Fuzzy Logic

LPS 30
TA: Jessica Gonzalez
j.gonzalez@uci.edu

Module Overview

1. Paradoxes in Logic
2. Rejecting the Law of Excluded Middle
3. Reimagining Truth Tables and Validity
4. Technological Applications
5. Practice Problems & Additional Resources

Paradoxes in Logic

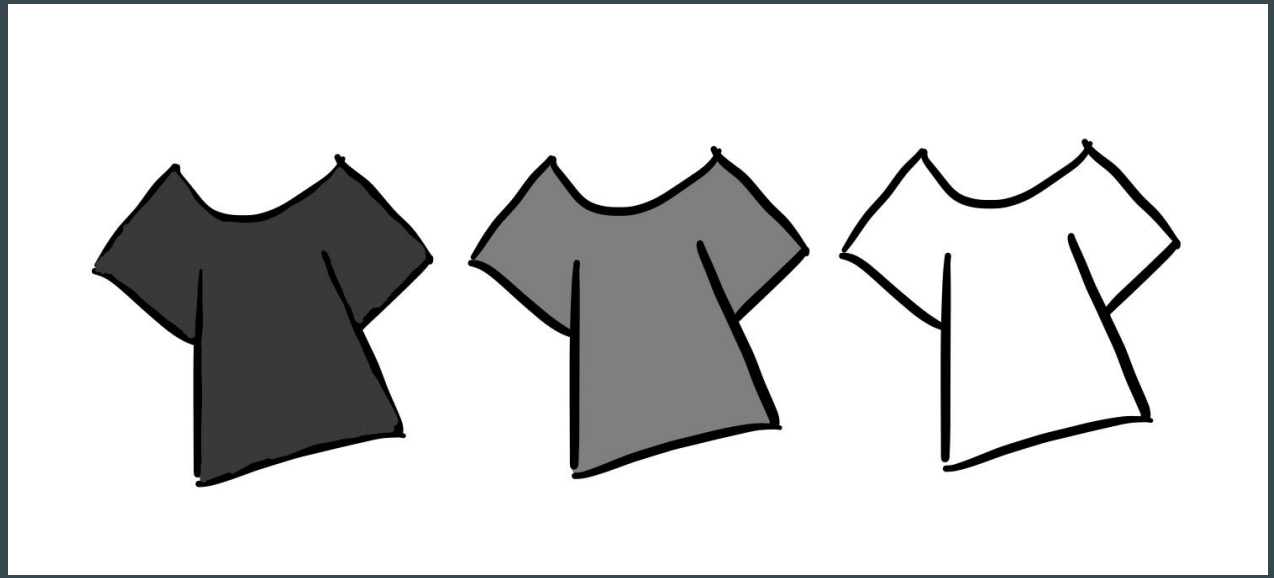
Imagine a class with 11 students,
where there is a 1” height difference between each student.

Paradoxes in Logic

If S_1 is tall, then S_2 is tall.

Is S_{11} tall?

Paradoxes in Logic



Rejecting the Law of Excluded Middle

The Law of Excluded Middle tells us: $p \vee \sim p$

Sentences are “crisply” true or false.

- Crisp sets: class attendance, pass/no pass
- Fuzzy sets: tallness/shortness, warmness

By rejecting the Law of Excluded Middle, fuzzy logic *extends* traditional logic.

Rejecting the Law of Excluded Middle

Reimagining Truth Tables and Validity

fuzzy set

S_1	6’
S_2	5’ 11”
S_3	5’ 10”
S_4	5’ 9”
S_5	5’ 8”
S_6	5’ 7”
S_7	5’ 6”
S_8	5’ 5”
S_9	5’ 4”
S_{10}	5’ 3”
S_{11}	5’ 2”

Reimagining Truth Tables and Validity

fuzzy set

Is S tall?

S_1	6’	1.0
S_2	5’ 11”	0.9
S_3	5’ 10”	0.8
S_4	5’ 9”	0.7
S_5	5’ 8”	0.6
S_6	5’ 7”	0.5
S_7	5’ 6”	0.4
S_8	5’ 5”	0.3
S_9	5’ 4”	0.2
S_{10}	5’ 3”	0.1
S_{11}	5’ 2”	0.0

Reimagining Truth Tables and Validity

Negation

The degree of truth of ~A = 1.0 - the degree of truth of A.

