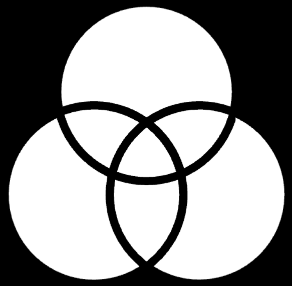
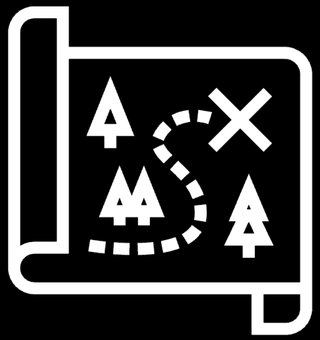
Venn Diagrams

Week 6 . Deeper dive.

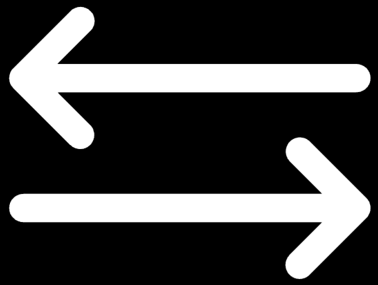
Types of models we will consider:



Venn diagrams

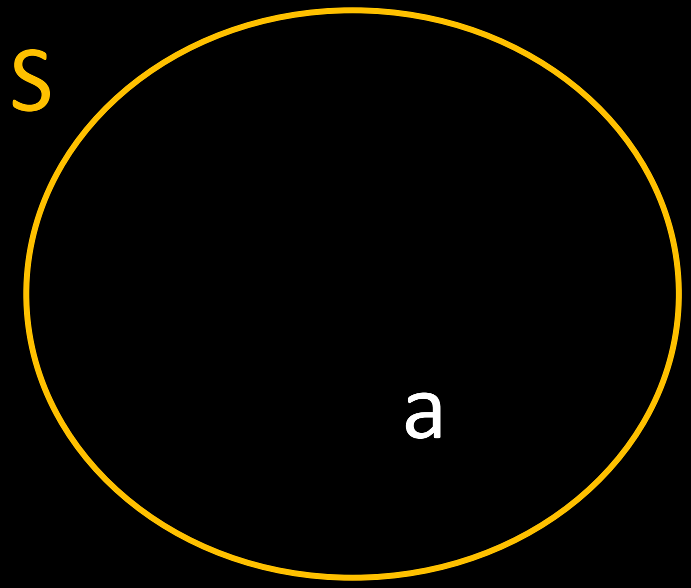


Map models



Arrow models

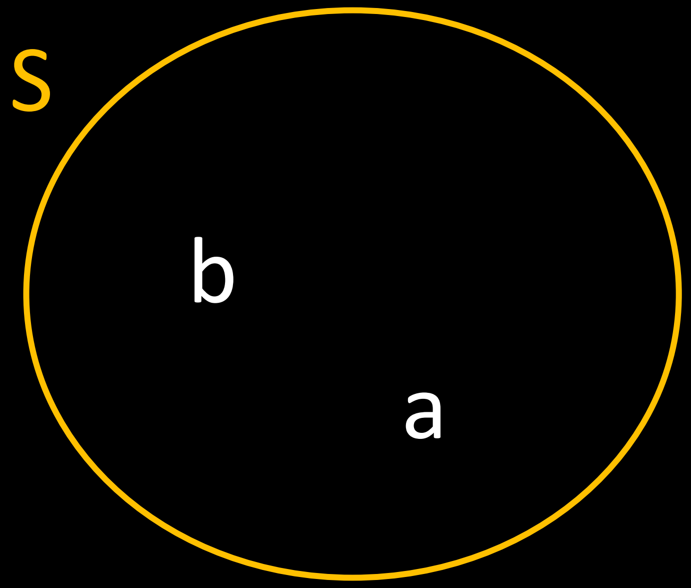
Reading Venn diagram models



Sa is true in the model

$$Sa$$

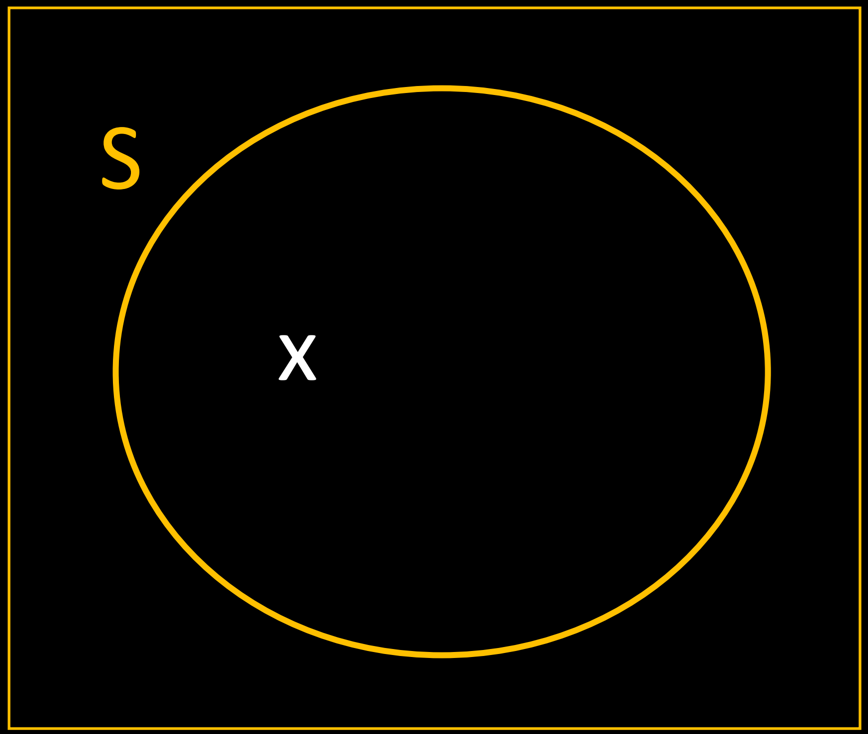
Reading Venn diagram models



