NHH

PROJECT 1

BAN402

CANDIDATES: 105 & 18

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Part A

Question 1

We aim to assist an agricultural company to determine the most feasible way to distribute fertilizers to reduce environmental impact following a new regional government initiative. To comply with the environmental regulations, the government has mandated a minimum reduction of 35 tons of pollutant type 1 (P1) and at least 40 tons of pollutant type 2 (P2). The company can employ three distinct locations - L1, L2 and L3 - to adhere to these new regulations. A linear optimization model was created to minimize cost and identify the optimal pollutant reduction strategy for each location. The mathematical formulation of this model can be expressed using the following elements:

Sets

$l \epsilon L$: Set of locations

$p \epsilon P$: Set of pollutants

Decision variable

x_L = km^2 per location where the new fertilizer is applied.

Objective function

Each location incurs a cost per square kilometer for applying the new fertilizer. These costs form the foundation of the objective function, which aims to minimize the total cost associated with emissions reduction while ensuring full compliance with the newly enacted regional regulations.

$$Min\ total\ cost = 19x_{L1} + 26x_{L2} + 35x_{L3}$$

Constraints

(1) $0.15x_{L1} + 0.05x_{L2} + 0.35x_{L3} \geq 35$ Minimum pollutant reduction for P1

(2) $0.20x_{L1} + 0.40x_{L2} + 0.25x_{L3} \geq 40$ Minimum pollutant reduction for P2

(3) $X_L \geq 0$ Non-negativity constraints

Optimal solution

	L1	L2	L3
x_L	161.538	0	30.769

Optimization results for fertilizer application across locations based on AMPL output

After executing the minimization model in AMPL and solving it, the total cost amounts to \$4,146.15. The optimal solution recommends applying the new fertilizer on 161.54 km² at location L1, no application at location L2, and 30.77 km² at location L3.

Location L1 has the lowest cost per km² of \$19 and is therefore extensively used to minimize overall costs. L1 is the most cost-effective option for reducing pollutant levels, achieving a reduction of 0.15 tons of pollutant P1 per km². In contrast, no fertilizer is applied at location L2. Despite its low cost compared to L3 (\$26 per km² versus \$35 per km²), L2 has limited effect on reducing pollutant P1, achieving only a reduction of 0.05 tons per km². As a result, the model concludes that excluding L2 completely is the most efficient way to minimize the total cost. While L3 incurs the highest cost per km², it offers the largest decrease in pollutant P1 at a rate of 0.35 tons per km². This 30.77 km² spot is designated for L3 to effectively achieve the pollution reduction goals while keeping costs from rising too high.

Question 2

Constraint (4) imposes a non-negativity requirement on the utilization of each location. The optimal solution, which recommends applying the new fertilizer to 161.54 km² at location L1, no application at Location L2, and 30.77 km² at Location L3, satisfies this condition.

Additionally, the model incorporates constraints for each pollutant, specifying minimum required reductions in emissions. Since this is a cost-minimization problem, the company is incentivized to surpass the mandated reduction thresholds, as any additional emissions reduction would result in increased costs. Hence, the goal of the solution is to adhere to, but not exceed, regulatory standards to reduce total costs.

By substituting the decision variables from the optimal solution into the constraints, we verify that all constraints are fully satisfied, thereby confirming the feasibility of the solution. The pollution reductions are precisely aligned with the minimum required thresholds, ensuring compliance without exceeding the mandated levels, thus optimizing resource allocation within the defined regulatory framework.

$$0.15 * 161.54\ tons + 0.05 * 0\ tons + 0.35 * 30.77\ tons = 35\ tons$$

$$0.20 * 161.54\ tons + 0.40 * 0\ tons + 0.25 * 30.77\ tons = 40\ tons$$

Question 3

To address how sensitive the optimal cost is to targets required by the government, we perform a sensitivity analysis centered on shadow prices. The shadow price reflects how much the optimal objective value will change if the right side of a constraint is adjusted by one unit. The following table displays the shadow prices of the pollutant reduction constraints.

Pollutant	**Shadow price**	**Current value**	**Lower bound**	**Upper bound**
P1	69.23	35	35	∞
P2	43.08	40	40	∞

Data obtained from sensitivity analysis in AMPL.

According to the sensitivity analysis, an increase in the required reduction of P1 would increase the optimal cost by approximately $69.23 per additional ton. In contrast, an increase in the required reduction of pollutant P2 results in a lower cost increase of approximately $43.08 per additional ton. This suggests that adjusting the constraint on P1 would incur higher costs than adjusting the constraint on P2. As a result, any further reduction in P1 would have a greater impact on the overall objective function compared to a reduction in P2.

The table also displays the upper and lower bounds within which range the shadow prices are valid. The lower bound related to P1 is 35 tons, and the lower bound related to P2 is 40 tons. These values are binding, and since the shadow prices are positive, we can infer that an increase in the targets required by the government will result in higher costs.

The upper bounds are infinite, since the company has no incentive to reduce emissions beyond the levels required by the government. Although there is no upper limit to the reduction of pollutants, the positive shadow prices indicate that any additional reduction beyond the current target will incur higher costs.

