

Ordinary Differential Equations

as an alternative to agent-based modelling

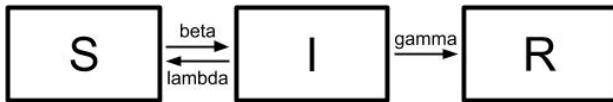
eX Modelo school

OpenMOLE

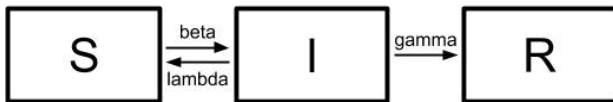
June 26, 2019

ODE systems

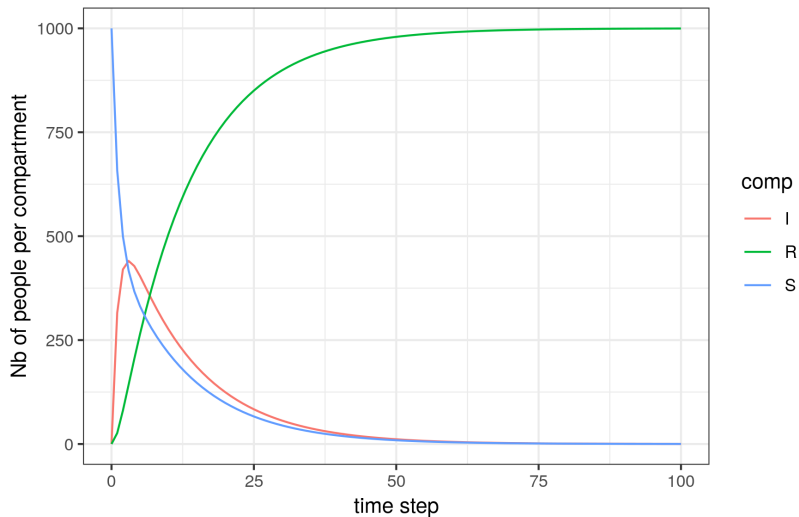
→ widely used to model transmission phenomena



- ▶ population split into compartments
- ▶ system of ordinary differential equations



$$\left\{ \begin{array}{l} \frac{dS}{dt} = -\beta S + \lambda I \\ \frac{dI}{dt} = \beta S - (\lambda + \gamma) I \\ \frac{dR}{dt} = \gamma I \end{array} \right.$$



ODE

Equation-based

Generic mechanisms

Population scale

Needs less resources

ABM

Individual-based

Precise mechanisms

Individual scale

Computationally expensive

A Zombie situation

How could we model the Zombie invasion?

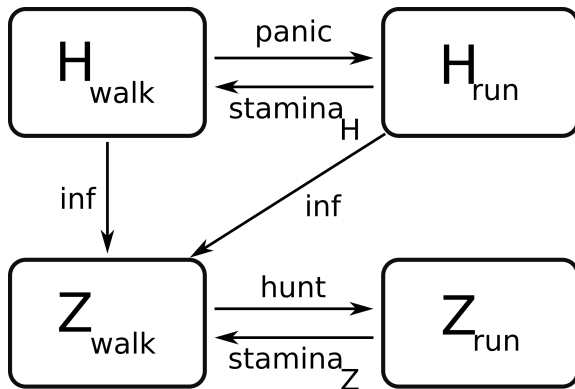
- ▶ Which mechanisms?
- ▶ Which parameters?

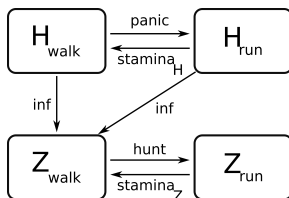
How could we model the Zombie invasion?

- ▶ Which mechanisms?
- ▶ Which parameters?

How can we assess our model's ability to reproduce the real data?

- ▶ Which metrics?
- ▶ Which fitness function?





