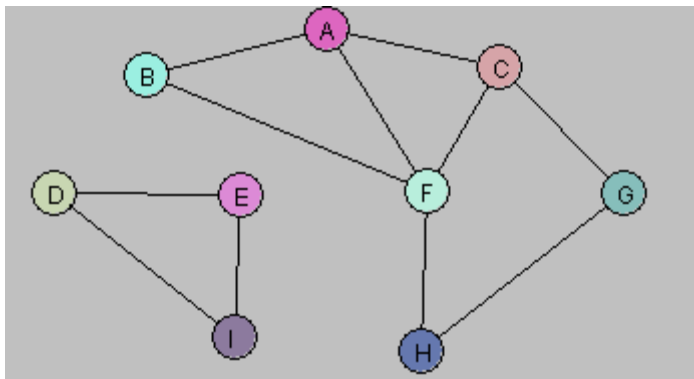
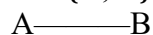


## Lab W3D3

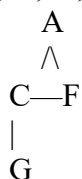
**Question 1. Induced Graphs.** Answer questions about the graph  $G = (V, E)$  displayed below.



A. Let  $U = \{A, B\}$ . Draw  $G[U]$ .



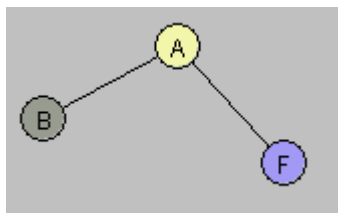
B. Let  $W = \{A, C, G, F\}$ . Draw  $G[W]$ .



C. Let  $Y = \{A, B, D, E\}$ . Draw  $G[Y]$ .



D. Consider the following subgraph  $H$  of  $G$ :



**Is there a subset  $X$  of the vertex set  $V$  so that  $H = G[X]$ ? Explain.**

A: There is **no** subset  $X$  of  $V$  such that  $H = G[X]$ . In the subgraph  $H$ , the vertices are  $\{A, B, F\}$  and the only edges shown are  $A \text{ --- } B$  and  $A \text{ --- } F$ .

However, in the original graph  $G$ , there is **also** an edge between  $B$  and  $F$ .

**E. Find a way to partition the vertex set  $V$  into two subsets  $V_1, V_2$  so that each of the induced graphs  $G[V_1]$  and  $G[V_2]$  is connected and  $G = G[V_1] \cup G[V_2]$ .**

A valid partition is:

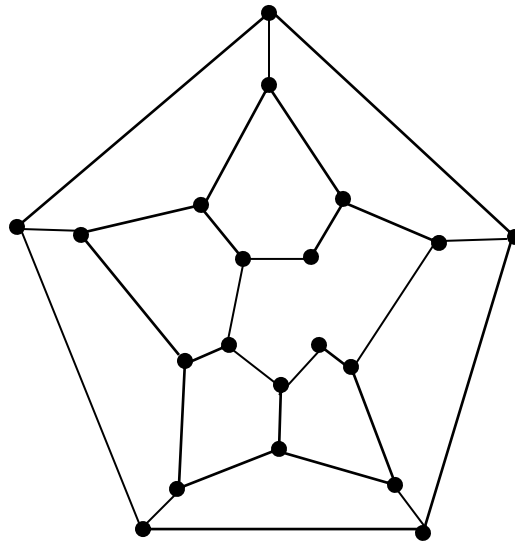
$V_1 = \{D, E, I\} \rightarrow$  connected graph(triangle)

$V_2 = \{A, B, C, F, G, H\} \rightarrow$  connected graph, since every vertex in this set is reachable from the others.

Together they cover all vertices of  $G$  and therefore

**$G = G[V_1] \cup G[V_2]$ .**

**Question 2.** The following graph has a Hamiltonian cycle. Find it.



DFS/backtracking

$\text{Adj}(4) = \{ \cancel{1}, 5, 12 \}$   
 $\text{Adj}(5) = \{ 2, \cancel{4}, 14 \}$

$\text{adj}(1) = \{ 2, 5, 6 \}$   
 $\text{adj}(2) = \{ 1, 3, 8 \}$   
 $\text{adj}(3) = \{ \cancel{4}, 4, 10 \}$  *visited*

$\text{Adj}(12) = \{ 13, \cancel{4}, 11 \}$   
 $\text{Adj}(14) = \{ \cancel{5}, 15, 13 \}$   
 $\text{Adj}(15) = \{ \cancel{14}, 16, 10 \}$

$\text{Adj}(13) = \{ 17, 12, 14 \}$   
 $\text{adj}(12) = \{ 13, 11, 4 \}$   
 $\text{Adj}(6) = \{ 7, 1, 15 \}$   
 $\text{Adj}(13) = \{ 9, 18 \}$

Visited:

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17 \rightarrow 18 \rightarrow 11 \rightarrow 10 \rightarrow 9 \rightarrow 8 \rightarrow 7 \rightarrow 20$

This graph does not contain a Hamilton Cycle