Using the information from the sensitivity analysis, we can calculate what the increased cost would be if the target value for P1 increases to 45, without running the model again. A target value of 45 is 10 tons more than the original target value. By multiplying the shadow price of P1 with the change in the target value, we get the following increased cost:

$$Increased\ cost = \$69.2308 * 10 = \$692.308$$

$$New\ total\ cost = \$4{,}146.15 + \$692.308 = \$4{,}838.46$$

If the target of P1 increases to 45 tons, we get a new total cost of $4,838.46.

Question 4

To assess how alterations in the cost coefficients impact the best solution, we will refer to the sensitivity analysis findings, focusing on the acceptable ranges for the cost coefficients x_{L1}, x_{L2} and x_{L3}. Sensitivity analysis allows us to determine how modifications in these coefficients impact the optimal solution and assesses the current solution's resilience to changes in cost parameters.

The allowable range is presented in the following table.

Variable	**Current cost ($/km^2)**	**Reduced cost ($)**	**Current optimal solution (km^2)**	**Allowable range ($)**	
				Smallest value	**Largest value**
$\mathbf{x_{L1}}$	19	0	162.538	15	20.3529
$\mathbf{x_{L2}}$	26	5.307	0	20.6923	∞

x_{L3}	35	0	30.7692	31.55	44.3333

Data obtained from sensitivity analysis in AMPL.

The results indicate that the cost at location L1, currently set at $19 per km^2, can increase up to $20.35 or decrease to $15 without altering the optimal solution. This implies that L1 is crucial in the current distribution because of its effectiveness and affordable price. Since the reduced cost for L1 is zero, it is already included in the optimal solution, so any small fluctuation in its cost will not affect the company´s decisions.

For location L2, the model excludes it from the optimal solution, due to its minimal contribution to pollutant P1 reduction by 0.05 ton per km^2. The reduced cost of $5.31 for L2 indicates that its current cost of $26 per km^2 would need to decrease by this amount to $20.69 per km^2 for it to be a part of the optimal solution. Unless there is a significant reduction in cost of applying fertilizer at L2, it will remain excluded from the solution.

Despite its high price of $35 per km^2, location L3 is included in the optimal solution because of its high pollutant reduction efficiency, particularly for P1 (by 0.35 ton per km^2). The acceptable range for L3 shows that it can rise to $44.33 before the optimal solution shifts. If the price of fertilizer in L3 went up to $40, the optimal allocation would remain the same. This demonstrates the importance of L3's effectiveness in reducing pollutants despite its higher cost. In optimization theory, it is important to understand the range of the cost coefficients to make informed decisions. In this case, L3's contribution is so vital that even a notable increase in cost would not exclude it from the optimal solution.

Overall, the company can withstand slight adjustments in the cost coefficients for L1 and L3. If the cost of L3 stays under $44.33, the best solution will still assign 162.538 km^2 to L1 and 30.7692 km^2 to L3. The sensitivity analysis demonstrates the effectiveness of the current allocation and reveals that the decisions are relatively insensitive to cost fluctuations unless there are substantial changes at L2 or L3.

Part B

Question 1

In this study, we address the optimization challenges of a company preparing to launch three new green fodder products denoted as Standard, Special and Ultra. These products are produced by using a blend of wheat, rye, grain, oats and corn. The objective is to develop a weekly blending plan that maximizes profit while meeting product demand, adhering to production capacity, and managing raw material constraints.

A linear programming model is formulated to capture the total profit, where decision variables represent production levels and raw material usage. The model includes constraints to ensure demand fulfillment, nutritional compliance, and raw material availability. The model has been mathematically formulated and subsequently implemented in AMPL, where it is solved using an advanced optimization solver. The mathematical formulation is as follows:

Indexes and sets

$p \in P$: Set of products

$i \in I$: Set of raw materials or inputs

$n \in N$: Set of nutrients

Parameters

r_p: Revenue per ton of product p

d_p: Demand for each product p

c_p: Production cost per ton of product p

z_i: Cost per ton of raw materials or input i

b: Total production capacity

s_i: Available supply for raw materials i

$q_{i,n}$: Content of nutrient n in input i

min_n^p: Minimum percentage of nutrient n required in product p

max_n^p: Maximum percentage of nutrient n in product p

Decision variables

x_p: Amount of product $p \in P$ produced in tons

$y_{p,i}$: Amount of input $i \in I$ used in products $p \in P$ (in tons)

Objective function

$$Max\ Total\ profit = \sum_{p \in P} (r_p * x_p) - \sum_{p \in P} (c_p * x_p) - \sum_{p \in P} \sum_{i \in I} (p_i * y_{i,p})$$

Where:

$\sum_{p \in P} (r_p * x_p)$: Total revenue for all products

$\sum_{p \in P} (c_p * x_p)$: Total production cost of all products

$\sum_{p \in P} \sum_{i \in I} (p_i * y_{p,i})$: Total cost of raw material used in production

Constraints

(1) $\sum_{p \in P} x_p \leq b$ $\qquad$ *Production capacity*

(2) $\sum_{p \in P} y_{p,i} \leq s_i, \quad \forall i \in I$ $\qquad$ *Raw material availability*

