

A dynamical model for the growth of a stand of Japanese knotweed

including mowing as a management technique

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OUTLINE OF THE PRESENTATION

- I) The Japanese knotweed: ecology and model.
- II) Simulation study of the model.

Particular case: the Japanese knotweed

Japanese knotweed



Rhizome



Stem: from 1 to 3 meters.

Rhizome: up to 8 cm diameter, length : 15 - 20 m, depth: 2 - 3 m, represents $\frac{2}{3}$ of the total biomass of the plant.

The rhizome withstands the cold and enables to spend the bad season burried in the ground.

Clonal development of the Japanese knotweed.

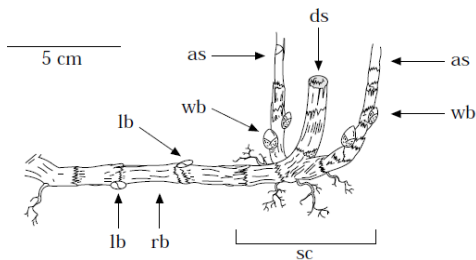
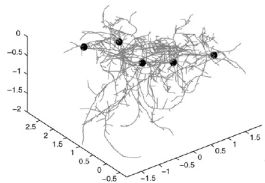


Figure: Diagram of the development of stems and buds along the rhizome, extracted from [Adachi et al., 1996] as: current aerial shoot, ds: dead aerial shoot, rb: rhizome, sc: shoot clump, lb: lateral bud, wb: winter bud.

References: [Adachi et al., 1996], [Dauer and Jongejans, 2013], [Price et al., 2002], [Beerling et al., 1994], [De Waal, 2001].

Litterature on modeling the Japanee knotweed evolution

- ▶ [Smith et al., 2007] have constructed a 3D correlated random walk for the development of the underground rhizome network.
- ▶ [Dauer and Jongejans, 2013] propose an "Integral Projection Model" (IPM), inspired by the "Matrix Population Model", for the dynamics of the plant at the level of a stand.
- ▶ [Gourley et al., 2016] have developed a model of biocontrol of the knotweed, based on the insect Psyllid *Aphalara itadori*, which feeds on the sap of the stems of the knotweed. Deterministic model that describes the evolution of the number of insects (larvae and adults) as well as the evolution of the total biomass of stems and rhizome.



Dynamical model for the knotweed

Objective: Describe the dynamics of Japanese knotweed at the local scale and the effects of mowing on it.

The mathematical formalism is the one of the **measure-valued stochastic processes**.

The model presented in this section is inspired by the work of [Fournier and Méléard, 2004] and [Tran, 2006].

The individuals, here the crowns (i.e. the places where the terminal buds are located and from which the stems sprout) are characterized by:

- ▶ their position (in the plan)
- ▶ a trait describing the underground biomass (i.e. that of the rhizome that is connected to the crown).

Events occurring in the model

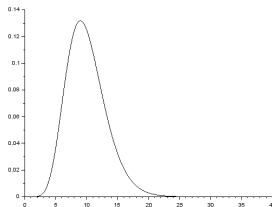
At each time, we calculate the next time at which there is an event. There are three possible events:

- ▶ a birth of a new crown: birth rate depending on positions, law for the dispersal distance and intra-specific competition zone
- ▶ a death of a crown: mortality rate depending on biomass
- ▶ the mowing of a proportion *proportionMowing* of individuals in the population. The effect of mowing is a decreasing function of the the biomass.
- ▶ Between those events, the biomass of each crown evolves in a deterministic way.

Birth and dispersal of the created individual

Birth rate depends on the neighbouring individuals present at a distance smaller than *distanceParent*. That allows us to account for the effects of apical dominance.

We choose a Gamma law with parameters (*shape*, *scale*) for the law of the dispersal distance of the daughter crown.



The new born individual must therefore be at a distance larger than *distanceCompetition* from its neighbours not to fall in the zone of intraspecific competition.

Effects of mowing on rhizome development are poorly known.

