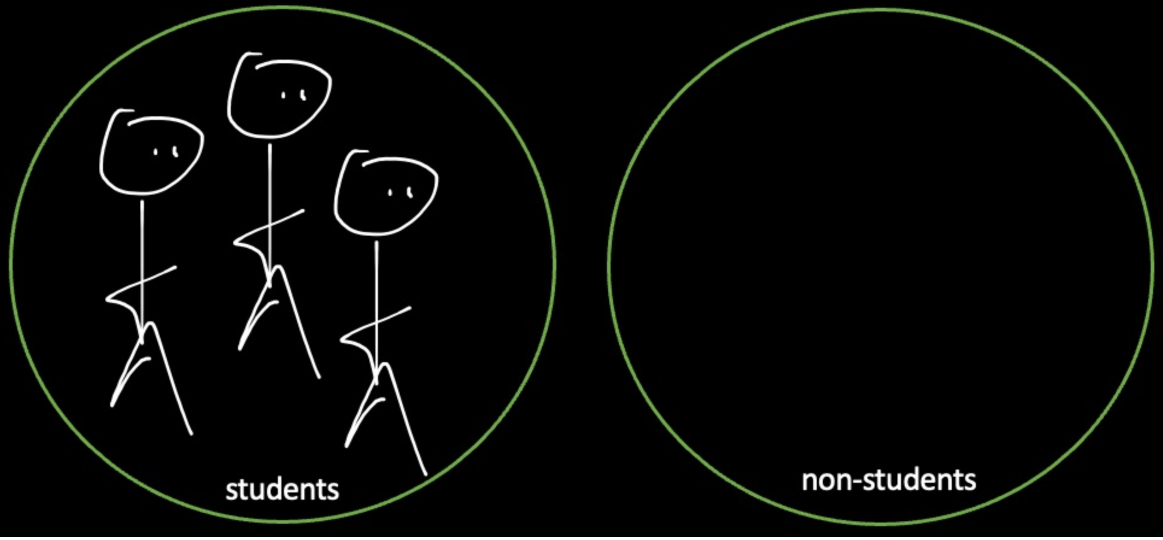
What are models?

Week 6 . Deeper dive.

Recall:

To say "everyone is a student", we write $\forall x S x$.

- $\forall x$ says that every individual can fill the role of x, and Sx says that the role of x is that it's a student.

