See https://stanford.edu/~rezab/amdm/notes/lecture8.pdf Section 9.2 For Problem Setup
Problem completed below:

$$V_c(n) = \sum_{t=c}^{n-1}(P(\text{not taking persons } c \text{ to } t-1) \cdot P(\text{taking person } t) \cdot E(\text{person } t))$$
$$+ P(\text{taking last person}) \cdot E(\text{last person})$$
$$= \left(\sum_{t=c}^{n-1} \frac{c-1}{t-1}\frac{1}{t}\frac{t}{t+1}\right) + \frac{c-1}{n-1}\frac{1}{2}$$
$$= \left((c-1)\sum_{t=c}^{n-1} \frac{1}{(t-1)(t+1)}\right) + \frac{c-1}{n-1}\frac{1}{2}$$
$$= (c-1)\sum_{t=c}^{n-1} \frac{\frac{1}{2}}{t-1} - \frac{\frac{1}{2}}{t+1}\Bigg) + \frac{c-1}{n-1}\cdot\frac{1}{2}$$
$$= \left(\frac{c-1}{2}\left(\frac{1}{c-1} - \frac{1}{c+1} + \frac{1}{c} - \frac{1}{c+2} + \frac{1}{c+1} - \frac{1}{c+3} + \ldots + \frac{1}{n-2} - \frac{1}{n}\right)\right) + \frac{c-1}{n-1}\cdot\frac{1}{2}$$
$$= \frac{c-1}{2}\left(\frac{1}{c-1} + \frac{1}{c} - \frac{1}{n-1} - \frac{1}{n}\right) + \frac{c-1}{n-1}\cdot\frac{1}{2}$$
$$= \frac{1}{2} + \frac{c-1}{2c} - \frac{c-1}{2(n-1)} - \frac{c-1}{2n} + \frac{c-1}{2(n-1)}$$
$$= \frac{cn}{2cn} + \frac{cn-n}{2cn} - \frac{c^2-c}{2cn}$$
$$= \frac{2cn - c^2 + c - n}{2cn}$$
$$= 1 - \frac{1}{2c} - \frac{c}{2n} + \frac{1}{2n}$$
$$\frac{d}{dc}V_c(n) = \frac{1}{2c^2} - \frac{1}{2n} = 0 \Rightarrow c = \sqrt{n}$$