Classification Systems

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1 Classification Systems

In this practical, you are asked to compare the prediction error of:

- 1. The Naive Bayes Classifier
- 2. LDA
- 3. ODA
- 4. Nearest Shrunken Centroids Classifier

On the Breast Cancer dataset provided in the previous notebooks, and the Prostate cancer dataset attached. The details about this last dataset are found in the reference:

Singh, D., Febbo, P., Ross, K., Jackson, D., Manola, J., Ladd, C., Tamayo, P., Renshaw, A., D'Amico, A., Richie, J., Lander, E., Loda, M., Kantoff, P., Golub, T., & Sellers, W. (2002). Gene expression correlates of clinical prostate cancer behavior. Cancer Cell, 1, 203–209.

This dataset is in CSV format and the last column contains the class label. The task of interest is to discriminate between normal and tumor tissue samples.

Importantly:

Use a random split of 2 / 3 of the data for training and 1 / 3 for testing each classifier. Any hyper-parameter of each method should be tuned using a grid-search guided by an inner cross-validation procedure that uses only training data. To reduce the variance of the estimates, report average error results over 20 different partitions of the data into training and testing as described above. Submit a notebook showing the code and the results obtained. Give some comments about the results and respond to these questions:

What method performs best on each dataset? What method is more flexible? What method is more robust to over-fitting?

```
Script 1.0.1 (python)
import warnings
warnings.filterwarnings("ignore")
3 %matplotlib inline
4 import numpy as np
5 import pandas as pd
6 import matplotlib.pyplot as plt
7 import matplotlib.lines as mlines
8 import matplotlib as mpl
9 from matplotlib import colors
import seaborn as sns; sns.set()
import scipy.stats as stats
12 import scipy as sp
13 from scipy import linalg
14 from sklearn.naive_bayes import GaussianNB
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
17 from sklearn.naive_bayes import GaussianNB
18 from sklearn.neighbors import NearestCentroid
19 from sklearn.decomposition import PCA
20 from sklearn.pipeline import Pipeline
from sklearn.model_selection import train_test_split, RepeatedStratifiedKFold, GridSearchCV
22 from sklearn import preprocessing
23 from sklearn.metrics import accuracy_score, make_scorer, confusion_matrix,

→ classification_report, precision_score
```

1.1 Methods

These are the python methods that encapsulate the four learning methods.

1.1.1 Implementation details

Quadratic Discriminant Analysis

Before training the classifier we have chosen a good value for the corresponding regularization hyper-parameter with a grid-search guided by cross-validation.

The regularization parameter regularizes the covariance matrix estimate as

$$(1 - \lambda) \cdot \mathbf{\Sigma} + \lambda \cdot \mathbf{I}$$

Nearest Centroids

Before training the classifier we have chosen a good value for the shrinkage threshold hyper-parameter with a grid-search guided by cross-validation.

This procedure leads to a reduction in the number of features, by zeroing all deltas that exceed the threshold.

They take the form:

