# WHAT ARE TRUTH TABLES?

WEEK 1. TOPIC INTRODUCTION

#### RECALL:

#### $(\neg C \land D)$

#### Translation manual:

C is "He's a white supremacist" and

D is "He's a racist."

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IS  $(\neg C \land D)$  TRUE OR FALSE?

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To answer, we'd need to know whether C is true or false and whether D is true or false. Then, we could determine whether  $(\neg C \land D)$  is true or false.

#### $\overline{\mathsf{IS}(\neg C \land D)}$ TRUE OR FALSE?

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But what if we don't know?

Is there a way we can represent all the possibilities?

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C is true and D is true

C is true and D is false

C is false and D is true

C is false and D is false

A truth table allows us to consider the truth value of a wff under all possible assignments of truth values to its atomic constituents.

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YOU CAN THINK OF EACH ROW OF THE TRUTH TABLE AS A POSSIBLE WORLD.

## $\overline{\mathsf{IS}\;(\neg C \land D)}\; \overline{\mathsf{TRUE}\;\mathsf{OR}\;\mathsf{FALSE?}}$

C is true and D is true
C is true and D is false
C is false and D is true
C is false and D is false

C is true and D is true
C is true and D is false
C is false and D is true
C is false and D is false

C is true and D is true
C is true and D is false
C is false and D is true
C is false and D is false

С	D	$\neg C$
Т	Т	F
Τ	F	F
F	Т	Т
F	F	Т



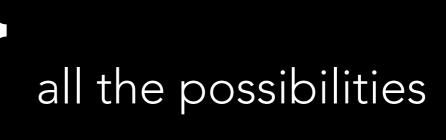
С	D	$\neg C$	$(\neg C \land D)$
Т	T	F	F
Т	F	F	F
F	Τ	Т	Т
F	F	Т	F

C is true and D is true
C is true and D is false
C is false and D is true
C is false and D is false

С	D	$\neg C$	$(\neg C \land D)$
Т	Т	F	F
Т	F	F	F
F	Т	Т	Т
F	F	Т	F

C is true and D is true
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С	D	$\neg C$	$(\neg C \land D)$
Т	Т	F	F
Т	F	F	F
F	Т	Т	Т
F	F	Т	F



С	D	$\neg C$	$(\neg C \land D)$
Т	T	F	F
Т	F	F	F
F	Т	Т	Т
F	F	Т	F



С	D	$\neg C$	$(\neg C \land D)$
Τ	Т	F	F
Τ	F	F	F
F	Т	Т	Т
F	F	Т	F



С	D	$\neg C$	$(\neg C \land D)$
Т	T	F	F
Т	F	F	F
F	Τ	Т	Т
F	F	Т	F



 Λ (and)
 A
 B
 A Λ B

 T
 T
 T

 T
 F
 F

 F
 T
 F

 F
 F
 F

∧ (and)	А	В	А∧В
			_
	T	T	Т
	Т	F	F
	F	Т	F
	F	F	F
V (or)	А	В	ΑVΒ
	Т	Т	Т
	Т	F	Т
	F	Т	Т
	F	F	F

∧ (and)	А	В	ΑΛВ	
	Т	Т	Т	
	Т	F	F	
	F	Т	F	
	F	F	F	
				you can see we're
V (or)	А	В	AVB	not using the "exclusive or" here!
	Т	Т	Т	
	Т	F	Т	
	F	Т	Т	

∧ (and)	А	В	А∧В
			_
	T	T	Т
	Т	F	F
	F	Т	F
	F	F	F
V (or)	А	В	ΑVΒ
	Т	Т	Т
	Т	F	Т
	F	Т	Т
	F	F	F

∧ (and)	А	В	ΑΛB		
	Τ	Т	Т		
	Τ	F	F	¬ (ned	gation)
	F	Т	F	•	<i>,</i>
	F	F	F	А	ΠА
				Τ	F
V (or)	А	В	ΑVΒ	F	Т
	Τ	Т	Т		
	Τ	F	Т		
	F	Т	Т		
	F	F	F		

→ (conditional)

 $\begin{array}{c|cccc} A & B & A \longrightarrow B \\ \hline T & T & T \\ \hline T & F & F \\ \hline F & T & T \\ \hline F & F & T \\ \end{array}$ 

→ (conditional)