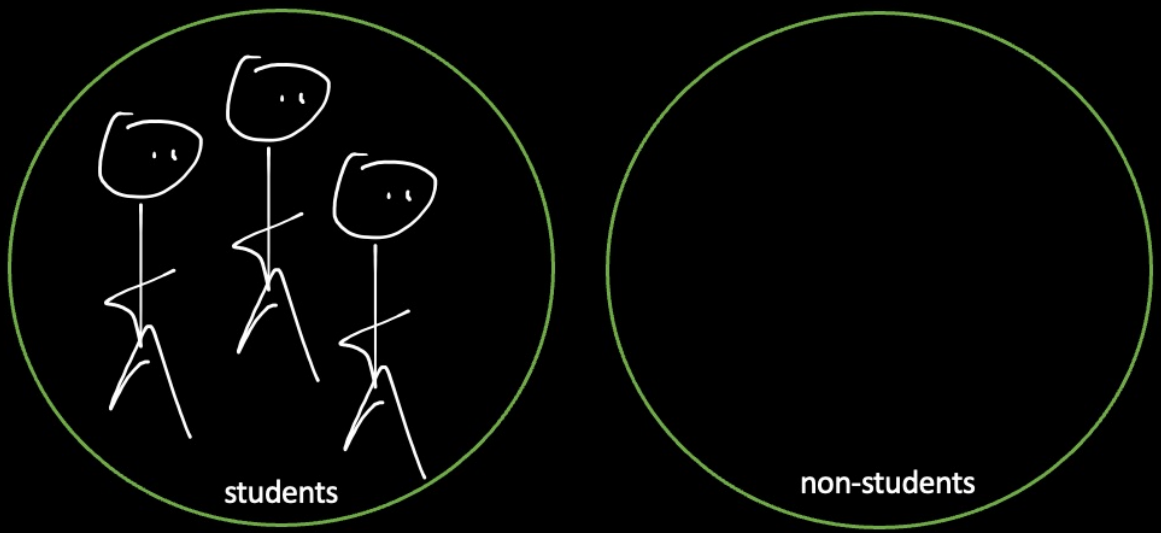


Models

- A model is a representation of a particular state of affairs.
    - Models are kind of like rows on a truth table.
    - Models help us reason: e.g., sometimes we can see that two formulas can’t be true in the same model, or that if one is true then the other must be true in a model.

Models

A model supplies a *domain of discourse (or just domain)*: the class of objects relative to which the names and predicate letters are interpreted. Notation: |M| where M is the model



We have three objects.
Let's call them a, b, and c.
We have one predicate "Sx"
meaning "x is a student."

To collect up a bunch of objects in an ordered list, we use curly braces: {…} and separate each object with commas.

So, in this example, we write our domain of discourse as: |M| = {a,b,c}

Models

Also supplies an interpretation of any non-logical symbols occurring some wffs of QL.

Symbol	*Interpretation*
name letter	individual object (e.g., the Moon)
zero-place predicate letter (sentence letter)	truth value (T or F)
one-place predicate letter	class of objects (e.g., the class of people)
n-place predicate letter ($n > 1$)	relation between n objects (e.g., the relation that holds between a pair of objects just in case the first is bigger than the second)

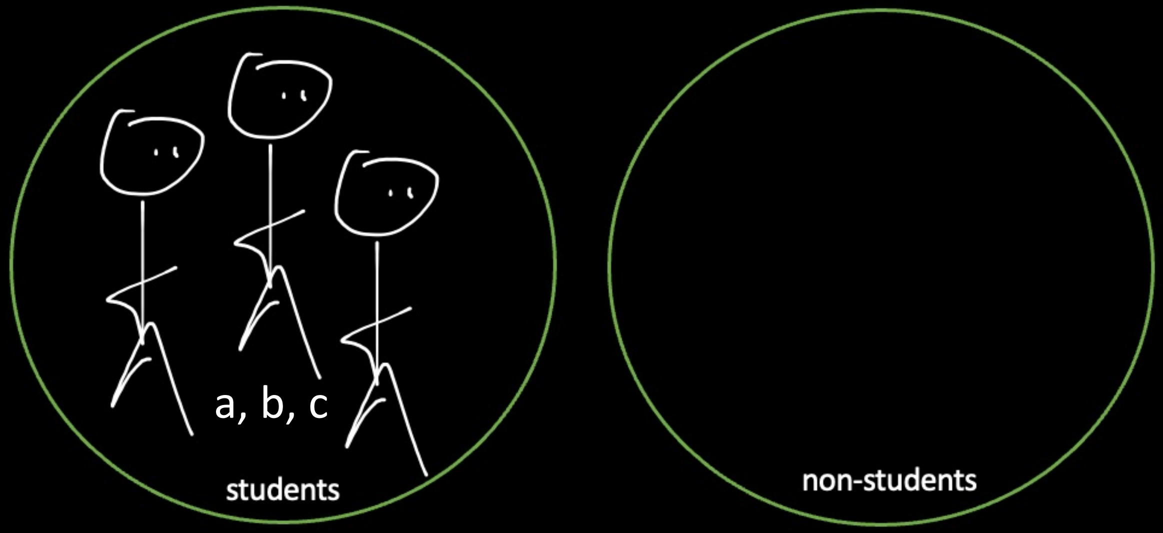
What does this look like?

Models

Also supplies an interpretation of any non-logical symbols occurring some wffs of QL.