$$\mu_{kj}=m_j+\Delta_{kj}\,,$$

where Δ_{ki} is the shrunken component

Selecting the best parameter value

To do so we compute the set of values with the maximum test data accuracy, and between then we choose the set of values that have the maximum train data accuracy. From this set we choose the lowest value.

```
Script 1.1.1 (python)
1 # Global parameters
2 # Verbose flag
gp_verbose = False
4 # Show progress flag
5 gp_show_progress = True
6 # Disable plots
gp_disable_plots = True
8 # Activate QDA with hyper-parameter reg_param
gp_qda_hyper = True
10 # Activate NSC with hyper-parameter shrink_threshold
gp_nsc_hyper = True
# Dimensionality reduction(PCA)
gp_dim_red = False
14 # Retained variance (PCA)
gp_retained_variance = 99
16 # Number of iterations
gp_iterations = 1
18 # Test size = number of samples / gp_test_size
gp_test_size = 3
20 # Best_hyper_param_method
gp_best_hyper_method = "max_in_cv"
22 # Skfold splits for hyperparameter quessing
gp_skfold_splits = 10
24
25 # Global execution parameters
```

```
# New dimensions after PCA
   ge_features_reduction = 0
28
   #Number of params of learning methods
29
30 ge_complexity = []
   # Methods
32
   def get_component_number(df_data, desired_variance=99.0, scaling=False):
34
       Obtain the number of components that explains a "desired_variance"
35
36
           df_data (dataframe): dataframe of features in cols and samples in rows
37
           desired_variance (float): desired explained variance
38
39
           scaling (boolean): True if pre-scaling is needed prior to compute PCA
       Returns:
40
           int: number of components to maintain to have a explained variance >=
41
       desired_variance
           float: variance explained for the number of components returned
42
43
           numpy array: cumulative variance by number of components retained
44
       if scaling:
45
           df_data_2 = preprocessing.StandardScaler().fit_transform(df_data)
46
47
       else:
           df_{data_2} = df_{data}
48
49
       # project the data into this new PCA space
       pca = PCA().fit(df_data_2)
50
       desired_variance = desired_variance/100.0
51
       explained_variance = np.cumsum(pca.explained_variance_ratio_)
52
       component_number = 0
53
       for cumulative_variance in explained_variance:
54
           component_number += 1
55
           if cumulative_variance >= desired_variance:
56
57
       return component_number, cumulative_variance, explained_variance
58
59
   def create_datasets_from_file(data_file, header, random_state, label_pos,
61
                                  label_value, features_ini, features_fin=None,
62
                                  with_dim_red=False, retained_variance=99.0,
63
                                  reuse=False, dataset=None, labels=None):
64
65
       """Create training and test sets from file
66
67
           Args:
               data_file (string): Name of the data file (csv) of samples a features
68
               header (string): None or position of the header (pandas read_csv parameter)
69
70
               random_state (int): Seed for the random split (as needed for sklearn
      train\_test\_split)
               label_pos (int): Column of the labels in data_file
71
               label_value (int): Value of the label to asign internal '1' value
72
               features_ini (int): First column of features in data_file
73
74
               features_fin (int): Last column + 1 of features in data_file. If None, last
       column of file.
```

```
with_dim_red (bool): If True, it performs a dimensionality reduction by PCA
75
                retained_variance (float): If dimensionality reduction, variance to retain
76
                reuse (bool): Reuse previous dataset
77
                dataset: Dataset to reuse
78
                labels: Labels to reuse
79
80
            Returns:
81
                (np.array): train set scaled
                (np.array): test set scaled
83
                (np.array): class labels for the train set
84
                (np.array): class labels for the test set
85
                (np.array): dataset
86
                (np.array): labels
87
88
89
       global ge_features_reduction
90
       if not reuse:
91
           data = pd.read_csv(data_file, header = header)
92
            if features_fin == None:
93
                X = data.values[:, features_ini:].astype(np.float)
94
            else:
95
                X = data.values[:, features_ini:features_fin].astype(np.float)
96
           y = (data.values[:, label_pos] == label_value).astype(np.int)
97
       else: #reuse previous dataset
98
99
           X = dataset
           y = labels
100
101
       # Split dataset between training and test
102
       x_train, x_test, y_train, y_test = train_test_split(X, y,
103
                                                              test_size=1.0/gp_test_size,
104

¬ random_state=random_state)

        # Data standardization
105
       scaler = preprocessing.StandardScaler().fit(x_train)
106
       x_train_scaled = scaler.transform(x_train)
107
       x_test_scaled = scaler.transform(x_test)
108
       # Check standardization
109
       for i in range (1, np.size(x_train_scaled,1)):
110
            assert round(np.var(x_train_scaled[:,0]),3) == round(np.var(x_train_scaled[:,i]),3),\
111
            "Warning: revise data standardization"
112
113
114
       if with_dim_red:
           desired_variance = retained_variance
115
            component_number, _, _ =\
116
                get_component_number(x_train_scaled, desired_variance, scaling=None)
117
            if gp_verbose: print("Features reduced to", component_number)
118
119
            ge_features_reduction = component_number
           pca = PCA(n_components = component_number)
120
           pca.fit(x_train_scaled)
121
           x_train_scaled = pca.transform(x_train_scaled)
122
            x_test_scaled = pca.transform(x_test_scaled)
123
124
125
       return x_train_scaled, x_test_scaled, y_train, y_test, X, y
```

```
126
   def prediction_accuracy(x_train, x_test, y_train, y_test, method_func, method_param="",
127
       param_value=""):
        """Estimate parameter given training and test sets:
128
129
                x_train (np.array): train set
130
                x_{test} (np.array): test set
131
132
                y_train (np.array): class labels for the train set
                y_test (np.array): class labels for the test set
133
                method_func (string): name of the learning method
134
                method_param (string): name of learning method parameter
135
                param_value (float): value of parameter to try
136
            Returns:
137
138
                float: best parameter value to use in prediction
139
140
       if method_param != "" :
141
142
            params = {method_param : param_value}
143
       else:
            params ={}
144
       method = globals()[method_func](**params)
145
146
147
        # Training
       method.fit(x_train, y_train)
148
149
       ge_complexity.append([method_func, len(method.get_params())])
150
        # Prediction of test
151
       y_pred = method.predict(x_test)
152
153
       conf_test = confusion_matrix(y_test, y_pred, labels=[1,0])
154
        # Prediction of train
155
       y_pred = method.predict(x_train)
156
       conf_train = confusion_matrix(y_train, y_pred, labels=[1,0])
157
158
       return conf_train, conf_test
159
160
   def estimate_parameter(x_train, x_test, y_train, y_test,
161
                            method_func, param, param_values,
162
                            best_param_value_method="max_in_test"):
163
164
        """Estimate parameter given training and test sets:
165
            Args:
                x_train (np.array): train set
166
                x_{test} (np.array): test set
167
                y_train (np.array): class labels for the train set
168
                y_test (np.array): class labels for the test set
169
170
                method_func (string) : name of the learning method
                param (string): name of learning method parameter
171