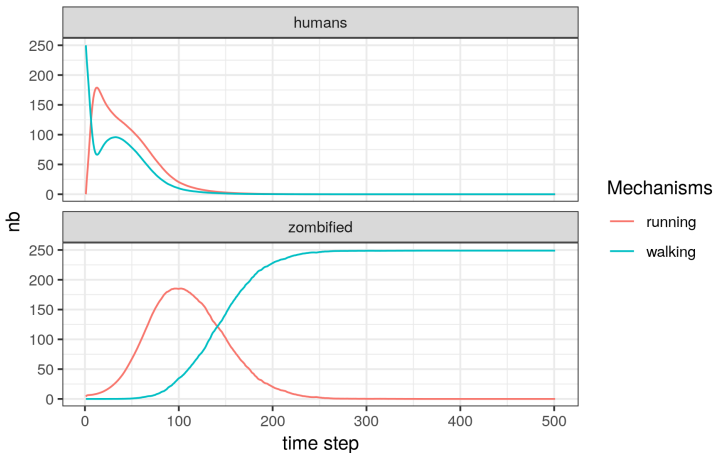
$$\begin{cases}
 \frac{dH_{walk}}{dt} = -(panic + inf) * H_{walk} + exhaustH * H_{run} \\
 \frac{dH_{run}}{dt} = panic * H_{walk} - (exhaustH + inf) * H_{run} \\
 \frac{dZ_{walk}}{dt} = inf * (H_{walk} + H_{run}) - hunt * Z_{walk} + exhaustZ * Z_{run} \\
 \frac{dZ_{run}}{dt} = hunt * Z_{walk} - exhaustZ * Z_{run}
 \end{cases}$$

$$\left\{ \begin{array}{lcl} N & = & H_{walk} + H_{run} + Z_{walk} + Z_{run} \\ panic & = & panic_0 * (Z_{walk} + Z_{run}) / N \\ hunt & = & hunt_0 * (H_{walk} + H_{run}) / N \end{array} \right.$$

Exploration

We have some real time series of zombie invasion
→ find the parameter values to best fit them

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→ find the parameter values to best fit them





Process

Process

- ▶ Embed the model in OpenMOLE

Process

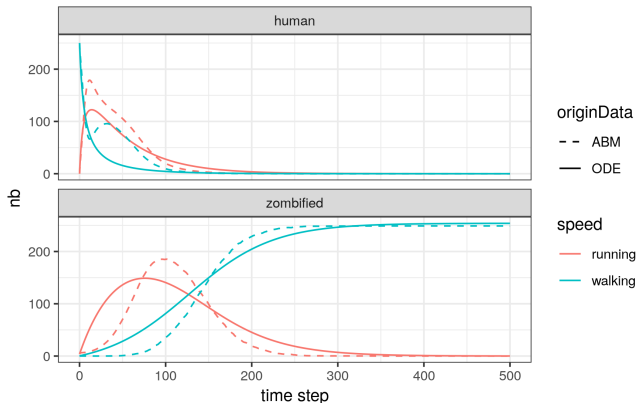
- ▶ Embed the model in OpenMOLE
- ▶ Define a fitness function

Process

- ▶ Embed the model in OpenMOLE
- ▶ Define a fitness function
- ▶ Write a calibration task

