

Theorems:

Proving something from nothing

Week 4 . Deep dive.

# Recall:

Consider  $\models (p \vee \neg p)$ . There aren't any premises!

$p$	$\models (p \vee \neg p)$
T	✓ T T F T
F	✓ F T T F

There are no premises. What row do we check?

Any argument with a tautology as a conclusion is valid.

Think about it as saying: " $(p \vee \neg p)$  will be valid no matter what the premises are."

**Theorem:** a wff of some formal system (e.g., sentential logic) which is the conclusion of some proof of that system that does not contain any non-hypothetical assumptions.

*Informally, a theorem is a statement that's provable from nothing!*

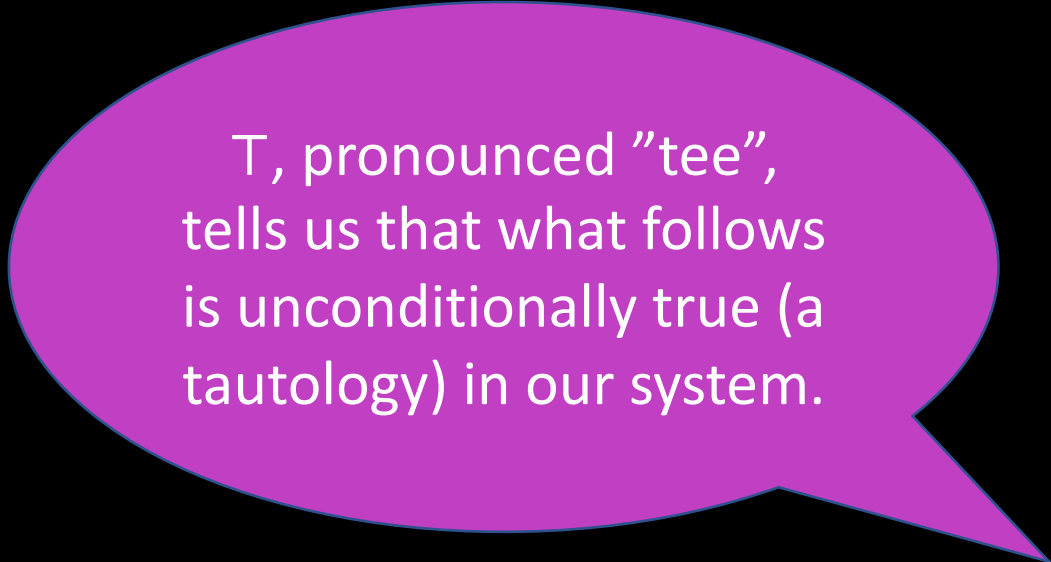
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Notation:

$$\vdash \sim(p \wedge \sim p)$$

*or*

$$\top \vdash \sim(p \wedge \sim p)$$



$\top$ , pronounced "tee", tells us that what follows is unconditionally true (a tautology) in our system.

Example:

$\vdash (p \wedge q) \rightarrow (p \vee q)$

1.	$(p \wedge q)$	: assumption
2.	$p$	: $E \wedge 1$
3.	$(p \vee q)$	: $I \vee 2$
4.	$(p \wedge q) \rightarrow (p \vee q)$	: $I \rightarrow$

Example:

$\vdash (p \rightarrow \sim \sim p)$

1. | p

: assumption

2. | |  $\sim p$

: assumption

3. | |  $\perp$

: E  $\sim$  1,2

4. |  $\sim \sim p$

: I  $\sim$

5.  $(p \rightarrow \sim \sim p)$

: I  $\rightarrow$