Day 2, Lecture 2

A tiny overview

Summary of TMLE

"Classical" causal inference estimators:

- consistency of g-formula estimators rely on correct specification of the outcome regression $f(a, x) = \mathbb{E}[Y \mid A = a, X = x]$
- consistency of ipw estimators rely on correct specification of the propensity score $\pi(a \mid x) = P(A = a \mid X = x)$
- both types of estimators work poorly with machine learning (cross-validation does not help)

TMLE (and other estimators based on the efficient influence curve):

- built-in bias correction
- double robustness in consistency
- inference based on the efficient influence curve

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TMLE (and other estimators based on the efficient influence curve):

- built-in bias correction
- double robustness in consistency
- inference based on the efficient influence curve
- even when incorporating machine learning (under conditions!!)

Summary of TMLE

TMLE is a two-step procedure:

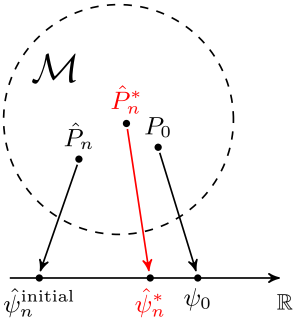
- Step 1 Construct initial estimator $\hat{P}_n$ for P.
- Step 2 Update the estimator $\hat{P}_n \mapsto \hat{P}_n^*$ such that $\hat{P}_n^*$ solves the efficient influence curve equation, i.e.,

$$\mathbb{P}_n \phi^*(\hat{P}_n^*) = \frac{1}{n} \sum_{i=1}^{n} \phi^*(\hat{P}_n^*)(O_i) \approx 0.$$

Step 1 = "initial estimation step"

Step 2 = "targeting step"

Summary of TMLE



- in contrast to the estimating equation (EE) estimator, TMLE is a substitution estimator;
- TMLE *may* show better small-sample performance in settings with unstable weights.

Summary of TMLE

$$\Psi(\hat{P}_n) - \Psi(P_0) = \mathbb{P}_n \phi^*(P_0) + o_P(n^{-1/2})$$
$$+ R(\hat{P}_n, P_0)$$
$$- \mathbb{P}_n \phi^*(\hat{P}_n)$$

- The role of the targeting step (Step 2):
  - Gaining double robustness in consistency.
  - Easier to get rid of second-order remainder and achieve asymptotic linearity.
- The role of the initial estimation step (Step 1):
  - This should be done well enough to get rid of the second-order remainder.
  - The second-order remainder tells us if/how machine learning estimators can be incorporated while still achieving asymptotic linearity.

Summary of TMLE

Specifically for the ATE:

$$\tilde{\Psi}(\hat{f}_n^*) - \tilde{\Psi}(f_0) = \mathbb{P}_n\tilde{\phi}^*(f_0, \pi_0) + o_P(n^{-1/2})$$

$$+ \ \tilde{R}(\hat{f}_n^*, \hat{\pi}_n, f_0, \pi_0) - \underbrace{\mathbb{P}_n\tilde{\phi}^*(\hat{f}_n^*, \hat{\pi}_n)}_{\text{=0, by targeting.}}$$

When $\mathbb{P}_n\tilde{\phi}^*(\hat{f}_n^*, \hat{\pi}_n) = 0$, recall that:

$$|\tilde{R}(\hat{f}_n^*, \hat{\pi}_n, f_0, \pi_0)| \leq \sum_{a=0,1} \delta^{-1} \|\pi_0(a \mid \cdot) - \hat{\pi}_n(a \mid \cdot)\|_{\mu_0} \|f_0(a \mid \cdot) - \hat{f}_n^*(a \mid \cdot)\|_{\mu_0}$$

What this tells us:

- Asymptotic linearity when π_0 and f_0 are estimated at rate at least $n^{-1/4}$.

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What this tells us:

- Asymptotic linearity when π_0 and f_0 are estimated at rate at least $n^{-1/4}$.

We often see this structure of the second-order remainder, but note that it has to be verified on a case-by-case basis.

Summary of TMLE

How can we perform estimation of π_0 and f_0 such as to achieve rate at least $n^{-1/4}$?

- Correctly specified parametric models
  - although consistency is guaranteed, inference cannot be based on the efficient influence curve when one is misspecified!
- There are no results on this being the case for generic implementations of, for example, random forests.
- Lasso, highly adaptive lasso (HAL), ...
- Loss-based "super learning"
  - oracle property: the super learner achieves the rate of convergence of the *best* estimator in its library.

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  - oracle property: the super learner achieves the rate of convergence of the *best* estimator in its library.
  - this is about minimizing expected loss; tuning is still important!

Summary of TMLE

$f(A, X) = \mathbb{E}_P[Y \mid A, X]$

A **loss function** $\mathscr{L}(f)(O)$ measuring the distance between an estimator f and the observed outcome Y, e.g., the negative log-likelihood:

$$\mathscr{L}(\hat{f}_n)(Y_i, A_i, X_i) = -(Y_i \log(\hat{f}_n(A_i, X_i)) + (1 - Y_i) \log(1 - \hat{f}_n(A_i, X_i))).$$

- The estimator $\hat{f}_n$ closest to the true f_0 minimizes the risk:

$$\mathbb{E}_{P_0}[\mathscr{L}(\hat{f}_n)(Y_i, A_i, X_i)].$$

- Loss-based super learning: Minimizing the cross-validated empirical risk with respect to the loss function $\mathscr{L}$ over the statistical model.

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Again, this does not usually by simple plug-in yield a good estimator for the particular feature of interest (the target parameter). But, combined with targeting, it is used to get rid of the second-order remainder.

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1. Scientific question $\Rightarrow$ causal parameter
2. Causal parameter $\Rightarrow$ statistical parameter
3. Statistical estimation problem = statistical parameter + statistical model
   - Efficient influence function
   - Second-order remainder
4. Identify relevant components that need targeting
   - Submodel + loss function
   - Targeting algorithm
5. Construct strong initial learners!!
   - *a priori* specified
6. Inference based on the efficient influence function

Summary of TMLE ... from a theoretical perspective.

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Summary of TMLE ... from a more applied perspective.

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3. Statistical estimation problem = statistical parameter + statistical model
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   - ▷ Submodel + loss function
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