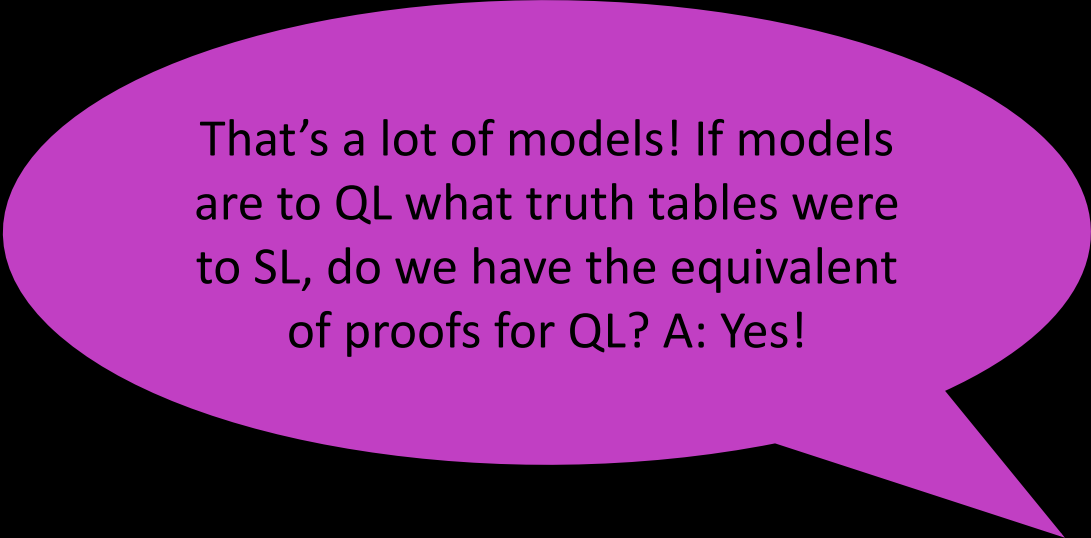


How do I do proofs in quantified logic?

Week 7. Topic Introduction

Recall

Validity via models: a valid argument is an argument in which the conclusion is true *in every model* in which the premises are true.



That's a lot of models! If models are to QL what truth tables were to SL, do we have the equivalent of proofs for QL? A: Yes!

*Our goal now is to introduce the proof
system for quantified logic*

Proof system for quantified logic

Our new proof system for QL has all of the rules from the old system SL, including derived rules, as well as **some new ones**.

Before introducing the new rules though, let's see exactly how we can use our old rules with formulas \forall for quantified logic


Show: $\neg Pa \wedge Qb \vdash Qb$

1. $\neg Pa \wedge Qb$:assumption
2. Qb : E \wedge 1

Show: $Pa \wedge \neg Pb, \neg Pb \rightarrow Qb \vdash Qb$

1. $Pa \wedge \neg Pb$:assumption
2. $\neg Pb \rightarrow Qb$:assumption
3. $\neg Pb$:E \wedge 1
4. Qb :E \rightarrow 2,3

Show: $\neg Pa \wedge Rb \vdash Ra$

1. $\neg Pa \wedge Rb$:assumption
2. Pa :E \wedge 1

We have that object b has property R. There's no way to go from that to object a having property R!

Rules in QL

- Our old rules and shortcuts from SL showed us what inferences we could draw based on the main connective of the formulas involved
- Ex: $E \wedge$ says we can take a formula like $(P \wedge Q)$ and infer P and we can infer Q . This holds even when 'P' is a formula in QL like Pa .
- What we now need for quantified logic are rules for our new operators: the quantifiers!