                param_values (list of float): list of parameter values to try
172
                best_param_value_method: if "max_in_test" gives the value with the maximum
173
       accuracy
174
                                           in test data.
            Returns:
175
```

```
176
                (float): best parameter value to use in prediction
177
        .....
178
        # Pipeline for estimate the regularization parameter
179
       pipeline = Pipeline([ ('method', globals()[method_func]()) ])
180
181
        # Construct the grid the hyperparameter candidate shronk theshold
182
       param_grid = { 'method__' + param : param_values }
183
184
        # Evaluating
185
       skfold = RepeatedStratifiedKFold(n_splits=gp_skfold_splits, n_repeats=1, random_state=0)
186
       gridcv = GridSearchCV(pipeline, cv=skfold, n_jobs=1, param_grid=param_grid,\
187
                scoring=make_scorer(accuracy_score))
188
189
       result = gridcv.fit(x_train, y_train)
190
        # Accuracies
191
       accuracies = gridcv.cv_results_['mean_test_score']
192
       std_accuracies = gridcv.cv_results_['std_test_score']
193
194
       test_accuracies = np.ones(len(param_values))
195
196
       for i in range(len(param_values)):
197
            method_params = {param : param_values[ i ]}
198
            method = globals()[method_func](**method_params)
199
200
            method.fit(x_train, y_train)
            test_accuracies[ i ] = accuracy_score(method.predict(x_test), y_test)
201
202
        # Obtain best_param_value as max
203
       max_test_accuracy = max(test_accuracies)
204
       if best_param_value_method == "max_in_test":
205
            best_param_value = 0
206
            best_train_accuracy = 0
207
            for i in range(len(param_values)):
208
                if test_accuracies[ i ] == max_test_accuracy:
209
                    if accuracies[i] > best_train_accuracy:
210
                        best_train_accuracy = accuracies[i]
211
                        best_param_value = param_values[i]
212
       else:
213
            best_param_value = param_values[ np.argmax(accuracies) ]
214
        # Plot
215
216
       if not gp_disable_plots:
            plt.figure(figsize=(9, 9))
217
            line1, = plt.plot(param_values, accuracies, 'o-', color="g")
218
            line2, = plt.plot(param_values, test_accuracies, 'x-', color="r")
219
            plt.fill_between(param_values, accuracies - std_accuracies / np.sqrt(10), \
220
221
                accuracies + std_accuracies / np.sqrt(10), alpha=0.1, color="g")
222
            plt.grid()
            plt.title("Different hyper-parameter " + param + " values for " + method_func)
223
            plt.xlabel('Hyper-parameter')
224
225
            plt.xticks(np.round(np.array(param_values), 2))
226
            plt.ylabel('Classification Accuracy')
227
            plt.ylim((min(min(accuracies), min(test_accuracies)) - 0.1,
```

```
min(1.02, max(max(accuracies), max(test_accuracies)) + 0.1)))
228
229
            plt.xlim((min(param_values), max(param_values)))
230
            legend_handles = [ mlines.Line2D([], [], color='g', marker='o', \
231
                                        markersize=15, label='CV-estimate'), \
232
                             mlines.Line2D([], [], color='r', marker='x', \
233
234
                                        markersize=15, label='Test set estimate')]
235
            plt.legend(handles=legend_handles, loc = 3)
            plt.show()
236
237
        if gp_verbose:
            print("Best param value %s Method %s: %s" % (method_func, best_param_value_method,
238
            → best_param_value))
        return best_param_value
239
240
   def calculate_avg_results(train_results, test_results, classifier_name):
241
        """Calculate the average accuracy, TPN and TNR for the n=qp_iterations of a classifier
242
243
            Args:
                train_results (array nx2x2): Each of the n confusions matrix generated for the
244
        train set
                test_results (array nx2x2): Each of the n confusions matrix generated for the
245
        test set
                classifier_name (string): Classifier name qp_iterations print the results
246
247
            Returns:
                (np.array 3): Average accuracy, TPR and TNR of the n iterations of the train set
248
249
                 (np.array 3): Average accuracy, TPR and TNR of the n iterations of the test set
                 (np.array 3): Stdev accuracy, TPR and TNR of the n iterations of the train set
250
251
                 (np.array 3): Stdev accuracy, TPR and TNR of the n iterations of the test set
252
        11 11 11
253
254
        measures_train = np.zeros(shape = (gp_iterations,3)) # Each row is an ex. and each
255
        \rightarrow column is the accuracy, TPR and TNR
        measures_test = np.zeros(shape =(gp_iterations,3))
256
257
        avg_results_train = np.zeros(3) # Each element is the average accuracy, the TPR and the
258
        \hookrightarrow TNR
        avg_results_test = np.zeros(3)
259
            TP in 0,0
260
261
            FN in 0.1
            TN in 1,1
262
263
           FP in 1,0
        for i in range(gp_iterations):
264
            # For train set
265
            TN = train_results[i,1,1]
266
            TP = train_results[i,0,0]
267
268
            FP = train_results[i,1,0]
            FN = train_results[i,0,1]
269
270
            measures\_train[i][0] = (TP + TN) / (TN + TP + FP + FN)
            measures_train[i][1] = (TP / (TP + FN))
271
272
            measures_train[i][2] = (TN / (TN + FP))
273
274
            #For the test set
```

```
TN = test_results[i,1,1]
275
            TP = test_results[i,0,0]
276
            FP = test_results[i,1,0]
277
           FN = test_results[i,0,1]
278
279
           measures\_test[i][0] = (TP + TN) / (TN + TP + FP + FN)
           measures_test[i][1] = (TP / (TP + FN))
280
281
            measures_test[i][2] = (TN / (TN + FP))
283
       avg_results_train = measures_train.mean(axis = 0)
284
       avg_results_test = measures_test.mean(axis = 0)
       std_results_train = measures_train.std(axis = 0)
285
286
       std_results_test = measures_test.std(axis = 0)
       if gp_verbose:
287
            print("\nResults for the ", classifier_name, " classifier\n")
288
289
            print('For the train set:')
290
           print('Prediction accuracy of train set is: %f' % avg_results_train[0])
291
            print('True postive rate of train set is: %f' % avg_results_train[1])
292
            print('True negative rate of train set is: %f\n' % avg_results_train[2])
293
294
           print('For the test set:')
295
           print('Predicion accuracy of train set is: %f' % avg_results_test[0])
296
            print('True postive rate of train set is: "f' " avg_results_test[1])
297
           print('True negative rate of train set is: %f\n' % avg_results_test[2])
298
299
       return avg_results_train, avg_results_test, std_results_train, std_results_test
300
301
   def print_parameters():
302
       print("Parameters")
303
       pdata = {'Parameters' : [gp_qda_hyper, gp_nsc_hyper,
304
                                  gp_dim_red, gp_retained_variance,
305
                                  gp_iterations, 100.0/gp_test_size,
306
                                  gp_best_hyper_method]}
307
       pdataf = pd.DataFrame(data = pdata, index = ['QDA hyper-parameter reg_param',
308
                                                        'NSC Hyper-parameter shrink_threshold',
309
                                                        'Dimensionality Reduction',
310
                                                        'Retained variance',
311
                                                        'Number of iterations',
312
                                                        'Test set size',
313
314
                                                        'Best hyperparameter select method'])
315
       display(pdataf)
       print("")
316
317
   def print_execution_data():
318
       print("Execution data")
319
320
       data = [ge_samples, ge_features, ge_features_reduction]