(3) $\sum_{i \in I} y_{p,i} = x_p, p \in P$ $\qquad$ *Material continuity*

(4) $\sum_{i \in I} (q_{i,n} * y_{i,p}) \geq min_n^p * x_p, \quad \forall p \in P, \quad \forall \in N$ $\qquad$ *minimum requirements of nutrients*

(5) $\sum_{i \in I} (q_{i,n} * y_{p,i}) \leq max_n^p * x_p, \quad \forall p \in p, \forall n \in N$ $\qquad$ *Maximum requirement of nutrients*

(6) $x_p \geq d_p, \quad \forall p \in P$ $\qquad$ *Marketing demand*

(7) $x_p, y_{p,i} \geq 0, \quad \forall p \in P, \forall i \in I \qquad Non-negativity\ constraints$

Optimal blending plan

Product	x_1 **Special**	x_2 **Standard**	x_3 **Ultra**
Tons produced	400	400	500

x_p: *Total amount of final product produced (in tons)*

Raw materials	x_1 **Special**	x_2 **Standard**	x_3 **Ultra**	**Sum**
y_1 **Corn**	0	0	0	0
y_2 **Grain**	0	0	285.714	285.714
y_3 **Oats**	50	25	196.429	271.429
y_4 **Rye**	0	0	0	0
y_5 **Wheat**	225	212.5	62.5	500

$y_{p,i}$: *Amount of raw materials used in production*

The optimal blending plan gives a total profit of NOK 9,852,860.

Each product`s nutrient constraints (for protein, carbohydrates, and vitamins) guide the choice of raw materials. For example, Ultra requires a high proportion of protein and carbohydrates, and thus utilities oats and grain as inputs, as they are cost-effective in meeting these nutrient requirements. Special has a higher vitamin demand, and utilities more wheat and oats. By strategically assigning raw materials based on nutrient content and cost, the model minimizes input costs while meeting the nutrient thresholds.

The production capacity constraint of 1300 imposes a critical limit on the overall blending strategy. The demand for Special and Standard is set at 400 tons each, while Ultra is produced at

500 tons, exceeding the minimum requirement of 350 tons. To maximize profit under these constraints, the model prioritizes inputs like grain and oats, which have lower costs (1,000 and 1,700 NOK/tons, respectively) compared to corn (2,500 NOK/tons). Corn is rich in nutrients but is used in limited quantities due to its higher cost, aligning with the cost minimization objectives.

Question 2

In this scenario, we were introduced to a new wheat supplier for HappyCattle, offering up to 400 tons of wheat at a price of NOK 1,540 per ton. This price is higher than the original supplier´s price of NOK 1,500 per ton. The objective is to determine how much wheat should be purchased from this new supplier while maximizing profit. Additionally, we will assess the differences in optimal decisions and compare the new results to the original scenario.

The objective function remains the same, aiming to maximize total profit by considering the cost of production and raw material purchases. However, the presence of a new supplier introduces another decision variable $y[p, y_6]$, which affects the total cost calculation. Accordingly, it will affect the supply constraint in the model.

$$\sum_{p \in P} y_{p,i} \leq s_i, \quad \forall i \in I \qquad \textit{Raw material availability}$$

With the new constraint:

$$y[p, y_6] \leq 400$$

After implementing the modifications and solving the model using AMPL, we obtained the following results:

Raw materials	x_1 **Special**	x_2 **Standard**	x_3 **Ultra**	**Sum**
y_1 **Corn**	0	0	0	0
y_2 **Grain**	0	0	0	63.64
y_3 **Oats**	38.64	25	0	0

y_4 **Rye**	0	0	0	0
y_5 **Wheat**	50	0	450	500
y_6 **Wheat_S2**	187.5	212.5	0	400

$y_{p,i}$: *Amount of raw materials used in production*

The total profit after introducing the new supplier is NOK 9,875,820. The change in profit (Z) is as follows:

$$\Delta Z = Z_{new} - Z_{old}$$
$$\Delta Z = 9{,}875{,}820 - 9{,}852{,}860$$
$$\Delta Z = 22{,}960$$

The profit increase of NOK 22,960 indicates a net positive effect on profitability despite the higher input costs. Wheat usage from the new supplier reached its supply limit of 400 tons, indicating that it was more profitable to utilize the new suppler despite the higher price. The total cost of purchasing wheat was slightly increased due to higher prices, but this was outweighed by the ability to meet production demands more efficiently, resulting in an overall profit increase.

Question 3

In this revised scenario, the demand for Standard increases to 500 tons, and its selling price increases to NOK 8,750 per ton. Additionally, the cost of oats decreases to NOK 1,400 per ton. To address these changes, the linear programming model needs to be modified to reflect the new demand, price and cost parameters.

$x_{Standard} = 500$ production capacity

$r_{Standard} = \frac{8750NOK}{ton}$ price change

$z_{oats} = \frac{1400NOK}{ton}$ cost adjustments for oats

Optimal production levels

Product	Original scenario (tons)	Modified scenario (tons)	Difference
x_1 **Special**	400	400	No change
x_2 **Standard**	400	500	+100 tons
x_3 **Ultra**	500	400	-100 tons
Total	1300 (full capacity)	1300 (full capacity)	No change

Changes in production capacity

The Standard product's demand has increased by 100 tons, in accordance with the new restriction in the revised scenario. This increase is offset by a 100-ton decrease in Ultra production, allowing the company to remain within the total production capacity limit of 1300 tons.

