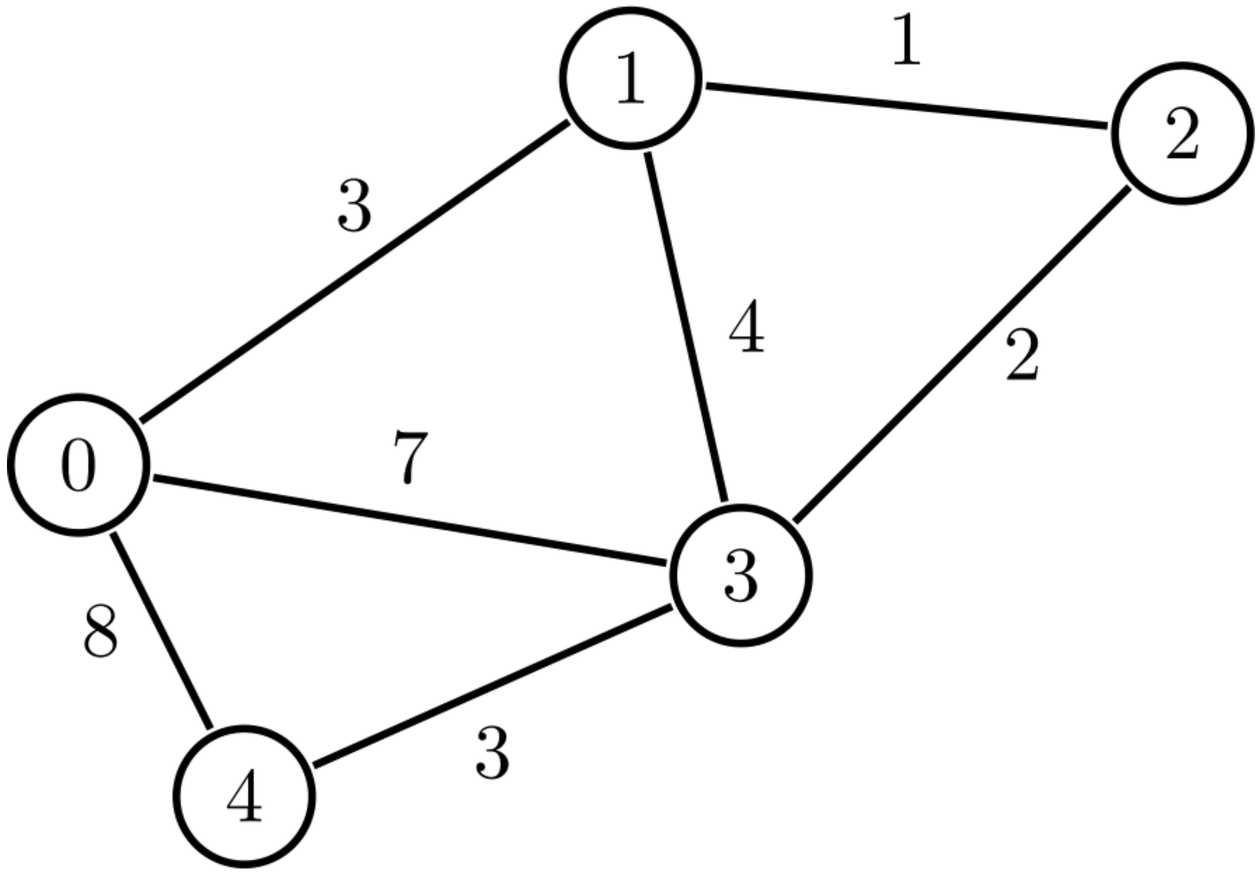
Edge vs. Path

Weight vs. Distance



dis(1, 3) = 3 < wt(1, 3).

