

Day 3, Practical 2, Hely's solution

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1 Simulation setting 1

Task 1.

```
library(data.table)
sim.fun <- function(n) {

  # baseline covariates
  X0.1 <- runif(n, -2, 2)
  X0.2 <- rnorm(n)
  X0.3 <- rbinom(n, 1, 0.2)

  # baseline treatment (randomized)
  A0 <- rbinom(n, 1, 0.5)

  # follow-up covariates
  X1.1 <- rbinom(n, 1, plogis(-0.7 + 0.3*X0.3 + 0.8*A0))
  X1.2 <- rbinom(n, 1, plogis(0.25 - 0.55*X0.3))

  # follow-up treatment
  A1 <- rbinom(n, 1, prob=plogis(0.9 - 5*(1-A0) - 4.7*X1.1 - 4.8*X1.2))

  # outcome
  Y <- rbinom(n, 1, prob=plogis(-0.9 - 0.2*A0 + 1.2*X1.1 - 0.1*A1 - 0.8*A1*(X1
    .1=0)))

  return(data.table(X0.1=X0.1, X0.2=X0.2, X0.3=X0.3,
    A0=A0,
    X1.1=X1.1, X1.2=X1.2,
    A1=A1,
    Y=Y))
}
```

Task 2:

```
sim.fun <- function(n, intervene=list()) {

  # baseline covariates
  X0.1 <- runif(n, -2, 2)
  X0.2 <- rnorm(n)
  X0.3 <- rbinom(n, 1, 0.2)
```

```

# baseline treatment (randomized)
if ("A0" %in% names(intervene)) {
  A0 <- intervene$A0
} else {
  A0 <- rbinom(n, 1, 0.5)
}

# follow-up covariates
X1.1 <- rbinom(n, 1, plogis(-0.7 + 0.3*X0.3 + 0.8*A0))
X1.2 <- rbinom(n, 1, plogis(0.25 - 0.55*X0.3))

# follow-up treatment
if ("A1" %in% names(intervene)) {
  A1 <- intervene$A1(X1.1)
} else {
  A1 <- rbinom(n, 1, prob=plogis(0.9 - 5*(1-A0) - 4.7*X1.1 - 4.8*X1.2))
}

# outcome
Y <- rbinom(n, 1, prob=plogis(-0.9 - 0.2*A0 + 1.2*X1.1 - 0.1*A1 - 0.8*A1*(X1.1==0)))

if (length(names(intervene))>0) {
  return(mean(Y))
} else {
  return(data.table(X0.1=X0.1, X0.2=X0.2, X0.3=X0.3,
                    A0=A0,
                    X1.1=X1.1, X1.2=X1.2,
                    A1=A1,
                    Y=Y))
}
}

```

```

set.seed(12)
ate.itt <- sim.fun(intervene=list(A0=1), n=1e6) -
  sim.fun(intervene=list(A0=0), n=1e6)
ate.static <- sim.fun(intervene=list(A0=1, A1=function(X1.1) 1), n=1e6) -
  sim.fun(intervene=list(A0=0, A1=function(X1.1) 0), n=1e6)
ate.dynamic <- sim.fun(intervene=list(A0=1, A1=function(X1.1) 1*(X1.1==0)), n=1e6) -
  sim.fun(intervene=list(A0=0, A1=function(X1.1) 0), n=1e6)
message(paste0("ITT:      ", ate.itt))
message(paste0("static:   ", ate.static))
message(paste0("dynamic:  ", ate.dynamic))

```

```

ITT:      -0.00931700000000002
static:   -0.063294
dynamic:  -0.050709

```

Task 3.

```

set.seed(15)
(sim.data <- sim.fun(1000))

```

	X0.1	X0.2	X0.3	A0	X1.1	X1.2	A1	Y
1:	0.40845618	-0.19620228	0	1	1	0	0	1
2:	-1.21982429	0.59503302	1	1	0	0	1	0
3:	1.86583493	-1.60888231	0	1	0	0	0	0
4:	0.60362212	0.04123507	0	1	0	0	1	0
5:	-0.53171243	-1.25139144	0	0	0	1	0	0

