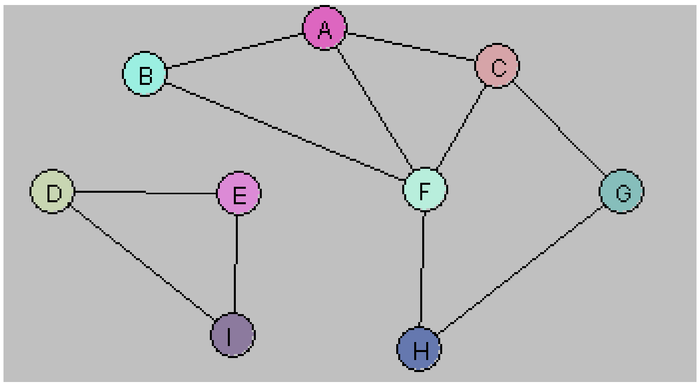
Lab W3D3

Quetion 1. *Induced Graphs.* Answer questions about the graph G = (V,E) displayed below.



A. Let U = {A, B}. Draw G[U].

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A———B
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B. Let W = {A, C, G, F}. Draw G[W].

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   A
  /\
C—F
|
G
```

C. **Let Y = {A, B, D, E}. Draw G[Y].**

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A———B          D———E
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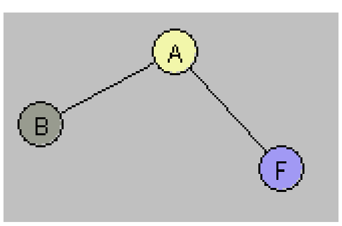
D. Consider the following subgraph H of G:



Is there a subset X of the vertex set V so that H = G[X]? Explain.

A: There is **no** subset X of V such that $H = G[X]$. In the subgraph H, the vertices are $\{A, B, F\}$ and the only edges shown are A__B and A___F.
However, in the original graph G, there is **also** an edge between B and F.

E. Find a way to partition the vertex set V into two subsets V_1, V_2 so that each of the induced graphs $G[V_1]$ and $G[V_2]$ is connected and $G = G[V_1] \cup G[V_2]$.

A valid partition is:

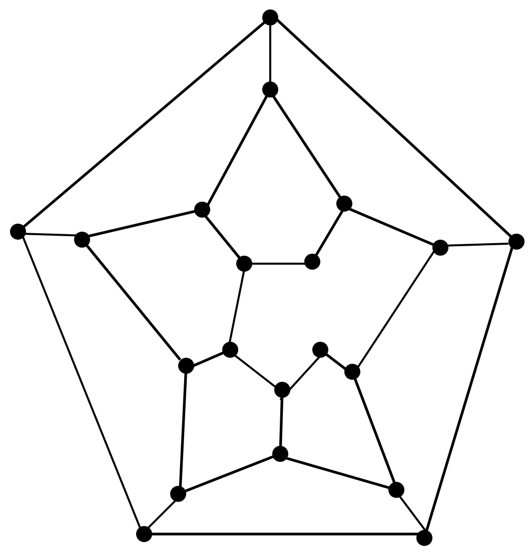
V1 = {D, E, I} ➔ connected graph(triangle)

V2 = {A, B, C, F, G, H} ➔ connected graph, since every vertex in this set is reacheable from the others.

Together they cover all vertices of G and therefore

$G = G[V_1] \cup G[V_2]$.

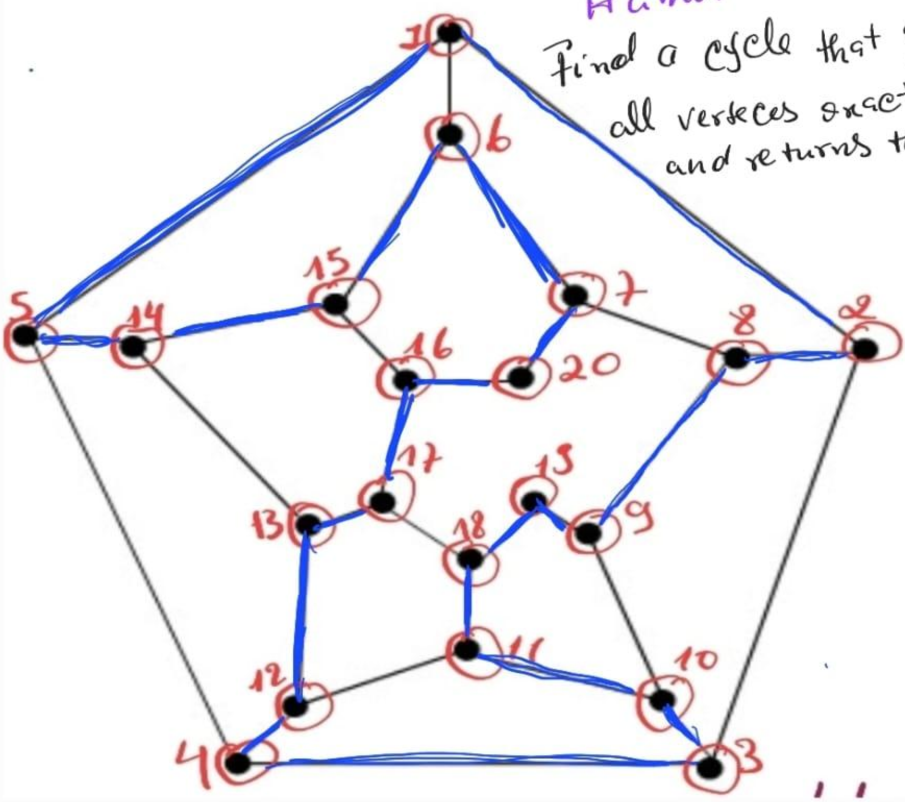
Question 2. The following graph has a Hamiltonian cycle. Find it.



Step 1: Identify all vertices

Hamilton Cycle

Find a cycle that goes over all vertices exactly once and returns to the start



1 -> 2 -> 8 -> 9 -> 19 -> 18 -> 11 -> 10 -> 3 -> 4 -> 12 -> 13 -> 17 -> 16 -> 20 -> 7 -> 6 -> 15 -> 14 -> 5 -> 1

This is an Hamilton Cycle

The graph has 20 vertices, which must all be visited exactly once in Hamilton Cycle