       index = ['Number of samples', 'Number of features', 'Features reduction']
321
       for [method, complexity] in ge_complexity:
322
            data.append(complexity)
323
            index.append("Params " + method)
324
       pdata = {'Execution Data' : data}
325
       pdataf = pd.DataFrame(data = pdata, index = index)
326
```

```
327
       display(pdataf)
       print("")
328
329
   def print_accuracies(accuracy_NBC, accuracy_LDA, accuracy_QDA, accuracy_NSC, metric):
330
       print(metric)
331
       d = \{\}
332
       for m in [accuracy_NBC, accuracy_LDA, accuracy_QDA, accuracy_NSC]:
333
            overfit_indicator = 1000 * round(abs(m[1] - m[0]) / (m[0] + 0.0000001), 4)
334
            #overfit_indicator = round(abs(m[1] - m[0]),3)
335
           m[0] = (round(m[0],2), round(m[2],2))
336
           m[1] = (round(m[1],2), round(m[3],2))
337
           m[2] = overfit_indicator
338
       d = {'NBC': accuracy_NBC[:3],
339
             'LDA': accuracy_LDA[:3],
340
             'QDA': accuracy_QDA[:3],
341
             'NSC': accuracy_NSC[:3]}
342
       df = pd.DataFrame(data = d, index = ['Train', 'Test', 'Overfit degree'])
343
344
       display(df)
       print("")
345
346
   def learn_dataset(data_file, header, random_state, label_pos,
347
348
                      label_value, features_ini, features_fin=None,
349
                      best_param_value_method="max_in_test",
                      with_dim_red=False, retained_variance=99.0):
350
351
        """Learn data sets from file, methods:
                1. The Naive Bayes Classifier
352
                2. LDA
353
                3. QDA
354
                4. Nearest Shrunken Centroids Classifier
355
            Args:
356
                data_file (string): Name of the data file (csv) of samples a features
357
                header (string): None or position of the header (pandas read_csv parameter)
358
                random_state (int): Seed for the random split of sets (as needed for sklearn
359
        train_test_split)
                label_pos (int): Column of the labels in data_file
360
                label_value (int): Value of the label to asign internal '1' value. We consider
361
       this label as
                the positive label in prediction validation. We asign malign or cancer status to
362
        this label.
363
                features_ini (int): First column of features in data_file
                features_fin (int): Last column + 1 of features in data_file. If None, last
364
       column of file
                best_param_value_method (str): if "max_in_test" gives the value with the maximum
365
       accuracy
                                           in test data
366
367
                with_dim_red (bool): If True, it performs a dimensionality reduction by PCA
                retained_variance (float): If dimensionality reduction, variance to retain
368
369
        .....
370
       global ge_features_reduction
371
372
       global ge_features
       global ge_samples
373
```

```
374
       global ge_complexity
       ge_features_reduction = 0
375
       nbc_train = np.zeros(shape=(gp_iterations,2,2))
376
       nbc_test = np.zeros(shape=(gp_iterations,2,2))
377
378
       lda_train = np.zeros(shape=(gp_iterations,2,2))
       lda_test = np.zeros(shape=(gp_iterations,2,2))
379
       qda_train = np.zeros(shape=(gp_iterations,2,2))
380
       qda_test = np.zeros(shape=(gp_iterations,2,2))
381
       nsc_train = np.zeros(shape=(gp_iterations,2,2))
382
383
       nsc_test = np.zeros(shape=(gp_iterations,2,2))
384
385
       for i in range(gp_iterations):
386
387
            ge_complexity = []
            if gp_show_progress: print("\nIteration: ",i)
388
            if i == 0:
389
                reuse = False
390
                X = None
391
392
                y = None
            else:
393
                reuse = True
394
            X_train_scaled, X_test_scaled, y_train, y_test, X, y = \
395
                create_datasets_from_file(data_file, header, random_state + i,
396
                                            label_pos, label_value, features_ini, features_fin =
397

    features_fin,

                                            with_dim_red = with_dim_red, retained_variance =
398

→ retained_variance,
                                            reuse = reuse, dataset = X, labels = y)
399
            ge_samples = X.shape[0]
400
            ge_features = X.shape[1]
401
            if gp_verbose: print(X_train_scaled.shape)
402
403
            if gp_verbose: print("NBC")
404
            # Naive Bayes accuracy
405
            nbc_train[i], nbc_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,
406

    y_train, y_test, "GaussianNB")

407
            # LDA accuracy
408
            if gp_verbose: print("LDA")
409
            lda_train[i], lda_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,
410

    y_train, y_test, "LinearDiscriminantAnalysis")

411
            # QDA estimate reg parameter
412
            if gp_verbose: print("QDA")
413
414
            if gp_qda_hyper:
415
                param_values = np.linspace(0, 1, 10).tolist()
                best_param_value = estimate_parameter(X_train_scaled, X_test_scaled, y_train,
416

    y_test,\

                                     "QuadraticDiscriminantAnalysis", "reg_param", param_values,\
417
                                     best_param_value_method)
418
419
                # QDA accuracy
420
                # Best parameter reg value according CV estimate
```

```
421
              qda_train[i], qda_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,

    y_train, y_test,

                                 "QuadraticDiscriminantAnalysis", "reg_param",
422
                                  → best_param_value)
          else:
423
              qda_train[i], qda_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,
424

    y_train, y_test,

                                 "QuadraticDiscriminantAnalysis")
425
           # Centroids
426
          if gp_verbose: print("NSC")
427
          if gp_nsc_hyper:
428
              # Best parameter shrink_threshold value according CV estimate
429
              param_values = np.linspace(0, 8, 20).tolist()
430
              best_param_value = estimate_parameter(X_train_scaled, X_test_scaled, y_train,
431

    y_test,

                                "NearestCentroid", "shrink_threshold", param_values,\
432
                                 best_param_value_method)
433
              # Centroids accuracy
434
              nsc_train[i], nsc_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,
435

    y_train, y_test,

                                                "NearestCentroid", "shrink_threshold",
436

→ best_param_value)

          else:
437
              nsc_train[i], nsc_test[i] = prediction_accuracy(X_train_scaled, X_test_scaled,
438

    y_train, y_test,

                                                "NearestCentroid")
439
       # Calculate and print the average results
440
      avg_results_train_NBC, avg_results_test_NBC, std_results_train_NBC, std_results_test_NBC
441
       avg_results_train_LDA, avg_results_test_LDA, std_results_train_LDA, std_results_test_LDA
442
       avg_results_train_QDA, avg_results_test_QDA, std_results_train_QDA, std_results_test_QDA
443
       avg_results_train_NSC, avg_results_test_NSC, std_results_train_NSC, std_results_test_NSC
444
       445
      print_parameters()
446
      i = 0
447
      for metric in ["Accuracy", "TPR", "TNR"]:
448
449
          print_accuracies([avg_results_train_NBC[i], avg_results_test_NBC[i],

    std_results_train_NBC[i], std_results_test_NBC[i]],

                       [avg_results_train_LDA[i], avg_results_test_LDA[i],
450

    std_results_train_LDA[i], std_results_test_LDA[i]],

                       [avg_results_train_QDA[i], avg_results_test_QDA[i],
451

→ std_results_train_QDA[i], std_results_test_QDA[i]],
452
                       [avg_results_train_NSC[i], avg_results_test_NSC[i],

→ std_results_train_NSC[i], std_results_test_NSC[i]], metric)
           i += 1
453
      print_execution_data()
454
```