- ▶ Hypothesis: reduction of underground biomass, rhizome resources used for growth of aerial parts [Gerber et al., 2010], [Rouifed et al., 2011]
- ▶ No influence of the distribution of the mowing dates, only the mowing number matters [Seiger et al., 1997]
- ▶ Modeling choice: after a mowing, the underground biomass is directly impacted and becomes $a.F(a)$, where $F(a) \in [0, 1]$. It is assumed that the effect of mowing is more important for low biomasses $\Rightarrow F$ **increasing**.

$$\forall a \in \mathbb{R}_+, F(a) = 1 - \exp(-\text{mowingParameter} * a)$$

Random mowing technique

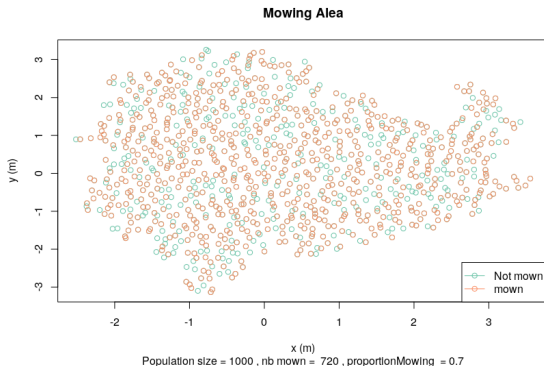


Figure: Represents the crowns that would be mown during a mowing event with the Random technique. A proportion of mown plants is imposed. The coordinates are in meters.

Side mowing technique

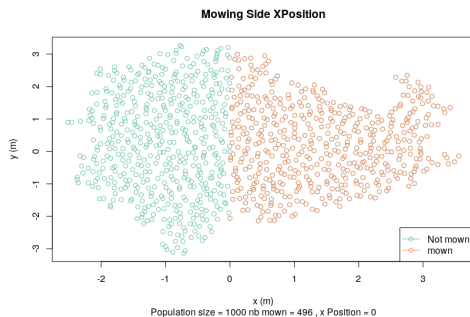


Figure: Represents the crowns that would be mown when imposing a *position* at the right of which all plants are mown.

Model Parameters

Variable	Description
Biomass	
K	maximal biomass (g)
L	growing rate for low biomasses
a_0	initial biomass of a born crown (g)
Mowing	
<i>mowingParameter</i>	effect of mowing
Mortality	
<i>deathParameterScaling</i>	for the small biomasses
<i>deathParameterDecrease</i>	decrease speed of the mortality rate
Birth	
<i>distanceParent</i>	distance of apical dominance (m)
<i>distanceCompetition</i>	intra specific competition distance (m)
\bar{b}	birth rate (ideal conditions)
(shape, scale)	Gamma law, dispersion of the created individual

Management parameters:

- ▶ initial population size: *InitialPopSize*
- ▶ mean number of mowing events a year: τ
- ▶ management project duration: T
- ▶ proportion of mown crowns: *proportionMowing*

Large population limit

Under boundary conditions over the birth and death rates, the stochastic differential equation admits a unique pathwise strong solution (and an upper bound to the growth of the population).

Objective: Study of a renormalisation of the Stochastic Differential Equation solution $(Z^n)_{n \in \mathbb{N}}$ associated with the dynamics.

- ▶ Simplification of the SDE: withdrawal the mowing term.
- ▶ Initial population size of order n , no modification of the interaction between individuals.

Result: law convergence of the sequence of renormalised processes toward a process solution of an integro-differential equation :

The techniques used for this result are those developed in [Tran, 2006].

ANALYSIS OF THE MODEL BY SIMULATIONS

CALIBRATION

Reminder of model parameters

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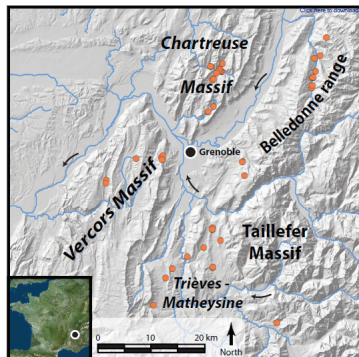


Figure: Extrait de [Martin, 2017].

48 stands of knotweeds in the French Alps (various altitudes).

Measurements carried out in 2008 and in 2015: on the stands themselves (outline, number of stems, ...) and on biotic and abiotic variables.

Variability in observed stands: size (less than $1m^2$ to $350m^2$), area (proximity of watercourse, road, forest, abandoned land)

F. Martin data- Calibration

- ▶ In model outputs, we have the final and initial population areas and sizes.
- ▶ In F. Martin's article, we use data on areas and densities of stands (so we have access to the size of the population) in 2008 and 2015.

stand	size 2008	area 2008	size2015	area 2015	τ	FullMow
1	261	14.525	112	10.815	0	0
2	1878	52.177	872	42.187	1	0
3	1063	75.899	1493	72.829	1	1
4	1771	104.203	852	49.273	2	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Method for the calibration

The OpenMOLE software proposes a method derived from genetic algorithms for the calibration of models.