The *interpretation function*, which we usually write capital ‘I’, tells us about two different sorts of linguistic items

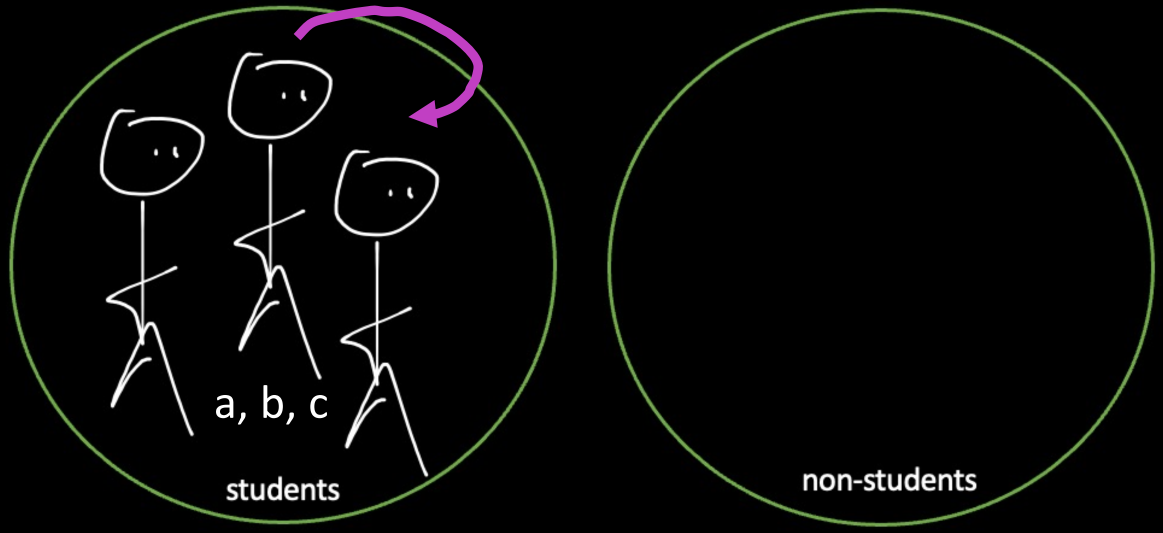
- For each **name**, it provides the object denoted by that name.
  - Ex: I(s) = Stella. This is the *interpretation of ‘s’*
- For each **predicate**, it provides its *extension*.
  - The extension of a predicate is the collection of objects that have this predicate as a property.



We have three objects.
Let’s call them a, b, and c.
We have one predicate “Sx”
meaning ”x is a student.”

The interpretation of our predicate is:
$I(S)=\{a,b,c\}$

What about for >1-place predicates?