Optimal raw material usage

Raw material	Original scenario (tons)	Modified scenario (tons)	Difference
y_1 **Corn**	0	0	No change
y_2 **Grain**	285.714	275	-10.714 tons
y_3 **Oats**	271.429	250	-21.429 tons
y_4 **Rye**	0	0	No change
y_5 **Wheat**	500	500	No change

$y_{p,i}$: *Changes in amount of raw materials used in production*

In the modified scenario, we observe that grain and oats usage decrease, even though the total production capacity of 1300 tons is still fully utilized. This can be explained by the substitution effect. The model optimizes the mix of raw materials by balancing nutrient requirements and costs. Wheat is the most cost-efficient for providing carbohydrates and protein, reducing the need for more expensive inputs like grain and oats. Additionally, it can be explained by the product mix shift. By reducing Ultra production, which has higher nutrient requirements, the model can allocate more resources to Standard, which require fewer nutrients. As a result, the total usage of raw material decreases even though the total production quantity remains the same.

Objective value and profit changes (in NOK)

Metric	Original scenario	Modified scenario	Difference
Total profit	9,852,860	9,950,000	+ 97,140
Capacity shadow price	8,057.14	8,185.71	+ 128.57
Wheat shadow price	128.571	42.8571	- 85.71

Changes in metrics obtained from results using AMPL

The total profit increases by NOK 97,140 due to the higher demand and selling price of Standard. The increase in capacity shadow price signifies that the marginal value of production capacity has risen. This suggests that the model is constrained by capacity, and additional capacity would yield higher profit. Meanwhile, the decrease in the shadow price for wheat indicates that the marginal value is reduced in the optimization model. This is primarily due to a decrease in cost of oats, which allows the model to achieve a more efficient balance of raw materials.

The modified scenario leads to a more profitable outcome for HappyCattle, driven by the increased production and higher selling price of Standard. Despite the cost reduction for Oats, the model reduces the usage of both Oats and Grain. Wheat remains the most cost-effective option for meeting the nutrient requirements. The shift in the product mix, favoring Standard over Ultra, allows the company to maintain full capacity while optimizing raw material usage, resulting in an increase in total profit.

Part C

Question 1

In this task, we will help the *FruitMix* company with minimizing their costs of transporting bananas. By creating and solving a linear programming model, we will determine the company's optimal shipping plan from two regions (R1 and R2) to 20 different markets (K1, K2, …, K20). To reach the markets, bananas can either be shipped directly from the regions, or through one of two ports (P1 and P2). A mathematical formulation of the model can be expressed using the following elements:

Indexes and sets

r $\in$ R: $Set\ of\ regions$

p $\in$ P: $Set\ of\ ports$

k $\in$ K: $Set\ of\ markets$

Parameters

S_r: $Weekly\ supply\ in\ region\ r\ (tons)$

C_{rp}: $Cost\ of\ transporting\ one\ ton\ of\ bananas\ from\ region\ r\ to\ port\ p$

C_{pk}: $Cost\ of\ shipping\ one\ ton\ of\ bananas\ from\ port\ p\ to\ market\ k$

C_{rk}: $Cost\ of\ shipping\ one\ ton\ of\ bananas\ directly\ from\ region\ r\ to\ market\ k$

D_k: $Weekly\ demand\ in\ market\ k\ (tons)$

Decision variables

x_{rp}: $Tons\ of\ bananas\ transported\ from\ region\ r\ to\ port\ p$

y_{pk}: $Tons\ og\ bananas\ shipped\ from\ port\ p\ to\ market\ k$

z_{rk}: $Tons\ of\ bananas\ shipped\ directly\ from\ region\ r\ to\ market\ k$

Objective function

The objective function seeks to minimize the total cost of transporting bananas through various routes.

$$Minimize\ Total\ Cost\text{:}\ \sum_{r\,\in\,R}\sum_{p\,\in P}(C_{rp} *\ x_{rp}) + \sum_{p\,\in\,P}\sum_{k\,\in K}(C_{pk} *\ y_{pk}) + \sum_{r\,\in\,R}\sum_{k\,\in K}(C_{rk} *\ z_{rk})$$

Constraints

$$(1) \sum_{p\in\mathrm{P}} x_{r_p} + \sum_{k\in\mathrm{K}} z_{rk} \leq\ S_r \quad \forall r\ \in R \quad Shipped\ amount\ cannot\ exceed\ supply$$

$$(2)\ \sum_{p\in P} y_{pk} + \sum_{r\in R} z_{rk} = D_k \quad \forall k \in K \quad \textit{Shipped amount is equal to market demand}$$

$$(3)\ \sum_{r\in R} x_{rp} = \sum_{k\in K} y_{pk} \quad \forall p \in P\ \textit{Amount transported to port is equal to amount shipped from ports}$$

$$(4)\ x_{rp}, y_{pk}, z_{rk} \geq 0\ \ \forall r\ \in R, \forall p \in P, \forall k \in K \quad Non-negativity\ constraint$$

