BUT WAIT...THAT'S NOT IN MY PREMISES? +OTHER WEIRD STUFF

WEEK 2 . DEEPER DIVE

Consider $\vDash (p \lor \neg p)$. There aren't any premises!

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p	$\vDash (p \lor \neg p)$
T	T T F T
F	F T T F

Consider $\vDash (p \vee \neg p)$. There aren't any premises!

p	$\vDash (p \vee \neg p)$
T	T T F T
F	F T T F

There are no premises. What row do we check?

Consider $\ \vDash (p \lor \neg p)$. There aren't any premises!

p	$\vDash (p \lor \neg p)$
T	✔ T T F T
F	✔ F T T F

There are no premises. What row do we check?

Consider $\vDash (p \lor \neg p)$. There aren't any premises!

p	$\vDash (p \lor \neg p)$
T	✔ T T F T
F	✔ F T T F

There are no premises. What row do we check?

Any argument with a tautology as a conclusion is valid.

Consider $\vDash (p \lor \neg p)$. There aren't any premises!

p	$\vDash (p \lor \neg p)$
T	✔ T T F T
F	✔ F T T F

There are no premises. What row do we check?

Any argument with a tautology as a conclusion is valid.

Think about it as saying: "$(p \lor \neg p)$ will be valid no matter what the premises are."

Consider $(p \wedge \neg p) \vDash p$

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p	$(p \wedge \neg \, p) \vDash p$
T	T F F T T
F	F F T F F

Consider $(p \wedge \neg p) \vDash p$

p	$(p \wedge \neg p) \vDash p$
T	T F F T T
F	F F T F F

Consider $(p \land \neg p) \vDash p$

p	$(p \land \neg \; p) \vDash p$
T	T F F T T
F	F F T F F

Our premise is a contradiction. What row do we check?

Consider $(p \wedge \neg p) \vDash p$

p	$(p \wedge \neg \ p) \vDash p$					
T	T	F	F	T	✔	T
F	F	F	T	F	✔	F

Our premise is a contradiction. What row do we check?

Consider $(p \land \neg p) \vDash p$

p	$(p \land \neg\ p) \vDash p$
T	T F F T ✔ T
F	F F T F ✔ F

Our premise is a contradiction. What row do we check?

Any argument with a contradiction as a premise is valid.

TRUTH TABLE SHORTCUT

p	q	$(p \rightarrow q)$,	$(p \rightarrow$	$\neg q)$	$\vDash$	$\neg p$
T	T	T	F	F		F
T	F	F	T	T		F
F	T	T	T	F		T
F	F	T	T	T		T

TRUTH TABLE SHORTCUT

p	q	$(p \rightarrow q)$,	$(p \rightarrow \neg q)$		$\vDash$	$\neg p$
T	T	T	F	F		F
T	F	F	T	T		F
F	T	T	T	F	✔	T
F	F	T	T	T	✔	T

TRUTH TABLE SHORTCUT

p	q	$(p \rightarrow q)$,	$(p \rightarrow \neg q)$	⊨	$\neg p$
T	T	T	F F	✓	F
T	F	F	T T	✓	F
F	T	T	T F	✓	T
F	F	T	T T	✓	T

TRUTH TABLE SHORTCOMINGS…

Suppose we're considering the argument:

P, Q, R, S, T ⊨ P

We'd have to check all the rows on which the premises are true.

So we'd need to write out the whole truth table. That's $2^5 = 32$ rows!

Surely there's an easier way?
The argument is so simple after all!