1996:	-0.72294993	-0.43541869	0	1	0	0	0	1
1997:	-0.13592934	-0.80204340	0	0	0	0	0	0
1998:	-1.67193359	-1.57095686	0	0	0	1	0	0
1999:	0.68115948	-0.72905589	0	1	0	0	1	0
2000:	0.04187752	0.10649384	0	0	0	0	0	1

```
##-- naive approach 1:
fit.glm1 <- glm(Y ~ A1 + X1.1 + X1.2 + A0 + X0.1 + X0.2 + X0.3,
               family=binomial, data=sim.data)
sim.data[, EY1:=predict(fit.glm1, type="response", newdata=copy(sim.data)[, ':(A0=1,
A1=1)])]
sim.data[, EY0:=predict(fit.glm1, type="response", newdata=copy(sim.data)[, ':(A0=0,
A1=0)])]
message("naive est1 = ", sim.data[, mean(EY1-EY0)])
##-- naive approach 2:
fit.glm2 <- glm(Y ~ A1 + A0 + X0.1 + X0.2 + X0.3,
               family=binomial, data=sim.data)
sim.data[, EY1:=predict(fit.glm2, type="response", newdata=copy(sim.data)[, ':(A0=1,
A1=1)])]
sim.data[, EY0:=predict(fit.glm2, type="response", newdata=copy(sim.data)[, ':(A0=0,
A1=0)])]
message("naive est2 = ", sim.data[, mean(EY1-EY0)])
```

```
naive est1 = -0.181898731224723
naive est2 = -0.241852261901675
```

Task 4. See Figure 1.

```
naive.est1 <- list()
naive.est2 <- list()

for (m in 1:500) {

  set.seed(15+m)
  sim.data <- sim.fun(n=1000)

  ##-- naive approach 1:
  fit.glm1 <- glm(Y ~ A1 + X1.1 + X1.2 + A0 + X0.1 + X0.2 + X0.3,
                 family=binomial, data=sim.data)
  sim.data[, EY1:=predict(fit.glm1, type="response", newdata=copy(sim.data)[, ':(A0=1,
A1=1)])]
  sim.data[, EY0:=predict(fit.glm1, type="response", newdata=copy(sim.data)[, ':(A0=0,
A1=0)])]
  naive.est1[[m]] <- sim.data[, mean(EY1-EY0)]

  ##-- naive approach 2:
```

```

fit.glm2 <- glm(Y ~ A1 + A0 + X0.1 + X0.2 + X0.3,
               family=binomial, data=sim.data)
sim.data[, EY1:=predict(fit.glm2, type="response", newdata=copy(sim.data)[, ':(
A0=1, A1=1)])]
sim.data[, EY0:=predict(fit.glm2, type="response", newdata=copy(sim.data)[, ':(
A0=0, A1=0)])]
naive.est2[[m]] <- sim.data[, mean(EY1-EY0)]
}

```

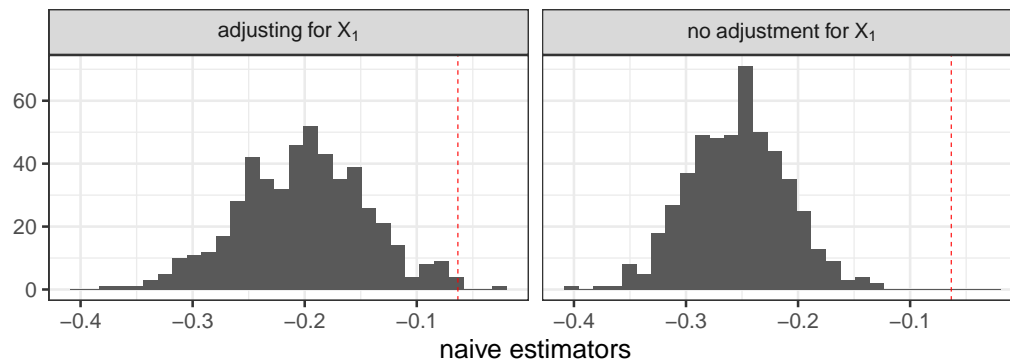


Figure 1

2 Simulation setting 2

Task 5.

```

library(data.table)
sim.fun2 <- function(n=1e6) {
  U <- rbinom(n, 1, prob=0.5)
  A <- rbinom(n, 1, prob=0.5)
  D2 <- D1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
  D2[D1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U)) [D1==0]
  return(data.table(A=A,D1=D1,D2=D2))
}

