Shortcuts:

When you're tired of proving the same thing over and over

Week 4 . Deep dive.

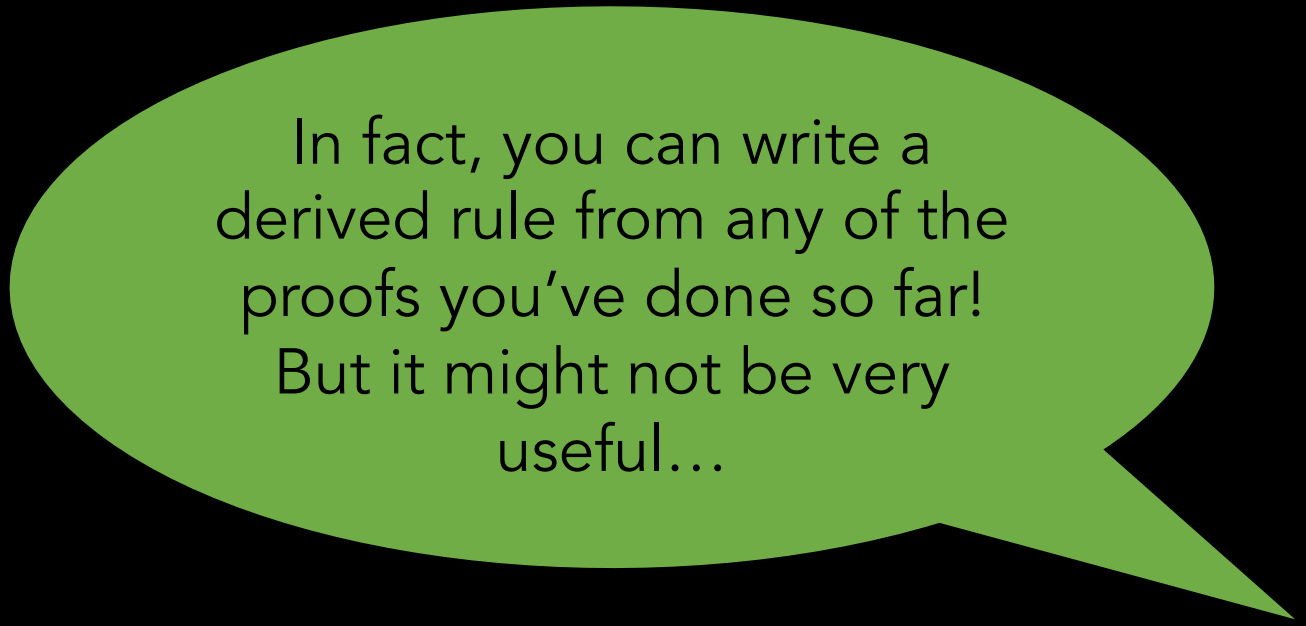
Shortcuts, aka Derived Rules

- The basic rules we learned last week tell us how the system works. They are the rules of the game.

- Derived rules let us skip through some steps.
- They're never essential, but can be very helpful!
  - In fact, they're all proven using the basic rules so we can always recreate their proofs.

Shortcuts, aka Derived Rules

Derived rules are helpful because proofs will often exhibit the same chain of reasoning. If we can capture that chain into a derived rule, we won't have to repeat it over and over.



Shortcuts, aka Derived Rules

There are lots of derived rules. Here are the ones we will learn:

- Double negation elimination (DNE)
- Contradiction (CON)
- Disjunctive syllogism (DS)
- DeMorgan's laws (DeM)
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Double negation elimination

From the wff $\sim\sim\phi$, infer ϕ.

This is what your book calls “negation elimination” (p. 83).

I won’t give the proof here as it is either very long or relies on other derived rules.

Write “DNE” + line number as the justification.

We’ll see this rule in action in a minute!

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Contradiction

From the wffs ϕ and $\sim\phi$, infer anything.

Often summarized as: “From contradiction, anything follows!”

Proof of $p, \sim p \vdash q$

1. p : assumption
2. $\sim p$: assumption
3. | $\sim q$: assumption
4. | $\bot$: E~1,2
5. $\sim\sim q$: I ~
6. q : DNE 5

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Disjunctive syllogism

From the wffs of the forms $(\phi \vee \psi)$ and $\sim\phi$, infer ψ.

Proof of $(p \vee q), \sim p \vdash q$

1.	$(p \vee q)$	: assumption
2.	$\sim p$	: assumption
3.	$\vert\ p$	: assumption
4.	$\vert\ q$	: CON 2,3
5.	$(p \rightarrow q)$	: I $\rightarrow$
6.	$\vert\ q$	: assumption
7.	$\vert\ q$	: repeat 6
8.	$(q \rightarrow q)$	: I $\rightarrow$
9.	q	: E v 1,5,8

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DeMorgan's Laws

$\sim(\phi \vee \psi) \leftrightarrow (\sim\phi \wedge \sim\psi)$.
$\sim(\phi \wedge \psi) \leftrightarrow (\sim\phi \vee \sim\psi)$.

If this looks familiar, you're right! On PS1, you gave a truth table for the first (hopefully you said it's a tautology!):

7.	$(\neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B))$	Tautology	Contradiction	Contingent

We'll have proofs for these in the practice problems this week!

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- Modus tollens (MT)

Modus Tollens

From wffs of the forms $(\phi \rightarrow \psi)$ and $\sim\psi$ infer $\sim\phi$

Proof of $(p \rightarrow q), \sim q \vdash \sim p$

1.	$(p \rightarrow q)$	: assumption
2.	~q	: assumption
3.	| p	: assumption
4.	| q	: E→ 1,3
5.	| ⊥	: E~2,5
6.	~p	: I ~

This one is famous so here's an example:

If the dog detects an intruder, the dog will bark.
The dog did not bark.
Therefore, no intruder was detected by the dog.

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  - From the wffs ϕ and $\sim\phi$, infer anything.
- Disjunctive syllogism (DS)
  - From the wffs of the forms $(\phi \vee \psi)$ and $\sim\phi$, infer ψ.
- DeMorgan's laws (DeM)
  - $\sim(\phi \vee \psi) \leftrightarrow (\sim\phi \wedge \sim\psi)$.
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  - From wffs of the forms $(\phi \rightarrow \psi)$ and $\sim\psi$ infer $\sim\phi$