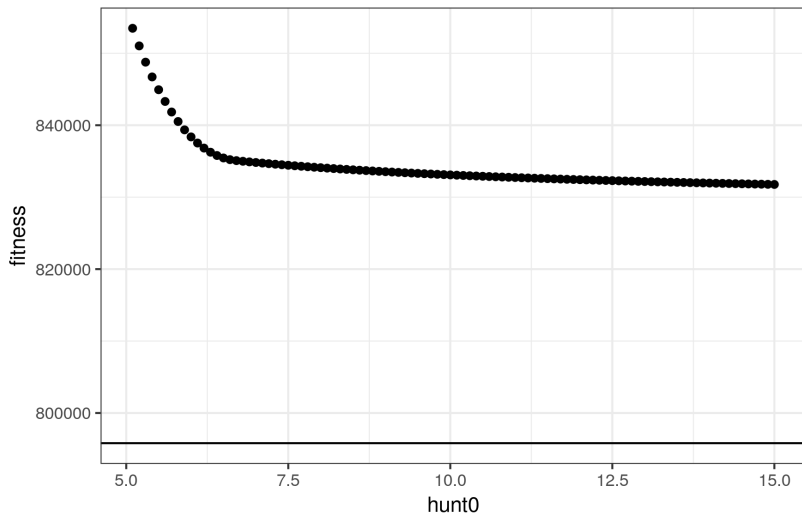
Parameter set

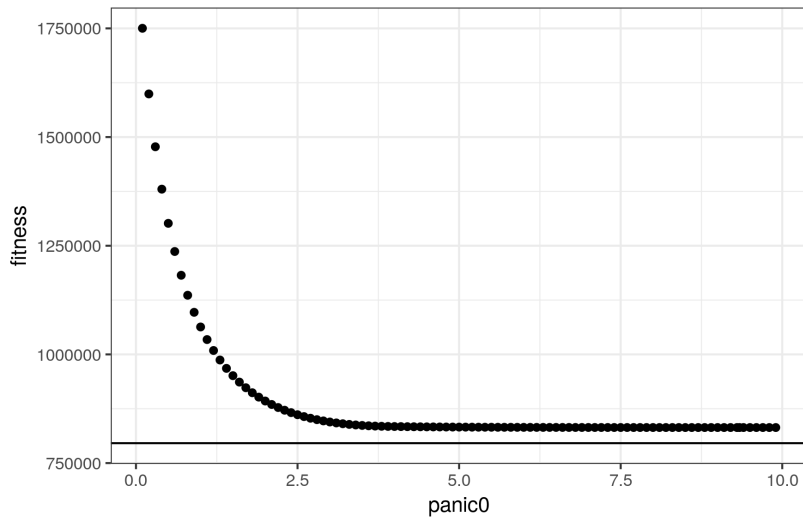
$panic_0$	7.25
$stamina_H$	0.99
$hunt_0$	10.15
$stamina_Z$	1.28
inf	0.02
fitness	780 394

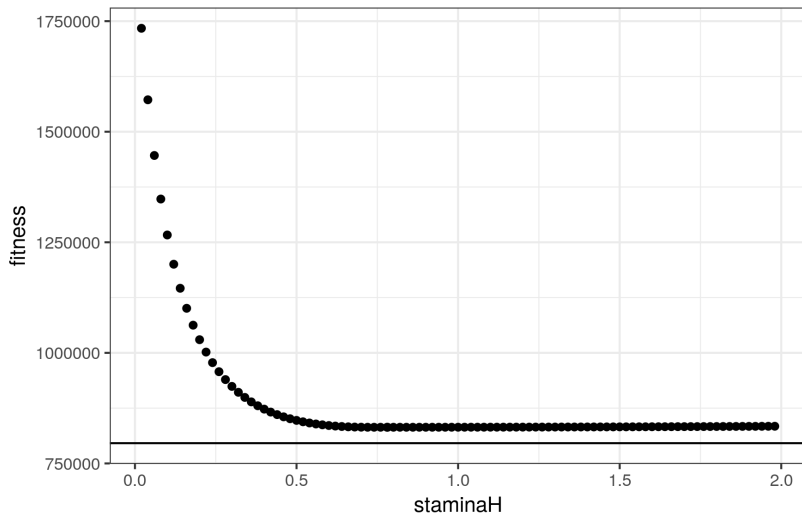


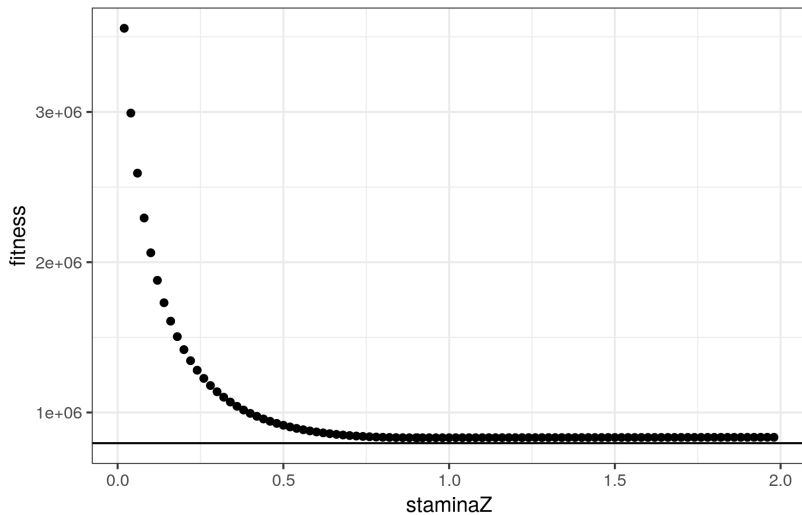
Profiles for each of the 5 parameters:

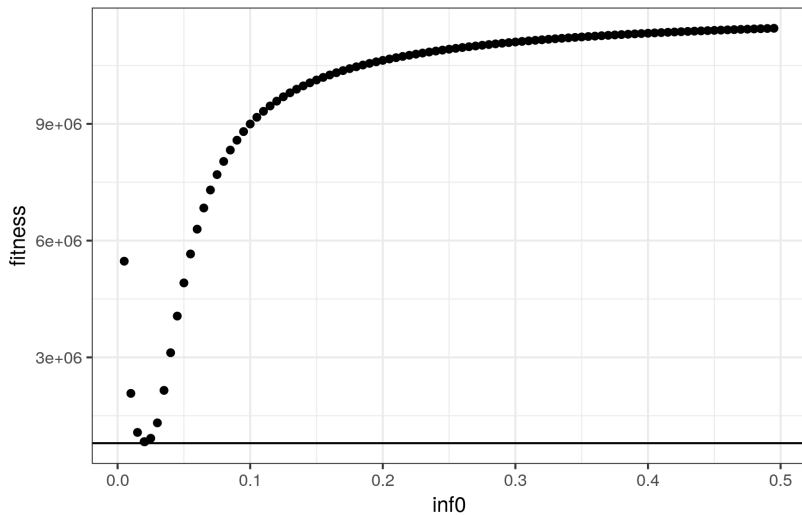
rightarrow $panic_0$, $stamina_H$, $hunt_0$, $stamina_Z$, inf_0











Adding complexity

What mechanisms could we add to better represent the complexity of our Zombie situation?

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The parcimony issue

- ▶ Do the new mechanisms really improve the fitness?
- ▶ Do we need them all?
- ▶ What are the best combinations?

Process

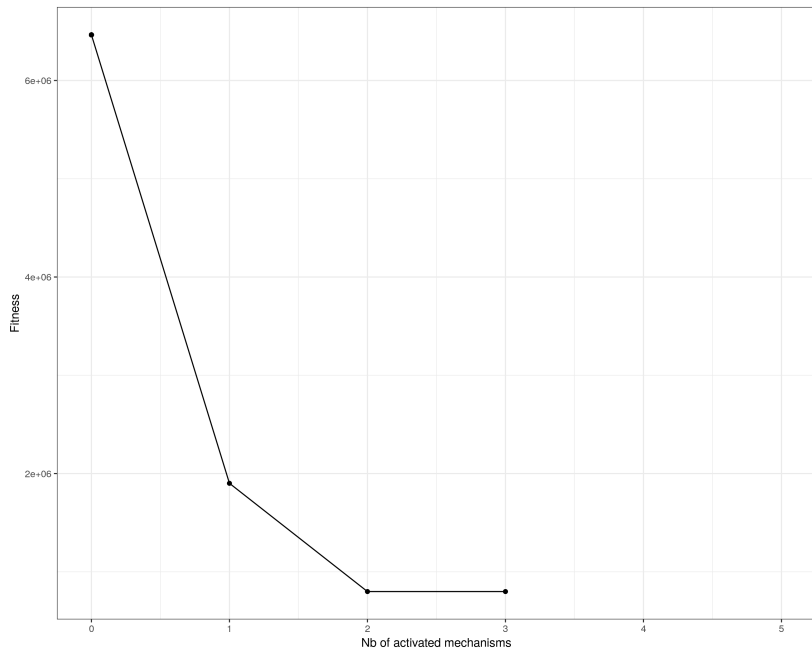
- ▶ Embed the model in OpenMOLE **DONE**

Process

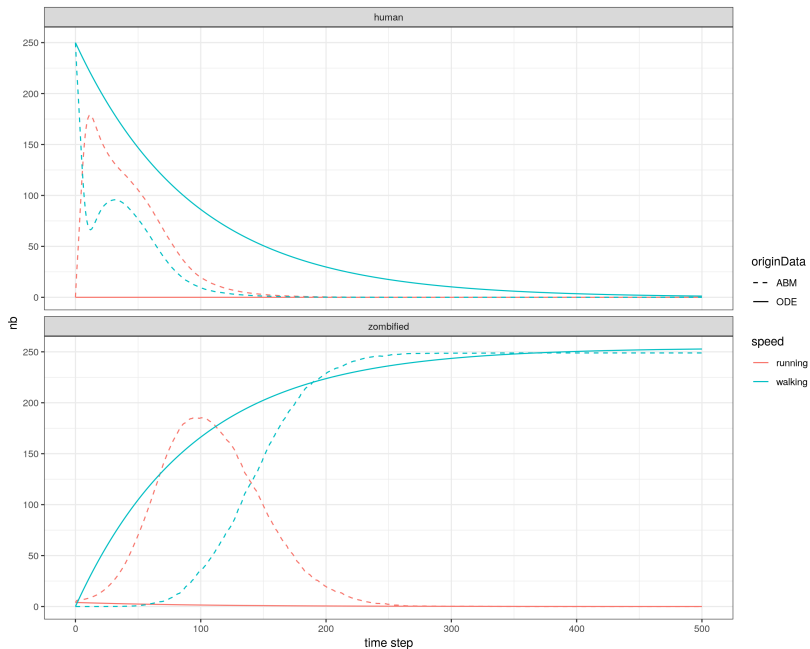
- ▶ Embed the model in OpenMOLE **DONE**
- ▶ Define a **second** fitness function

