Ordinary Differential Equations

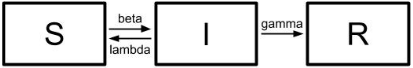
as an alternative to agent-based modelling

Module

June 20, 2019

ODE systems

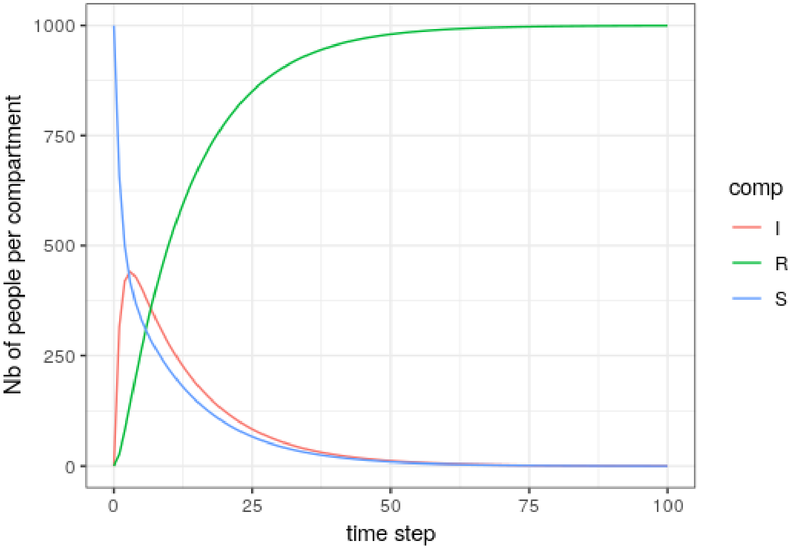
→ widely used to model transmission phenomena



- population split into compartments
- system of ordinary differential equations

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}t} = -\beta S + \lambda I \\ \dfrac{\mathrm{d}I}{\mathrm{d}t} = \beta S - (\lambda + \gamma) I \\ \dfrac{\mathrm{d}R}{\mathrm{d}t} = \gamma I \end{cases}$$

SIR model dynamics



ODE vs ABM

ODE	ABM
Equation-based	Individual-based
Generic mechanisms	Precise mechanisms
Population scale	Individual scale
Needs less resources	Computationally expensive

A Zombie situation

How could we model the Zombie invasion?

- Which mechanisms?
- Which parameters?

OpenMOLE

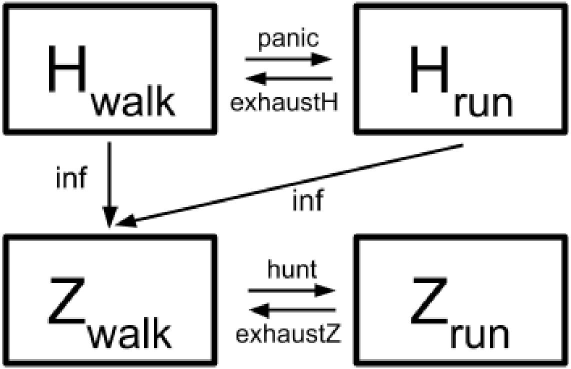
How could we model the Zombie invasion?

- Which mechanisms?
- Which parameters?

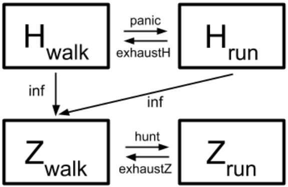
How can we assess our model's ability to reproduce the real data?

- Which metrics?
- Which fitness function?

A very simple ODE model



A very simple ODE model



$$
\begin{cases}
\dfrac{\mathrm{d}H_{walk}}{\mathrm{d}t} = -(panic + inf) * H_{walk} + exhaustH * H_{run} \\
\dfrac{\mathrm{d}H_{run}}{\mathrm{d}t} = panic * H_{walk} - (exhaustH + inf) * H_{run} \\
\dfrac{\mathrm{d}Z_{walk}}{\mathrm{d}t} = inf * (H_{walk} + H_{run}) - hunt * Z_{walk} + exhaustZ * Z_{run} \\
\dfrac{\mathrm{d}Z_{run}}{\mathrm{d}t} = hunt * Z_{walk} - exhaustZ * Z_{run}
\end{cases}
$$

Exploration

First step: Calibrate

We have some real time series of zombie invasion
$\rightarrow$ find the parameter values to best fit them

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- Embed the model in OpenMOLE
- Define a fitness function

First step: Calibrate

We have some real time series of zombie invasion
$\rightarrow$ find the parameter values to best fit them

Process

- Embed the model in OpenMOLE
- Define a fitness function
- Write a calibration task

Parameter set

Second step: Profiles

Adding complexity

What mechanisms could we add to better represent the complexity of our Zombie situation?

