**Name:_____ ID: _____**

**Midterm Exam**

**Please make sure your writing is visible (use pen or dark pencil) and legible.**

**There are 7 questions and 45 points. Time limit : 2 hours.** Use the space provided for answers.

---

**Question 1.**

**(a) [2 Points]** What is time complexity of `questionOne`? Please Explain in 2 sentences.

```
int questionOne(int n)
{
  int count = 0;
  for (int i = n; i > 0; i /= 2)
     for (int j = 0; j < i; j++)
        count += 1;
  return count;
}
```

**(b) [2 Points]** What is the time complexity of `questionTwo`? Please Explain in 2 sentences.

```
int questionTwo(int n)
{
  int count = 0;
  for (int i = 0; i < n; i++)
     for (int j = i; j > 0; j--)
        count = count + 1;
  return count;
}
```

**(c) [1 Points]**

Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. which of the following is ALWAYS TRUE?

(A) A(n) is in $\Omega$(W(n)) //Big-Omega

(B) A(n) is in $\Theta$(W(n)) //Theta

(C) A(n) is in O(W(n)) //Big-O

(D) A(n) is in o(W(n)) //Little-o

**Question 2 [5 points] Short answer questions. Each question is worth 1 point.**

(a) What is the value of 10 Choose 5?

(b) What is the average time complexity of Quick Select?

(c) Assume that the Bubble sort algorithm you wrote took 20 units of time to sort an array of size 1000. How much time it may take if your array size is 4000 based on what you have studied in this course.

(d) What is the minimum height required for binary tree to have 2024 leaves?

(e) Draw a **perfect skip list** **having 8 values.** You are free to pick 8 integer values between 10 and 50.

**Question 3. [10 points] Search Problem.**

You are given an n x n integer 2d-array M[0 .. n – 1, 0 .. n – 1] with the following properties:

(1) The number of rows is equal to the number of columns.
(2) Each row is in ascending order.
(3) Each column is in ascending order.

See the example below:

| | | | | |
|---|---|---|---|---|
| 5 | 7 | 10 | 12 | 14 |
| 6 | 9 | 13 | 15 | 18 |
| 16 | 17 | 22 | 25 | 26 |
| 21 | 23 | 27 | 29 | 40 |
| 24 | 28 | 31 | 35 | 41 |

Let us call such arrays “Sorted 2-D arrays” for the purpose of this question.

(a) [6 points] Write a search algorithm “Search2D(M, key)” such that if key is present in the Sorted 2-D array, your algorithm will print both row and column numbers as a pair. If the key is not present in the Sorted 2-D array, it will print “Not Found”.

***Example:*** *For the Sorted 2-D array shown above, if the key is 23, it will print (3, 1). However, if the key is 34, your algorithm will print “Not Found”.*

If your algorithm is correct, it will be graded on time complexity (on scale: 1, 2, 3) and space complexity (on scale: 1, 2, 3). Thus, the maximum possible points is 6.

(b) [1 + 1 points] What is the time complexity of your algorithm? Please explain your claim.

(c) [1 + 1 points] What is the space complexity of your algorithm? Please explain your claim.

**Question 4. [10 points] Dynamic Programming Problem. If you write the recursive formula and do not solve the problem iteratively, you will get 3 points.**

You are climbing a staircase. It takes **10** steps to reach the top.
Each time you can either climb 1 or 2 or 3 steps. In how many distinct ways can you climb to the top? You must solve this problem through dynamic programming using tabular method.

## Question 5. [ 5 points] Amortized Cost analysis.

**Data structure : ArrayList with size quadrupling strategy.**

(a) What is the actual cost of resize?

Consider a sample instance. Assume resize just happened and current size of the array is 16. You are adding. Then when array is full, you resize it again.

(b) What is the Amortized_Cost(add)?

**Meaning of the word Quadrupling : Four times.**

**Question 6 [5 Points]** What are the time complexities of four algorithms (applying Master theorem)? **In order to get credit, you must simplify all expressions. If Master theorem is not applicable, please write it so to get full credit.**

For recurrences that arise from Divide-And-Conquer algorithms (like Binary Search), there is a general formula that can be used.

**Theorem.** Suppose $T(n)$ satisfies

$$T(n) = \begin{cases} d & \text{if } n = 1 \\ aT(\lceil \frac{n}{b} \rceil) + cn^k & \text{otherwise} \end{cases}$$

where $k$ is a nonnegative integer and $a, b, c, d$ are constants with $a > 0, b > 1, c > 0, d \geq 0$. Then

$$T(n) = \begin{cases} \Theta(n^k) & \text{if } a < b^k \\ \Theta(n^k \log n) & \text{if } a = b^k \\ \Theta(n^{\log_b a}) & \text{if } a > b^k \end{cases}$$

**Algorithm 1**

T(1) = 10

T(n) = 2T(n/2) + 12n + 9

**Algorithm 2**

T(1) = 10

T(n) = 4T(n/2) + 15 $n^2$

**Algorithm 3**

$T(1) = 10$

$T(n) = 4T(n/2) + 12$

**Algorithm 4**

$T(1) = 10$

$T(n) = 2T(n - 2) + T(n - 4) + 6$

**Algorithm 5**

$T(1) = 10$

$T(n) = 2T(n/4) + 15$

**Question 7[5 Points]**

Illustrate QuickSelect(A, 8) where A = {39, 3, 15, 11, 14, 2, 27, 26, 38, 100, 110}. Please use “median of the three” to pick your pivot and show details as in your class notes. (otherwise, no credit).

# THE END OF THE EXAM

**Have a nice weekend! Table below is for the Professor to use.**

| Qn. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|
| Max. | 5 | 5 | 10 | 10 | 5 | 5 | 5 | 45 |
| score | | | | | | | | |