Optimal shipping plan

By implementing and solving the model in AMPL, the optimal shipping plan has been computed. The results are displayed in the following tables:

	P1		**P2**		**Sum**	
	Tons	**Cost**	**Tons**	**Cost**	**Tons**	**Cost**
R1	168	\$5,544	0	\$0	168	\$5,544
R2	0	\$0	152	\$5,320	152	\$5,320
Sum	168	\$5,544	152	\$5,320	320	\$10,864

Tons of bananas transported from regions to ports

The total transportation cost from regions to ports amounts to \$10,864. A total of 320 tons of bananas are transported, where 168 tons are transported from R1 to P1, and 152 tons are transported from R2 to P2. The solution avoids partially discharged shipments between regions and ports to ensure that each region ships only to its nearest and most cost-effective port. The cost per ton for the two routes are relatively consistent where R1 to P1 has a cost of \$33 per ton and R2 to P2 has a cost of \$35 per ton. This indicates that the transportation cost is nearly uniform across the two regions. The minimal variation in costs per ton suggests that the model has considered geographical factors, transportation distances and port handling efficiencies to balance the overall costs.

	K1	**K2**	**K3**	**K4**	**K5**	**K6**	**K7**	**K8**	**K9**	**K10**	**K11**	**K12**	**K13**	**K14**	**K15**	**K16**	**K17**	**K18**	**K19**	**K20**	**Sum**
P1	15	23	0	16	0	0	0	0	16	0	0	30	0	25	26	0	0	0	17	0	168
P2	0	0	19	0	26	0	21	14	0	0	20	0	0	0	0	0	25	27	0	0	152
R1	0	0	0	0	0	0	0	0	0	19	0	0	0	0	0	0	0	0	0	13	32
R2	0	0	0	0	0	13	0	0	0	0	0	0	27	0	0	32	0	0	0	0	72
Sum	15	23	19	16	26	13	21	14	16	19	20	30	27	25	26	32	25	27	17	13	424

	$135	$276	$285	$432	$702	$468	$567	$336	$336	$532	$1140	$990	$1296	$825	$1014	$1184	$675	$810	$357	$416	$12776

Tons of bananas shipped from ports and regions to markets

The total transportation cost from ports and regions to markets amounts to $12,776. A total of 320 tons out of 424 passes through ports P1 and P2, counting for approximately 75% of the total shipments. This reliance on ports allows the company to benefit from economies of scale, where large shipments reduce the unit cost of transportation. Using centralized hubs like ports is particularly cost effective for long-distance transportation compared to smaller, fragmented shipments sent directly to the markets.

Direct shipping from regions to markets have only been applied to 5 out of the 20 markets, including K6, K10, K13 and K16. These markets account for 104 tons, or about 25% of the total shipment volume. For these markets, shipping directly from the regions was determined to be the most cost-effective option. Several factors likely influenced this decision, such as the proximity of the regions to the market, inefficiency of certain transportation networks from the port to these markets, or smaller demands. In these cases, direct shipping could be more practical than passing through a port.

In summary, the optimal shipping plan has a total cost of $23,640. This is the most cost-effective option while ensuring that all market demands are met in accordance with the given constraints.

Question 2

The port authorities are considering a renovation project for port P2, which will require temporarily closure of the port. As a result, some banana shipments will need to be rerouted to alternative pathways to reach the market. To develop the new optimal shipping plan, we will introduce two constraints to our model. These constraints will ensure that no bananas are being transported to nor shipped from port P2, while identifying the most cost-effective alternative.

New constraints:

$$(5)\ x_{r,P2} = 0\ \forall r\ \in R \quad No\ transport\ from\ regions\ to\ port\ P2$$

$$(6)\ y_{P2,k} = 0\ \forall K\ \in K\ \ No\ shipping\ from\ port\ P2\ to\ markets$$

New optimal shipping plan

By solving the adjusted model in AMPL, we extract the following shipping plan:

	P1		**P2**		**Sum**	
	Tons	**Cost**	**Tons**	**Cost**	**Tons**	**Cost**
R1	168	$5544	0	$0	168	$5544
R2	19	$798	0	$0	19	$798
Sum	187	$6342	0	0$	187	$6342

Amount transported from regions to ports

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20	Sum
P1	15	23	19	16	0	0	0	0	16	0	0	30	0	25	26	0	0	0	17	0	187
P2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R1	0	0	0	0	0	0	0	0	0	19	0	0	0	0	0	0	0	0	0	13	32
R2	0	0	0	0	26	13	21	14	0	0	20	0	27	0	0	32	25	27	0	0	205
Sum	15	23	19	16	26	13	21	14	16	19	20	30	27	25	26	32	25	27	17	13	424
	$135	$276	$456	$432	$1976	$468	$2184	$1456	$336	$532	$1840	$990	$1296	$825	$1014	$1184	$2800	$3348	$357	$416	$22321

Amount shipped from ports and regions to markets

The revised shipping plan shows that 187 tons are shipped through port P1, while direct shipment from R1 and R2 accounts for 237 tons. The new optimal shipping plan has a total cost of $28,663. By limiting the flexibility of the transportation network, the total cost has increased by $5,023. Rerouting of shipments through port P1 and increased direct shipments from the regions, particularly R2, have caused the overall cost per ton to rise.

