


# What are proofs?

Week 3 . Topic Introduction

Thus far, we have been checking the validity of arguments using truth tables.

This week, we begin to learn a new method: proofs.



Why do we need  
a new method  
when truth tables  
work just fine?

# Recall:

## TRUTH TABLE SHORTCOMINGS...

Suppose we're considering the argument:

$$P, Q, R, S, T \models P$$

We'd have to check all the rows on which the premises are true.

So we'd need to write out the whole truth table. That's  $2^5 = 32$  rows!

Surely there's an easier way?

The argument is so simple after all!

Besides practical reasons, proofs tend to give a better understanding of the argument.

Think back to the practice problems last week: once the arguments became more complex, did you really understand why the arguments were (in)valid?

# Proofs: the basics

A proof is a sequence of wffs of a formal system, each member of which is an assumption or an axiom, or is derived from previous wffs by the inference rules of that system.

To start writing our own proofs, we need to know the inference rules of our system: sentential logic! The proof system we're learning is called "natural deduction."

# Proofs: the basics


Proofs give us another notion of validity: deductive validity.

An argument is **deductively valid** just in case we can *prove* its conclusion from its premise(s) .

Deductive validity gets a fancy symbol too:  $\vdash$

If we want to say  $A, B, \dots, C \therefore D$  is deductively valid, we write:

$$A, B, \dots, C \vdash D.$$

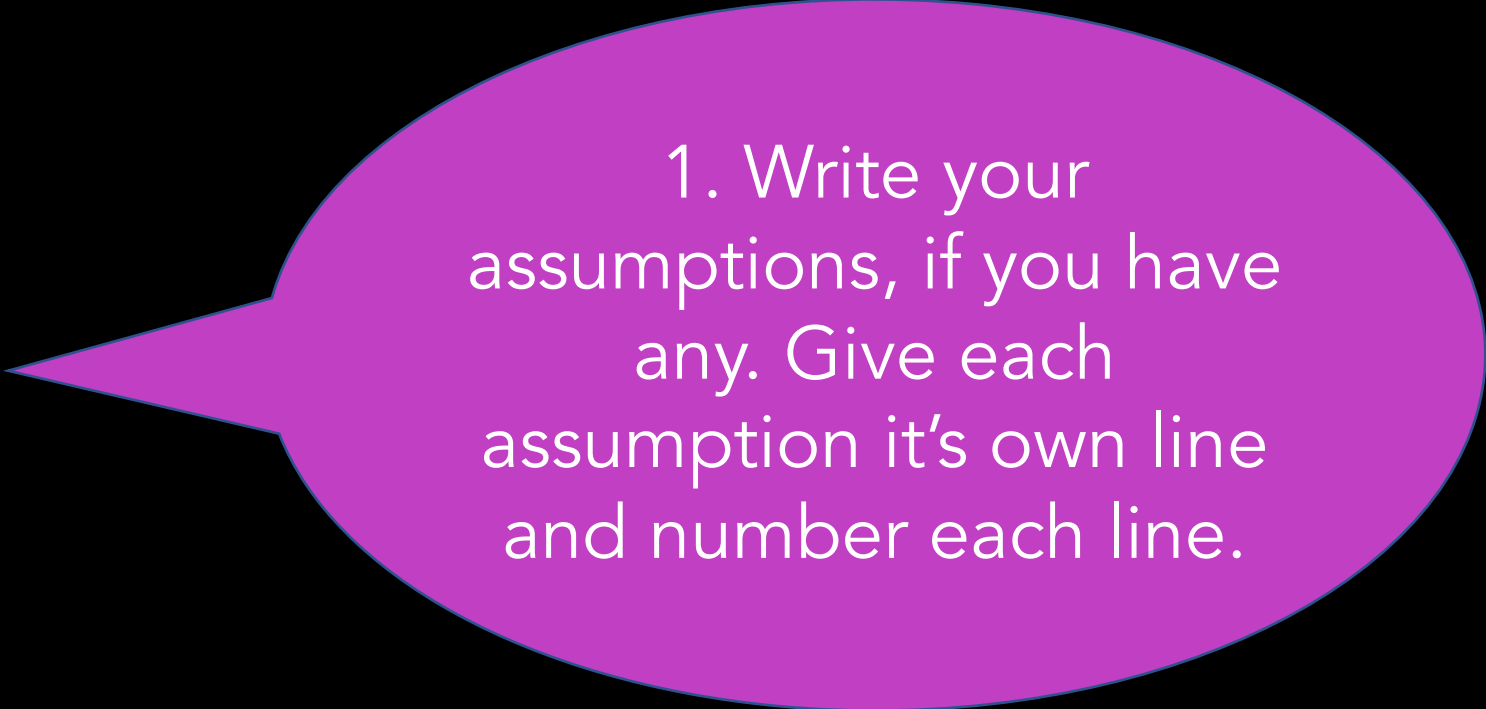


What are the  
inference  
rules...?

# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

1.  $(A \wedge B)$



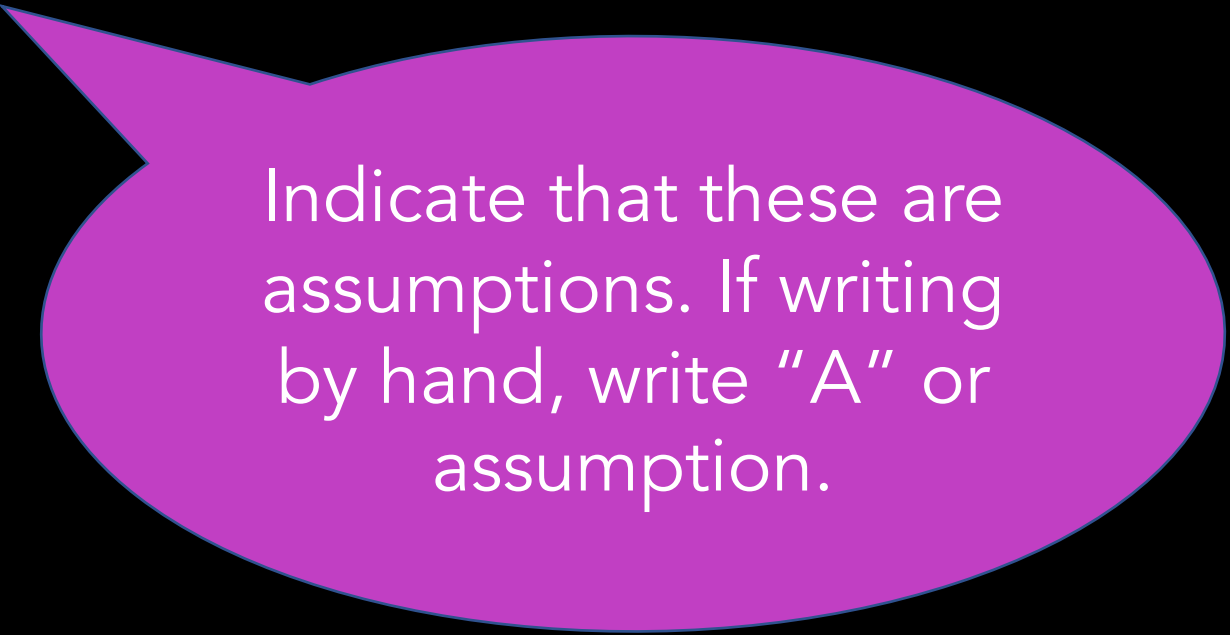
1. Write your assumptions, if you have any. Give each assumption its own line and number each line.



# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

1.  $(A \wedge B)$  Assumption

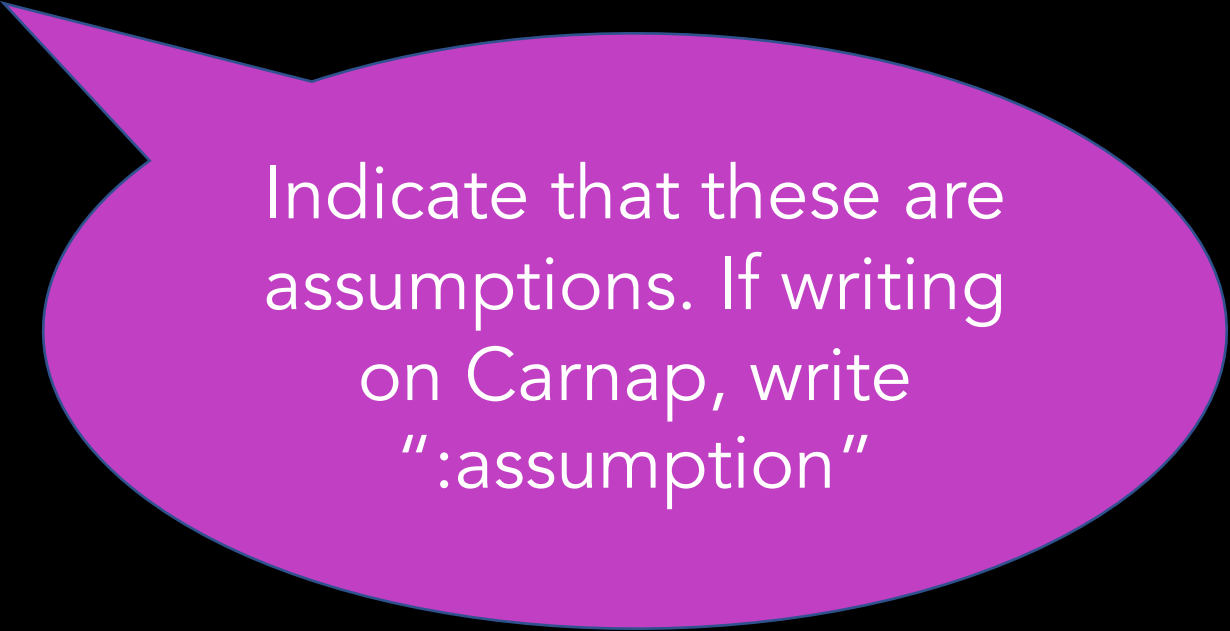


Indicate that these are assumptions. If writing by hand, write "A" or assumption.

# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

1.  $(A \wedge B)$ :assumption



Indicate that these are assumptions. If writing on Carnap, write ":assumption"

# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

1.  $(A \wedge B)$  Assumption

2.  $A$   $E \wedge, 1$

2. Use the inference rules to derivate the conclusion desired. For each line, indicate the inference rule and the lines you're using the inference rule on.

# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

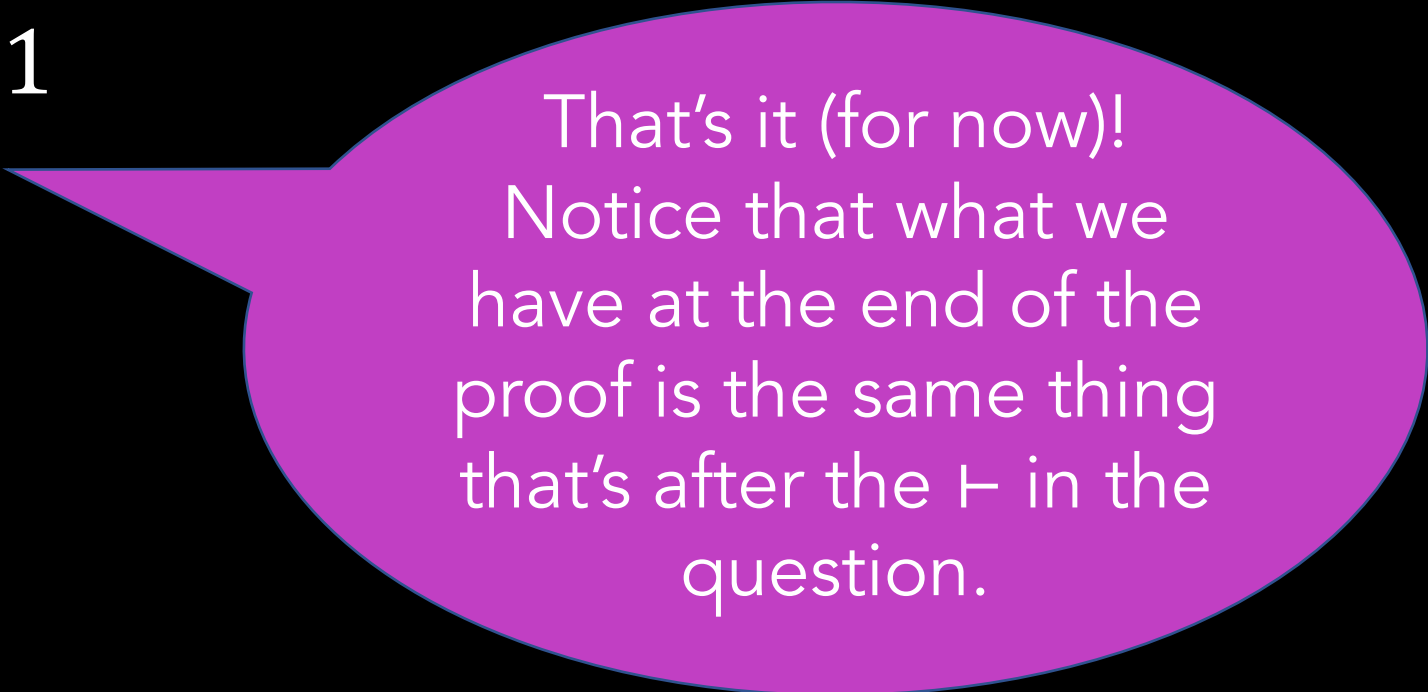
1.  $(A \wedge B)$ : assumption
2.  $A$  :E/ $\wedge$ , 1

If on Carnap, write the inference rule and line numbers after a colon.

# Proofs: how to set them up

Ex. Provide a proof for:  $(A \wedge B) \vdash A$

1.  $(A \wedge B)$ : assumption
2.  $A$  :E/ $\wedge$ , 1



That's it (for now)!  
Notice that what we  
have at the end of the  
proof is the same thing  
that's after the  $\vdash$  in the  
question.