HOW DO I TRANSLATE ENGLISH INTO (SENTENTIAL) LOGIC? PART 2

WEEK 1 . DEEPER DIVE

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harder translations . well-formed formulas . main connective

RECALL:

CONNECTIVES ARE OPERATORS WHICH COMBINE ONE OR MORE SENTENCES TO GENERATE COMPLEX SENTENCES.

THERE ARE 5: $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\neg$

SO HOW DO WE KNOW WHICH CONNECTIVE TO USE?

INDICATOR WORDS

$\wedge$	AND, ALTHOUGH, BUT, UNLESS
$\vee$	OR, EITHER…OR
$\rightarrow$	IF…THEN, ONLY IF
$\leftrightarrow$	IF AND ONLY IF, NECESSARY AND SUFFICIENT, JUST IN CASE
$\neg$	IT'S NOT THE CASE THAT…

HOW SHOULD I INTERPRET $\neg C \wedge D$?

Translation manual:

C is "He's a white supremacist" and

D is "He's a racist."

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Does it mean:

“He’s not a white supremacist but he’s a racist”?

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Does it mean:

“He’s not a white supremacist but he’s a racist”?

Or does it mean:

“It’s false that he’s a white supremacist and he’s a racist”?

HOW DO I COMPLEX UNAMBIGUOUS SENTENCES WITH CONNECTIVES?

Use the rules for well-formed formulas (wffs)

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   $(\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi),$ and $(\phi \leftrightarrow \psi)$

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4. *and nothing else!*

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4. *and nothing else!*

WHY'D I SWITCH FROM "A" TO "ϕ"?

"A" IS A SENTENCE LETTER WHEREEAS "ϕ" IS A WELL-FORMED FORMULA ITSELF. THIS MEANS IT COULD BE "A" BY RULE 1, BUT IT COULD ALSO BE "$(A \wedge B)$"!

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$(\neg C \wedge D)$: "He's not a white supremacist but he's a racist"

$\neg(C \wedge D)$: "It's false that he's a white supremacist and he's a racist"

EXAMPLES

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$\sim R$ and $(P \leftrightarrow (P \land Q))$ are wff so $(\sim R \rightarrow (P \leftrightarrow (P \land Q)))$ is as well.

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$\sim R$ is a wff but $(P \leftrightarrow P \wedge Q)$ is not! So $(\sim R \rightarrow (P \leftrightarrow P \wedge Q))$ is not.

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ASK YOURSELF: IS THIS UNIQUELY READABLE?

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