The algorithm explores the parameter space to find the minimal distance between the observations and the simulations (the algorithm manages the stochasticity of the model).

As distance between simulations and observations, we take for each stand and each type (area or size) the distance: $\frac{|simu - data|}{data}$.



Reference : [Romain Reuillon, 2013]

The result of the calibration

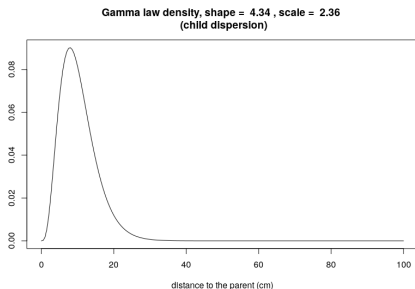
Variable	Valeur Calibration
K	12.72
L	0.26
distanceCompetition	0.15
distanceParent	0.20
shape	4.34
scale	2.36
deathParameterDecrease	2.32
deathParameterScaling	1.12
mowingParameter	0.11
bbar	0.18
a0	1.73
delta	26.06
evolution.samples	79

Agreement between calibrated values and experts' statements

- The parameters *distanceCompetition* and *distanceParent*, close to experts' statements.

Variable	Calibration value	experts' statements
distanceCompetition	0.15	≈ 0.15
distanceParent	0.20	≈ 0.15

- The law for the dispersal of individuals $\Gamma(shape = 4.34, scale = 2.36)$, in compliance with experts' statements.



Agreement between calibrated values and experts' statements

- The ratio of the values of K (maximum biomass), and a_0 the biomass of the individuals at birth, which is 7.5 (a ratio of about 10 is expected).

Variable	Calibration value	Calibration values ratio	Experts' statements ratio
K	12.72	7.5	≈ 10
a_0	1.73		

- Mortality rate:

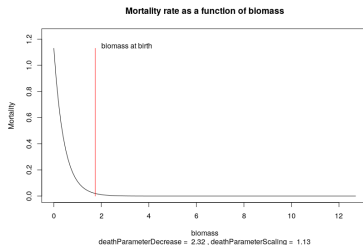


Figure: Mortality rate of a crown according to its biomass. We note that a crown that is not mown keeps a very low mortality rate, which agrees with field observations.

ANALYSIS OF THE MODEL BY SIMULATIONS

INFLUENCE OF MOWING PARAMETERS

Law of the outputs, replications

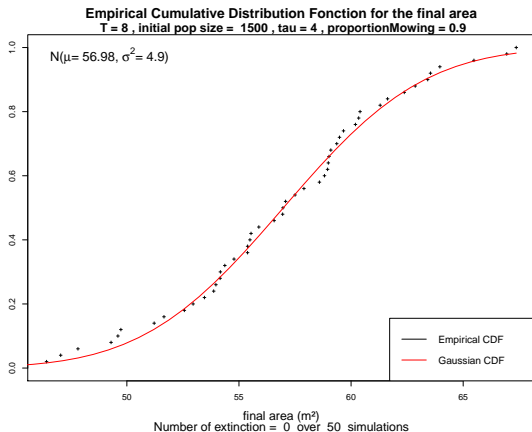


Figure: Probability Distribution Function of the final area, obtained with an initial population size = 1500, $\tau = 4$, *proportionMowing* = 0.9, and $T = 8$.

Comparison of initial and final area densities.

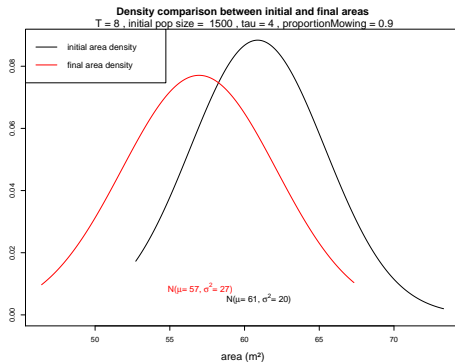


Figure: Density of the Gaussian laws of the initial and final areas, obtained from the empirical averages and variances. Initial population size = 1500, $\tau = 4$, $\text{proportionMowing} = 0.9$, and $T = 8$.

Influence of T on the final average area

We plot the regression curves obtained for different τ .
[InitialPopSize = 1500 and *proportionMowing* = 0.9 fixed].

