

Q1 Adjacency Matrix

A	B	C	D	E	F	G	H	I
0	1	1	1	0	0	0	0	0
1	0	1	0	0	1	0	1	0
2	1	1	0	1	1	1	0	0
3	1	0	1	0	1	0	0	2
4	0	0	1	2	0	1	1	0
5	0	1	1	0	1	0	1	1
6	0	0	0	1	1	0	1	2
7	0	1	0	0	0	1	2	0
8	0	0	0	1	0	0	1	0

Q2 Find the shortest path from A to all other vertices using  
BFS Algorithm

$$\text{1. A} \left\{ \begin{array}{l} A-B: 22 \\ A-C: 3 \\ A-D: 12 \end{array} \right.$$

$$\text{4. B} \left\{ \begin{array}{l} B-C: 57 \\ B-F: 58 \\ B-H: 56 \end{array} \right.$$

$$\text{2. C} \left\{ \begin{array}{l} C-D: 13 \\ C-E: 74 \\ C-F: 51 \end{array} \right.$$

$$\text{3. D} \left\{ \begin{array}{l} D-E: 45 \\ D-I: 42 \end{array} \right.$$

$$\text{5. E} \left\{ \begin{array}{l} E-F: 63 \\ E-G: 68 \end{array} \right.$$

$$\text{6. F} \left\{ \begin{array}{l} F-G: 90 \\ F-H: 75 \end{array} \right.$$

$$\text{8. G} \left\{ \begin{array}{l} G-H: 93 \\ G-I: 83 \end{array} \right.$$

$$\text{7. H} \left\{ \begin{array}{l} H-I: 75 \end{array} \right.$$

(4) • I

dis:

A	B	C	D	E	F	G	H	I
0	22	3	12	45	51	68	56	42

path: A → B → C → D → E → F → G → H → I  
 $\{ \} \{ A-B \} \{ A-C \} \{ A-D \} \{ A-B-E \} \{ A-C-F \} \{ A-D-E-G \} \{ A-B-H \} \{ A-B-E \}$

Q3 What is the time complexity?

Q4 Find a minimum spanning tree using Kruskal's algorithm

$$L = \left[ \begin{array}{l} (C, D), (A, C), (E, F), (H, I), (\cancel{G, I}), (G, I), (A, B), (E, G), (F, H), (G, H), \\ (B, I), (B, E), (B, H), (B, C), (B, F), (F, G), (C, F), (C, E) \end{array} \right]^{25}$$

$$\text{Union-Find}: \{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{F\}, \{G\}, \{H\}, \{I\} \Rightarrow n = 9$$

MST edges = 9 - 1 = 8

pick(C, D)  $\Rightarrow$  find(C)  $\neq$  find(D). Union(C, D)

$$\{A\}, \{B\}, \{C, D\}, \{E\}, \{F\}, \{G\}, \{H\}, \{I\}$$

pick(A, C)  $\Rightarrow$  find(A)  $\neq$  find(C). Union(A, C)

$$\{A, C, D\}, \{B\}, \{E\}, \{F\}, \{G\}, \{H\}, \{I\}$$

pick(E, F)  $\Rightarrow$  find(E)  $\neq$  find(F). Union(E, F)

$$\{A, C, D\}, \{B\}, \{E, F\}, \{G\}, \{H\}, \{I\}$$

pick(H, I)  $\Rightarrow$  find(H)  $\neq$  find(I). Union(H, I)

$$\{A, C, D\}, \{B\}, \{E, F\}, \{G\}, \{H, I\}$$

pick(G, I)  $\Rightarrow$  find(G)  $\neq$  find(I). Union(G, I)

$$\{A, C, D\}, \{B\}, \{E, F\}, \{G, H, I\}$$

pick(A, B)  $\Rightarrow$  find(A)  $\neq$  find(B). Union(A, B)

$$\{A, B, C, D\}, \{E, F\}, \{G, H, I\}$$

pick(E, G)  $\Rightarrow$  find(E)  $\neq$  find(G). Union(E, G)

$$\{A, B, C, D\}, \{E, F, G, H, I\}$$

pick(B, I)  $\Rightarrow$  find(B)  $\neq$  find(I). Union(B, I)

pick(A, B, C, D, E, F, G, H, I) // we can stop here since we have n-1 edges

Q3. What is the time complexity?

Q6. DAG adjacency matrix

P	Q	R	S	T	U
0	1	0	6	7	6
0	0	1	4	0	0
0	0	0	2	0	1
0	0	0	0	3	2
0	0	0	0	0	2
0	0	0	0	0	0

rows(i)

columns(j)

Wt of  $i \rightarrow j$  is  $\Theta$  if there is no edges

Q7. Shortest Path from P to U using DAG algorithm

dist:  $\begin{matrix} P & Q & R & S & T \\ 0 & 1 & 2 & 3 & 4 & 7 \end{matrix}$

Path:  $\begin{matrix} P & Q & R & S & T \\ 0 & 1 & 2 & 3 & 4 & 7 \end{matrix}$

Final Result:

$$\text{dist}[U] = 3$$

Shortest path: P-Q-R-U

Q8. DAG's algorithm Time Complexity

$$O(V+E) \text{ or } O(n+m)$$

Q9. Can you use Dijkstra's algorithm to find the shortest path from P to U?

Yes, because all vertices have positive weights and the graph is a directed graph with non-negative weights

Q10. Shortest Path using Dijkstra's algorithm

$$\text{P-Q-R-U}$$
  
Time complexity:  $O(V^2)$