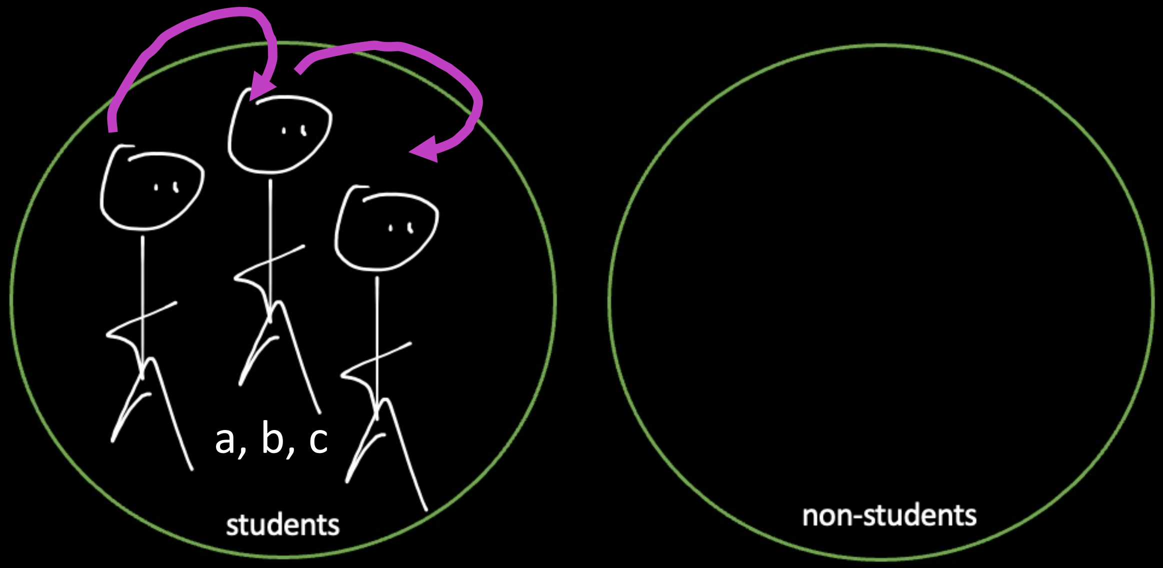


To collect up a bunch of objects in an ordered list, we use angled brackets “< >” separating each object inside with commas.

We have three objects.
Let’s call them a, b, and c.
We have two-place predicate “Lxy” meaning “x loves y.”
From the above, we know Lbc

The interpretation of our predicate is:

I(L)={<b,c>}



We separate each set of angled brackets within the curly braces with commas.

From the above, we know Lbc and Lab

The interpretation of our predicate is now:

I(L)={<a,b>,<b,c>}

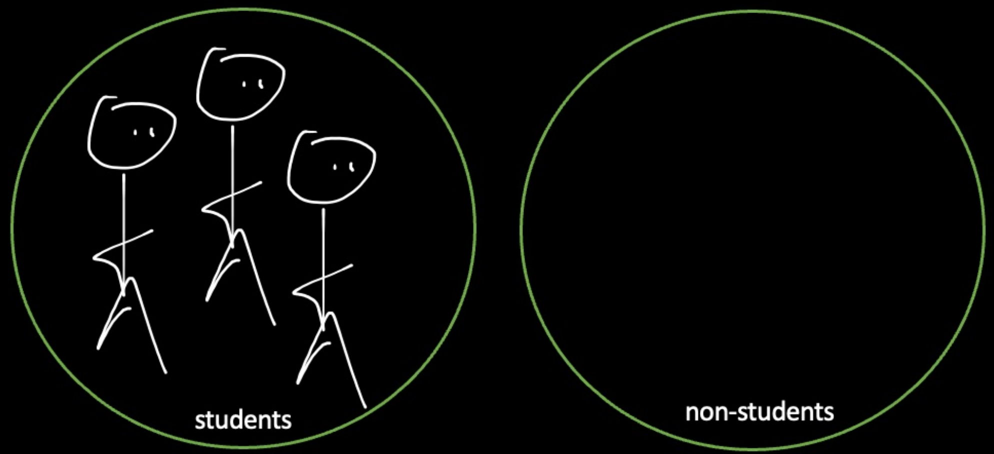
One more notational thing:

As noted, we use curly braces for a collection of unordered objects.
And we use the inclusion symbol '∈' for **membership**

- **e.g., a** $\in$ {a,b,c}
- e.g., Helen $\in$ {Helen, Josiah, Jessica}
- **e.g., d** $\notin$ {a,b,c}

Truth in models…

Given a property S and a name h, the proposition Sh is **true in a model** M if and only if: I(h) ∈ I(S)

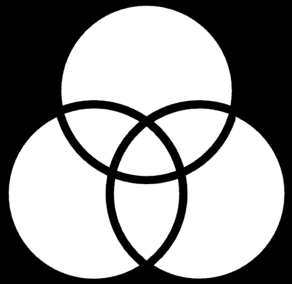


h = Helen, j = Jessica, o = Josiah
I(S)={Helen, Jessica, Josiah}
I(h) = Helen

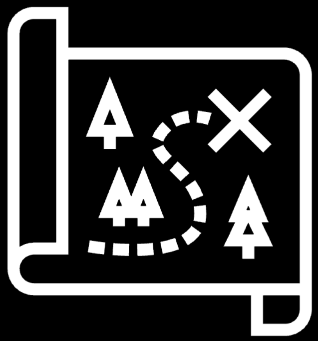
The above is saying that "Helen is a student" is true in the model iff I(h) ∈ I(S)