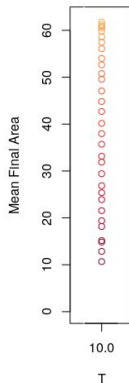
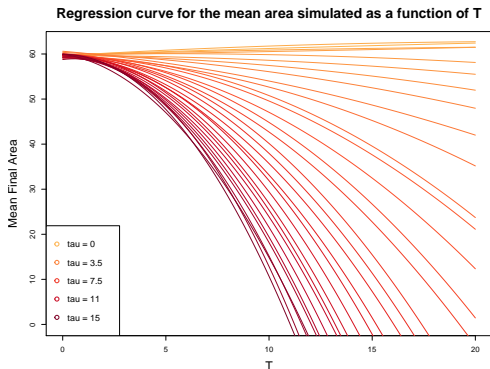


Figure: Quadratic regression curves of the mean final area as a function of T , for different τ .

Figure: Cross section.

Summary of the influences of the management parameters on the average values of the outputs

Initial / Final	Param	Average Area	Average Size
Initial	InitialPopSize	linear ↗	linear ↗
Final	InitialPopSize	linear ↗	linear ↗
Final	T	linear ↗ (τ weak)	linear ↗
		quadratic ↘ (τ high)	exponential ↘
Final	τ weak	linear ↗ or ↘	linear ↘
Final	τ high	linear ↘	exponential ↘

We have the more general result:

Result

For $\tau \gtrsim 2$:

$$\text{Final Size} = \text{Initial Size} \times \exp(-T.(\tau - a)/b),$$

with $a, b \in \mathbb{R}$ constants,

and,

$$\text{Final Area} = \max(c \times \tau \times t^2 + 0.04 \times \text{Initial Size}, 0)$$

with $c \in \mathbb{R}$ constant.

Important remark for the formulas

Formulas obtained for the mean output quantities are still relevant for direct outputs.

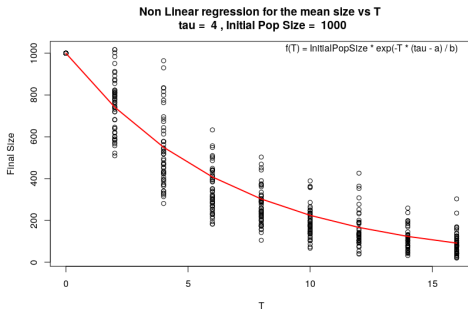


Figure: Black circles represent stand sizes resulting of 50 replications with $\tau = 4$, initial population size = 1000, and varying T . The red line is the function of T defined by Equation for the mean final size. It has been found with a regression on a far bigger set of points than the subset selected to plot this example.

Take an interest in two other questions managers face:

- ▶ At the scale of a landscape with several knotweed stands, how to distribute the mowing effort (intensity / frequency) between the different stands?
→ Modeling the dispersion due to mowing.
- ▶ Compare mechanical engineering (mowing) vs ecological engineering (willow)?
→ Model the influence of the shadow on the dynamics of knotweed.

Pose viability problems in stochastic formalism.

OpenMOLE USER CASE

OpenMOLE USER CASE, joint work with G.Chérel

On Gitlab:

https://gitlab.iscpif.fr/gcherel/renouee/blob/master/README_en.html

and on OpenMOLE blog:

<https://blog.openmole.org/>

More details can be found in:

[Lavallée et al., 2019a] Quels apports de la modélisation pour l'aide à la gestion de la renouée du Japon. *Sciences Eaux et Territoires*.

[Lavallée et al., 2019b] A stochastic individual based model for the growth of a stand of japanese knotweed including mowing as a management technique. *Submitted to Ecological Modelling*,

<https://arxiv.org/abs/1902.06971>

Thanks for your attention!

Varying the mean number of mowing events

Question : We vary the mean number of mowing events each year. Which management strategy minimizes the final area?

We set the total number of mowing events over the all period of management equal to N_{tot} , and total number of mowing events over a year equal to τ_{max} . The aim is to find (τ_1, \dots, τ_T) which minimizes the final area.

The set of admissible configurations is defined by

$$X := \{(\tau_1, \dots, \tau_T) \in [0; \tau_{max}]^T \mid \sum_{i=1}^T \tau_i = N_{tot}\}$$

Varying the mean number of mowing events

Example: $T = 4$, $N_{tot} = 5 \times 4$, $\tau_{max} = 15$, $(5, 5, 5, 5) \in X$,
 $(8, 6, 4, 2) \in X$. $\#(X) = 1631$.

Practical issue: Using reasonable size of configuration set, we can perform a *sample* (DirectSampling and filter), and deal with stochasticity by hand.