Sa and Sb are both true in the model

$$Sa \land Sb$$

Reading Venn diagram models



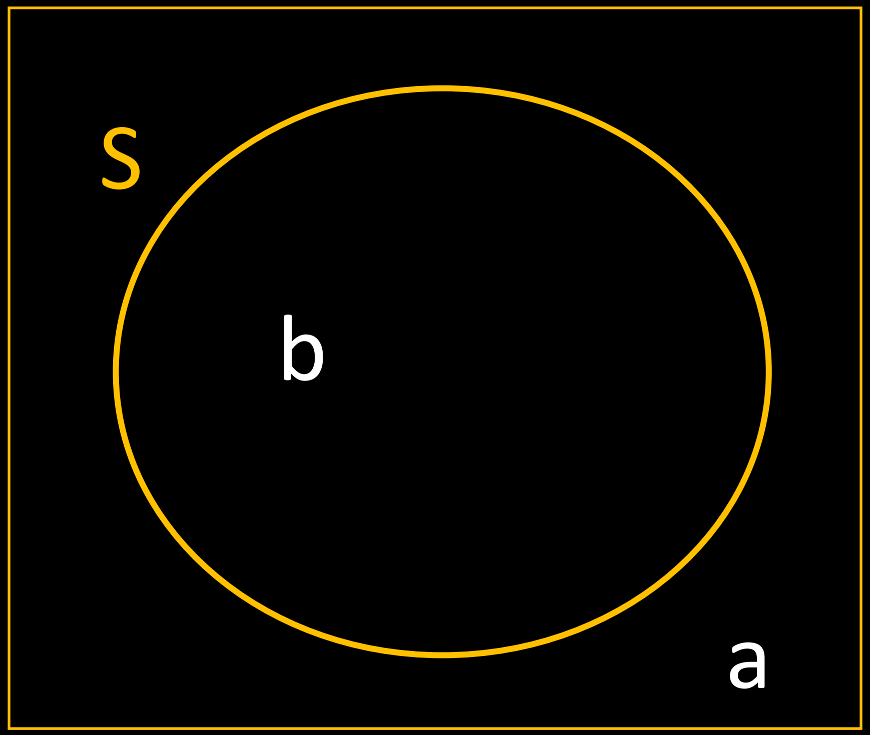
There is at least one object with property S.

$$\exists x \, Sx$$

Is there anything that isn't an S? unclear!

If an area is empty, we must not assume that the corresponding set has members.

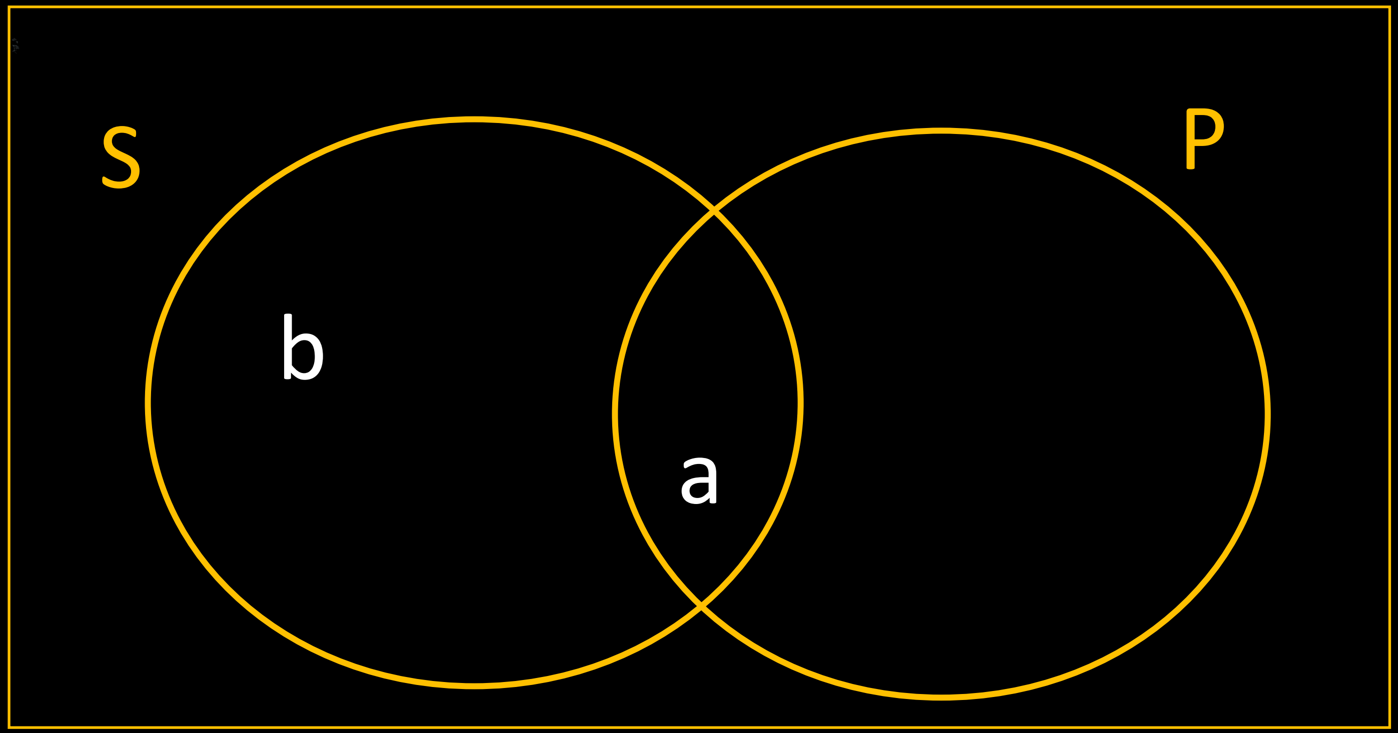
Reading Venn diagram models



Sb is true in the model but Sa is not.