```

Task 6.

```

set.seed(100)
head(dat2 <- sim.fun2())

```

```

  A D1 D2
1: 1  1  1
2: 1  0  1
3: 0  1  1
4: 0  1  1
5: 1  1  1
6: 1  1  1
6.1:

```

```
summary(fit.D1 <- glm(D1~A, data=dat2, family=binomial))$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.3346503	0.002869386	116.6279	0
A	-0.9172592	0.004113874	-222.9672	0

(We may note that the coefficient in front of A corresponds to:)

```
p.A0 <- 0.5*(plogis(1.3-1.8)+plogis(1.3))
p.A1 <- 0.5*(plogis(1.3-1.8-1.1)+plogis(1.3-1.1))
log((p.A1/(1-p.A1)) / (p.A0/(1-p.A0)))
```

```
[1] -0.9098137
```

(And the intercept coefficient to:)

```
qlogis(p.A0)
```

```
[1] 0.3297059
```

6.2:

```
summary(fit.D2 <- glm(D2~A, data=dat2[D1==0,], family=binomial))$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-0.6884959	0.004631504	-148.65492	0
A	0.2961096	0.005865281	50.48515	0

6.3:

```
dat2[, id:=1:N]
dat21 <- copy(dat2)[, k:=1]
dat21$D <- dat21$D1
dat22 <- dat2[dat2$D1==0,][, k:=2]
dat22$D <- dat22$D2
head(dat2.stacked <- rbind(dat21, dat22)[, t:=factor(k)][order(id)])
```

	A	D1	D2	id	k	D	t
1:	0	1	1	1	1	1	1
2:	1	0	1	2	1	0	1
3:	1	1	1	3	1	1	1
4:	0	0	0	4	1	0	1
5:	1	0	1	5	1	0	1
6:	1	0	1	6	1	0	1

6.4:

```
summary(fit.D <- glm(D~A+t, data=dat2.stacked, family=binomial))$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.1322593	0.002595677	50.95367	0
A	-0.5117185	0.003311977	-154.50546	0
t2	-0.3365993	0.003509668	-95.90632	0

```
summary(fit.D <- glm(D~A*t, data=dat2.stacked, family=binomial))$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.3271974	0.002866385	114.1498	0
A	-0.9103901	0.004112876	-221.3512	0
t2	-1.0156933	0.005446742	-186.4772	0
A:t2	1.2064997	0.007163607	168.4207	0

6.5:

```
set.seed(10)
U <- rbinom(n, 1, prob=0.5)
A <- 1
D2.1 <- D1.1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
D2.1[D1.1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1.1==0]
A <- 0
D2.0 <- D1.0 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.0*A))
D2.0[D1.0==0] <- rbinom(n, 1, prob=plogis(2.0-3.9*U))[D1.0==0]
message(paste0("Counterfactual risk difference: ", mean(D2.1-D2.0)))
```

Counterfactual risk difference: -0.106

```
set.seed(10)
U <- rbinom(n, 1, prob=0.5)
A <- 1
D2.1 <- D1.1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
D2.1[D1.1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1.1==0]
A <- 0
D2.0 <- D1.0 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.0*A))
D2.0[D1.0==0] <- rbinom(n, 1, prob=plogis(2.0-3.9*U))[D1.0==0]
message(paste0("Counterfactual hazard ratio at 2nd time-point: ", mean(D2.1[D1.1==0])/
  mean(D2.0[D1.0==0])))
```

Counterfactual hazard ratio at 2nd time-point: 1.21615895286972

2.1 Treatment switching

Task 7.

```
library(data.table)
sim.fun2 <- function(n=1e6) {
  U <- rbinom(n, 1, prob=0.5)
  A1 <- A <- rbinom(n, 1, prob=0.5)
  D2 <- D1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
  D2[D1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1==0]
  A1[D1==0] <- rbinom(n, 1, prob=plogis(1.2+0.5*A))[D1==0]
  return(data.table(A=A, Aswitch=1*(A1!=A), D1=D1, D2=D2))
}
```

