# Hints for small exercise (Lecture 2, Day 2)

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First recall that

$$\operatorname{expit}(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}, \qquad \operatorname{logit}(x) = \log\left(\frac{x}{1-x}\right),$$

1. Show that $\operatorname{expit}(\operatorname{logit}(x)) = x$.
2. Show that $\operatorname{expit}(-\operatorname{logit}(x)) = 1 - x$.
3. Show that $\log(\operatorname{expit}(x)) = -\log(1 + e^{-x})$.
4. Show that $\log(1 - \operatorname{expit}(x)) = -\log(1 + e^{x})$.
5. Compute $\frac{d}{dx}\log(\operatorname{expit}(x))$ and $\frac{d}{dx}\log(1 - \operatorname{expit}(x))$.
6. Derive the derivatives with respect to $\varepsilon$ of the composite functions $\log(\operatorname{expit}(\operatorname{logit}(x) + \varepsilon h))$ and $\log(1 - \operatorname{expit}(\operatorname{logit}(x) + \varepsilon h))$.
7. Set $\varepsilon = 0$ in the expressions for the derivatives of 5.
8. Applying these steps to $\mathscr{L}\big(f_\varepsilon\big)$ now gives:

$$\begin{aligned}\frac{d}{d\varepsilon}\bigg|_{\varepsilon=0} \mathscr{L}\big(f_\varepsilon\big)(O) &= YH(A,X)(1 - f(A,X)) - (1-Y)H(A,X)f(A,X) \\ &= YH(A,X) - H(A,X)f(A,X) = H(A,X)\big(Y - f(A,X)\big).\end{aligned}$$