

# Lab W1D2 — Question 1: Comparing Algorithms (Third Largest)

**Problem:** Given an array A ( $n \geq 3$ ), return the third largest value. Duplicates are allowed (e.g., 20,20,20  $\rightarrow$  3rd = 20).

## Algorithm 1 — Three Passes ( $\Theta(n)$ )

Idea: Pass 1 gets  $\text{max}_1$ ; pass 2 gets  $\text{max}_2 \leq \text{max}_1$ ; pass 3 gets  $\text{max}_3 \leq \text{max}_2$ . Time  $\Theta(n)$ , Space  $O(1)$ .

## Algorithm 2 — One Pass (max, preMax, prePreMax) ( $\Theta(n)$ )

Idea: Maintain top-3 in one scan; use  $\geq$  to handle duplicates. Time  $\Theta(n)$ , Space  $O(1)$ .

Correctness: loop invariant ensures (max, preMax, prePreMax) are top-3 of the prefix.

## Algorithm 3 — Ordered Dictionary (TreeMap) ( $O(n \log n)$ )

Idea: Build frequency map (sorted), then walk keys descending until 3rd element. Time  $O(n \log n)$ , Space  $O(u)$  ( $u \leq n$ ).

Algorithm	Best	Average	Worst	Space
Three Passes	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$O(1)$
One Pass	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$O(1)$
TreeMap	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(n)$

**Empirical Results:** OnePass and ThreePasses scale linearly ( $\Theta(n)$ ); TreeMap grows faster ( $O(n \log n)$ ).

As  $n$  increases, TreeMap visibly separates from linear curves, matching theory.

**Recommendation:** Use Algorithm 2 (one pass) for efficiency and simplicity. Algorithm 3 only if ranked queries are needed.

**Chart:**