CS 455/655 --Mobile Sensor Networks
Lecture Notes 11: Consensus-2
Weighted Average Consensus

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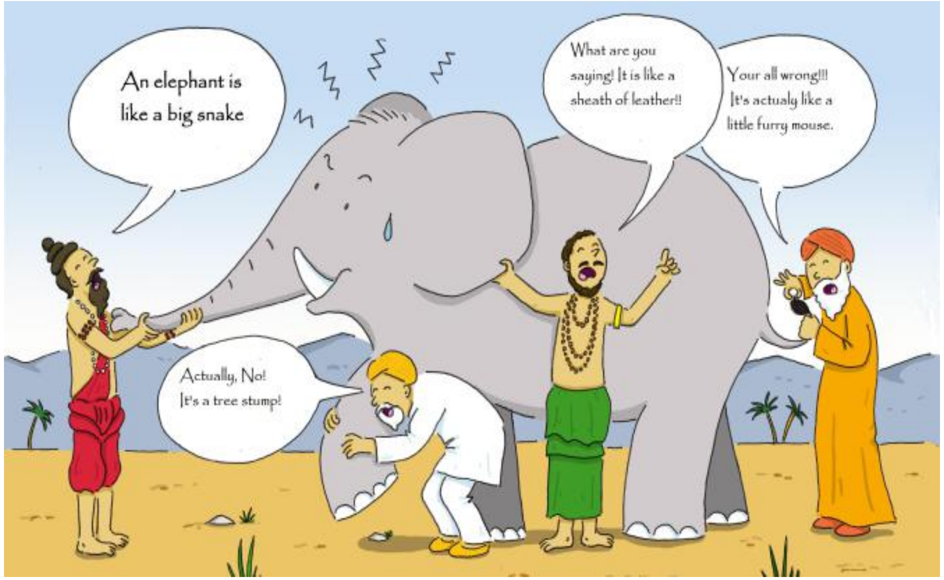
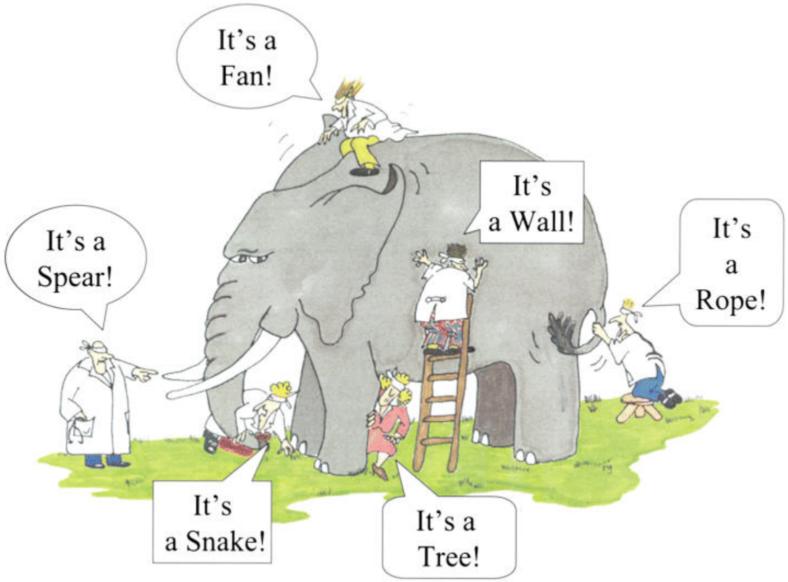
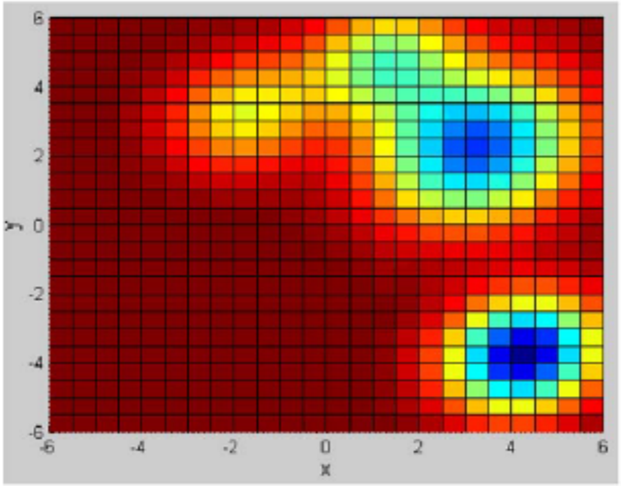
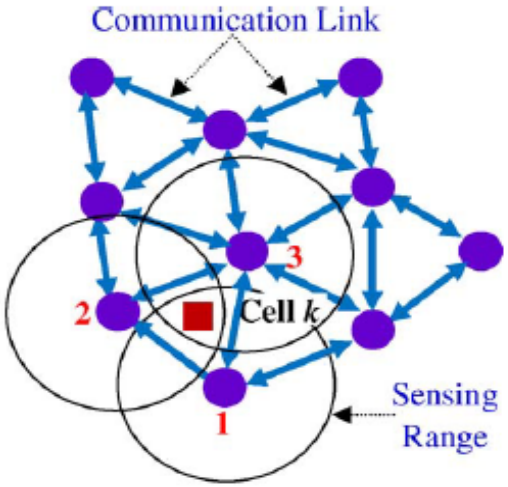
Consensus Filter for Scalar Field Mapping









