

Question 3. (a) $k = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$k=4$

Step 1:

• $J_0 = 0, \text{Ind} = 8, \text{mid} = 4 \rightarrow \text{triple } \{4, 5, 6\} \rightarrow \{1, 2, 3\} \rightarrow \text{front} = 5$

• Partition: $L = \{1, 2, 3, 4\}$ (4), $E = \{5\}$ (1), $G_L = \{6, 7, 8, 9\}$ (4)

Since: $k=4 \leq 4 - |L|$, recursion in L with
value $k \rightarrow k=4$

Step 2 (in $[1, 2, 3, 4]$) $\rightarrow k=4$ $\rightarrow \text{front} = 2$

• $J_0 = 0, \text{Ind} = 3, \text{mid} = 2 \rightarrow \text{triple } \{1, 2, 4\} \rightarrow \text{front} = 2$

• Partition:

$L_1 = \{1, 3\}$ (2), $E_1 = \{2\}$ (1), $G_{L_1} = \{4\}$ (1)

$|L_1| + |E_1| = 2$

Since: $k=4 > |L_1| + |E_1| = 2$
recursion in G_{L_1} with $k=4-2=2$

Step 3 (in $\{3, 4\}$ with $k=2$): $\{3, 4\} \rightarrow \{3, 4\} \rightarrow \text{front} = 3$

• $J_0 = 0, \text{Ind} = 2, \text{mid} = 0 \rightarrow \{3, 4\}$

• Partition:

$L_2 = \{3\}$ (1), $E_2 = \{3\}$ (1), $G_{L_2} = \{4\}$ (1)

$|L_2| + |E_2| = 2$, recursion in G_{L_2} with
 $k=2-2=0=1$

Since $k=2 > |L_2| + |E_2| = 1$ \rightarrow answer = 4

• Subscript w/ 4th one element $\{4\} \rightarrow$ answer = 4

Q.3) (b) $A = \{8, 7, 6, 5, 4, 3, 2, 1, 9\}$, $k = 5$

Step 1:

$i_0 = 0, i_{hi} = 8, mid = 4 \rightarrow \text{triple } \{8, 9, 4\} \Rightarrow \{4, 8, 9\} \rightarrow \text{front} = 8$

• Partition: $L = \{7, 6, 5, 4, 3, 2, 1\} (7), E = \{8\} (1), G = \{9\} (3)$

Since $k = 5 \leq |L| = 7 \rightarrow \text{recursion on } L \text{ with } k = 5$

Step 2 (in $\{7, 6, 5, 4, 3, 2, 1\}$ with $k = 5$): $\text{front} = 4$

$i_0 = 0, i_{hi} = 6, mid = 3 \rightarrow \text{triple } \{1, 4, 7\}$

$A_{1,0,3} = 7 \quad A_{1,3,5} = 2 \quad A_{1,mod,3} = 4$

• Partition:

$L_1 = \{3, 2, 1\} (3), E_1 = \{4\} (1), G_1 = \{7, 6, 5\} (3)$

$|L_1| = 3 + |E_1| = 1 = 4$

- $k = 5 > |L_1| = 3 + |E_1| = 1$

Recursion on G_1 with $k = 5 - 4 = 1$

Step 3 (in $\{7, 6, 5\}$ with $k = 1$): $\text{front} = 6$

$i_0 = 0, i_{hi} = 2, mid = 1 \rightarrow \text{triple } \{5, 6, 7\}$

$A_{1,0,1} = 7 \quad A_{1,1,2} = 6 \quad A_{1,mod,1} = 5$

$L_2 = \{5\} (1), E_2 = \{6\} (1), G_2 = \{7\} (1)$

• Partition: $L_2 = \{5\} (1), E_2 = \{6\} (1), G_2 = \{7\} (1), k = 1$

- Since $k = 1 \leq |L_2| = 1 \rightarrow \text{recursion on } L_2 (|L_2|, k = 1)$

$\rightarrow \text{answer} = 5$

Q3 (c) $A = \{9, 1, 8, 2, 7, 3, 6, 4, 5\}, K = 6$

Step 1:

• $lo = 0, hi = 8, mid = 4 \rightarrow$ triple $\{5, 7, 9\} \rightarrow \text{first} = 2$

• Partition:

$L = \{1, 2, 3, 6, 4, 5\} (6), E = \{7\} (1), G_1 = \{9, 8\} (2)$
 $L = \{1, 2, 3, 6, 4, 5\} (6), E = \{7\} (1), G_1 = \{9, 8\} (2), K = 6$

• $K = 6 \leq |L| = 6$ recursion in L

Step 2 (in $\{1, 2, 3, 6, 4, 5\}, K = 6$)

• $lo = 0, hi = 5, mid = 2 \rightarrow$ triple $\{1, 3, 5\} \rightarrow \text{first} = 3$

• Partition:

$L_2 = \{1, 2\} (2), E_2 = \{3\} (1), G_2 = \{6, 4, 5\} (3)$

• $K = 6 > |L_2| + |E_2| = 2 + 1 = 3$

• \downarrow recursion in G_2 with $K = 6 - 3 = 3$

Step 3 ($\{6, 4, 5\}, K = 3$):

• $lo = 0, hi = 2, mid = 1 \rightarrow$ triple $\{4, 5, 6\} \rightarrow \text{first} = 5$

• Partition:

$L_3 = \{4\} (1), E_3 = \{5\} (1), G_3 = \{6\} (1)$

• $K = 3 > |L_3| + |E_3| = 1 + 1 = 2 \rightarrow$ recursion in G_3 with $K = 3 - 2 = 1$

• G_3 is correct ($G_3 = \{6\} \rightarrow$ only one element \rightarrow answer = 6)