Is it? Yes!: I(h) = Helen ∈ I(S) since I(S) ={Helen, Jessica, Josiah}

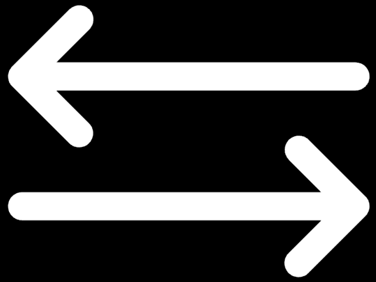
Types of models we will consider:



Venn diagrams

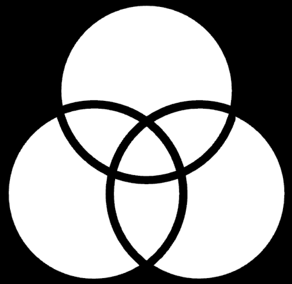


Map models



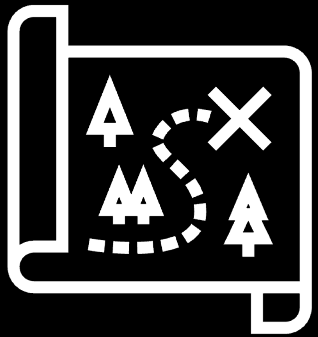
Arrow models

Types of models we will consider:

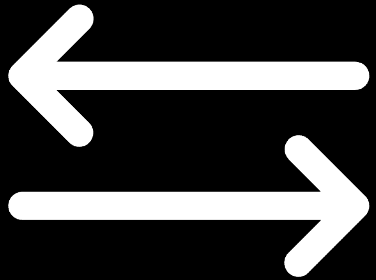


Venn diagrams

good for representing objects with properties



Map models



Arrow models

good for representing objects with properties and relations between objects

Why are we talking about models…?

Validity via models: a valid argument is an argument in which the conclusion is true *in every model* in which the premises are true.

By 'every model' we mean each model which interprets the vocabulary in the argument

wait…*every model??*

Validity via models: a valid argument is an argument in which the conclusion is true *in every model* in which the premises are true.

Yes…remember a model is like a row on a truth table and we have to check every row in which the premises were true when we were checking for validity!

wait…*every model*??

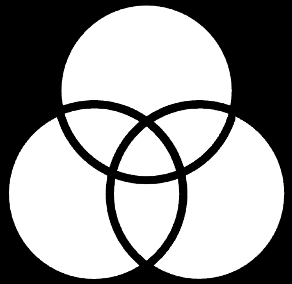
BUT unlike truth tables, here we'd have infinitely many models… which will take a very long time…

wait…*every model*??

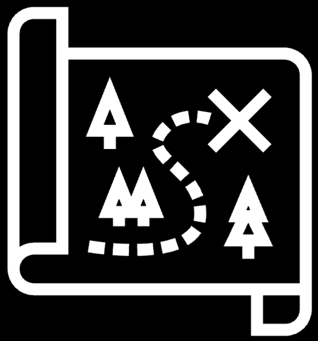
Validity via models: a valid argument is an argument in which the conclusion is true *in every model* in which the premises are true.

Instead, we'll often use models as *counterexamples!* Remember an argument was invalid if we found a row on which the premises were true but the conclusion wasn't. We often want to give these kinds of counterexamples to show an argument is invalid!

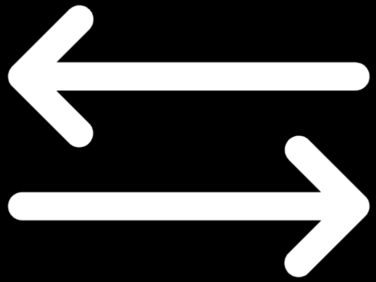
To come:



Venn diagrams



Map models



Arrow models