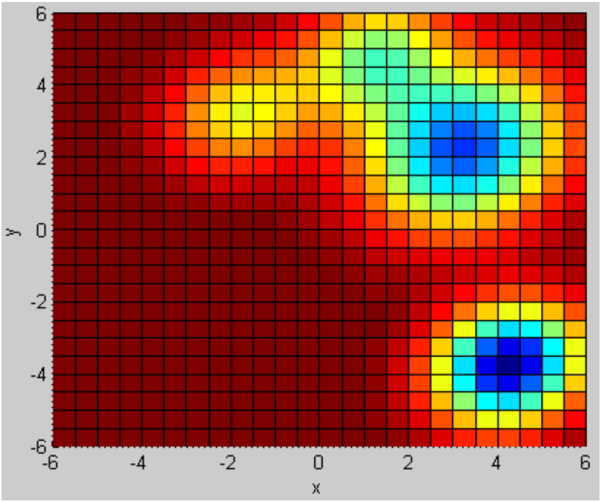
Model of the Scalar Field:

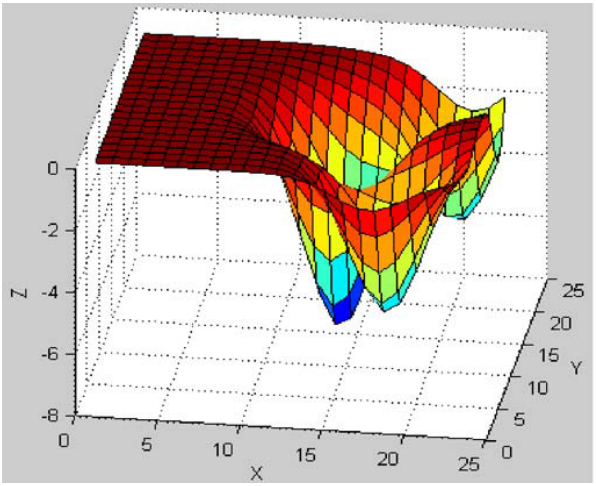
$$F = \Theta\Phi^T \quad \text{here } \Theta = [\theta_1, \theta_2, \ldots, \theta_K], \text{ and } \Phi = [\phi_1, \phi_2, \ldots, \phi_K]$$

$$\phi_j = \frac{1}{\sqrt{det(C_j)(2\pi)^2}} e^{\frac{1}{2}(x-\mu_x^j)C_j^{-1}(y-\mu_y^j)^T}, j \in [1, 2, \ldots, K]$$

here $[\mu_x^j \ \mu_y^j]$ is the mean of the distribution of function ϕ^j

C_j is covariance matrix $\quad C_j = \begin{bmatrix} (\sigma_x^j)^2 & c_j^o \sigma_x^j \sigma_y^j \\ c_j^0 \sigma_x^j \sigma_y^j & (\sigma_y^j)^2 \end{bmatrix}$





det: determinant of the matrix
Each Gaussian kernel can represent an oil leak or chemical leak