1.2 Breast cancer

Script 1.2.1 (python) gp_disable_plots = True gp_verbose = False gp_dim_red = False 4 gp_retained_variance = 99 5 gp_qda_hyper = True gp_nsc_hyper = True 7 gp_iterations = 20 gp_show_progress = False 9 gp_best_hyper_method = "max_in_cv" gp_skfold_splits = 10 11 learn_dataset(data_file = './data/wdbc.csv', header = None, random_state=0, 13 label_pos=1, label_value="M", features_ini = 2, features_fin = None, best_param_value_method = gp_best_hyper_method, 14 with_dim_red = gp_dim_red, retained_variance = gp_retained_variance) 15 16 17 gp_dim_red = True learn_dataset(data_file = './data/wdbc.csv', header = None, random_state=0, label_pos=1, label_value="M", features_ini = 2, features_fin = None, 19 best_param_value_method = gp_best_hyper_method, 20 with_dim_red = gp_dim_red, retained_variance = gp_retained_variance) 21

Output

Parameters

	Parameters
QDA hyper-parameter reg_param	True
${\tt NSC\ Hyper-parameter\ shrink_threshold}$	True
Dimensionality Reduction	False
Retained variance	99
Number of iterations	20
Test set size	33.3333
Best hyperparameter select method	max_in_cv

Output

Accuracy

```
NBC LDA QDA NSC
Train (0.94, 0.01) (0.97, 0.01) (0.97, 0.01) (0.94, 0.01)
Test (0.93, 0.01) (0.96, 0.01) (0.97, 0.01) (0.94, 0.01)
Overfit degree 5.4 7.5 9.4 7
```

TPR

	NBC	LDA	QDA	NSC
Train	(0.9, 0.01)	(0.92, 0.01)	(0.95, 0.01)	(0.9, 0.02)
Test	(0.9, 0.03)	(0.89, 0.03)	(0.93, 0.03)	(0.9, 0.04)
Overfit degree	10.1	23.3	14.7	3.6

Output

 ${\tt TNR}$

	NBC	LDA	QDA	NSC
Train	(0.97, 0.01)	(1.0, 0.0)	(0.99, 0.0)	(0.97, 0.01)
Test	(0.95, 0.02)	(1.0, 0.0)	(0.98, 0.02)	(0.96, 0.02)
Overfit degree	14.3	0	6.8	9.2

Output

Execution data

	Execution Data
Number of samples	569
Number of features	30
Features reduction	0
Params GaussianNB	2
Params LinearDiscriminantAnalysis	6
Params QuadraticDiscriminantAnalysis	5
Params NearestCentroid	2

Output

Parameters

	Parameters
QDA hyper-parameter reg_param	True
${\tt NSC\ Hyper-parameter\ shrink_threshold}$	True
Dimensionality Reduction	True
Retained variance	99
Number of iterations	20
Test set size	33.3333
Best hyperparameter select method	max_in_cv

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.9, 0.01)	(0.96, 0.0)	(0.98, 0.01)	(0.93, 0.01)
Test	(0.89, 0.02)	(0.96, 0.01)	(0.97, 0.01)	(0.93, 0.02)
Overfit degree	10.2	7.4	8.8	5.7

