

# Comp 141 Probabilistic Robotics Homework 1:

## Kalman Filter

### 1 Kalman Filter: Prediction

Code available at: <https://github.com/helenlu66/KalmanFilter.git>.

A balloon drone has encountered a glitch in its program and needs to reboot its on-board computer. While rebooting, the drone is helpless and cannot issue motor commands. To help the drone, you'll need some understanding of the Kalman filter algorithm.

The drone operates in a 1-D world where  $x_t$  is the position at time  $t$ , while  $\dot{x}_t$  and  $\ddot{x}_t$  are the velocity and acceleration. For simplicity, assume that  $\Delta t = 1$ .

Due to random wind fluctuations, at each new time step, your acceleration is set randomly accordingly to the distribution  $\mathcal{N}(\mu_{wind}, \sigma_{wind}^2)$ , where  $\mu_{wind} = 0.0$  and  $\sigma_{wind}^2 = 1.0$ .

**Question 1.1:** The minimal state vector for the Kalman Filter is the state vector for the Kalman Filter is  $\begin{bmatrix} x_t & \dot{x}_t \end{bmatrix}$

**Question 1.2:** The state transition model:

$$\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot [0]$$

where matrix A is

$$\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

and matrix B is 0 since there's no control signal

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

state transition probability function  $p(x_t|u_t, x_{t-1}) = \det(2\pi R_t^{-1})^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1} (x_t - A_t x_{t-1} - B_t u_t)\right)$   
where R is

$$\begin{bmatrix} \frac{1}{4}\Delta t^4 & \frac{1}{2}\Delta t^3 \\ \frac{1}{2}\Delta t^3 & \Delta t^2 \end{bmatrix} \sigma_{wind}^2$$

=

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

**Question 1.3:** assuming the initial time step is  $t=0$ ,

$$\mu_{t1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t1} = \begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$$

$$\mu_{t2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t2} = \begin{bmatrix} 2.5 & 2.0 \\ 2.0 & 2.0 \end{bmatrix}$$

$$\mu_{t3} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t3} = \begin{bmatrix} 8.75 & 4.5 \\ 4.5 & 3.0 \end{bmatrix}$$

$$\mu_{t4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t4} = \begin{bmatrix} 21.0 & 8.0 \\ 8.0 & 4.0 \end{bmatrix}$$

$$\mu_{t5} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sigma_{t5} = \begin{bmatrix} 41.25 & 12.5 \\ 12.5 & 5.0 \end{bmatrix}$$

**Question 1.4:** the uncertainty ellipse of the joint posterior over  $x$  and  $\dot{x}$  in a diagram where  $x$  is the horizontal and  $\dot{x}$  is the vertical axis for  $t=1$  to  $t=5$