Measurement Model:

Each sensor make a measurement at cell k *th*:

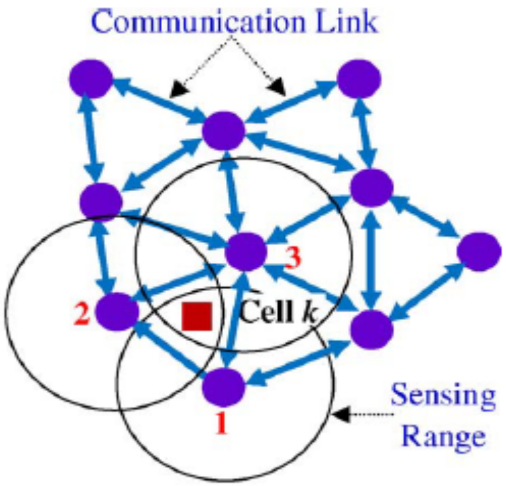
$$m_i^k(t) = O_i^k(t)[F^k(t) + n_i^k(t)]$$

here $n_i^k(t)$ is the Gaussian noise with zero mean and variance $V_i^k(t)$ at time step t.

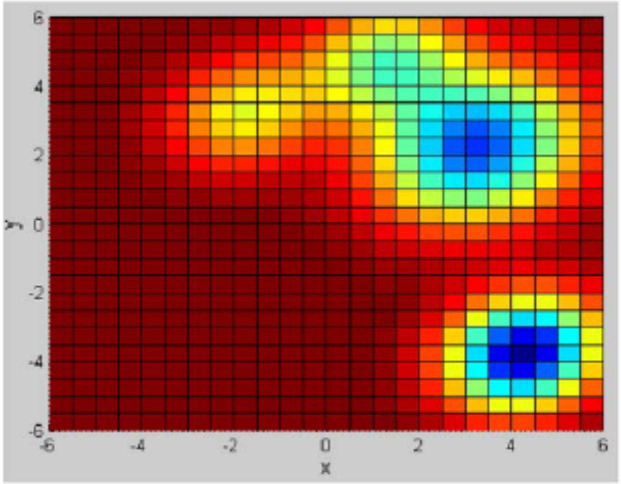
n_i^k is uncorrelated noise which satisfies $Cov(n_i^k(s), n_i^k(t)) = \begin{cases} V_i^k, & if \quad s = t \\ 0, & otherwise, \end{cases}$

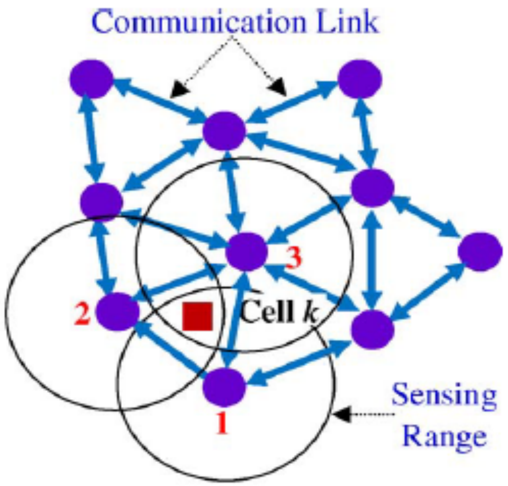
$O_i^k(t)$ is the observability of sensor node i at cell k

$$O_i^k(t) = \begin{cases} 1, & if \quad \|q_i(t) - q_c^k\| \le r_i^s \\ 0, & otherwise, \end{cases}$$



