

Comp 141 Probabilistic Robotics Homework 1:

Kalman Filter

1 Kalman Filter: Prediction

A balloon drone has encountered a glitch in its program and needs to reboot its on-board computer. While rebooting, the drone is helpless and cannot issue motor commands. To help the drone, you'll need some understanding of the Kalman filter algorithm.

The drone operates in a 1-D world where x_t is the position at time t , while \dot{x}_t and \ddot{x}_t are the velocity and acceleration. For simplicity, assume that $\Delta t = 1$.

Due to random wind fluctuations, at each new time step, your acceleration is set randomly accordingly to the distribution $\mathcal{N}(\mu_{wind}, \sigma_{wind}^2)$, where $\mu_{wind} = 0.0$ and $\sigma_{wind}^2 = 1.0$.

Question 1.1: The minimal state vector for the Kalman Filter is the state vector for the Kalman Filter is $(x_t \ \dot{x}_t \ \ddot{x}_t)$

Question 1.2: The state transition model:

$$\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where matrix A is

$$\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

and matrix B is

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

state transition probability function $p(x_t | u_t, x_{t-1}) = \det(2\pi R_t^{-1})^{-\frac{1}{2}} \exp \left(-\frac{1}{2} (x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1} (x_t - A_t x_{t-1} - B_t u_t) \right)$
where R is

$$\begin{bmatrix} \frac{1}{4} \Delta t^4 & \frac{1}{2} \Delta t^3 \\ \frac{1}{2} \Delta t^3 & \Delta t^2 \end{bmatrix} \sigma_{wind}^2$$

=

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

Question 1.3: assuming the initial time step is $t=0$,

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$$

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 2.5 & 2.0 \\ 2.0 & 2.0 \end{bmatrix}$$

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 8.75 & 4.5 \\ 4.5 & 3.0 \end{bmatrix}$$

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 21.0 & 8.0 \\ 8.0 & 4.0 \end{bmatrix}$$

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 41.25 & 12.5 \\ 12.5 & 5.0 \end{bmatrix}$$

Question 1.4: For each value of t in the previous question, plot the joint posterior over x and \dot{x} in a diagram where x is the horizontal and \dot{x} is the vertical axis. For each posterior, you are asked to plot the uncertainty ellipse which is the ellipse of points that are one standard deviation away from the mean. Some additional information about uncertainty ellipses and how to calculate them using MATLAB or C++ can be found here: <http://www.visiondumny.com/2014/04/draw-error-ellipse-representing-covariance-matrix/>.