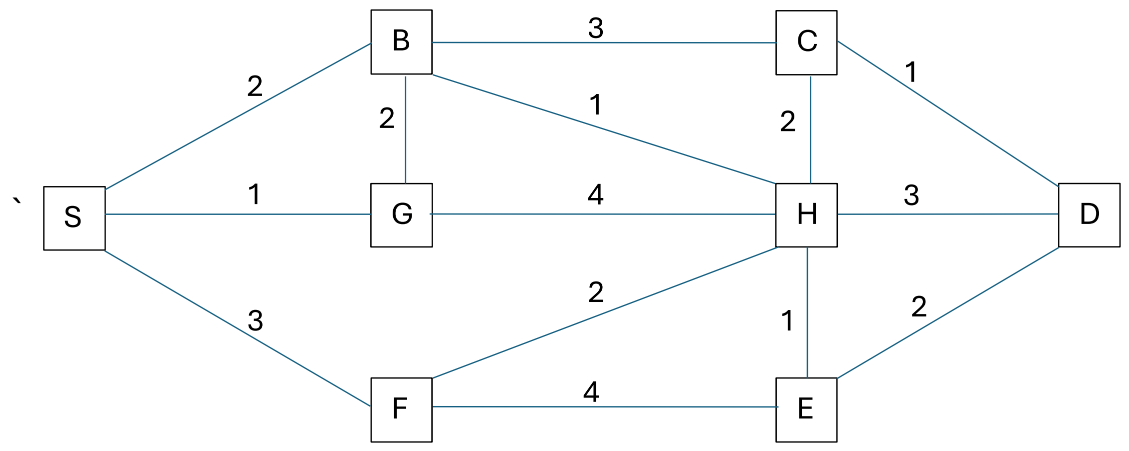
dis(x, y) = min{ path(x, y)}.

dis(x, y) <= wt(x, y) for all x and y.

Dijkstra’s Shortest Path Greedy Algorithm O(m log n)

Prerequisites

1. Graph is undirected.
2. Weights are positive.



Summary

Repeat steps 1 to 3.

1. Add one vertex at a time, (say “**w**”)
2. Compute distances to **unvisited vertices** that are adjacent to **w.**
3. Pick the vertex with minimum cost. **Best First Approach.** Not DFS or BFS. That is what makes this a greedy approach.

Initialization

dis[S] =0. Path[S] = { }.

Visited = {S}. Unvisited = {B, C, D, E, F, G, H}.

Compute the distance to all **unvisited** vertices that are adjacent to S.

dis[B] = dis[S] + wt(S, B) = 0 + 2 = 2.

dis[G] = dis[S] + wt(S, G) = 0 + 1 = 1.

dis[F] = dis[S] + wt(S, F) = 0 + 3 = 3.

Pick the vertex that can be reached with minimum cost. (**Greedy approach.**).

Pick G.

Delete G from the list of unvisited vertices.

Add G to the list of visited vertices.

Value of G will never change. Value of G is finalized.

dis[G] = dis[S] + wt(S, G) = 1.

Path[G] = path[S] $\cup$ {(S, G)} = {(S, G)}.

Visited = {S, G}. Unvisited = {B, C, D, E, F, H}.

Compute the distance to all **unvisited** vertices that are adjacent to G.

dis[B] = dis[G] + wt(G, B) = 1 + 2 = 3. (This is larger than dis[B]. Ignore.)

dis[H] = dis[G] + wt(G, H) = 1 + 4 = 5.

Pick the vertex that can be reached with minimum cost from S or G.

Pick B.

Delete B from the list of unvisited vertices.

Add B to the list of visited vertices.

Value of B will never change. Value of B is finalized.

dis[B] = dis[S] + wt(S, B) = 2.

Path[B] = path[S] $\cup$ {(S, B)} = {(S, B)}.

Visited = {S, G, B}.   Unvisited = {C, D, E, F, H}.

Compute the distance to all **unvisited** vertices that are adjacent to B.

dis[C] = dis[B] + wt(B, C) = 2 + 3 = 5.

dis[H] = dis[B] + wt(B, H) = 2 + 1 = 3.

Pick the vertex that can be reached with minimum cost from S, G, or B.

Pick H.

Delete H from the list of unvisited vertices.

Add H to the list of visited vertices.

Value of H will never change. Value of H is finalized.

dis[H] = dis[B] + wt(B, H) = 3.