Noise variance model:





$$V_i^k(t) = \begin{cases} \frac{\|q_i(t)-q_c^k\|^2+c_v}{(r_i^s)^2}, & if \quad \|q_i(t)-q_c^k\| \le r_i^s \\ 0, & otherwise, \end{cases}$$

Weighted Average Consensus

- Each sensor node makes the observation at cell *k* at time step *t* based on its own confidence (weight).
- Need to find an agreement among the estimates at cell *k* from all sensor nodes in the network.

$$x_i^k(l+1) = w_{ii}^k(t)x_i^k(l) + \sum_{j \in N_i(t)} w_{ij}^k(t)x_j^k(l). \qquad (7)$$

$$x_i^k(l=0) = m_i^k(t)$$

$$N_i(t) = \{j \in \vartheta : \|q_j - q_i\| \leq r, \vartheta = \{1, 2, \ldots, n\}, j \neq i\}$$

Weight Design

Weighted Average

$$E^k(t) = \frac{\sum_{i=1}^{n} w_{ii}(t) m_i(t)}{\sum_{i=1}^{n} w_{ii}(t)}. \tag{8}$$

If (7) converges, we have $E_1^k(t) = E_2^k(t) = \ldots = E_n^k(t) = E^k(t)$; here, $E_i^k(t)$, with $i = 1, \ldots, n$, is the estimate of the field at cell k at time step t of the sensor node i. Therefore, our goal is to let

$$\lim_{l \to \infty} \left(x_i^k(l) - E^k(t) \right) \to 0. \tag{9}$$

We can write (9) in the following matrix form:

$$\lim_{l \to \infty} \mathbf{x}^k(l) = E^k(t)\mathbf{1}. \tag{10}$$

Here, $\mathbf{x}^k(l) = [x_1^k(l), x_2^k(l), \ldots, x_n^k(l)]_{n\times 1}^T$, and $\mathbf{1} = [1, 1, \ldots, 1]_{n\times 1}^T$.

Weight Design

$$x_i^k(l+1) = w_{ii}^k(t)x_i^k(l) + \sum_{j \in N_i(t)} w_{ij}^k(t)x_j^k(l). \qquad (7)$$

We can also write (7) in the following matrix form:

$$\mathbf{x}^k(l+1) = \mathbf{w}^k(t)\mathbf{x}^k(l) \qquad (11)$$

with the initial condition $\mathbf{x}^k(0) = \mathbf{m}^k(t)$; here, $\mathbf{m}^k(t) = [m_1^k(t), m_2^k(t), \ldots, m_n^k(t)]_{n\times 1}^T$.

To make (11) converge to $E^k(t)$, we need

$$\mathbf{w}^k(t) = \frac{1}{n}\mathbf{1}\mathbf{1}^T. \qquad (12)$$

Weight Design

In order to achieve this, we need to ensure that the sum of all weights including the vertex and edge weights at each node is equal to 1 or

$$w_{ii}^{k}(t) + \sum_{j \in N_i(t)} w_{ij}^{k}(t) = 1. \tag{13}$$

To satisfy this, we can design the weights of the consensus filters as follows:

Weight design 1: From (13), the vertex weight at node i is obtained as

$$w_{ii}^{k}(t) = 1 - \sum_{j \in N_i(t)} w_{ij}^{k}(t). \tag{14}$$

Weight Design 1

Here, $w_{ij}^k(l)$ is defined as

$$w_{ij}^k(t) = \frac{c_1^w}{V_i^k(t) + V_j^k(t)}, \quad i \neq j, \quad j \in N_i(t). \qquad (15)$$

Here, c_1^w is a designed factor. If none of the sensor nodes i and j observes cell k $(O_i^k(t) = O_j^k(t) = 0)$, then to avoid dividing by zero, the edge weight $w_{ij}^k(l)$ is set to zero.