$$Sb \wedge \neg Sa$$

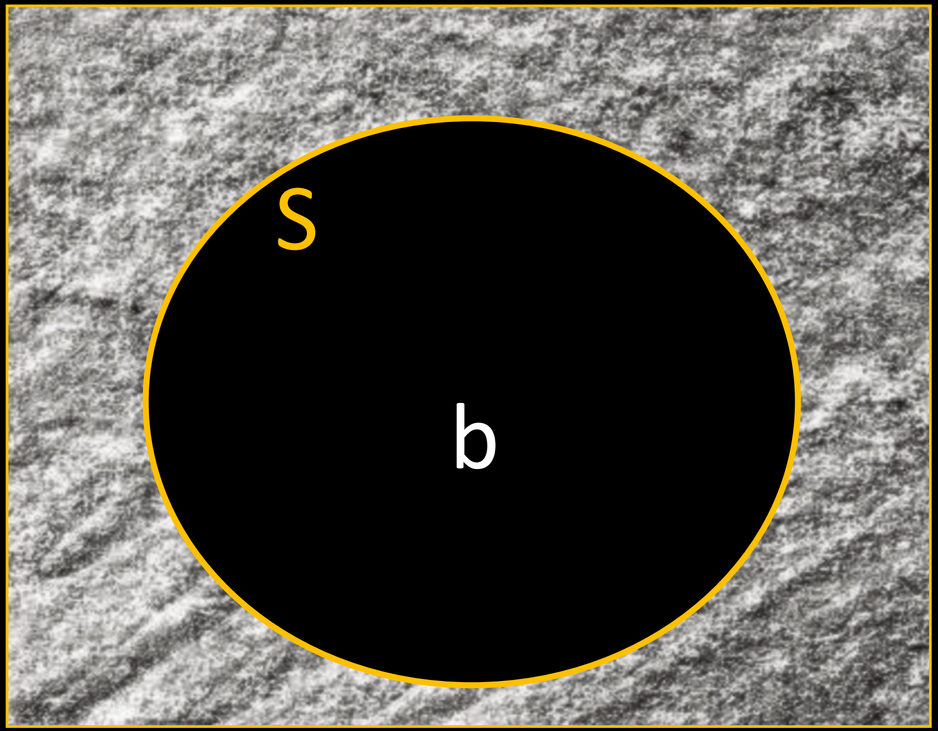
Reading Venn diagram models



Sb is true in the model.
Sa and Pa are both true in the model.

$$Sb \ \wedge (Sa \wedge Pa)$$

Reading Venn diagram models



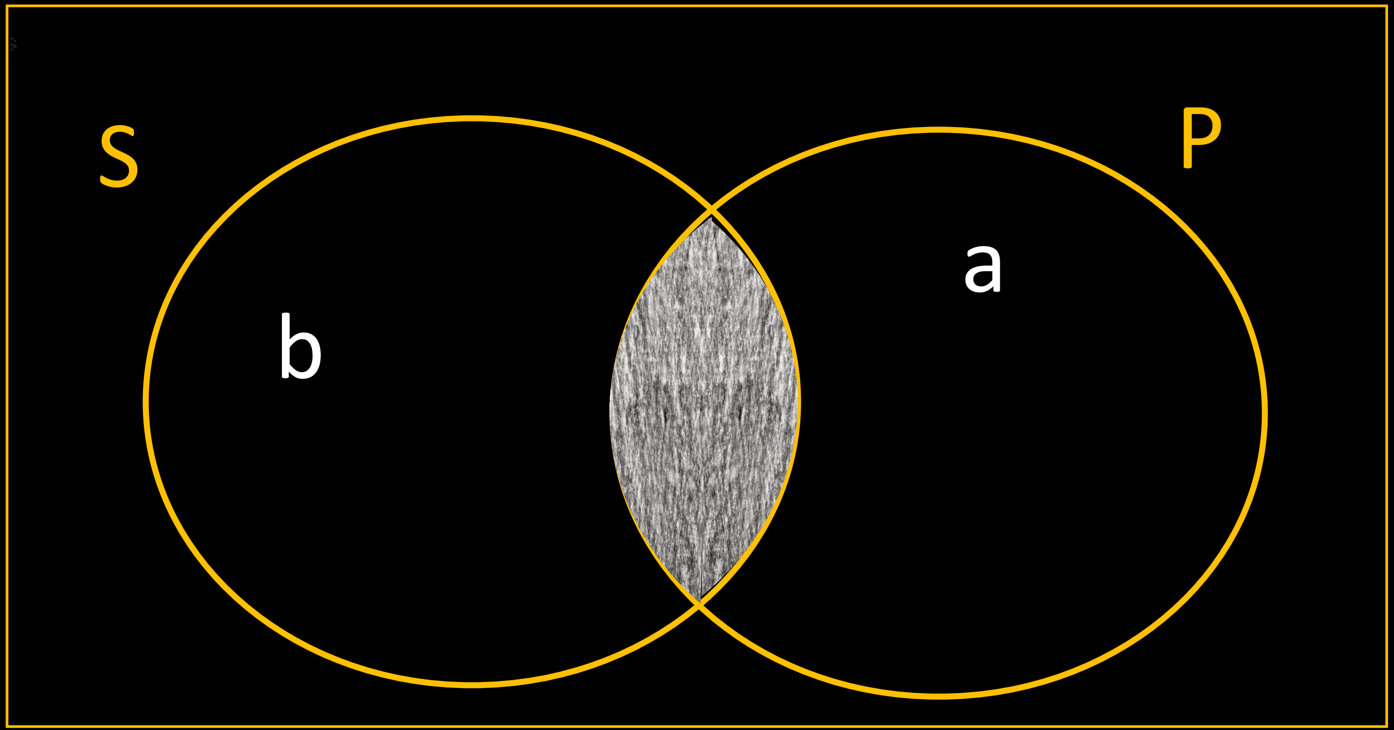
Sb is true in the model and there is nothing that is not an S in the model (we've "blocked out" everything else by shading it in).