Output

TPR

		NBC		LDA	QDA	NSC
Train	(0.84,	0.02)	(0.91,	0.01)	(0.95, 0.02)	(0.87, 0.01)
Test	(0.83,	0.04)	(0.89,	0.02)	(0.94, 0.03)	(0.87, 0.03)
Overfit degree		5.4		18.6	17.2	4

Output

TNR

	NBC	LDA	QDA	NSC
Train	(0.94, 0.01)	(1.0, 0.0)	(0.99, 0.0)	(0.97, 0.01)
Test	(0.93, 0.03)	(0.99, 0.01)	(0.99, 0.01)	(0.96, 0.02)
Overfit degree	13.1	2.1	4.6	7.6

Output

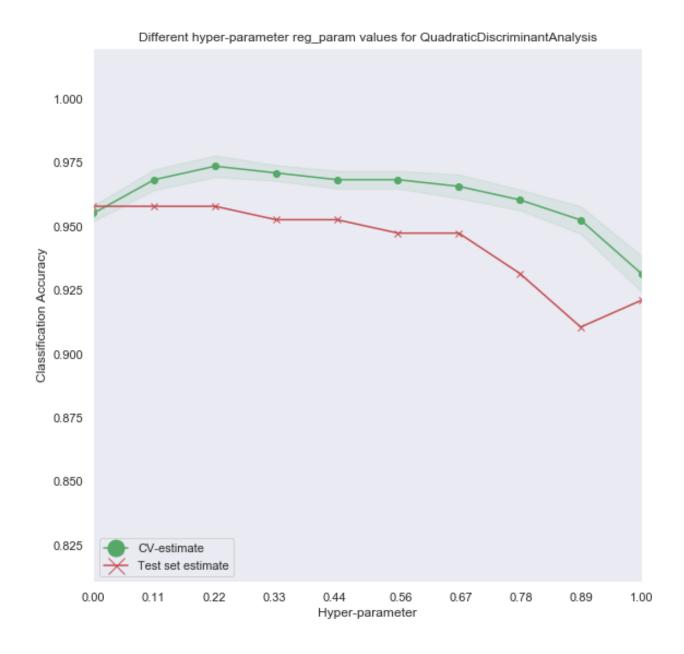
Execution data

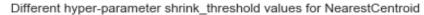
	Execution Data
Number of samples	569
Number of features	30
Features reduction	17
Params GaussianNB	2
Params LinearDiscriminantAnalysis	6
Params QuadraticDiscriminantAnalysis	5
Params NearestCentroid	2

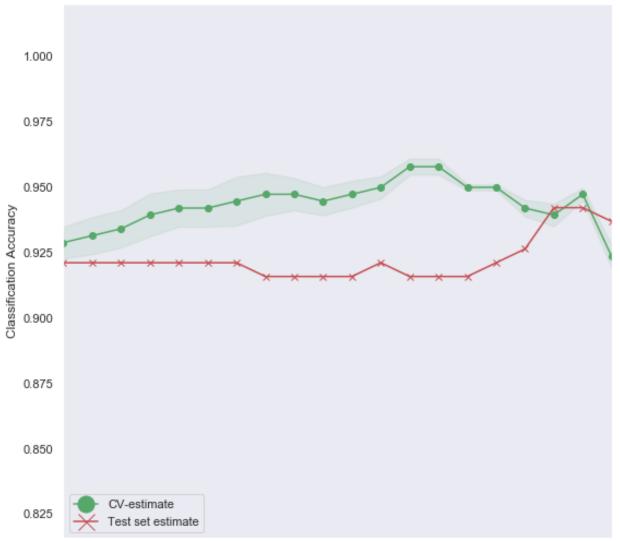
1.2.1 Hyper-parameter guessing plots

71 1 0 01

```
Script 1.2.2 (python)
gp_disable_plots = False
gp_verbose = False
gp_dim_red = False
4 gp_retained_variance = 99
5 gp_qda_hyper = True
gp_nsc_hyper = True
gp_iterations = 1
gp_show_progress = False
9 gp_best_hyper_method = "max_in_cv"
ge_features_reduction = 0
11
learn_dataset(data_file = './data/wdbc.csv', header = None, random_state=0,
                label_pos=1, label_value="M", features_ini = 2, features_fin = None,
13
14
                best_param_value_method = gp_best_hyper_method,
                with_dim_red = gp_dim_red, retained_variance = gp_retained_variance)
15
```







0.00 0.42 0.84 1.26 1.68 2.11 2.53 2.95 3.37 3.79 4.21 4.63 5.05 5.47 5.89 6.32 6.74 7.16 7.58 8.00 Hyper-parameter

 ${\tt Parameters}$

	Parameters
QDA hyper-parameter reg_param	True
${\tt NSC\ Hyper-parameter\ shrink_threshold}$	True
Dimensionality Reduction	False
Retained variance	99
Number of iterations	1
Test set size	33.3333

Best hyperparameter select method max_in_cv

Output

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.95, 0.0)	(0.96, 0.0)	(0.98, 0.0)	(0.94, 0.0)
Test	(0.9, 0.0)	(0.97, 0.0)	(0.96, 0.0)	(0.92, 0.0)
Overfit degree	49.9	11	18.8	30.5

Output

TPR

	NBC	LDA	QDA	NSC
Train	(0.9, 0.0)	(0.91, 0.0)	(0.94, 0.0)	(0.91, 0.0)
Test	(0.85, 0.0)	(0.93, 0.0)	(0.91, 0.0)	(0.87, 0.0)
Overfit degree	55.2	18.4	34.6	46.3

Output

TNR

	NBC	LDA	QDA		NSC
Train	(0.97, 0.0)	(1.0, 0.0)	(1.0, 0.0)	(0.97,	0.0)
Test	(0.93, 0.0)	(1.0, 0.0)	(0.98, 0.0)	(0.94,	0.0)
Overfit degree	49.5	4.3	12.2		24.2