Task 8.

```
set.seed(100)
head(dat2 <- sim.fun2())
```

```
      A Aswitch D1 D2
1: 1      0  1  1
2: 1      1  0  1
3: 0      0  1  1
4: 0      0  1  1
5: 1      0  1  1
6: 1      0  1  1
```

```
summary(glm(D2~A+Aswitch, family=binomial, data=dat2[A==1]))$coefficients
```

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.5770818	0.003103637	185.93729	0
Aswitch	-0.9519854	0.009647494	-98.67696	0

Task 9.

```
A1 <- A <- rep(1, n)
D2.11 <- D1.11 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
D2.11[D1.11==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1.11==0]
A1[D1.11==0] <- 1

A1 <- A <- rep(1, n)
D2.10 <- D1.10 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
D2.10[D1.10==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1.10==0]
A1[D1.10==0] <- 0

mean(D2.11 - D2.10)
```

```
[1] -0.000303
```

2.2 Right-censoring

Task 10.

```
library(data.table)
sim.fun2 <- function(n=1e6) {
  U <- rbinom(n, 1, prob=0.5)
  A <- rbinom(n, 1, prob=0.5)
  D2 <- D1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
  C1 <- rbinom(n, 1, prob=plogis(1.3))
  D2[D1==0 & C1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1==0 & C1==0]
  return(data.table(A=A,D1=D1,C1=C1,D2=D2))
}
```

Task 11.

```
set.seed(100)
head(dat2 <- sim.fun2())
```

```

  A D1 C1 D2
1: 1  1  1  1
2: 1  0  0  0
3: 0  1  1  1
4: 0  1  1  1
5: 1  1  0  1
6: 1  1  1  1

```

11.1:

```
summary(fit.D1 <- glm(D1~A, data=dat2, family=binomial))$coefficients
```

```

              Estimate Std. Error   z value Pr(>|z|)
(Intercept)  0.3346503  0.002869386  116.6279      0
A            -0.9172592  0.004113874  -222.9672      0

```

11.2:

```
summary(fit.D2 <- glm(D2~A, data=dat2[D1==0 & C1==0,], family=binomial))$coefficients
```

```

              Estimate Std. Error   z value   Pr(>|z|)
(Intercept) -0.6803444  0.01002981 -67.83221 0.000000e+00
A             0.2937851  0.01269486  23.14206 1.747762e-118

```

11.3:

```

dat2[, id:=1:.N]
dat21 <- copy(dat2)[, k:=1]
dat21$D <- dat21$D1
dat22 <- dat2[dat2$D1==0 & dat2$C1==0,][, k:=2]
dat22$D <- dat22$D2
head(dat2.stacked <- rbind(dat21, dat22)[, t:=factor(k)][order(id)])

```

```

  A D1 C1 D2 id k D t
1: 1  1  1  1  1 1 1 1
2: 1  0  0  0  2 1 0 1
3: 1  0  0  0  2 2 0 2
4: 0  1  1  1  3 1 1 1
5: 0  1  1  1  4 1 1 1
6: 1  1  0  1  5 1 1 1

```

11.4:

```
summary(fit.D <- glm(D~A+t, data=dat2.stacked, family=binomial))$coefficients
```

```

              Estimate Std. Error   z value Pr(>|z|)
(Intercept)  0.2777191  0.002789808   99.54772      0
A            -0.8004605  0.003888827 -205.83597      0
t2           -0.3092774  0.006580645  -46.99805      0

```

```
summary(fit.D <- glm(D~A*t, data=dat2.stacked, family=binomial))$coefficients
```


	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.3346503	0.002869386	116.62786	0
A	-0.9172592	0.004113874	-222.96724	0
t2	-1.0149947	0.010432186	-97.29454	0
A:t2	1.2110443	0.013344786	90.75037	0

11.5:

```
set.seed(10)
U <- rbinom(n, 1, prob=0.5)
A <- 1
D2.1 <- D1.1 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.1*A))
D2.1[D1.1==0] <- rbinom(n, 1, prob=plogis(2.1-3.9*U))[D1.1==0]
A <- 0
D2.0 <- D1.0 <- rbinom(n, 1, prob=plogis(1.3-1.8*U-1.0*A))
D2.0[D1.0==0] <- rbinom(n, 1, prob=plogis(2.0-3.9*U))[D1.0==0]
mean(D2.1-D2.0)
```

[1] -0.106