$$Sb \land \neg\exists x \neg Sx$$

or equivalently

$$Sb \land \forall x Sx$$

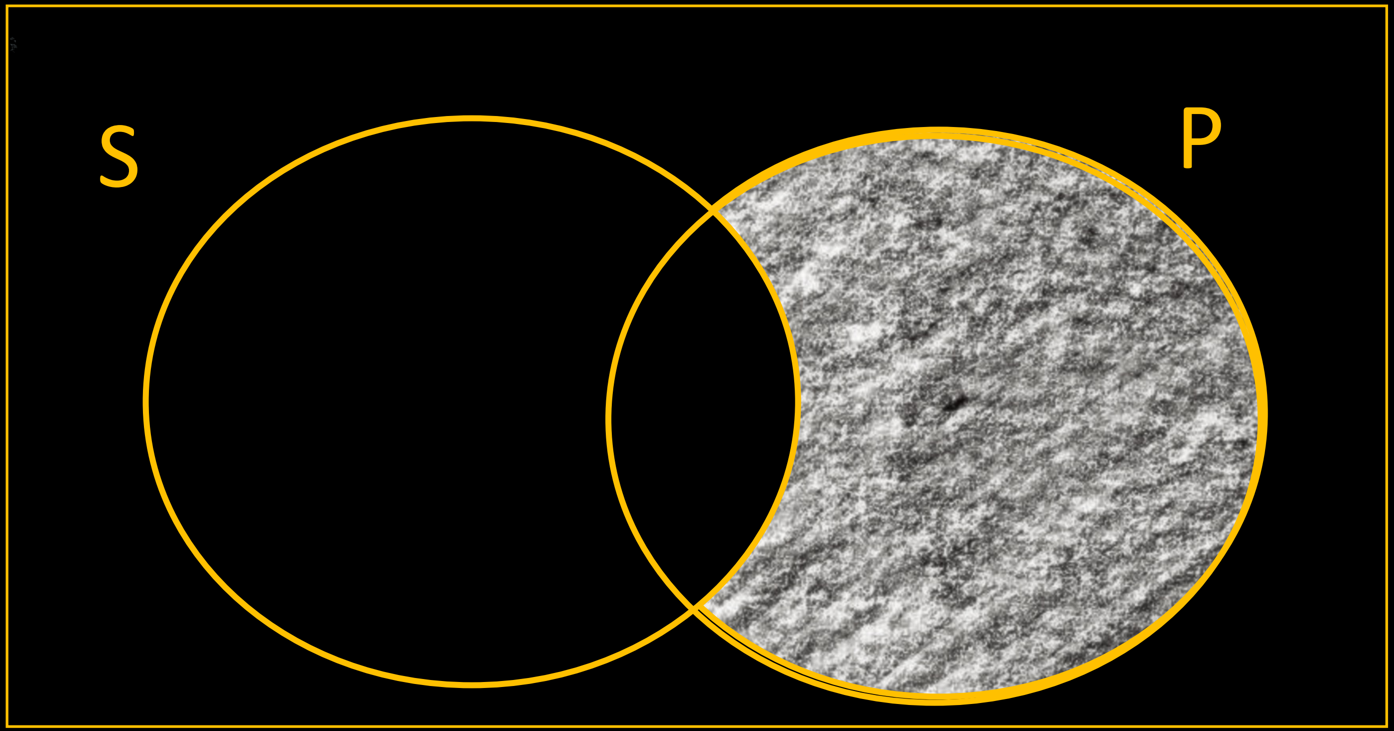
Reading Venn diagram models



Sb is true in the model,
Pb is true in the model,
and there is nothing that
has both properties.

$$(Sb \wedge Pa) \wedge \neg\exists x(Sx \wedge Px)$$

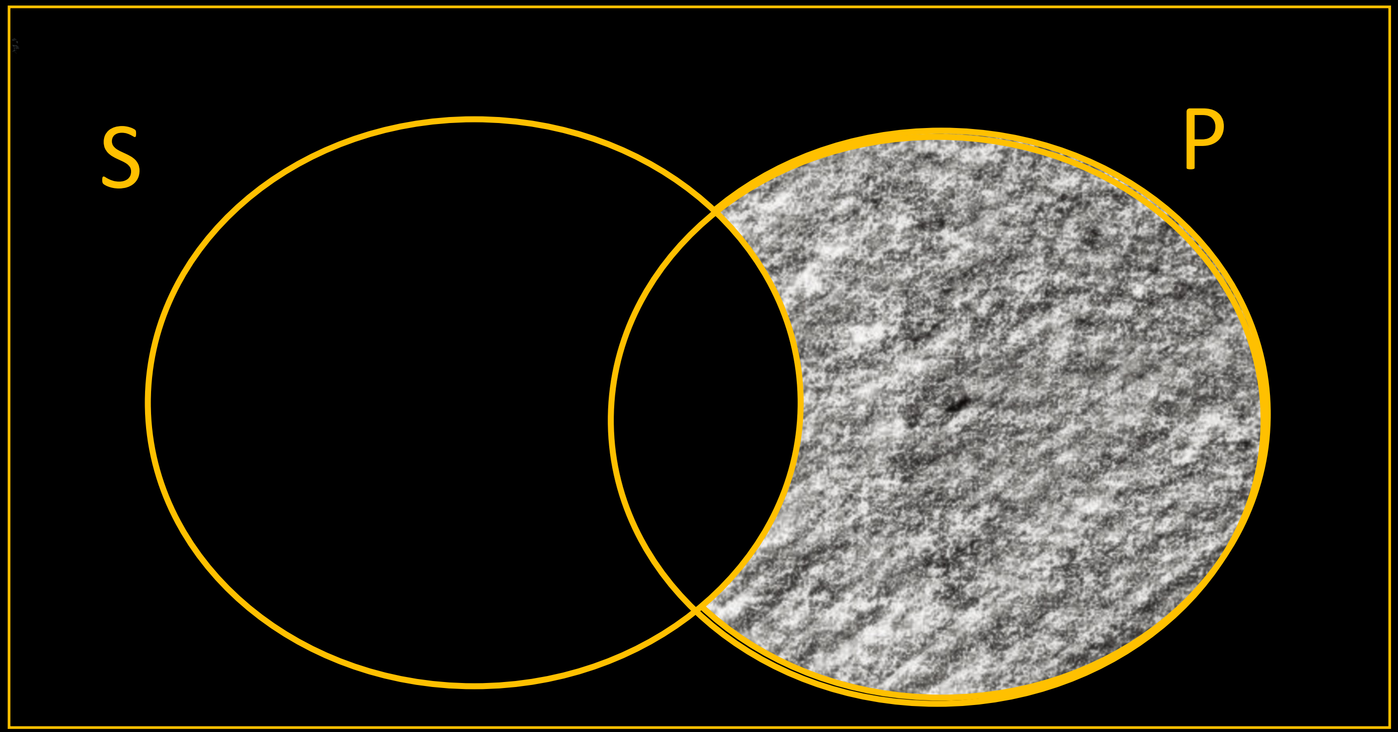
Reading Venn diagram models



No non-S is a P.
or, equivalently,
All Ps are Ss.

$\forall x\ (Px \rightarrow Sx)$
or, equivalently,
$\neg\exists x(Px\ \wedge \neg Sx)$