Output

Execution data

	Execution Data
Number of samples	569
Number of features	30
Features reduction	0
Params GaussianNB	2
Params LinearDiscriminantAnalysis	6
Params QuadraticDiscriminantAnalysis	5
Params NearestCentroid	2

1.3 Prostate cancer

```
Script 1.3.1 (python)
1 # Prostate Cancer
gp_disable_plots = True
gp_verbose = False
4 gp_dim_red = False
5 gp_retained_variance = 99
6 gp_qda_hyper = True
gp_nsc_hyper = True
8 gp_iterations = 20
gp_best_hyper_method = "max_in_cv"
10 ge_features_reduction = 0
gp_skfold_splits = 5
12
learn_dataset(data_file = './data/prostate.csv', header = 0, random_state = 1,
                 label_pos = -1, label_value = 1, features_ini = 0, features_fin = -1,
14
                 best_param_value_method = gp_best_hyper_method,
15
16
                 with_dim_red = gp_dim_red, retained_variance = gp_retained_variance)
17
gp_best_hyper_method = "max_in_cv"
19 gp_dim_red = True
gp_retained_variance = 99
21
  ge_features_reduction = 0
learn_dataset(data_file = './data/prostate.csv', header = 0, random_state = 1,
                 label_pos = -1, label_value = 1, features_ini = 0, features_fin = -1,
23
                 best_param_value_method = gp_best_hyper_method,
24
                 with_dim_red = gp_dim_red, retained_variance = gp_retained_variance)
25
26
gp_best_hyper_method = "max_in_cv"
gp_dim_red = True
29 gp_retained_variance = 80
30 ge_features_reduction = 0
31 learn_dataset(data_file = './data/prostate.csv', header = 0, random_state = 1,
                label_pos = -1, label_value = 1, features_ini = 0, features_fin = -1,
32
                 best_param_value_method = gp_best_hyper_method,
33
                with_dim_red = gp_dim_red, retained_variance = gp_retained_variance)
34
```

Output

Parameters

Dimensionality Reduction False
Retained variance 99
Number of iterations 20
Test set size 33.3333
Best hyperparameter select method max_in_cv

Output

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.72, 0.06)	(0.86, 0.02)	(0.14, 0.25)	(0.91, 0.02)
Test	(0.64, 0.12)	(0.85, 0.04)	(0.64, 0.1)	(0.89, 0.05)
Overfit degree	106.3	17.9	3557.9	26.5

Output

TPR

	NBC	LDA	QDA	NSC
Train	(0.68, 0.07)	(0.83, 0.03)	(0.12, 0.24)	(0.88, 0.03)
Test	(0.66, 0.12)	(0.8, 0.07)	(0.62, 0.18)	(0.85, 0.07)
Overfit degree	29.6	36.8	4243.1	26.7

Output

TNR

	NBC	LDA	QDA	NSC
Train	(0.76, 0.08)	(0.89, 0.03)	(0.16, 0.27)	(0.95, 0.03)
Test	(0.62, 0.18)	(0.9, 0.08)	(0.63, 0.23)	(0.93, 0.06)
Overfit degree	178.1	16.7	2863.6	19.8

Output

Execution data

Number of samples Execution Data
Number of features 12625

Features reduction	0
Params GaussianNB	2
Params LinearDiscriminantAnalysis	6
Params QuadraticDiscriminantAnalysis	5
Params NearestCentroid	2

Parameters

	Parameters
QDA hyper-parameter reg_param	True
${\tt NSC\ Hyper-parameter\ shrink_threshold}$	True
Dimensionality Reduction	True
Retained variance	99
Number of iterations	20
Test set size	33.3333
Best hyperparameter select method	max_in_cv

Output

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.92, 0.04)	(1.0, 0.0)	(0.87, 0.15)	(0.75, 0.08)
Test	(0.74, 0.12)	(0.9, 0.04)	(0.71, 0.09)	(0.72, 0.11)
Overfit degree	200.6	100	181.1	41.3

Output

TPR

	NBC	LDA	QDA	NSC
Train	(0.92, 0.05)	(1.0, 0.0)	(0.83, 0.19)	(0.75, 0.08)
Test	(0.95, 0.05)	(0.87, 0.07)	(0.71, 0.14)	(0.73, 0.12)
Overfit degree	40.9	128.7	140.4	22.2

Output

TNR

	NBC	LDA	QDA	NSC
Train	(0.93, 0.05)	(1.0, 0.0)	(0.92, 0.13)	(0.74, 0.1)
Test	(0.48, 0.29)	(0.93, 0.07)	(0.7, 0.19)	(0.69, 0.17)
Overfit degree	483.8	66.9	235.8	62.9

Execution data

	Execution Data
of samples	102
of features	12625
es reduction	63
GaussianNB	2
LinearDiscriminantAnalysis	6
QuadraticDiscriminantAnalysis	5
NearestCentroid	2
	of features es reduction GaussianNB LinearDiscriminantAnalysis QuadraticDiscriminantAnalysis

Output

 ${\tt Parameters}$

	Parameters
QDA hyper-parameter reg_param	True
NSC Hyper-parameter shrink_threshold	True
Dimensionality Reduction	True
Retained variance	80
Number of iterations	20
Test set size	33.3333
Best hyperparameter select method	max_in_cv

Output

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.89, 0.06)	(0.96, 0.02)	(0.98, 0.01)	(0.72, 0.08)
Test	(0.78, 0.08)	(0.85, 0.05)	(0.87, 0.05)	(0.67, 0.12)
Overfit degree	116.8	110.6	116.8	76.3