The original plan had more options for distributing the shipments, allowing the company to choose the most cost-effective routes. With port P2 unavailable, the company is forced to use less efficient, more expensive routes. In particular, those involving direct shipments. This demonstrates the importance of flexibility of the transportation network, as losing these key pathways have risen the costs significantly.

Question 3a

In this task, we will assist the company in developing an optimal shipping plan tailored to the new scenario. This scenario implies that all the bananas need to pass through one of the ports, and that 1% of the shipment will be disposed at the port. To account for these changes, we need to make some adjustments to our original model.

Considering all the bananas need to pass through either port P1 or port P2, there is no longer need for parameter C_{rk}, (cost of shipping one ton of bananas directly from region r to market k).

Additionally, the model now only has two decision variables: x_{rp} (tons of bananas transported from region r to port p) and y_{pk} (tons of bananas shipped from port p to market k).

These adjustments lead to the following updated objective function:

$$Minimize\ Total\ Cost: \sum_{r \in R} \sum_{p \in P} (C_{rp} * x_{rp}) + \sum_{p \in P} \sum_{k \in K} (C_{pk} * y_{pk})$$

We must also modify the constraints to ensure that no bananas can be shipped directly from regions to markets, while accounting for the disposal of 1% of the bananas at the ports. The adjusted constraints can be expressed as follows:

$$(1) \sum_{p \in \mathrm{P}} x_{r_p} \leq S_r \quad \forall r \in R \quad Amount\ transported\ from\ regions\ to\ ports\ cannot\ exceed\ supply$$

$$(2) \sum_{\mathrm{p} \in \mathrm{P}} y_{pk} = D_k \quad \forall k \in K \quad Amount\ shipped\ from\ ports\ to\ markets\ is\ equal\ to\ market\ demand$$

$$(3) \sum_{\mathrm{r} \in \mathrm{R}} x_{rp} * 0.99 = \sum_{k \in \mathrm{K}} y_{pk} \quad \forall p \in P \qquad Only\ 99\%\ reach\ the\ markets$$

$$(4)\ x_{rp}, y_{pk}, \geq 0 \quad \forall r \in R, \forall p \in P, \forall \in K \qquad Adjusted\ non-negativity\ constraint$$

New optimal shipping plan

	P1		**P2**		**Sum**	
	Tons	**Cost**	**Tons**	**Cost**	**Tons**	**Cost**
R1	200	\$6 600	0	\$0	200	\$6 600
R2	2.02	\$84,84	226.26	\$7 919.1	228.28	\$8 003.94
Sum	202.02	\$6 684,84	226.26	\$7 919.1	428.28	\$14 603,94

Amount transported from regions to ports

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20	Sum
P1	15	23	0	16	0	0	0	0	16	19	0	30	0	25	26	0	0	0	17	13	200
P2	0	0	19	0	26	13	21	14	0	0	20	0	27	0	0	32	25	27	0	0	224
Sum	15	23	19	16	26	13	21	14	16	19	20	30	27	25	26	32	25	27	17	13	424
	$135	$276	$285	$432	$702	$312	$567	$336	$336	$570	$1140	$990	$405	$825	$1014	$192	$675	$810	$357	$390	$10749

Amount shipped from ports to markets

The optimal shipping plan in this scenario yields a total cost of $25,353. As anticipated, the modified plan is more expensive compared to the original, with a cost increase of $1,713.

One factor contributing to the increased cost is that *FruitMix* now needs to transport 1% additional tons of bananas to reach total market demand. The company is required to transport slightly more than 428 tons of bananas, whereas the original shipment requirement was only 424 tons.

Another factor can be explained by the shift in the shipping routes. Bananas that were previously shipped directly from regions must now take less cost-effective routes due to the new imposed constraint in the model. For instance, the cheapest route to market K10 was originally directly from R1, with a cost of $28 per ton. However, this is no longer an option, and given that the bananas are now shipped from R1 through P1, the cost is now $66 ($30 + $33) per ton.

Question 3b

Under circumstances of limited inspection capacity at the ports, we need to ship some bananas through a different port to reach all market demands. For our model to account for this scenario, we need to introduce a new parameter:

$$L_p: Maximum\ tons\ of\ bananas\ at\ port\ p$$

This parameter is used in the context of a new constraint that ensures no exceeding of capacity.

$$\sum_{r \in \mathrm{R}} x_{r_p} \leq L_p \quad \forall p \in P \quad Tons\ transported\ from\ regions\ to\ ports\ cannot\ exceed\ capacity\ limit$$

These adjustments lead to the following results:

New optimal shipping plan

	P1		P2		Sum	
	Tons	**Cost**	**Tons**	**Cost**	**Tons**	**Cost**
R1	175	$5,775	3.28	$147.6	178.28	$5,922.6
R2	0	$0	250	$8,750	250	$8,750
Sum	175	$5,775	253.28	$8,897.6	428.28	$14,672.6

Amount transported from regions to ports

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	K16	K17	K18	K19	K20	Sum
P1	15	23	0	14.25	0	0	0	0	16	19	0	30	0	0	26	0	0	0	17	13	173
P2	0	0	19	1.75	26	13	21	14	0	0	20	0	27	25	0	32	25	27	0	0	251
Sum	15	23	19	16	26	13	21	14	16	19	20	30	27	25	26	32	25	27	17	13	424
	$135	$276	$285	$495	$702	$312	$567	$336	$336	$570	$1140	$990	$405	$1650	$1014	$192	$675	$810	$357	$390	$11637

Amount shipped from ports to markets

The total cost of this shipping plan is $26,310. Conversely, the total cost was $25,353 in the infinite capacity scenario, representing an increase of $956 in total costs. The cost increase is due to limited capacity constraints imposed on the ports, including a limit of 175 tons at port P1 and 275 tons at port P2.