Path[H] = path[B] $\cup$ {(B, H)} = {(S, B), (B, H)}.

Visited = {S, G, B, H}. Unvisited = {C, D, E, F}.

Compute the distance to all **unvisited** vertices that are adjacent to H.

dis[C] = dis[H] + wt(H, C) = 3 + 2 = 5.

dis[F] = dis[H] + wt(H, F) = 3 + 2 = 5.

dis[E] = dis[H] + wt(H, E) = 3 + 1 = 4.

dis[D] = dis[H] + wt(H, D) = 3 + 3 = 6.

Pick the vertex that can be reached with minimum cost from S, G, B, or H.

Pick F.

Delete F from the list of unvisited vertices.

Add F to the list of visited vertices.

Value of F will never change. Value of F is finalized.

dis[F] = dis[S] + wt(S, F) = 3.

Path[F] = path[S] $\cup$ {(S, F)} = {(S, F)}.

Visited = {S, G, B, H, F}.   Unvisited = {C, D, E}.

Compute the distance to all **unvisited** vertices that are adjacent to F.

dis[E] = dis[F] + wt(F, E) = 3 + 4 = 7. (This is larger dis[E]. Ignore.)

Pick the vertex that can be reached with minimum cost from S, G, B, H or F.

Pick E.

Delete E from the list of unvisited vertices.

Add E to the list of visited vertices.

Value of E will never change. Value of E is finalized.

dis[E] = dis[H] + wt(H, E) = 4.

Path[E] = path[H] $\cup$ {(H, E)} = {(S, B), (B, H), (H, E)}.

Visited = {S, G, B, H, F, E}.   Unvisited = {C, D}.

Compute the distance to all **unvisited** vertices that are adjacent to E.

dis[D] = dis[E] + wt(E, D) = 4 + 2 = 6.

Pick the vertex that can be reached with minimum cost from S, G, B, H, F or E.

Pick C.

Delete C from the list of unvisited vertices.

Add C to the list of visited vertices.

Value of C will never change. Value of C is finalized.

dis[C] = dis[B] + wt(B, C) = 5.

Path[C] = path[B] $\cup$ {(B, C)} = {(S, B), (B, C)}.

Visited = {S, G, B, H, F, E, C}.   Unvisited = {D}.

Compute the distance to all **unvisited** vertices that are adjacent to C.

dis[D] = dis[C] + wt(C, D) = 5 + 1 = 6.

Pick the vertex that can be reached with minimum cost from S, G, B, H, F, E or C.

Pick D.

Value of D will never change. Value of D is finalized.

dis[D] = dis[C] + wt(C, D) = 6.

Path[D] = path[C] $\cup$ {(C, D)} = {(S, B), (B, C), (C, D)}.

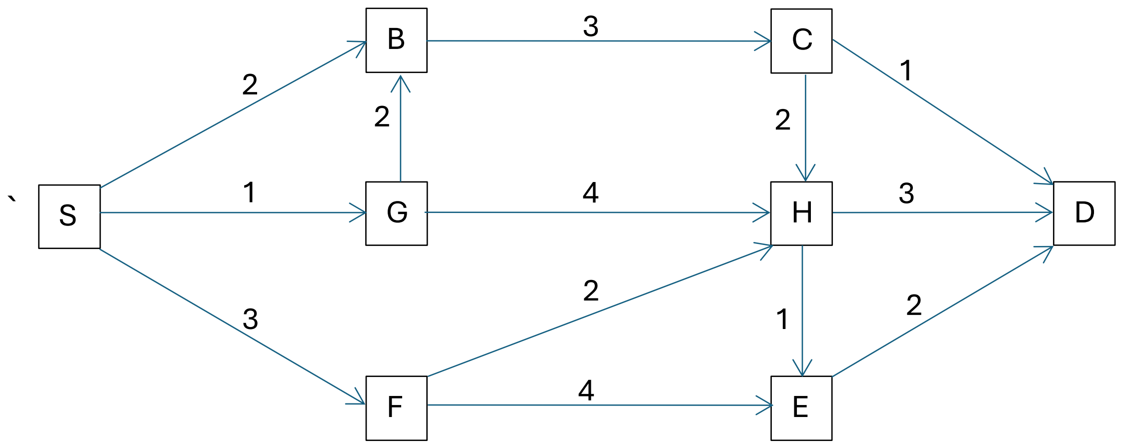
Visited = {S, G, B, H, F, E, D}.   Unvisited = { }.