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Example: $T = 8$, $N_{tot} = 5 \times 8$.

More general situation: N_{tot} varies.

Then we use NSGA2 algorithms, which deals with stochasticity, considering real numbers as inputs parameters, with 2 objectives: minimizing the invaded area at time T and the total number of

mowing events $\sum_{i=1}^T \tau_i$. We obtain a Pareto front.

We introduce the following notations

- ▶ $\chi = \mathbb{R}^2 \times \mathbb{R}_+$.
- ▶ $\mathcal{M}_F(\chi)$ [resp. $\mathcal{P}(\chi)$] the set of finite measure (resp. probability measure) on X .
- ▶ $\mathcal{M} \subset \mathcal{M}_F(\chi)$: finite point measures whose point mass is 1 or 0.

$$\mathcal{M} = \left\{ \sum_{i=1}^n \delta_{x_i, a_i} , n \geq 0, (x_1, a_1), \dots, (x_n, a_n) \in \chi \right\}$$

In the model, a crown is represented by a Dirac mass $\delta_{(x,a)}$, with $(x, a) \in \chi$.

The set of all crowns at time t is described by the measure $Z_t \in \mathcal{M}$.

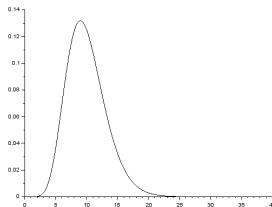
Birth and dispersal of the created individual

Birth rate of the form:

$$b(x, Z) = \bar{b} \cdot 1_{\{\sum_{y \in V(Z)} 1_{\{|x-y| \leq \text{distanceParent}\}} \leq 3\}}$$

where $V(Z) := \{x \in \mathbb{R}^2, Z(\{x\} \times \mathbb{R}_+) > 0\}$ is the set of the crowns present in the population Z .

An individual with trait (x, a) produces an individual in x' at rate $b(x, Z)$. We choose a Gamma law with parameters (*shape*, *scale*) for the law of the dispersal distance of the daughter crown.



We consider that the individual created is really born if it is born in the following area (**intra spécifique competition**) :

$$C_{x,Z} = \{z \in \mathbb{R}^2 \text{ tq } \forall y \in V(Z) \setminus \{x\}, |y-z| > \text{distanceCompetition}\}$$

- ▶ Hypothesis: the mortality rate is independent of the position x of an individual.
- ▶ An individual living at time t with a biomass $a(t)$ dies at the rate $m(a(t))$.
- ▶ The mortality m is a **decreasing function of the biomass**: an individual with a low biomass, either because it has just been created or because it has undergone mowing, will have a higher rate of mortality.

The rhizome biomass associated with a crown evolves (in the absence of a mowing or mortality event) according to the following equation:

$$\frac{da(t)}{dt} = L(K - a(t)) \quad (1)$$

where L is the low biomass growth rate and K is the asymptotic biomass.

We are able to solve explicitly Equation (1).

We denote by A_b the flow: $t \rightarrow A_b(t, t_0, a_0)$ associated with the ordinary differential equation (1).

Effects of mowing on rhizome development are poorly known.

- ▶ Hypothesis: reduction of underground biomass, rhizome resources used for growth of aerial parts [Gerber et al., 2010], [Rouifed et al., 2011]
- ▶ No influence of the distribution of the mowing dates, only the mowing number matters [Seiger et al., 1997]
- ▶ Modeling choice: after a mowing, the underground biomass is directly impacted and becomes $a.F(a)$, where $F(a) \in [0, 1]$. It is assumed that the effect of mowing is more important for low biomasses $\Rightarrow F$ **increasing**.

$$\forall a \in \mathbb{R}_+, F(a) = 1 - \exp(-\text{mowingParameter} * a)$$

Poisson Point Measure

An application $M : \Omega \times E \rightarrow \mathbb{R}_+$ is a **random measure** if

- ▶ $\omega \rightarrow M(\omega, A)$ is a random variable for each $A \in E$
- ▶ $A \rightarrow M(\omega, A)$ is a measure on (E, \mathcal{E}) for each $\omega \in \Omega$. We denote $M(A)$ this random variable.

The term "random measure" means that M is a random variable that associate a measure M_ω to each event $\omega \in \Omega$.