With the limited capacity, we observe that two markets are now supplied from a different port. In the infinite capacity scenario, market K4 shipped all 16 tons through port P1 at a cost of $27 per ton. However, the market now receives 1.75 out of 16 tons from port P2 at a cost of $63 per ton. The capacity constraint on port P1 forces the shipment to be partially discharged at port P1 and port P2 in order to meet market demand. Market K14 is also supplied from a different port compared to the infinite capacity scenario. All 25 tons of bananas were originally shipped from port P1 at a cost of $33 per ton. These are now all shipped from port P2 at a cost of $66 per ton.

In conclusion, the limited capacity scenario illustrates the impact of binding constraints on the optimization process. When capacity limits are reached, the model is forced to find alternative routes that are less cost-effective, leading to an increase in total costs. The occurrences of partially discharging shipments, and the rerouting of markets such as K14, highlight the inefficiencies that arise when capacity is constrained.

Part D

Question 1

In the article "Estimating Effectiveness of Identifying Human Trafficking via Data Envelopment Analysis," the authors applied Data Envelopment Analysis (DEA)**.** This method is based on linear programming to assess the efficiency of border stations managed by Love Justice International (LJI)**.** DEA allowed the authors to construct an optimization framework that evaluates the relative efficiency of decision-making units (DMUs). This involved comparing their resource consumption (inputs such as staff and working hours) to their operational outcomes (outputs such as the number of potential trafficking victims intercepted and the completeness of forms) (Dimas et al., 2023, p. 410).

The DEA model identified inefficiencies by benchmarking each DMU against the most efficient units. This comparison enabled the model to reveal "best practice frontiers," highlighting stations that operated efficiently within their resource constraints. Moreover, they identified inefficient DMUs which failed to maximize their outputs given their input levels (Dimas et al., 2023, p. 412).

Linear programming helped provide executable insights. The DEA model indicated areas where resource reallocation, increased staff training, or procedural adjustments could improve operational efficiency. For instance, stations with low report completeness were flagged, and the model suggested that improvements in staff training or process optimization could enhance their performance (Dimas et al., 2023, p. 413).

This approach allowed for data-driven decision modeling, providing LJI with concrete recommendations to optimize resource allocation and station operations. It effectively bridged the gap between theoretical optimization and practical intervention in a resource-constrained setting. As a result, it functioned as a guide for LJI towards higher operational effectiveness (Dimas et al., 2023, p. 414).

Question 2

In the model presented in Table A.1 of the article, the objective is to assess the relative efficiency of multiple decision-making units (DMUs). This is achieved by optimizing the ratio of outputs to inputs for each DMU while ensuring that the same efficiency standard applies across all DMUs. The question arises as to whether this model could be infeasible, even when all parameters (inputs and outputs) are positive.

One possible cause of infeasibility in the model is if the border stations vary widely in terms of their performance or operating environment. The DEA model assumes that all DMUs are comparable in terms of the process being analyzed. If some stations operate under immensely different conditions, it may be difficult for the model to assign meaningful weights to inputs and outputs, causing infeasibility. This issue relates to model misspecification, where failing to account for significant differences in scale or operating environments can lead to infeasible solutions.

However, the author addressed this potential issue by applying homogeneity criteria to ensure that the border stations were comparable. They focused solely on stations with similar levels of traffic and operational characteristics, thereby reducing the risk of infeasibility caused by non-comparability (Dimas et al., 2023, p. 413)

Another potential source of infeasibility in the DEA model could arise from overly restrictive weight constraints. If the model imposes limits on the weights assigned to inputs (such as the number of staff) or outputs (such as the number of intercepted victims), it could become impossible to obtain a feasible solution. This occurs because excessively high or low weight restrictions could disrupt the balance of efficiency constraints across all DMUs. This could potentially prevent the model from accurately comparing and optimizing the performance of each station.

This issue is often encountered when external factors impose rigid constraints on decision variables. However, the DEA model in the article is designed to offer flexibility in assigning weights, enabling it to determine the most appropriate weight distribution for each station. This flexibility minimizes the risk of infeasibility by ensuring each station is evaluated under the most optimal conditions (Dimas et al., 2023, p. 412-413). Additionally, the authors implemented cross-efficiency analysis to help identify any outliers that could distort the evaluation and potentially lead to infeasibility.

Lastly, infeasibility could arise due to inconsistent relationship between inputs and outputs across different border stations. For instance, a station that inputs many staff and hours worked but generates very few intercepted victims could create a scenario where the model struggles to assign appropriate weights to make the station efficient. Conversely, a station that intercepts a disproportionately high number of victims and minimal inputs could cause infeasibility for other stations, as the model might not be able to reconcile such wide disparities in performance.