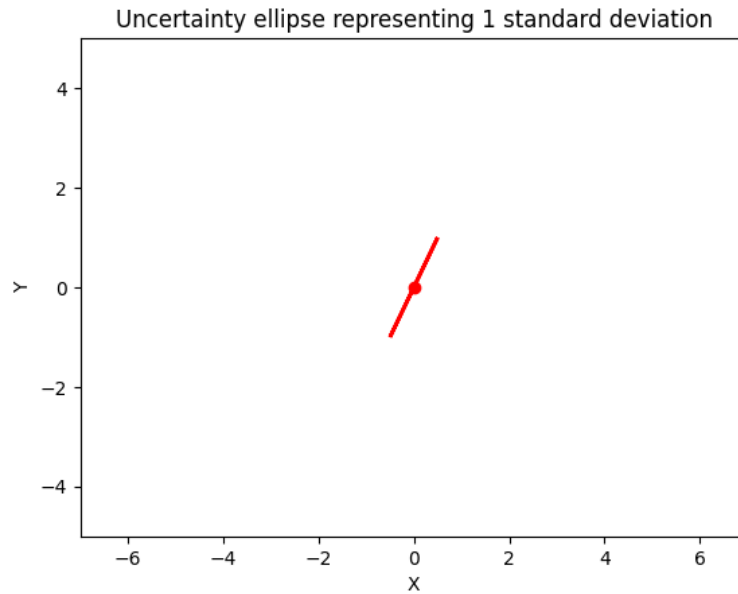


Figure 1: Uncertainty Ellipse at $t=1$

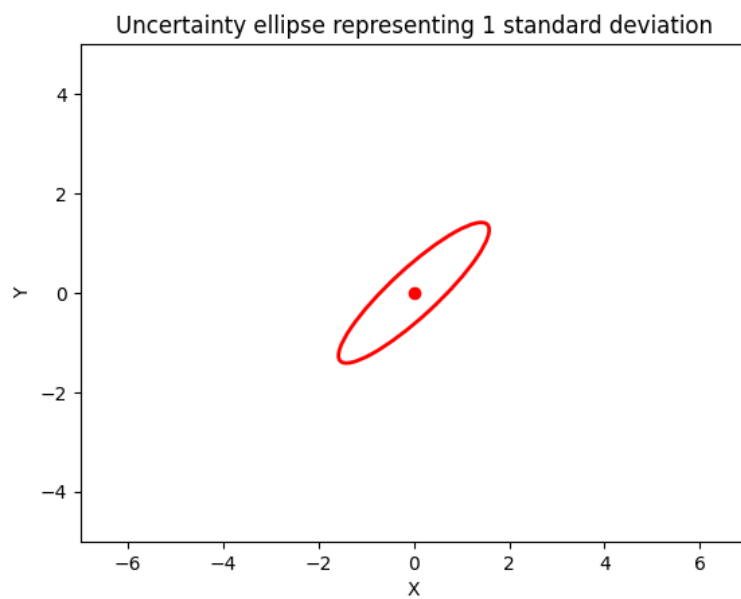


Figure 2: Uncertainty Ellipse at $t=2$

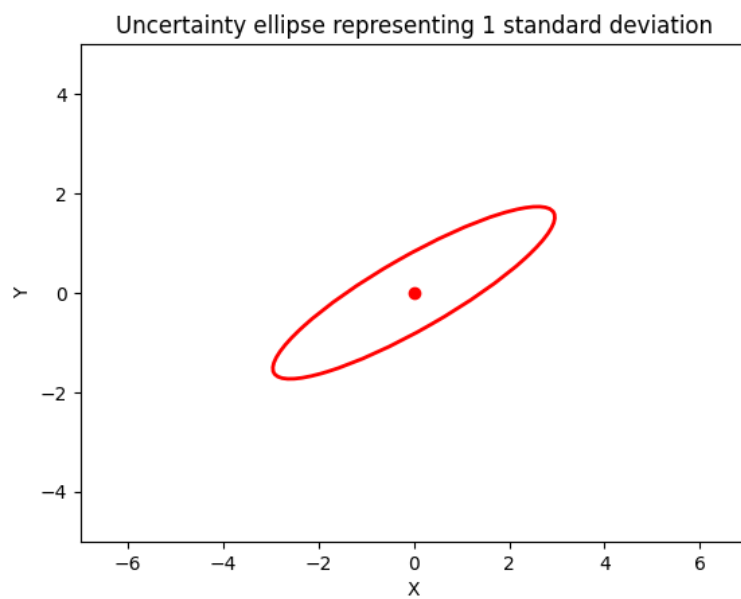


Figure 3: Uncertainty Ellipse at $t=3$

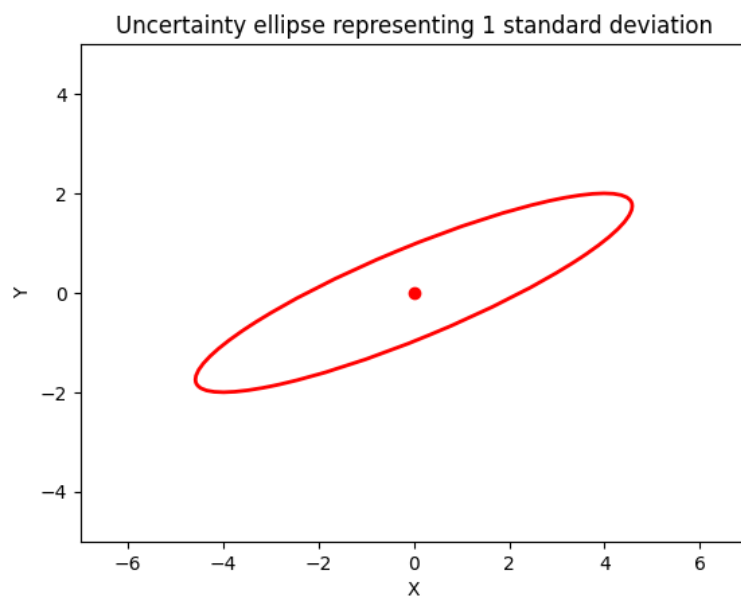


Figure 4: Uncertainty Ellipse at $t=4$

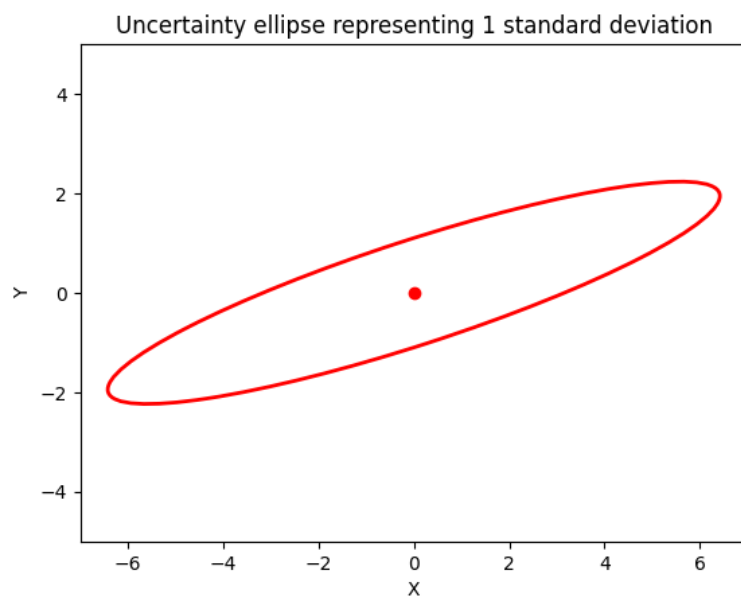


Figure 5: Uncertainty Ellipse at $t=5$

2 Kalman Filter: Measurement

Prediction alone will result in greater and greater uncertainty as time goes on. Fortunately, your drone has a GPS sensor, which in expectation, measures the true position. However, the measurement is corrupted by Gaussian noise with covariance $\sigma_{gps}^2 = 8.0$.

Question 2.1: the measurement model:

$$C = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 8.0 \end{bmatrix}$$

Question 2.2: Implement the measurement update. Suppose at time $t = 5$, the drone's computer has rebooted and we query our sensor for the first time to obtain the measurement $z = 10$. State the parameters of the Gaussian estimate before and after incorporating the measurement. Afterwards, implement the sensor modal to randomly sample the true position, corrupted with noise σ_{gps}^2 .

Question 2.3: All of a sudden, the sky gets cloudy which may cause the sensor to fail and not produce a measurement with probability $p_{gps-fail}$. For three different values of this probability (e.g., 0.1, 0.5, and 0.9), compute and plot the expected error from the true position at time $t = 20$. You may do so by running up to N simulations and use the observed errors to obtain the expected error empirically.

3 Kalman Filter: Movement

Question 3.1 The drone is now fully operational and can not only take measurements, but also issue motor commands in the form of acceleration commands to its propeller. For example, a command of 1.0 will increase the drone's velocity by 1.0. Revisit Question 1.3 to provide the matrix B . If at time $t - 1$, the drone's position and velocity are 5.0 and 1.0, compute the mean estimate for the state at time t given a motor command of 1.0. Your answer should be based on the constants provided but also include a random variable due to the wind effects. State the distribution of that random variable.

4 Extra Credit

Now, formulate both the prediction and measurement steps in the 2-D case. Construct a plot showing the true position and the position tracked by the Kalman filter over the first 30 time steps.

What to turn in: A PDF document with the answers to the questions, along with the code implementation and a README file that describes what to run in order to get the results in your PDF. You can use a language of your choice.