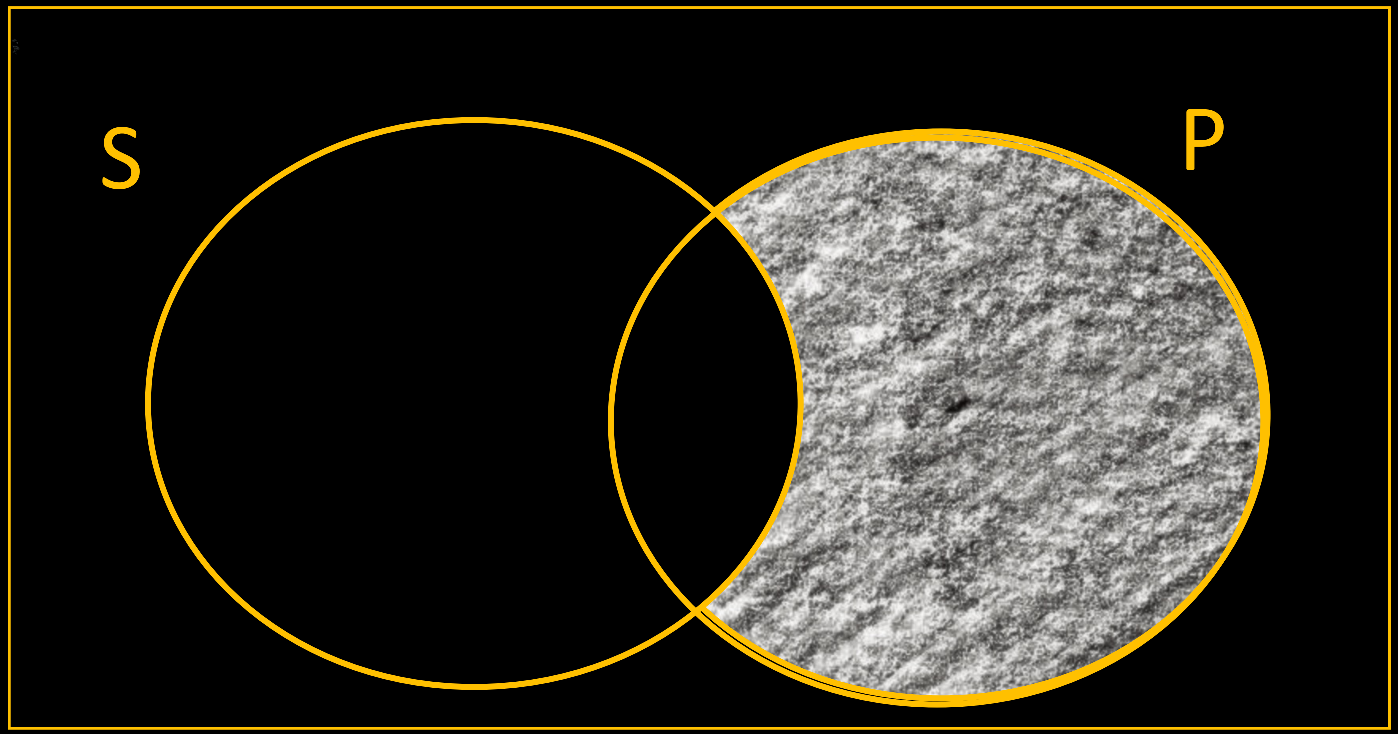
Reading Venn diagram models



Can I say "All Ps are Ss"?

Yes. This doesn't mean that there is a P necessarily. Just that if there were one, it would also be an S!

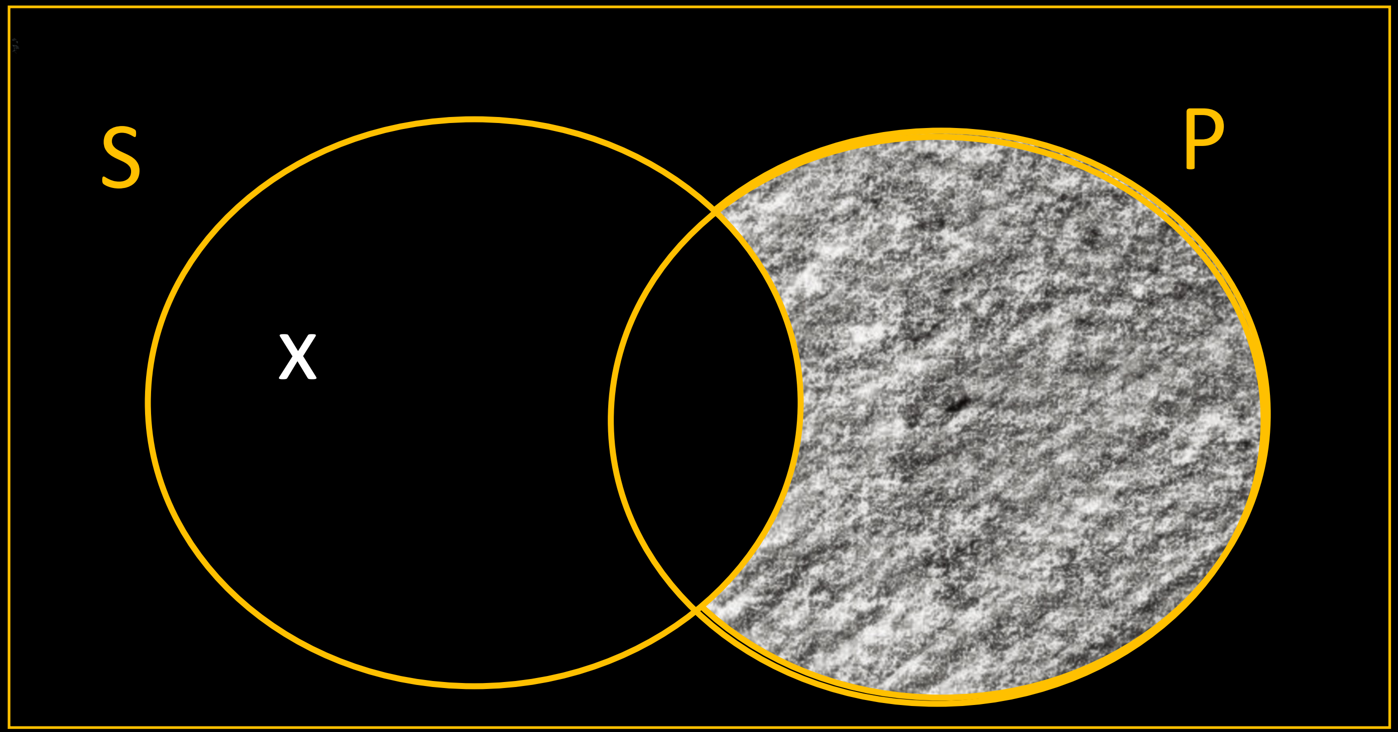
Reading Venn diagram models



Can I say
$\exists x\ Sx$?

No! Not unless we're told that the set has members.