Therefore, we have the following form of weight design:

$$w_{ij}^k(t) = \begin{cases} \frac{c_1^w}{V_i^k(t)+V_j^k(t)}, & \text{if } i \neq j, j \in N_i(t), \\ 1 - \sum_{j \in N_i(t)} w_{ij}^k(t), & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases} \qquad (16)$$

Weight Design 1

Now, we need to find c_1^w to satisfy (13). We know that $\min(V_i^k(t)) = \min(\|q_i(t) - q_c^k\|^2 + c_v/(r_i^s)^2) = c_v/(r_i^s)^2$ if $\|q_i(t) - q_c^k\| = 0$. Hence, we have

$$
min\left(V_i^k(t)\right) + min\left(V_j^k(t)\right)
= \begin{cases} \frac{2c_v}{(r^s)^2}, & \text{if } \left(r_i^s = r_j^s = r^s\right) \\ \frac{c_v}{\left(r_i^s\right)^2} + \frac{c_v}{\left(r_j^s\right)^2}, & \text{otherwise.} \end{cases} \qquad (17)
$$

To satisfy (13), we need

$$
0 < \sum_{j \in N_i(t)} w_{ij}^k(t) < 1 \Rightarrow 0 < \sum_{j \in N_i(t)} \frac{c_1^w}{V_i^k(t) + V_j^k(t)} < 1
$$

$$
0 < c_1^w < \frac{V_i^k(t) + V_j^k(t)}{|N_i(t)|}. \qquad (18)
$$

Weight Design 1

Here, $|N_i(t)|$ is the number of neighbors of the sensor node i at time t, and from (17) and (18), we can select c_1^w as

$$\begin{cases} 0 < c_1^w < \frac{2c_v}{(r_i^s)^2|N_i(t)|}, & \text{if } r_i^s = r_j^s = r^s \\ 0 < c_1^w < \frac{1}{|N_i(t)|}\left(\frac{c_v}{\left(r_i^s\right)^2} + \frac{c_v}{\left(r_j^s\right)^2}\right), & \text{otherwise.} \end{cases} \tag{19}$$

Weight Design 2

$$\boxed{w_{ii}^k(t) + \sum_{j \in N_i(t)} w_{ij}^k(t) = 1. \qquad (13)}$$

Weight design 2: From (13), by assigning the same value to all edge weights, we obtain

$$w_{ij}^k(t) = \frac{1 - w_{ii}^k(t)}{|N_i(t)|}. \qquad (20)$$

Here, $w_{ii}^k(t)$ is defined as

$$w_{ii}^k(t) = \frac{c_2^w}{V_i^k(t)} \qquad (21)$$

where c_2^w is a designed factor. If the sensor node i does not observe cell k ($O_i^k(t) = 0$), then the vertex weight $w_{ii}^k(t)$ is set to zero.

Weight Design 2

Therefore, we have the following weight design:

$$w_{ij}^k(t) = \begin{cases} \frac{c_2^w}{V_i^k(t)}, & \text{if } i = j \\ \frac{1-w_{ii}^k(t)}{|N_i(t)|}, & \text{if } i \neq j, j \in N_i(t) \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

Now, we discuss how to select constant c_2^w. In order to satisfy (13), we need the following condition:

$$0 < \frac{c_2^w}{V_i^k(t)} < 1. \quad (23)$$

Since $\min(V_i^k(t)) = c_v/(r_i^s)^2$ when $\|q_i(t) - q_c^k\| = 0$, we have

$$0 < \frac{c_2^w}{\frac{c_v}{(r_i^s)^2}} < 1 \Rightarrow 0 < c_2^w < \frac{c_v}{(r_i^s)^2}. \quad (24)$$

Summary: Weight Design 1 and 2

$$
w_{ij}^k(l) = \begin{cases} \frac{c_1^w}{V_i^k(t)+V_j^k(t)}, & if \quad i \neq j, j \in N_i(t), \\ 1 - \sum_{j \in N_i(t)} w_{ij}^k(l), & if \quad i = j, \\ 0, & otherwise. \end{cases}
$$