To mitigate this, the authors carefully selected the inputs and outputs to be used in the DEA model. They included measures such as the completeness of interception reports (IRFs) and victim interview forms (VIFs). This was to account for qualitative aspects of outputs, which help balance the model´s assessment of stations with differing performance levels (Dimas et al., 2023, p. 414). By including these additional outputs, the model can adjust the differences in the number of victims intercepted, reducing the risk of infeasibility.

Despite the potential causes of infeasibility, the DEA model in the article is unlikely to be infeasible. The flexibility in assigning weights to inputs and outputs allows the model to evaluate each station under optimal conditions. By optimizing the ratio of outputs to inputs for each station, the DEA model ensures the efficiency scores remain within the bounds of the model´s constraints. By using flexible weight assignments and applying the homogeneity criteria, the authors ensured that the model could effectively evaluate the efficiency of the border stations.

Question 3

The second constraint in the DEA model ensures that the total weighted output of each DMU, minus its efficiency score (denoted as v), is less than or equal to the total weighted input. The constraint is formulated as follows:

$$\sum_{m \in M} w_m x_{mj} - v \geq 0, \forall j \in J,$$

Where w_m represents the weights assigned to inputs, x_{mj} is the amount of input m used by DMU, j, and v is the efficiency score for the DMU being evaluated.

The constraint ensures that the weighted sum of inputs adjusted by the efficiency score v is at least as large as the weighted sum of outputs for each DMU, thus keeping the problem bounded. It ensures that a valid efficiency score, constrained between 0 and 1, can be determined for each DMU, maintaining the model's integrity and feasibility within the set parameters.

If the second constraint is modified by replacing $-v$ with $+v$ it would be expressed mathematically as follows:

$$\sum_{m \in M} w_m x_{mj} + v \geq 0, \forall j \in J,$$

In this form, the efficiency score v is now added to the weighted sum of inputs rather than being substracted.

Modifying the constraint by replacing $-v$ with $+v$ could make the model unbounded. This is because the optimization would no longer limit v within a feasible range, allowing the solution to increase v without bounds. The term $+v$ would no longer properly constrain the efficiency frontier, which is supposed to limit the relative performance of DMUs. In the DEA framework, v acts as a scaling factor that ensures no DMU exceeds an efficiency score of 1. By adding v rather than subtracting it, the model could allow DMUs to have efficiency score greater than 1. This would disrupt the balance between inputs and outputs, undermining the DEA model´s ability to assess the relative efficiency of DMUs. As a result, the model could become unbounded, allowing efficiency score to increase without limit.

Question 4

An agent suggests that the difference between the weights given the outputs should not be greater than a limit a. In the DEA model, the weights assigned to the outputs are represented as decision variables u_n (where n indexes the outputs). The agent´s condition essentially adds a new set of constraints that control the maximum allowable difference between the weights of any two outputs. This ensures that no single output weight is disproportionately larger or smaller than others.

To incorporate the agent´s suggestion and keep the model linear, we can create two separate linear constraints:

$$u_i - u_j \leq a, \forall i, j \in N \; with \; i \neq j$$

$$u_j - u_i \leq a, \forall i, j \in N \; with \; i \neq j$$

Where u_i is the weight assigned to output i and u_j is the weight assigned to another output j. These two constraints ensure that the difference between any two output weights within the limit a, which maintains balance between the importance of each output.

Since these additional constraints are linear inequalities, they maintain the linear nature of the original DEA model, meaning the linear programming framework is preserved. These constraints will ensure that the weights given to the outputs are balanced within the limit α, addressing the agent’s suggestion without changing the fundamental structure of the DEA model.

APPENDIX

The part and questions answered in this report have been addressed using AMPL and the corresponding files is included in the submission. To facilitate the review, the final chapter of this report provides a detailed list of all files associated with each part and question.

PART A

Question 1-3

- Part_A.mod
- Part_A.dat
- Part_A.run

PART B

Question 1-3

- Part_B_Q123.mod

Question 1

- Part_B_Q1.dat
- Part_B_Q1.run

Question 2

- Part_B_Q2.dat
- Part_B_Q2.run

Question 3

- Part_B_Q3.dat
- Part_B_Q3.run

PART C

Question 1

- Part_C_Q1.mod
- Part_C_Q1.dat
- Part_C_Q1.run

Question 2

- Part_C_Q2.mod

- Part_C_Q2.dat
- Part_C_Q2.run

Question 3a

- Part_C_Q3a.mod
- Part_C_Q3a.dat
- Part_C_Q3a.run

Question 3b

- Part_C_Q3b.mod
- Part_C_Q3b.dat
- Part_C_Q3b.run

PART D

Dimas, L. G., Khalkhali, E. M., Bender, A., Maass, L. K., Konrad, A. R., Blom, S. J., Zhu, J., Trapp, C. A. (2023). Estimating Effectiveness of Identifying Human Trafficking via Development Analysis. Informs Journal on Applied Analytics 53(6):408-424. https://doi.org/10.1287/inte.2023.1162