

Question 3. (a) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$K=4$$

Step 1:

- $lo=0, hi=8, mid=4 \rightarrow$ triple $\{1, 9, 5\} \Rightarrow \{1, 5, 9\} \rightarrow pivot = 5$
- Partition: $L = \{1, 2, 3, 4\} (4), E = \{5\} (1), G = \{6, 7, 8, 9\} (4)$

since: $K=4 \leq 4 \leftarrow |L|$, recursion in L with
same $K \rightarrow K=4$

Step 2 (in $[1, 2, 3, 4]$) $\rightarrow K=4$

- $lo=0, hi=3, mid=1 \rightarrow$ triple $\{1, 2, 4\} \rightarrow pivot = 2$

• partition:

$$L_1 = [1] (1), E_1 = [2] (1), G_1 = [3, 4] (2)$$

$$\text{since: } K=4 > |L_1| + |E_1| = 2$$

recursion in G_1 with $K \leftarrow 4 - 2 = 2$

Step 3 (in $[3, 4]$ with $K=2$):

- $lo=0, hi=1, mid=0 \rightarrow \{3, 4, 3\} \rightarrow \{3, 3, 4\} \rightarrow pivot = 3$

• partition:

$$L_2 = [] (0), E_2 = [3] (1), G_2 = [4] (1)$$

$$\text{since } K=2 > |L_2| + |E_2| = 1, \text{ recursion in } G_2 \text{ with } K \leftarrow 2 - 1 = 1$$

- subarray with one element $[4] \rightarrow answer = 4$

Q.3) (5) $A = \{8, 7, 6, 5, 4, 3, 2, 1, 9\}$, $k = 5$

Step 1:

• $lo = 0, hi = 8, mid = 4 \rightarrow \text{tuple } \{8, 9, 4\} \Rightarrow \{4, 8, 9\} \rightarrow \text{pivot} = 8$

• Partition:

$L = [7, 6, 5, 4, 3, 2, 1] (7), E = [8] (1), G = [9] (1)$

since $k = 5 \leq |L| = 7 \rightarrow \text{recursion in } L \text{ with } k = 5$

Step 2 (in $[7, 6, 5, 4, 3, 2, 1]$ with $k = 5$):

• $lo = 0, hi = 6, mid = 3 \rightarrow \text{tuple } \{1, 4, 7\} \rightarrow \text{pivot} = 4$

$\downarrow \quad \downarrow \quad \downarrow$
 $A[lo] = 7 \quad A[mid] = 1 \quad A[hi] = 4$

• Partition:

$L_1 = [3, 2, 1] (3), E_1 = [4] (1), G_1 = [7, 6, 5] (3)$

- $k = 5 > (|L_1| = 3 + |E_1| = 1) = 4$

\downarrow
Recursion in G_1 with $k = 5 - 4 = 1$

Step 3 (in $[7, 6, 5]$ with $k = 1$)

• $lo = 0, hi = 2, mid = 1 \rightarrow \text{tuple } \{5, 6, 7\} \rightarrow \text{pivot} = 6$

$\downarrow \quad \downarrow \quad \downarrow$
 $A[lo] = 7 \quad A[mid] = 5 \quad A[hi] = 6$

• Partition: $L_2 = [5] (1), E = [6] (1), G_2 = [7] (1)$

- since $k = 1 \leq |L_2| = 1 \rightarrow \text{recursion in } L_2 (|L_2| = 1, k = 1)$
 $\rightarrow \text{answer} = 5$

Q3 (c) $A = \{9, 1, 8, 2, 7, 3, 6, 4, 5\}, K = 6$

Step 1:

• $lo = 0, hi = 8, mid = 4 \rightarrow \text{Triple } \{5, 7, 9\} \rightarrow pivot = 7$

• Partition:

$L = [1, 2, 3, 6, 4, 5](6), E = [7](1), G = [9, 8](2)$

• $K = 6 \leq |L| = 6$ recursion in L ($[1, 2, 3, 6, 4, 5], K = 6$)

Step 2 (in $[1, 2, 3, 6, 4, 5], K = 6$)

• $lo = 0, hi = 5, mid = 2 \rightarrow \text{triple } \{1, 3, 5\} \rightarrow pivot = 3$

partition:

$L_2 = [1, 2](2), E_2 = [3](1), G_2 = [6, 4, 5](3)$

• $K = 6 > |L_2| + |E_2| = 2 + 1 = 3$

\downarrow
recursion in G with $K \leftarrow 6 - 3 = 3$

Step 3 ($[6, 4, 5], K = 3$):

• $lo = 0, hi = 2, mid = 1 \rightarrow \text{Triple } \{4, 5, 6\} \rightarrow pivot = 5$

• Partition: $L_3 = [4](1), E = [5](1), G = [6](1)$

• $K = 3 > |L| + |E| = 2 \rightarrow \text{recursion in } G([6], K = 1)$
 $K = 3 - 2$

• subarray $G = [6] \rightarrow$ only one element $\rightarrow \text{answer} = 6$