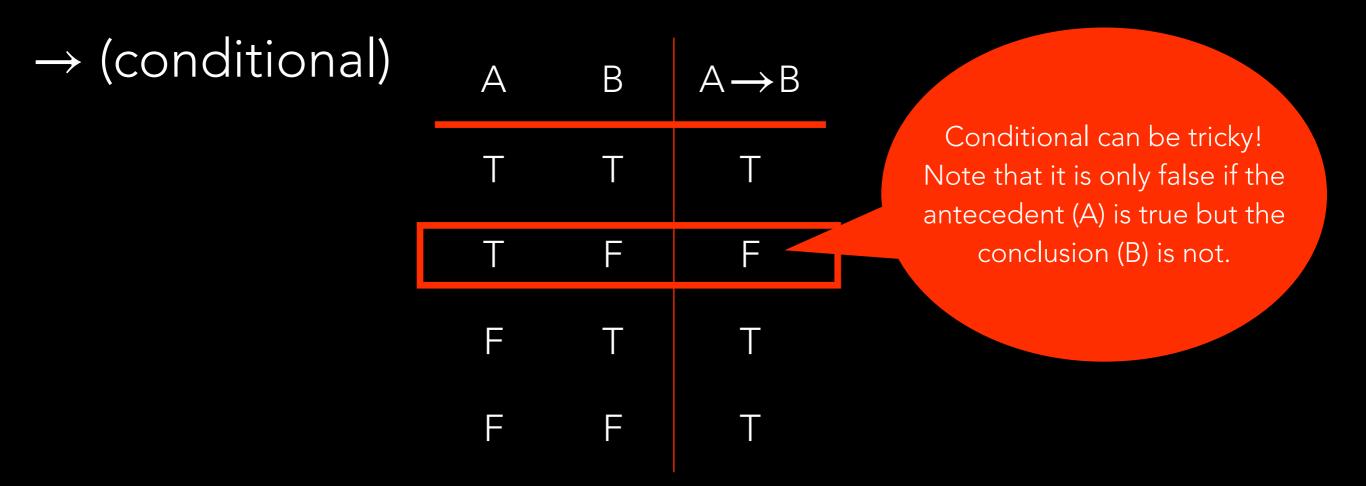
A B  $A \rightarrow B$ T T T

T F T

F T T

Conditional can be tricky!

Note that it is only false if the antecedent (A) is true but the conclusion (B) is not.



Example: Suppose I tell my sister
"If you do your homework, I'll give you a cookie."

To understand implication, think: "When does she have the right to be mad?" When she does her homework but I don't give her a cookie!

But I haven't said anything about the case where she doesn't do her homework. Maybe she goes and makes cookies herself!

→ (conditional)

 $\begin{array}{c|cccc} A & B & A \longrightarrow B \\ \hline T & T & T \\ \hline T & F & F \\ \hline F & T & T \\ \hline F & F & T \end{array}$ 

Example: Suppose I have the following script:

```
| if x :
| a = 2
| print(a)
```

Think of implication like you've triggered the "if" condition. If x is True, the script will print "2." If x is False, the script might still print "2" because a was already 2 but it might print something else.

→ (conditional)

 $\begin{array}{c|cccc} A & B & A \longrightarrow B \\ \hline T & T & T \\ \hline T & F & F \\ \hline F & T & T \\ \hline F & F & T \\ \end{array}$ 

→ (conditional)  $A \rightarrow B$ B ↔ (biconditional)  $A \rightarrow B \rightarrow A ((A \rightarrow B^{AA})^{ARE} \Lambda^{TH} (B \rightarrow A))$ 

→ (conditional)	А	В	Д	$\rightarrow$ B		
	Т	Т		Τ		
	Т	F		F		
	F	Т		Т		
	F	F		Τ		This is like $((A \rightarrow B) \land (B \rightarrow A))$
↔ (biconditional)		^	1			
↔ (biconditional)		А	В	$A \rightarrow B$	$B \rightarrow A$	$((A \rightarrow B^{AT})^{ARE} / (B \rightarrow A))$
↔ (biconditional)				A → B T		$((A \rightarrow B^{\text{HAT}})^{\text{ARE T}} \land (B \rightarrow A))$
↔ (biconditional)		Т	Τ			$((A \rightarrow B^{\text{HAT}})^{\text{ARE TATH TABLE}}B \rightarrow A))$ $T$
↔ (biconditional)		T	T F	Т	T	Т

→ (conditional)	Α	В	A	$\rightarrow$ B		
	Т	Т		Т		
	Τ	F		F		
	F	Т		Т		
	F	F		Т		$\wedge$
↔ (biconditional)						
		А	В	$A \rightarrow B$	$B \longrightarrow A$	$((A \rightarrow B^{A})^{ARE} A^{T} A^{TH} A^{RE} B \rightarrow A))$
		Т	Т	Т	Т	Т
		Т	F	F	Т	F
		F	Т	Т	F	F
		F	F	Т	Т	Т

→ (conditional)

 $\begin{array}{c|cccc} A & B & A \longrightarrow B \\ \hline T & T & T \\ \hline T & F & F \\ \hline F & T & T \\ \hline F & F & T \end{array}$ 



← (biconditional)

А	В	A↔B
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

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