THE END

Dijkstra’s Dynamic Programing Algorithm O(n + m)

Prerequisites

1. Graph is directed.
2. Graph is acyclic.
3. Negative weights allowed.



Summary

Step 1.

Perform topological ordering (or topological sort) of all vertices.

Repeat

1. Pick the vertex **w** based on the topological ordering.
2. Compute distances to **w** for each **incoming edge**.
3. Compute the minimum.

Topological Ordering: SFGBCHED

Initialization

dis[S] =0.   path[S] = { }.

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dis[F] = dis[S] + wt(S, F) = 3. ⬅

Path[F] =path[S] ∪ {(S, F)} = {(S, F)}.

---

dis[G] = dis[S] + wt(S, G) = 1. ⬅

path[G] =path[S] ∪ {(S, G)} = {(S, G)}.

---

dis[B] = min{ dis[S] + wt(S, B) = 0 + 2 = 2 ⬅

                    dis[G] + wt(G, B) = 1 + 2 = 3 }

path[B] =path[S] ∪ {(S, B)} = {(S, B)}.

---

dis[C] = dis[B] + wt(B, C) = 2 + 3 = 5. ⬅

path[C] =path[B] ∪ {(B, C)} = {(S, B), (B, C)}.

---

dis[H] = min{ dis[C] + wt(C, H) = 5 + 2 = 7

                    dis[G] + wt(G, H) = 1 + 4 = 5 ⬅

                    dis[F] + wt(F, H) = 3 + 2 = 5 }

path[H] =path[G] ∪ {(G, H)} = {(S, G), (G, H)}.

---

dis[E] = min{ dis[H] + wt(H, E) = 5 + 1 = 6 ⬅

                    dis[F] + wt(F, E) = 3 + 4 = 7 }

path[E] =path[H] ∪ {(H, E)} = {(S, G), (G, H), (H, E)}.

---

dis[D] = min{ dis[C] + wt(C, D) = 5 + 1 = 6 ⬅

dis[H] + wt(H, D) = 5 + 3 = 8

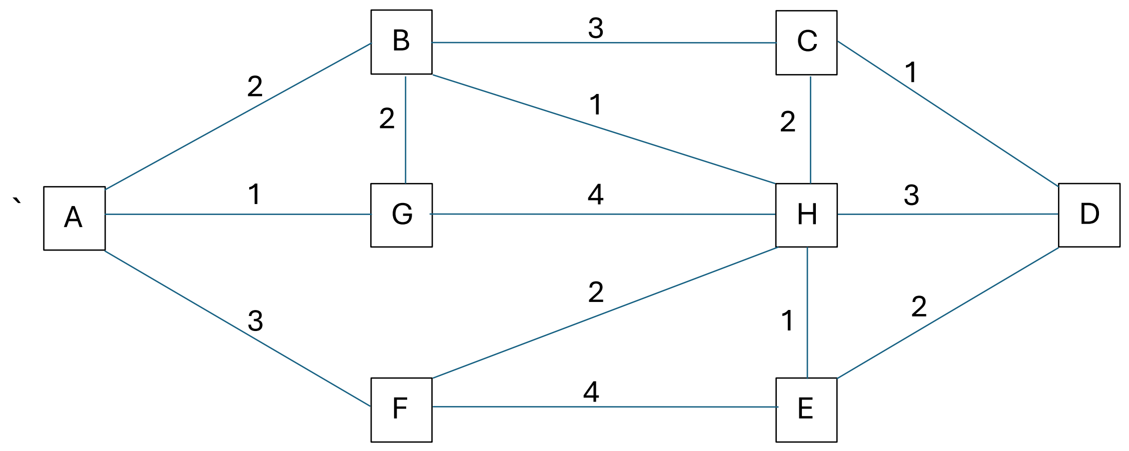
dis[E] + wt(E, D) = 6 + 2 = 8 }

path[D] =path[C] $\cup$ {(C, D)} = {(S, B), (B, C), (C, D)} **THE END**

Kruskal's Minimum Spanning Tree Algorithm O(m log n)

