How do I do proofs in quantified logic?

Week 7. Topic Introduction

Recall

Validity via models: a valid argument is an argument in which the conclusion is true *in every model* in which the premises are true.

That's a lot of models! If models are to QL what truth tables were to SL, do we have the equivalent of proofs for QL? A: Yes!

Our goal now is to introduce the proof system for quantified logic

Proof system for quantified logic

Our new proof system for QL has all of the rules from the old system SL, including derived rules, as well as some new ones.

Before introducing the new rules though, let's see exactly how we can use our old rules with formulas vfor quantified logic

Show: $\neg Pa \land Qb \vdash Qb$

1. $\neg Pa \land Qb$:assumption
2. Qb : E $\land$ 1

Show: $Pa \land \neg Pb, \neg Pb \rightarrow Qb \vdash Qb$

1. $Pa \land \neg Pb$:assumption
2. $\neg Pb \rightarrow Qb$:assumption
3. $\neg Pb$:E $\land$1
4. Qb :E$\rightarrow$ 2,3

Show: $\neg Pa \wedge Rb \vdash Ra$

1. $\neg Pa \wedge Rb$:assumption
2. Ra :E $\wedge$1

We have that object b has property R. There's no way to go from that to object a having property R!

Rules in QL

- Our old rules and shortcuts from SL showed us what inferences we could draw based on the main connective of the formulas involved

- Ex: E ∧ says we can take a formula like (P ∧ Q) and infer P and we can infer Q. This holds even when 'P' is a formula in QL like Pa.

- What we now need for quantified logic are rules for our new operators: the quantifiers!