(R_1) h(R_2))$

Problem 3

a) Counterexample

Let M be non-regular.

If $L = \phi$ then $L \cup M = M$, which is non-regular

b) Counterexample

Let M be non-regular

If $L = \Sigma^*$ then $L \cap M = M$, which is non-regular

c) Counterexample

Let L be non-regular

Let $M = \Sigma^*/L$, the complement of L which is also non-regular (proven in Homework 2)

$L \cup M = \Sigma^*$, which is regular

d) Let $L = \{0^n 1^n \mid n \geq 0\}$

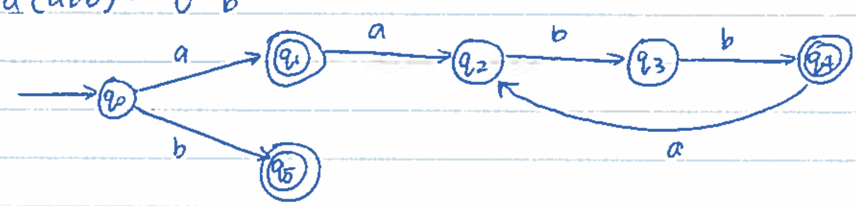
Let $M = \{1^n 0^n \mid n \geq 0\}$

Both L and M are non-regular (proven in Textbook pg. 80)

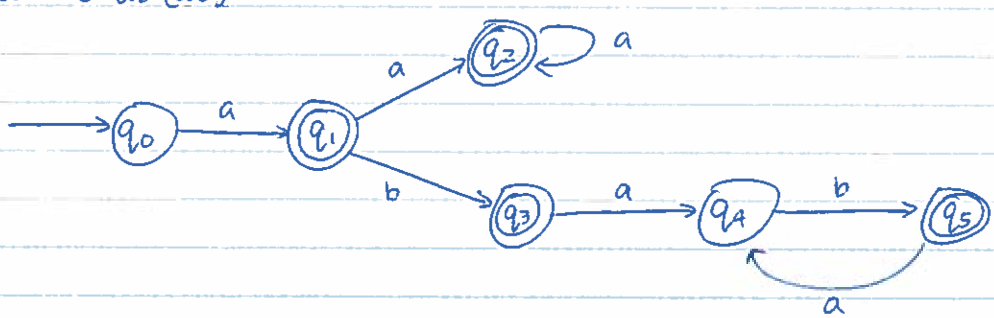
$L \cap M = \{\varepsilon\}$, which is regular

Problem 4

a) $a(abb)^* \cup b$



b) $aa^* \cup ab(ab)^*$



Problem 5

$\Sigma = \{a, b, c\}$

$L = \{ w = a^k b^m c^n \mid k+m+n \text{ is even} \}$

Consider $S = \{ a\varepsilon, ab, abb, abbb, \dots \} = \{ ab^k \mid k \geq 0 \}$

S contains infinitely many strings

To show that S contains pairwise distinguishable strings, consider two strings ab^i and ab^j with $j = i + 1$

Only one of $ab^i \cdot c$ and $ab^j \cdot c$ is in L, because the lengths of the two strings differ by one – that is, they are consecutive. Since consecutive integers are represented by $2k$ and $2k+1$, only one is even-lengthed.

$\therefore$ By Thm from Lecture 8, slide 30, L is not regular

Problem 6

$\Sigma = \{1\}$

$L = \{ 1^p \mid p \text{ is prime} \}$

Proof by contradiction:

Assume L is regular.

Let n be the pumping length given by the pumping lemma.

Let s be the string 1^k such that k is a prime number $> n$.

Then s can be split into xyz, satisfying the conditions of the pumping lemma.

Now consider $xy^{k+1}z$.

$$|xy^{k+1}z| = |xyz| + |y^k| = k + |y|k = k(\underbrace{1 + |y|})$$

Cannot be 1 because it is prime

$|y|$ cannot be 0 by condition 2 of pumping lemma, so expression is > 1

Therefore, $|xy^{k+1}z|$ is a composite number.

Contradiction.

Problem 7

$\Sigma = \{1\}$

$L = \{1^n \mid n \text{ is not a perfect square}\}$

First, will prove that $\overline{L} = \{1^n \mid n \text{ is a perfect square}\}$ is not regular.

Proof by contradiction.

Assume $\overline{L}$ is regular.

Let p be the pumping length given by the pumping lemma.

Let s be the string 0^{p^2}.

Then s can be split into xyz, satisfying the conditions of the pumping lemma.

By condition 3, $|xy| \leq p$.

Let us say $|y| = k$.

Then $|xyz| = (p^2 - k) + k$

Then $|xyyz| = (p^2 - k) + 2k$

$= p^2 + k$

Since $k \leq p$, $p^2 + k \leq p^2 + p$

$< p^2 + 2p + 1$

$= (p+1)^2$

But $p^2 + k > p^2$ since $|y|$ cannot equal 0.

Then $p^2 < \underbrace{p^2 + k} < (p+1)^2$

$\therefore$ Not a perfect square.

Contradiction.

Therefore, L is also non-regular because the class of regular languages is closed under complement (proven in HW 2)

Problem 8

a) $\Sigma = \{0, 1\}$

$L = \{ 0^n 1^m 0^n \mid m, n \geq 0 \}$

Proof by contradiction.

Assume L is regular.

Let p be the pumping length given by the pumping lemma

Choose s to be the string $0^p 1 0^p$

Then s can be split into xyz, satisfying conditions of pumping lemma

By condition 3, $|xy| \leq p$, so y can only consist of 0's.

In this case, $xyyz$ has more 0's before the 1 than after the 1, violating the condition that the number of 0's at the beginning is equal to the number of 0's at the end.

Contradiction.

b) $L = \{ w \mid w \text{ is not a palindrome} \}$

Consider $S = \{ \varepsilon b, ab, aab, aaab, \ldots \} = \{ a^k b \mid k \geq 0 \}$

S contains infinitely many strings

To show that S contains pairwise distinguishable strings, consider two strings $a^i b$ and $a^j b$ with $i \,!= j$.

$a^i b . a^i$ is not in L, but $a^j b . a^i$ is in L.

$\therefore$ By Thm from Lecture 8, slide 30, L is not regular