Prerequisite

1. Graph is undirected.
2. Graph has positive weights.



Summary

Step 1. Sort the edges by weight and keep it in a list **L.**

Step 2. Initialize a Union-Find data structure with vertices such that each vertex is a singleton.

Step 3. Repeat

```
        Pick next edge (x, y) from the list L.
        if (Find(x) != Find(y)) Union (x, y)
        else delete the edge (x, y) from the list L.
```

Output: L contains all the edges of the minimum spanning tree. The sum of all weights of edges in L gives the weight of MST.

List of edges sorted by weight.

(A, G)

(C, D)

(H, E)

(B, H)

(A, B)

(B, G)

(F, H)

(H, C)

(D, E) ...

Initialize Union-Find : {A}, {B}, {C}, {D}, {E}, {F}, {G}, {H}

Find(A) ≠ Find(G). Union(A, G)

{A, G}, {B}, {C}, {D}, {E}, {F}, {H}

Find(C) ≠ Find(D). Union(C, D)

{A, G}, {B}, {C, D}, {E}, {F}, {H}

Find(H) ≠ Find(E). Union(H, E)

{A, G}, {B}, {C, D}, {E, H}, {F}

Find(B) ≠ Find(H). Union(B, H)

{A, G}, {B, E, H}, {C, D}, {F}

Find(A) ≠ Find(B). Union(A, B)

{A, B, E, G, H}, {C, D}, {F}

Find(B) == FindG). Delete(B, G).

Find(F) ≠ Find(H). Union(F, H)

{A, B, E, F, G, H}, {C, D}

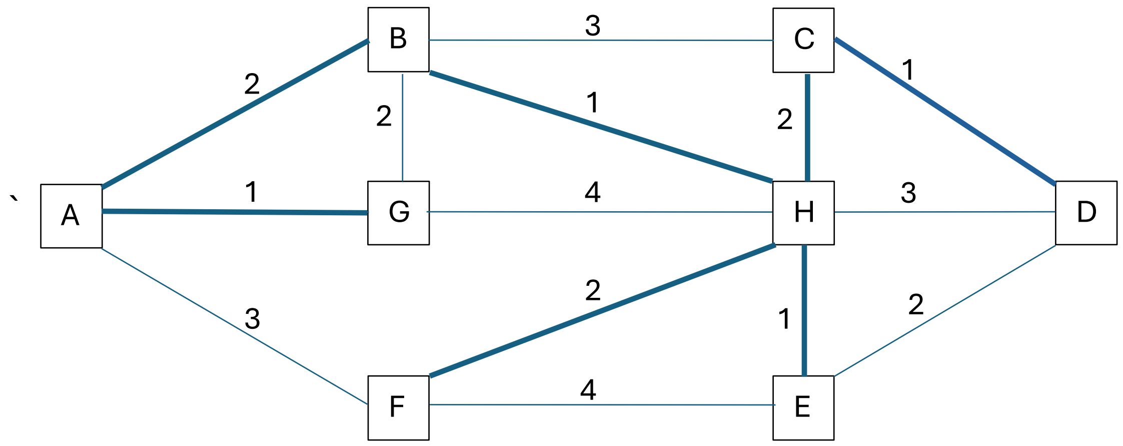
Find(F) ≠ Find(C). Union(F, C)

{A, B, C, D, E, E, F, G, H}

Find(F) == Find(D) Delete(B, G).

The same will happen to remaining edges. Edges of the spanning tree are

(A, G)	1
(C, D)	1
(H, E)	1
(B, H)	1
(A, B)	2
(F, H)	2
(H, C)	2



Total weight of the spanning tree = 1+1+1+1+2+2+2 = 10. THE END