Définition

Let (E, \mathcal{E}) be a measurable set and ν be a measure on (E, \mathcal{E}) . A random measure N on (E, \mathcal{E}) is a Poisson random measure with intensity ν if :

- *for each $A \in E$, the random $N(A)$ has a Poisson law with parameter $\nu(A)$.*
- *for each $A_1, \dots, A_n \in \mathcal{E}$ disjoints, random variables $(N(A_1), \dots, N(A_n))$ are independents, for all $n \geq 2$.*

Stochastic Differential Equation

$$\begin{aligned}
 Z_t = & \sum_{i=1}^{N_0} \delta_{(X_i(Z_0), A_b(t, 0, A_i(Z_0)))} \\
 & + \int_0^t \int_{\mathbb{N}^*} \int_{\mathbb{R}_+} \int_{\mathbb{R}^2} \mathbf{1}_{\{i \leq N_{s-}\}} \delta_{(X_i(Z_s) + z, A_b(t, s, a_0))} \\
 & \times \mathbf{1}_{\{\theta \leq b(X_i(Z_s), Z_s)\}} \mathbf{1}_{\{X_i(Z_s) + z \in C_{X_i(Z_s), Z_s}\}} \times M_1(ds, di, d\theta, dz) \\
 & + \int_0^t \int_{[0,1]^{N^*}} \sum_{i=1}^{N_{s-}} \mathbf{1}_{y_i \leq \text{proportionMowing}} \\
 & (\delta_{(X_i(Z_s), A_b(t, s, A_i(Z_{s-})).F(A_i(Z_{s-})))} - \delta_{(X_i(Z_s), A_b(t, s, A_i(Z_s)))}) M_2(ds, dy) \\
 & - \int_0^t \int_{\mathbb{N}^*} \int_{\mathbb{R}_+} \mathbf{1}_{\{i \leq N_{s-}\}} \mathbf{1}_{\{\theta \leq m(A_i(Z_s))\}} \delta_{(X_i(Z_s), A_b(t, s, A_i(Z_s)))} M_3(ds, di, d\theta).
 \end{aligned}$$

$M_1(ds, di, d\theta, dz)$ is a Poisson point measure on

$E_1 := \mathbb{R}_+ \times \mathbb{N}^* \times \mathbb{R}_+ \times \mathbb{R}^2$ with intensity $\nu_1 = ds \otimes n(di) \otimes d\theta \otimes D(dz)$.

Large population limit

Objective: Study of a renormalisation of the Stochastic Differential Equation solution $(Z^n)_{n \in \mathbb{N}}$ associated with the dynamics.

- Simplification of the SDE: withdrawal the mowing term.
- Initial population size of order n , no modification of the interaction between individuals.

Result: law convergence of the sequence of renormalised processes toward a process solution of an integro-differential equation :

$$\begin{aligned} \langle \xi_t, f_t \rangle &= \langle \xi_0, f_0 \rangle + \int_0^t \int_{\mathcal{X}} \left[v(a) \nabla_a f_s(x, a) + \frac{df_s}{ds}(x, a) \right. \\ &+ \left. \int_{\mathbb{R}^2} f_s(x+z, a_0) b(x, \xi_s) \cdot h(x, z, \xi_s) d(dz) - f_s(x, a) m(a) \right] \xi_s(dx, da) ds. \end{aligned}$$

The techniques used for this result are those developed in [Tran, 2006].

Law of the output, with extinction

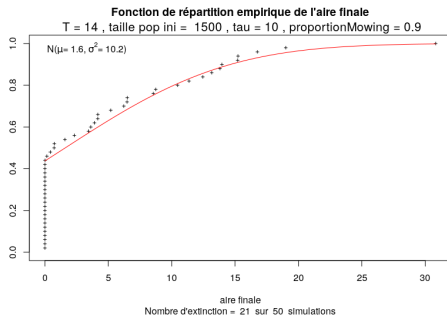


Figure: Empirical Cumulative Distribution Function of a Gaussien law with empirical mean and variance of the final area, for an initial population size = 1500, $\tau = 10$, *proportionMowing* = 0.9, et $T = 14$. There are 21 extinctions (over 50 simulations).

Method to assess formulas

Sobol sampling of 5000 points with $\tau \in [0; 15.0]$, $T \in [0; 20]$, and $initialPopSize \in [100; 1500]$.

	Mean Area	Mean Size
Regression tool	<i>lm</i>	<i>nls</i>
Correlation	> 0.99	> 0.99
Residual standard error	2.23	26.12
95 % confident interval	$c \in [-0.0342; -0.0336]$	$a \in [0.90; 0.94]$
	$d \in [0.0960; 0.0998]$	$b \in [20.46; 20.77]$



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