Study our model's parcimony

Recall the ODE system:

$$\begin{cases} \dfrac{\mathrm{d}H_{walk}}{\mathrm{d}t} & = & -(panic0 * \dfrac{Z_{walk} + Z_{run}}{N} + inf) * H_{walk} + exhaustH * H_{run} \\ \dfrac{\mathrm{d}H_{run}}{\mathrm{d}t} & = & panic0 * \dfrac{Z_{walk} + Z_{run}}{N} * H_{walk} - (exhaustH + inf) * H_{run} \\ \dfrac{\mathrm{d}Z_{walk}}{\mathrm{d}t} & = & inf * (H_{walk} + H_{run}) - hunt0 * \dfrac{H_{walk} + H_{run}}{N} * Z_{walk} + exhaustZ * Z_{run} \\ \dfrac{\mathrm{d}Z_{run}}{\mathrm{d}t} & = & hunt0 * \dfrac{H_{walk} + H_{run}}{N} * Z_{walk} - exhaustZ * Z_{run} \end{cases}$$

Where $N := H_{walk} + H_{run} + Z_{walk} + Z_{run}$

First-order **nonlinear** (autonomous) ordinary differential equation... a priori no explicit solution, hence numerical solutions.

Let's note $X(t) := (H_{walk}(t), H_{run}(t), Z_{walk}(t), Z_{run}(t))$
We have the following Cauchy problem:

$$\begin{cases} X'(t) & = & F(X(t)) \\ X(0) & = & (x_0, y_0, z_0, w_0) \quad \text{initial condition} \end{cases}$$

where

$$(x_0, y_0, z_0, w_0) \in \mathbb{R}_+^*$$

and

$$\begin{array}{rccl} F & : & \mathbb{R}^4 & \rightarrow & \mathbb{R}^4 \\ & & (x, y, z, w) & \mapsto & F(x, y, z, w) := ((panic0 * \dfrac{z + w}{x + y + z + w} + inf) * x + exhaustH * y, \\ & & & & \ldots, \ldots, \ldots) \end{array}$$

Cauchy-Lipschitz theorem : Existence and uniqueness to a solution of the Cauchy problem.

Total population size

Summing the 4 equations, we have:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\mathrm{d}H_{walk}}{\mathrm{d}t} + \frac{\mathrm{d}H_{run}}{\mathrm{d}t} + \frac{\mathrm{d}Z_{walk}}{\mathrm{d}t} + \frac{\mathrm{d}Z_{run}}{\mathrm{d}t} = 0$$

So $N(t)$ is constant:

$$N(t) = N(0), \text{ for all } t.$$

We find back that the population size (human + zombies) is constant : natural !

Let's note $H := H_{walk} + H_{run}$
Summing the first equations, we have:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}H_{walk}}{\mathrm{d}t} + \frac{\mathrm{d}H_{run}}{\mathrm{d}t} = -inf(H_{walk}(t) + H_{run}(t)) = -infH(t)$$

First-order linear ordinary differential equation with constant coefficient: explicit solution !

$$H(t) = H(0) * e^{-inf*t}$$

Likewise,

$$Z(t) := Z_{walk} + Z_{run} = N - H(t)$$

Equilibrium points

Definition: The point $x \in \mathbb{R}^4$ is an equilibrium point for the differential equation $X' = F(X)$ if $F(X) = 0$.

Here, there is a unique equilibrium point: $(0, 0, 0, N(0))$: all the population is composed of walking zombies.

Theses 2 facts : $N(t)$ constant and solutions for H and Z are still valid for the system with additional mechanism.
Not able to change the shape of the solutions of the equation of H and Z with such system modifications.