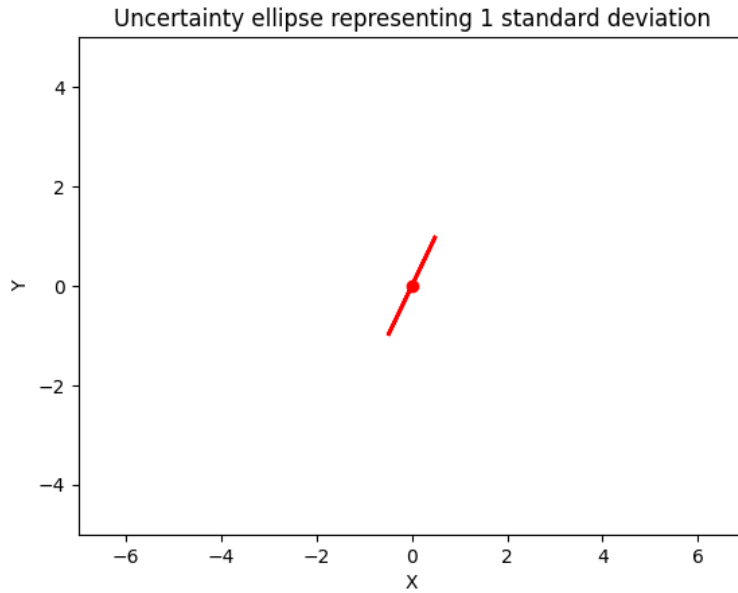


Figure 1: Uncertainty Ellipse at  $t=1$

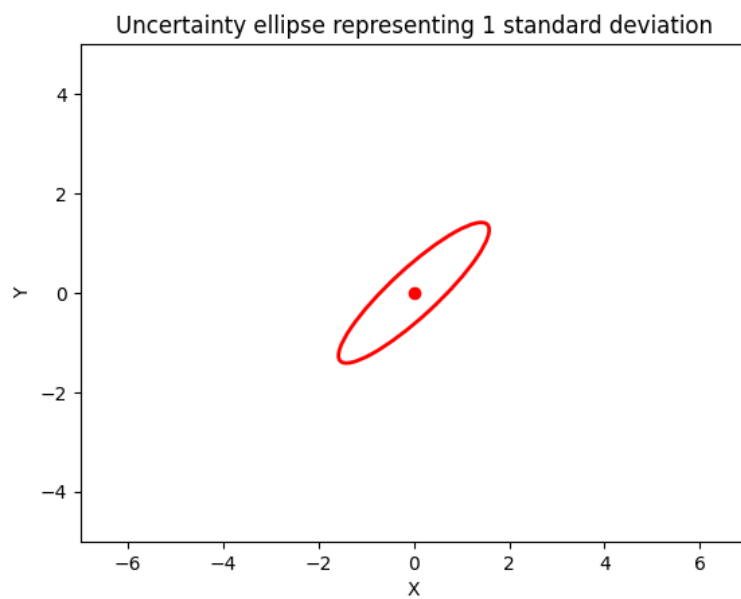


Figure 2: Uncertainty Ellipse at  $t=2$

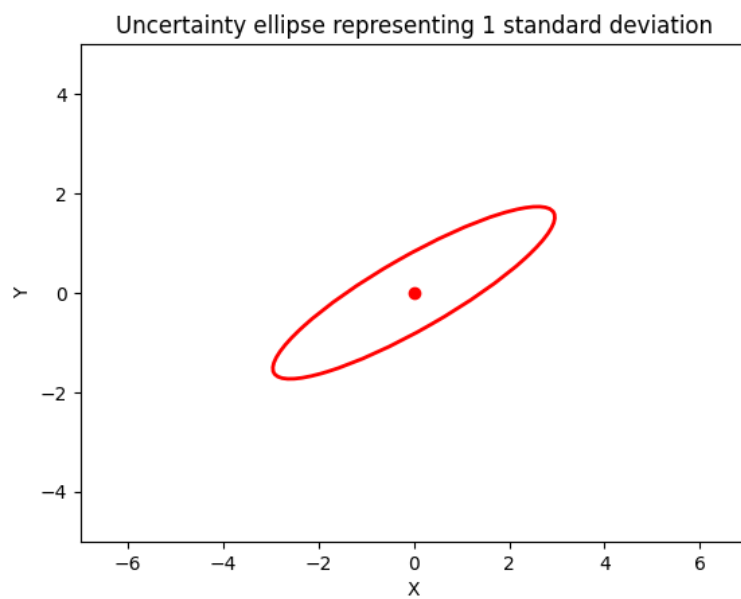


Figure 3: Uncertainty Ellipse at  $t=3$

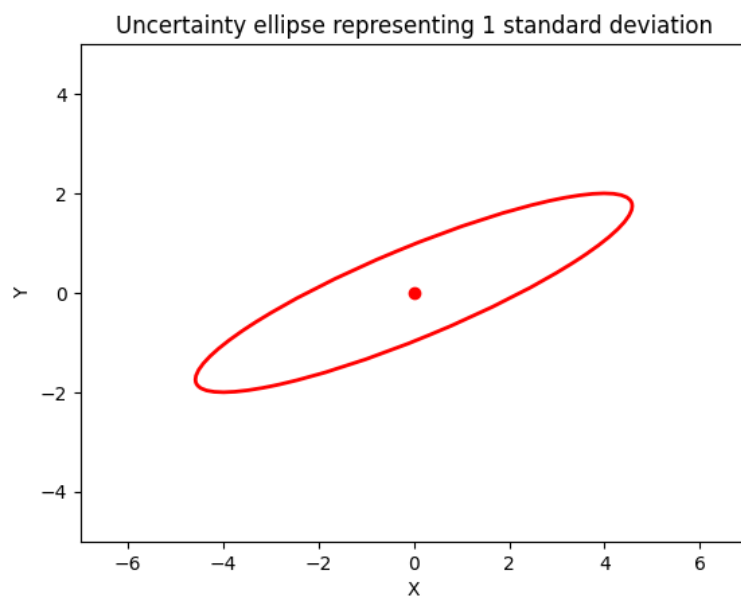


Figure 4: Uncertainty Ellipse at  $t=4$

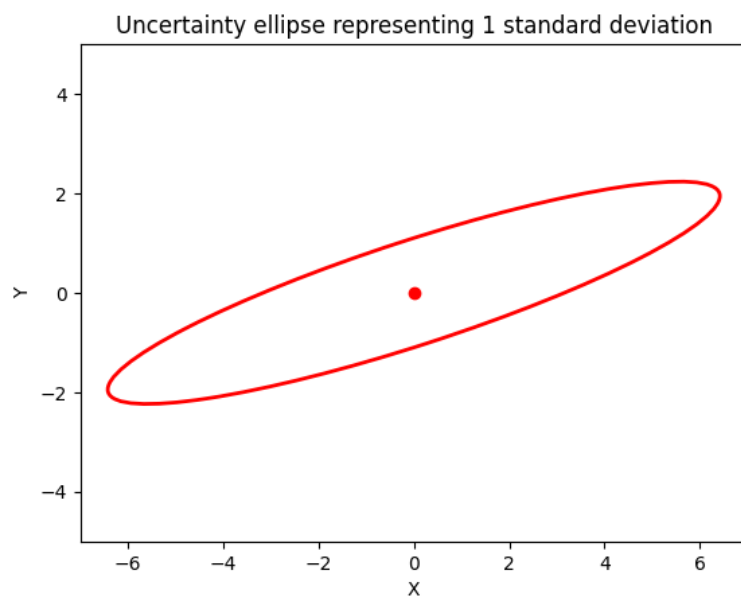


Figure 5: Uncertainty Ellipse at  $t=5$

## 2 Kalman Filter: Measurement

Prediction alone will result in greater and greater uncertainty as time goes on. Fortunately, your drone has a GPS sensor, which in expectation, measures the true position. However, the measurement is corrupted by Gaussian noise with covariance  $\sigma_{gps}^2 = 8.0$ .

**Question 2.1:** the measurement model:

$$C = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 8.0 \end{bmatrix}$$

**Question 2.2:** Suppose at time  $t = 5$ , we obtain  $z = 10$ . Then:

$$\sigma_{t5} = \begin{bmatrix} 41.25 & 12.5 \\ 12.5 & 5. \end{bmatrix}$$

$$\mu_{t5} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$\sigma_{t5} = \begin{bmatrix} 6.70050761 & 2.03045685 \\ 2.03045685 & 1.82741117 \end{bmatrix}$$

$$\mu_{t5} = \begin{bmatrix} 8.37563452 \\ 2.53807107 \end{bmatrix}$$

The simulated positions of the drone from  $t=1$  to  $t=5$  are:

$$x_{t1} = 0.0$$

$$x_{t2} = -0.27283527$$

$$x_{t3} = 0.49668245$$

$$x_{t4} = 3.00267159$$

$$x_{t5} = 6.22960415$$

The measurement at  $t=5$  is:

$$z_{t5} = 8.58188962$$

with a 37.75% error compared to ground truth

**Question 2.3:** at time  $t = 20$ , the expected error in terms of distance from the true position for each sensor fail probability is:

for  $p_{gps-fail} = 0.1$ , expected distance from true position = 2.10

for  $p_{gps-fail} = 0.5$ , expected distance from true position = 3.88

for  $p_{gps-fail} = 0.9$ , expected distance from true position = 16.14

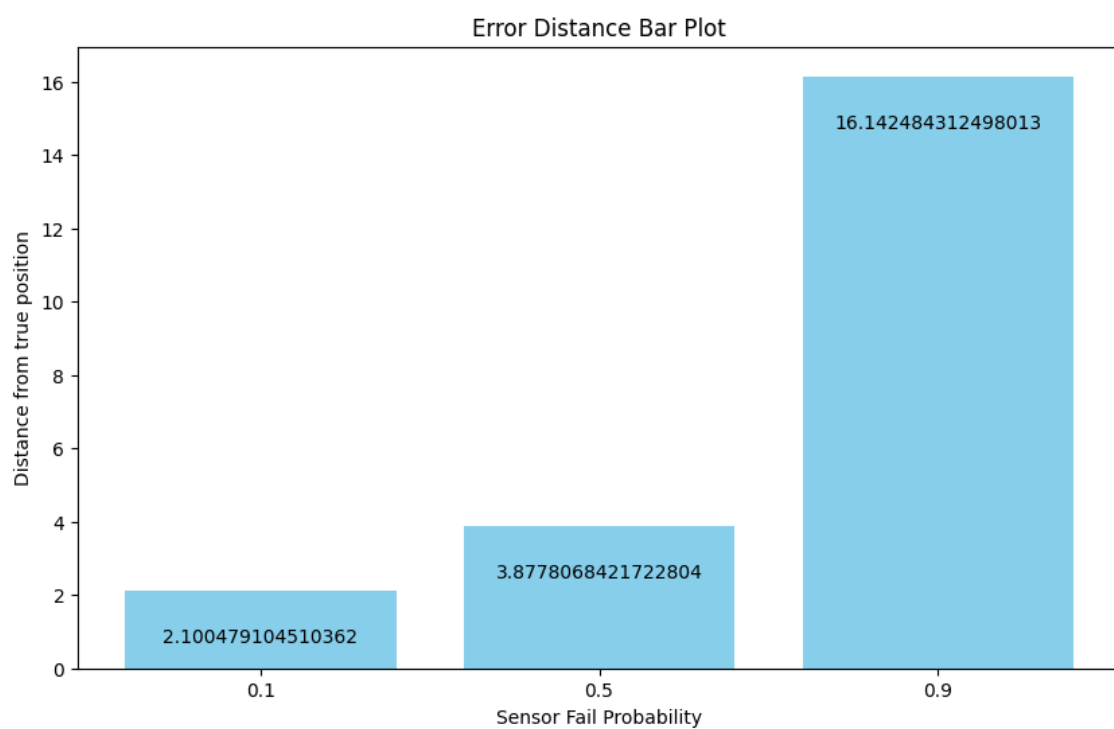


Figure 6: Error Distance Bar Plot for Each Sensor Fail Probability

### 3 Kalman Filter: Movement

**Question 3.1** The drone is now fully operational and can not only take measurements, but also issue motor commands in the form of acceleration commands to its propeller. For example, a command of 1.0 will increase the drone's velocity by 1.0. Revisit Question 1.3 to provide the matrix  $B$ . If at time  $t - 1$ , the drone's position and velocity are 5.0 and 1.0, compute the mean estimate for the state at time  $t$  given a motor command of 1.0. Your answer should be based on the constants provided but also include a random variable due to the wind effects. State the distribution of that random variable. The updated B matrix is:

$$\begin{bmatrix} 0.5 & 0.5 \\ 1.0 & 1.0 \end{bmatrix}$$

The u matrix is:

$$\begin{bmatrix} 1.0 \\ \mathcal{N}(\mu_{wind}, \sigma_{wind}^2) \end{bmatrix}$$

where the distribution of the random variable is  $\mathcal{N}(\mu_{wind}, \sigma_{wind}^2)$

The estimated  $\mu$  at time  $t$  is

$$\begin{bmatrix} 6.55246225 \\ 2.1049245 \end{bmatrix}$$

### 4 Extra Credit