Day 2, Lecture 2

A tiny overview

Summary of TMLE

$$\Psi(\hat{P}_n) - \Psi(P_0) = \mathbb{P}_n \phi^*(P_0) + o_P(n^{-1/2})$$
$$+ R(\hat{P}_n, P_0)$$
$$- \mathbb{P}_n \phi^*(\hat{P}_n)$$

- The role of the targeting step (Step 2):
  - Gaining double robustness in consistency.
  - Easier to get rid of second-order remainder.
- The role of the initial estimation step (Step 1):
  - This should be done well enough to get rid of the second-order remainder.

Summary of TMLE

$$\tilde{\Psi}(\hat{f}_n^*) - \tilde{\Psi}(f_0) = \mathbb{P}_n \tilde{\phi}^*(f_0, \pi_0) + o_P(n^{-1/2})$$

$$+ \ \tilde{R}(\hat{f}_n^*, \hat{\pi}_n, f_0, \pi_0) - \underbrace{\mathbb{P}_n \tilde{\phi}^*(\hat{f}_n^*, \hat{\pi}_n)}_{\text{=0, by targeting.}}$$

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For the ATE, when $\mathbb{P}_n\tilde{\phi}^*(\hat{f}_n^*, \hat{\pi}_n) = 0$, recall that:

$$|\tilde{R}(\hat{f}_n^*, \hat{\pi}_n, f_0, \pi_0)| \leq \sum_{a=0,1} \delta^{-1} \|\pi_0(a \mid \cdot) - \hat{\pi}_n(a \mid \cdot)\|_{\mu_0} \|f_0(a \mid \cdot) - \hat{f}_n^*(a \mid \cdot)\|_{\mu_0}$$

What this tells us:

- Asymptotic linearity when π_0 and f_0 are estimated at rate at least $n^{-1/4}$.

Summary of TMLE

How can we perform estimation of π_0 and f_0 such as to achieve rate at least $n^{-1/4}$?

- Correctly specified parametric models
  - although consistency is guaranteed, inference cannot be based on the efficient influence curve when one is misspecified!
- There are no results on this being the case for generic implementations of, for example, random forests.
- Lasso, highly adaptive lasso (HAL), ...
- Loss-based "super learning"
  - oracle property: the super learner achieves the rate of convergence of the *best* estimator in its library.

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  - this is about minimizing expected loss; tuning is still important!

Summary of TMLE

$$f(A, X) = \mathbb{E}_P[Y \mid A, X]$$

A **loss function** $\mathscr{L}(f)(O)$ measuring the distance between an estimator f and the observed outcome Y, e.g., the negative log-likelihood:

$$\mathscr{L}(\hat{f}_n)(Y_i, A_i, X_i) = -(Y_i \log(\hat{f}_n(A_i, X_i)) + (1 - Y_i)\log(1 - \hat{f}_n(A_i, X_i))).$$

- The estimator $\hat{f}_n$ closest to the true f_0 minimizes the risk:

$$\mathbb{E}_{P_0}[\mathscr{L}(\hat{f}_n)(Y_i, A_i, X_i)].$$

- Loss-based super learning: Minimizing the cross-validated empirical risk with respect to the loss function $\mathscr{L}$ over the statistical model.

Summary of TMLE

This is all about constructing a good estimator for the conditional expectation f;

- does not necessarily yield a good estimator for the particular feature of interest, the target parameter.
- but, it needs to be done well enough to get rid of the second-order remainder.

This is Step 1.

Overview of TMLE

1. Scientific question $\Rightarrow$ causal parameter
2. Causal parameter $\Rightarrow$ statistical parameter
3. Statistical estimation problem = statistical parameter + statistical model
   - Efficient influence function
   - Second-order remainder
4. Identify relevant components that need targeting
   - Submodel + loss function
   - Targeting algorithm
5. Construct strong initial learners!!
   - *a priori* specified
6. Inference based on the efficient influence function

Overview of TMLE . . . from a more applied perspective.

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