$or,$

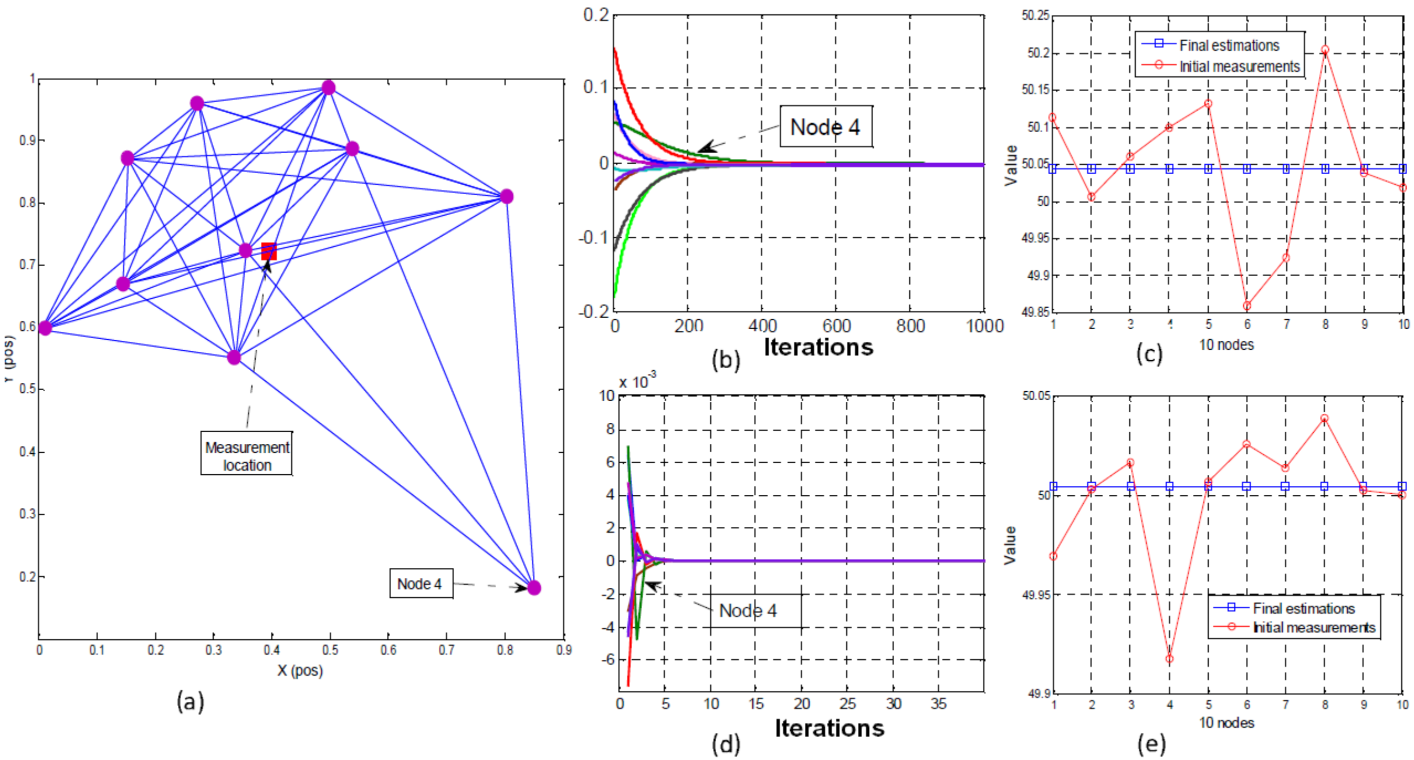
$$
w_{ij}^k(l) = \begin{cases} \frac{c_2^w}{V_i^k(t)}, & if \quad i = j, \\ \frac{1-w_{ii}^k(l)}{|N_i(t)|}, & if \quad i \neq j, j \in N_i(t), \\ 0, & otherwise. \end{cases}
$$

Weighted Average Consensus

$$
w_{ij}^k(l) = \begin{cases} \frac{c_1^w}{V_i^k(t)+V_j^k(t)}, & if \quad i \neq j, j \in N_i(t), \\ 1 - \sum_{j \in N_i(t)} w_{ij}^k(l), & if \quad i = j, \\ 0, & otherwise. \end{cases}
$$