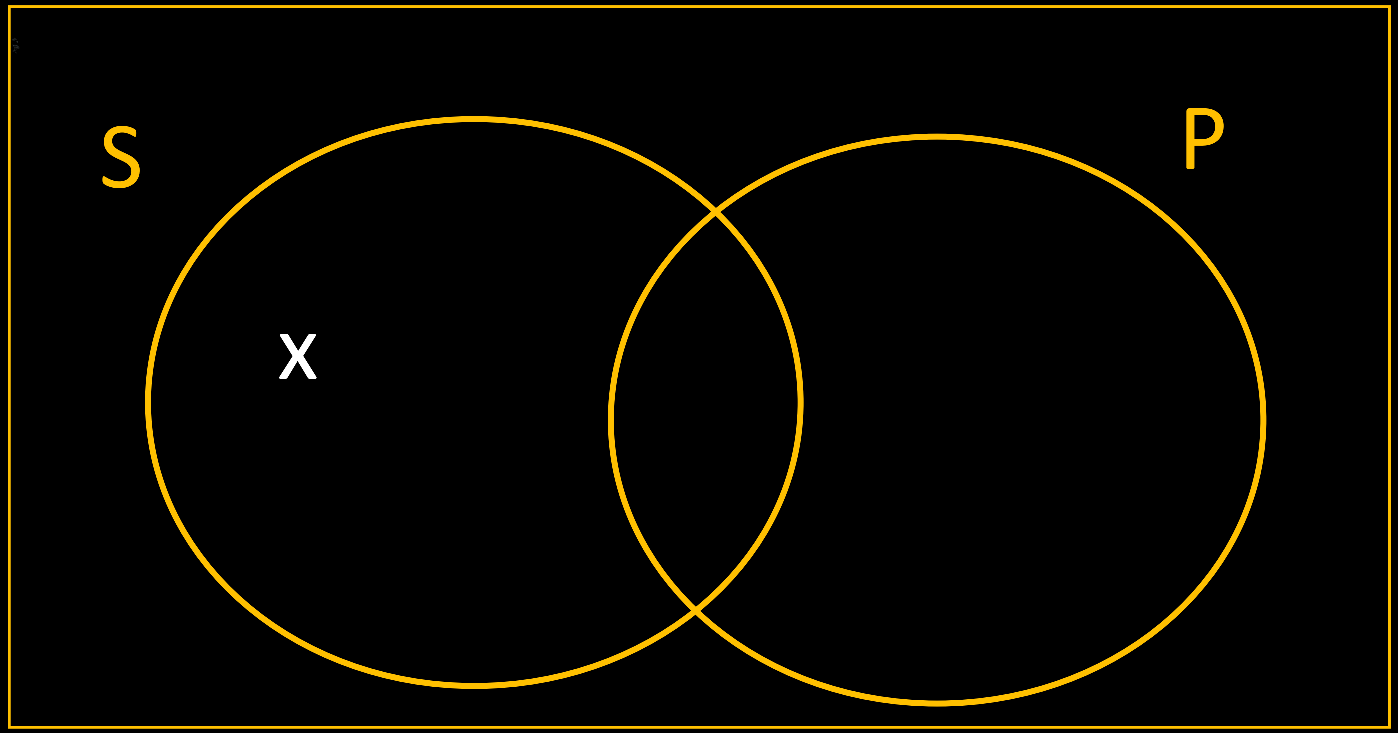
Reading Venn diagram models



Can I say
$\exists x\ Sx$?

Yes! The "x" tells us that there is at least one object with property S.

Practice: Are the following true or false?



$\forall x\,(Px \rightarrow Sx)$ false

$\forall x\,(Sx \rightarrow Px)$ false

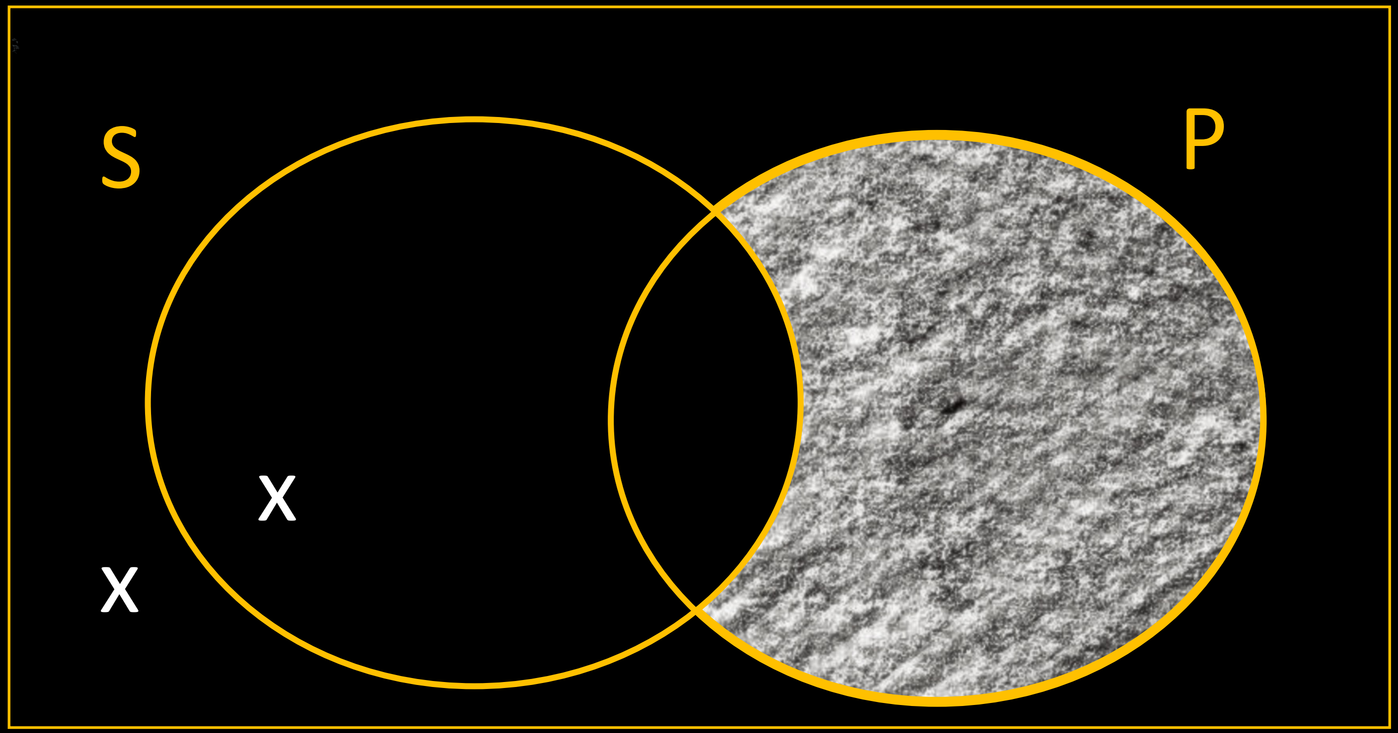
$\exists x Sx$ true

$\exists x \neg Sx$ false

$\exists x Px$ false

$\exists x \neg Px$ true

Practice: Are the following true or false?