TPR

	NBC	LDA	QDA	NSC
Train	(0.83, 0.1)	(0.93, 0.04)	(0.97, 0.02)	(0.7, 0.09)
Test	(0.66, 0.13)	(0.82, 0.07)	(0.88, 0.07)	(0.69, 0.12)
Overfit degree	200.4	122.7	91.3	11.2

Output

TNR

	NBC	LDA	QDA	NSC
Train	(0.94, 0.03)	(0.98, 0.02)	(0.99, 0.01)	(0.75, 0.08)
Test	(0.93, 0.07)	(0.9, 0.1)	(0.86, 0.11)	(0.64, 0.2)
Overfit degree	15.2	82.4	133.7	137.2

Output

Execution data

	Execution Data
Number of samples	102
Number of features	12625
Features reduction	19
Params GaussianNB	2
Params LinearDiscriminantAnalysis	6
Params QuadraticDiscriminantAnalysis	5
Params NearestCentroid	2

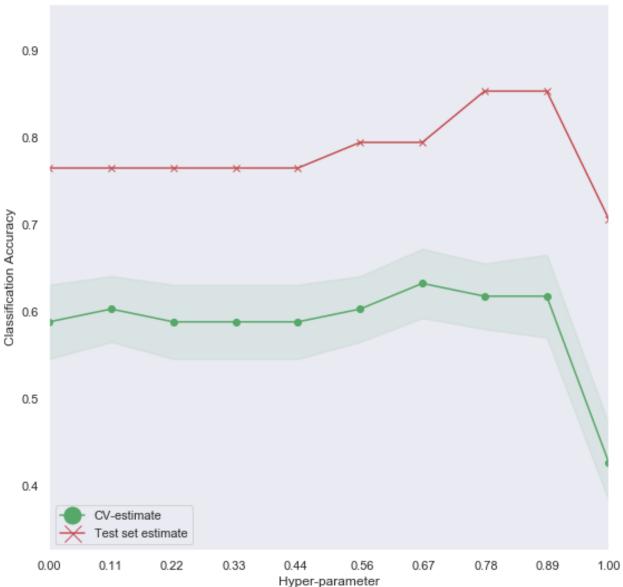
Output

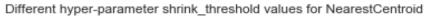
1.3.1 Hyper-parameter guessing plots

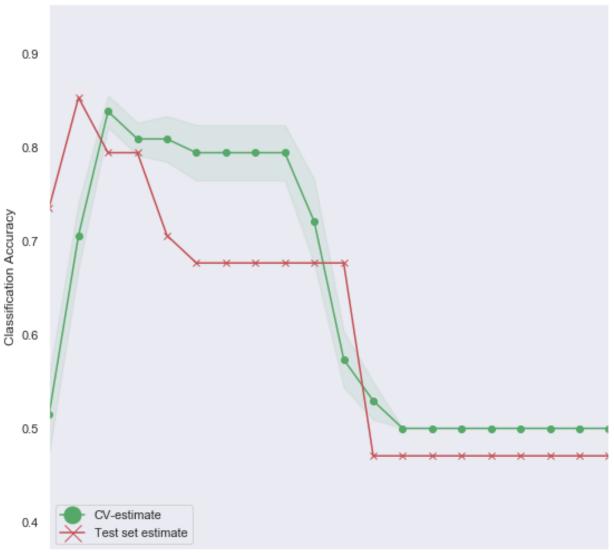
Script 1.3.2 (python)

gp_disable_plots = False
gp_verbose = False









0.00 0.42 0.84 1.26 1.68 2.11 2.53 2.95 3.37 3.79 4.21 4.63 5.05 5.47 5.89 6.32 6.74 7.16 7.58 8.00 Hyper-parameter

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Parameters

_			
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QDA hyper-parameter reg_param True NSC Hyper-parameter shrink_threshold True

Dimensionality Reduction True
Retained variance 99
Number of iterations 1
Test set size 33.3333
Best hyperparameter select method max_in_cv

Output

Accuracy

	NBC	LDA	QDA	NSC
Train	(0.94, 0.0)	(1.0, 0.0)	(0.93, 0.0)	(0.85, 0.0)
Test	(0.82, 0.0)	(0.91, 0.0)	(0.79, 0.0)	(0.79, 0.0)
Overfit degree	125	88.2	142.9	69

Output

TPR

	NBC	LDA	QDA	NSC
Train	(0.97, 0.0)	(1.0, 0.0)	(0.91, 0.0)	(0.85, 0.0)
Test	(0.94, 0.0)	(0.83, 0.0)	(0.94, 0.0)	(0.72, 0.0)
Overfit degree	26.9	166.7	35.8	153.3

Output

TNR

	NBC	LDA	QDA	NSC
Train	(0.91, 0.0)	(1.0, 0.0)	(0.94, 0.0)	(0.85, 0.0)
Test	(0.69, 0.0)	(1.0, 0.0)	(0.62, 0.0)	(0.88, 0.0)
Overfit degree	246	0	335.9	25.9

Output

Execution data

Number of samples Execution Data
Number of features 12625

Features reduction		
Params GaussianNB	2	
Params LinearDiscriminantAnalysis	6	
Params QuadraticDiscriminantAnalysis	5	
Params NearestCentroid	2	

Output			

1.4 Conclusions

We observe that **QDA** performs very poorly in the prostate dataset, given the high dimensionality of this dataset (more than 12625) and the relatively low number of samples (around 100), which do not ease the accurate computation of the covariance matrices.

We perform previously a dimensionality reduction by PCA, were we see that we improve this result.

NSC performs in this case much better due to the reduced number of parameters and the feature selection properties of this classifier and more consistently between both cases (prostate and breast).