Process

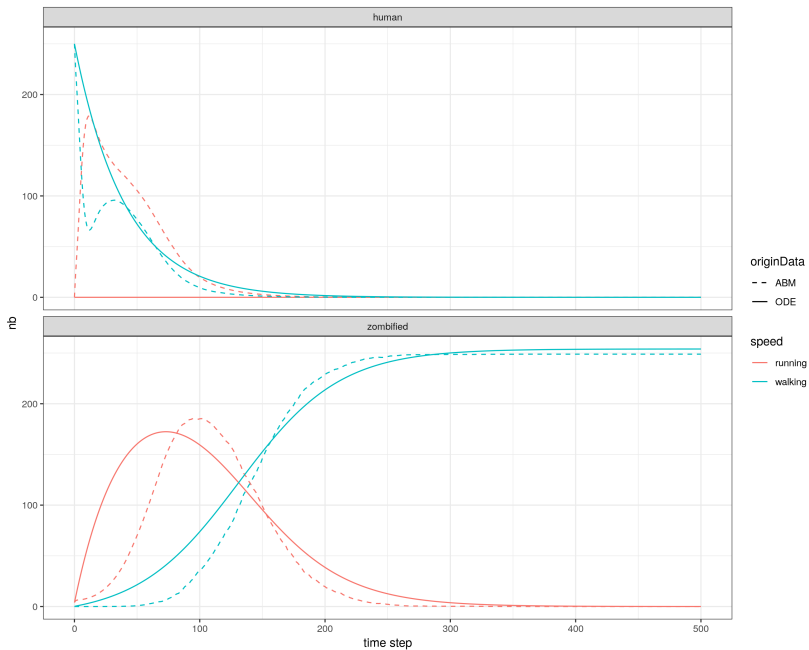
- ▶ Embed the model in OpenMOLE **DONE**
- ▶ Define a **second** fitness function
- ▶ **Modify** the calibration task