S_1 is tall = 1.0	$\sim S_1$ is tall = 0.0
S_4 is tall = 0.7	$\sim S_4$ is tall = 0.3
S_6 is tall = 0.5	$\sim S_6$ is tall = 0.5
S_{11} is tall = 0.0	$\sim S_{11}$ is tall = 1.0

Reimagining Truth Tables and Validity

Conjunction

The degree of truth of A $\wedge$ B = the **minimum** degree of truth of A and B.

S_1 and S_3 are tall	=	1.0 $\wedge$ 0.8	=	0.8
S_2 and S_6 are tall	=	0.9 $\wedge$ 0.5	=	0.5
S_{11} and S_5 are tall	=	0.0 $\wedge$ 0.6	=	0.0

Reimagining Truth Tables and Validity

Disjunction

The degree of truth of A $\vee$ B = the **maximum** degree of truth of A and B.

S_1 and S_3 are tall	=	1.0 $\vee$ 0.8	=	1.0
S_2 and S_6 are tall	=	0.9 $\vee$ 0.5	=	0.9
S_{11} and S_5 are tall	=	0.0 $\vee$ 0.6	=	0.6

Reimagining Truth Tables and Validity

Contradictions

Consider the contradiction (p ∧ ~p).

S_1 is tall and not tall = 1.0 ∧ 0.0 = 0.0

S_3 is tall and not tall = 0.8 ∧ 0.2 = 0.2

S_6 is tall and not tall = 0.5 ∧ 0.5 = 0.5

S_{11} is tall and not tall = 0.0 ∧ 1.0 = 0.0

The truth value of a contradiction will range between 0.0 - 0.5

Reimagining Truth Tables and Validity

Tautologies

Consider the tautology (p ∨ ~p).

S_1 is tall or not tall	=	1.0 ∨ 0.0	=	1.0
S_3 is tall or not tall	=	0.8 ∨ 0.2	=	0.8
S_6 is tall or not tall	=	0.5 ∨ 0.5	=	0.5
S_{11} is tall or not tall	=	0.0 ∨ 1.0	=	1.0

The truth value of a tautology will range between 0.5 - 1.0

Reimagining Truth Tables and Validity

Conditionals

The degree of truth of $A \rightarrow B = 1.0 - (A - B)$ if A is greater than B, otherwise 1.

If S_1 is tall, then S_3 is tall	=	$1.0 \rightarrow 0.8$	=	0.8
If S_2 is tall, then S_6 is tall	=	$0.9 \rightarrow 0.5$	=	0.6
If S_{11} is tall, then S_5 is tall	=	$0.0 \rightarrow 0.6$	=	1.0
If S_1 is tall, then S_{11} is tall	=	$1.0 \rightarrow 0.0$	=	0.0

Reimagining Truth Tables and Validity

Validity

A completely (100%) fuzzy valid argument is one that does not allow for a loss of truth in going from the premises to the conclusion.

Technological Applications

Fuzzy logic is used in technology such as:

- Cement kilns
- Steel production
- Shower heads
- Photography equipment
- Car transmissions
- Blood pressure meters
- Fraud detection
- Facial pattern recognition
- Air conditioners
- Washing machines
- Vacuum cleaners
- Anti-lock braking systems
- Subway systems
- Unmanned helicopters
- Weather forecasting systems
- Medical diagnosis
- Stock trading

Practice Problems & Additional Resources

Practice Problems

- Fuzzy Logic Practice Problems

Additional Resources

- Ronald C. Pine's *Essential Logic: Basic Reasoning Skills for the 21st Century* (Chapter 12)
- Harpreet Singh, Madan M. Gupta, Thomas Meitzler, Zeng-Guang Hou, Kum Kum Garg, Ashu M. G. Solo, Lotfi A. Zadeh, "Real-Life Applications of Fuzzy Logic", *Advances in Fuzzy Systems*, vol. 2013, Article ID 581879, 3 pages, 2013.https://doi.org/10.1155/2013/581879 Link: https://www.hindawi.com/journals/afs/2013/581879/
- Eugenia Cheng's *The Art of Logic in an Illogical World* (pp. 71, 200, 258)
- Fuzzy Logic - Computerphile