$or,$

$$
w_{ij}^k(l) = \begin{cases} \frac{c_2^w}{V_i^k(t)}, & if \quad i = j, \\ \frac{1-w_{ii}^k(l)}{|N_i(t)|}, & if \quad i \neq j, j \in N_i(t), \\ 0, & otherwise. \end{cases}
$$



Average Consensus

- Each sensor node has its own confidence of the measurement of the value of the scalar field at each cell at each time step *t.*
- Need to find an agreement among the confidences of sensor nodes.

$$y_i^k(l+1) = w_{ii}^k(l)y_i^k(l) + \sum_{j \in N_i(t)} w_{ij}^k(l)y_j^k(l)$$

$$y_i^k(l=0) = w_{ii}^k(t)$$

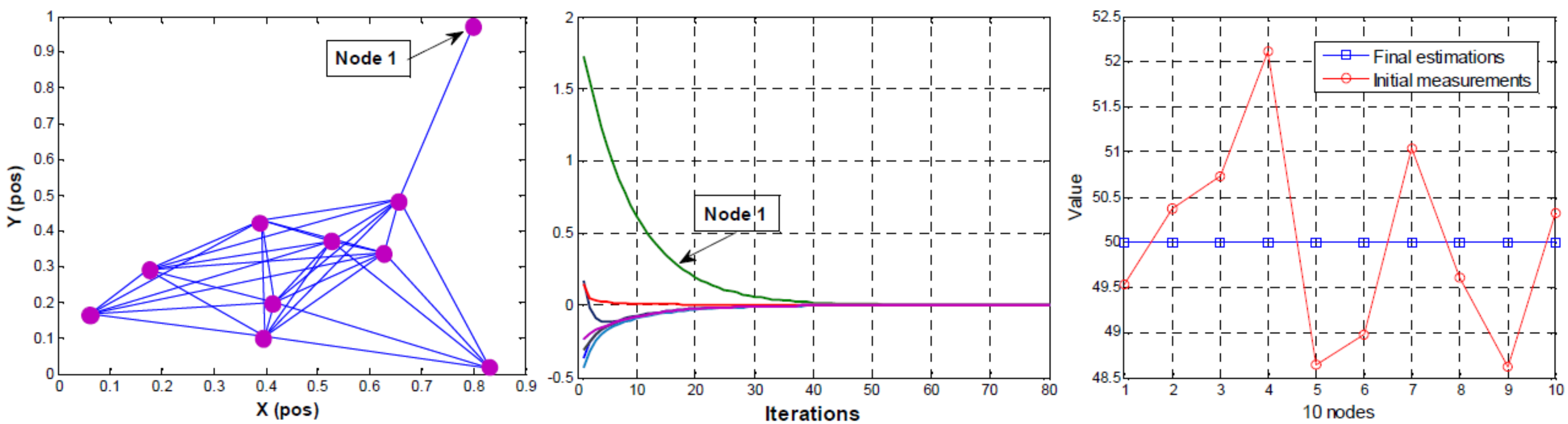
Using Metropolis weight (Xiao *et al.* 2005)

$$w_{ij}^k(l) = \begin{cases} \frac{1}{1+max(|N_i(t)|,|N_j(t)|)}, & if \quad i \neq j, j \in N_i(t) \\ 1 - \sum_{j \in N_i(t)} w_{ij}^k(l), & if \quad i = j, \\ 0, & \quad otherwise. \end{cases}$$

Average Consensus

$$y_i^k(l+1) = w_{ii}^k(l) y_i^k(l) + \sum_{j \in N_i(t)} w_{ij}^k(l) y_j^k(l) \qquad y_i^k(l=0) = w_{ii}^k(t)$$

$$w_{ij}^k(l) = \begin{cases} \frac{1}{1+max(|N_i(t)|,|N_j(t)|)}, & if \quad i \neq j, j \in N_i(t) \\ 1 - \sum_{j \in N_i(t)} w_{ij}^k(l), & if \quad i = j, \\ 0, & \quad otherwise. \end{cases}$$



(a) (b) (c)