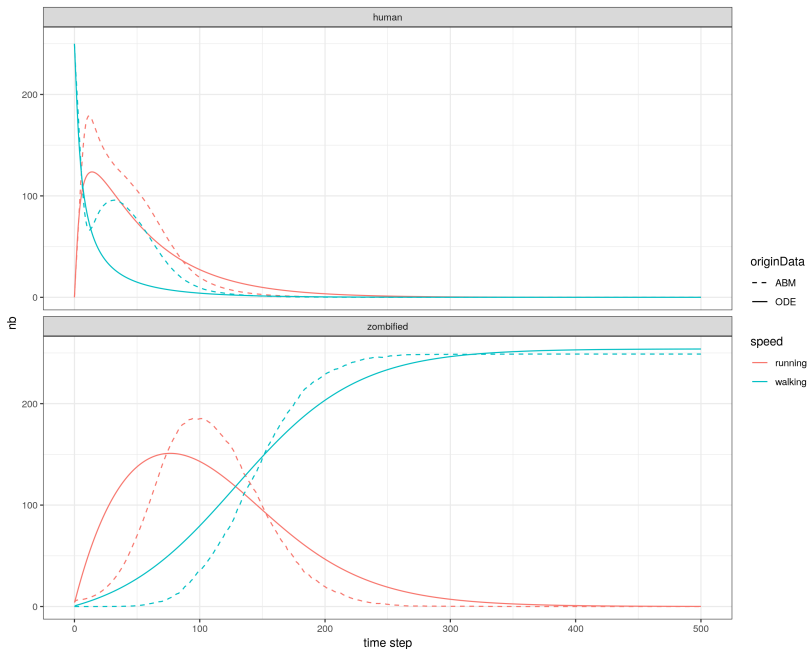
Dynamics for 0 mechanism activated



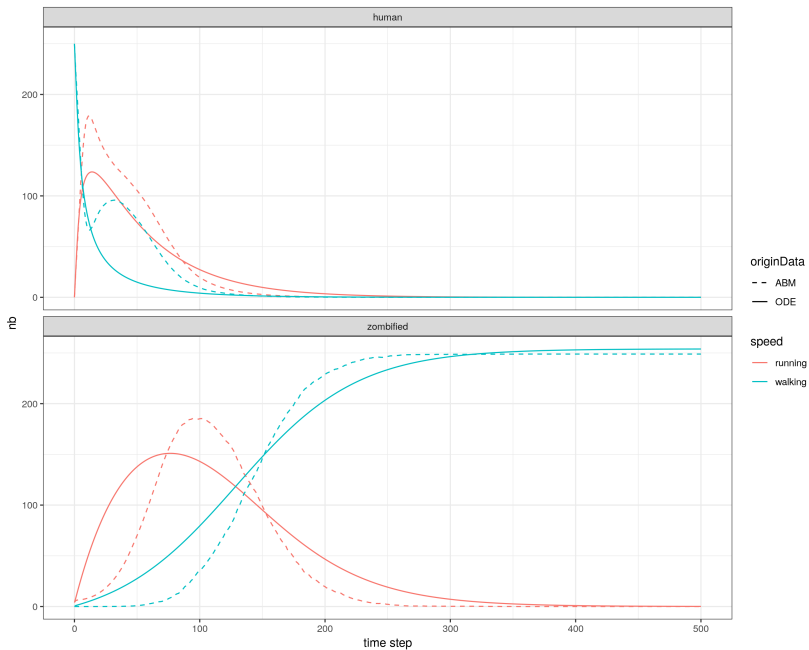
Dynamics for 1 mechanism activated



Dynamics for 2 mechanisms activated



Dynamics for 3 mechanisms activated



Some mathematics

Recall the ODE system:

$$\left\{ \begin{array}{lcl} \frac{dH_{walk}}{dt} & = & -(panic0 * \frac{Z_{walk} + Z_{run}}{N} + inf) * H_{walk} + exhaustH * H_{run} \\ \frac{dH_{run}}{dt} & = & panic0 * \frac{Z_{walk} + Z_{run}}{N} * H_{walk} - (exhaustH + inf) * H_{run} \\ \frac{dZ_{walk}}{dt} & = & inf * (H_{walk} + H_{run}) - hunt0 * \frac{H_{walk} + H_{run}}{N} * Z_{walk} + exhaustZ * Z_{run} \\ \frac{dZ_{run}}{dt} & = & hunt0 * \frac{H_{walk} + H_{run}}{N} * Z_{walk} - exhaustZ * Z_{run} \end{array} \right.$$

Where $N := H_{walk} + H_{run} + Z_{walk} + Z_{run}$

First-order **nonlinear** (autonomous) ordinary differential equation... a priori no explicit solution, hence numerical solutions.

Summing the 4 equations, we have:

$$\frac{dN}{dt} = \frac{dH_{walk}}{dt} + \frac{dH_{run}}{dt} + \frac{dZ_{walk}}{dt} + \frac{dZ_{run}}{dt} = 0$$

So $N(t)$ is constant:

$$N(t) = N(0), \text{ for all } t.$$

We find back that the population size (human + zombies) is constant: natural !

Let's note $H := H_{walk} + H_{run}$

Summing the first equations, we have:

$$\frac{dH}{dt} = \frac{dH_{walk}}{dt} + \frac{dH_{run}}{dt} = -inf * (H_{walk}(t) + H_{run}(t)) = -inf * H(t)$$

First-order linear ordinary differential equation with constant coefficient: explicit solution !

$$H(t) = H(0) * e^{-inf * t}$$

Likewise,

$$Z(t) := Z_{walk} + Z_{run} = N - H(t)$$

Definition: The point $x \in \mathbb{R}^4$ is an *equilibrium point* for the differential equation $X' = F(X)$ if $F(X) = 0$.

For example, points of the form $(0, 0, 0, N)$ are equilibrium points. They correspond to a population composed of walking zombies.

Theses 2 facts :

- ▶ $N(t)$ constant
- ▶ solutions for H and Z
... are still valid for the system with additional mechanism.

→ Not able to change the shape of the solutions of the equation of H and Z with such system modifications.