$\exists x \neg Sx$ true

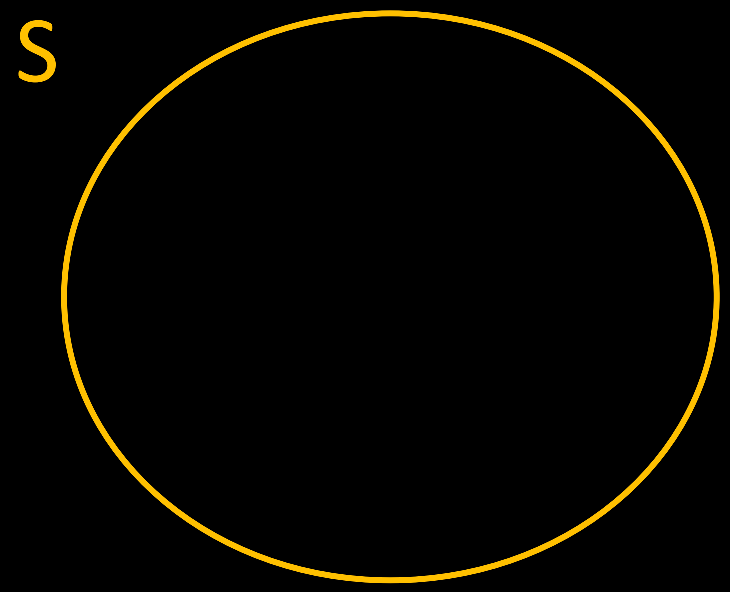
$\forall x(Sx \rightarrow Px)$ false

$\forall x(Px \rightarrow Sx)$ true

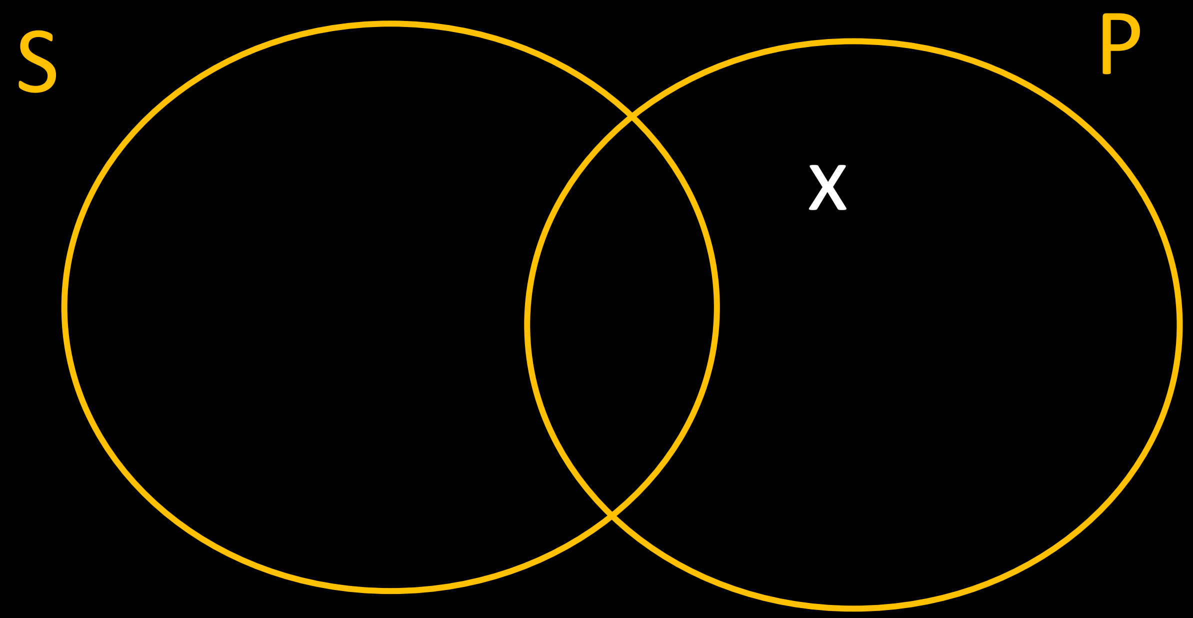
$\forall x(\neg Px \rightarrow Sx)$ false

$\forall x(\neg Sx \rightarrow \neg Px)$ true

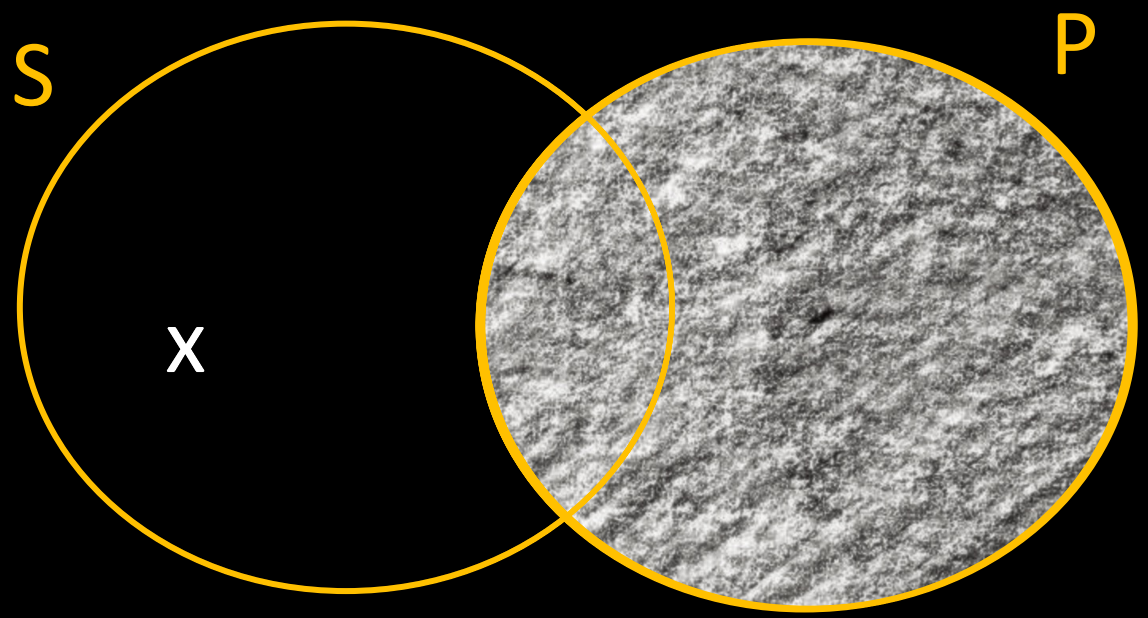
Draw a model that makes $\exists x\ (Sx \wedge \neg Px)$ false:



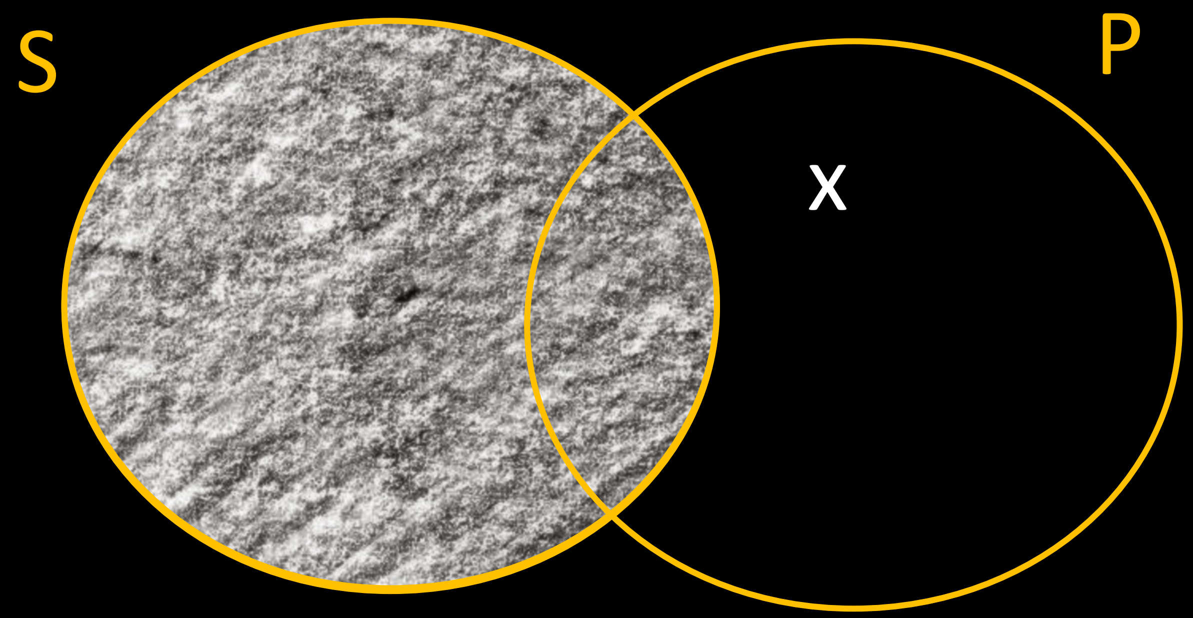
Draw a *non-empty* model that makes $\exists x\,(Sx \wedge \neg Px)$ false:



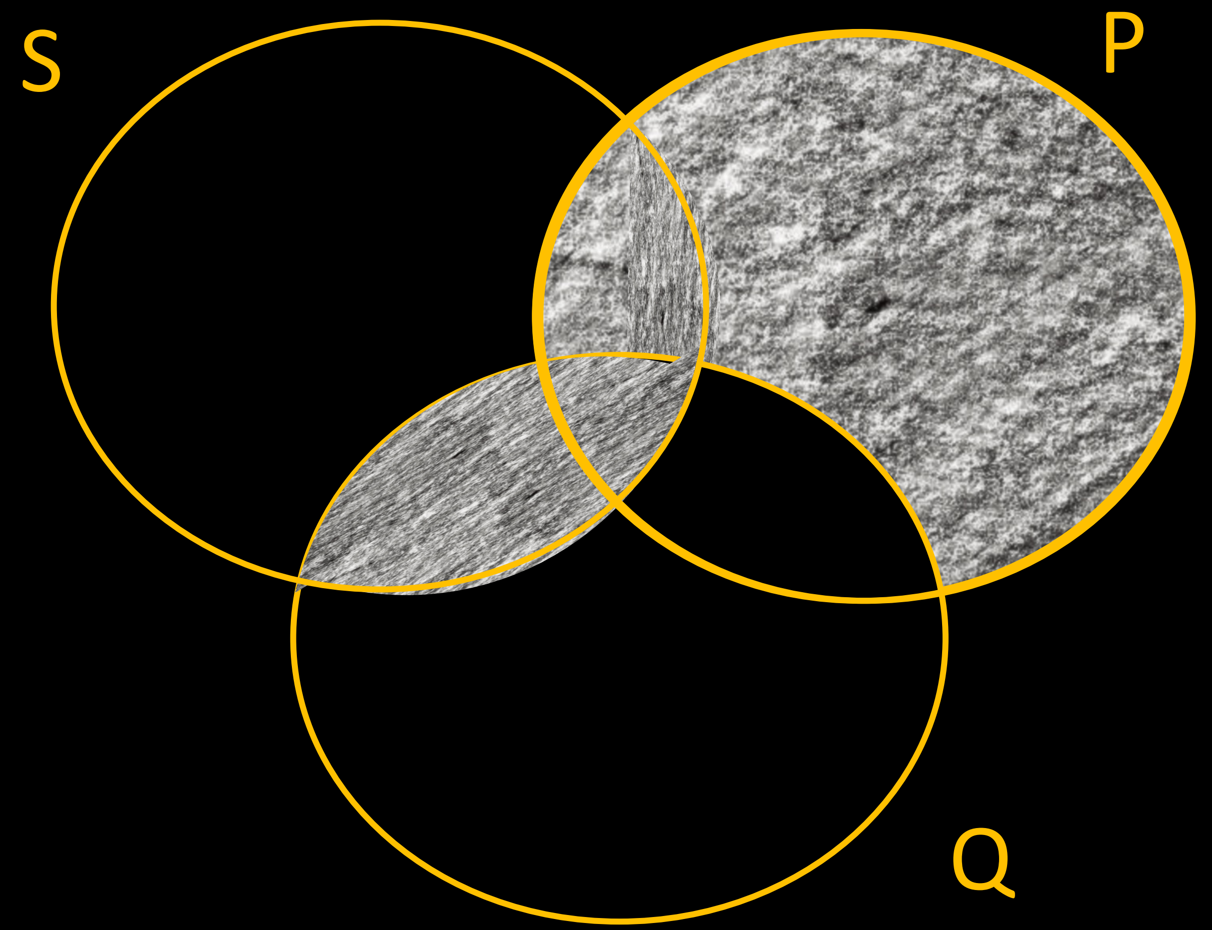
Draw a non-empty model that makes $\forall x\,(Sx \land \neg Px)$ true:



Draw a non-empty model that makes $\forall x\,(Sx \wedge \neg Px)$ false:



Using Venn diagram models for arguments

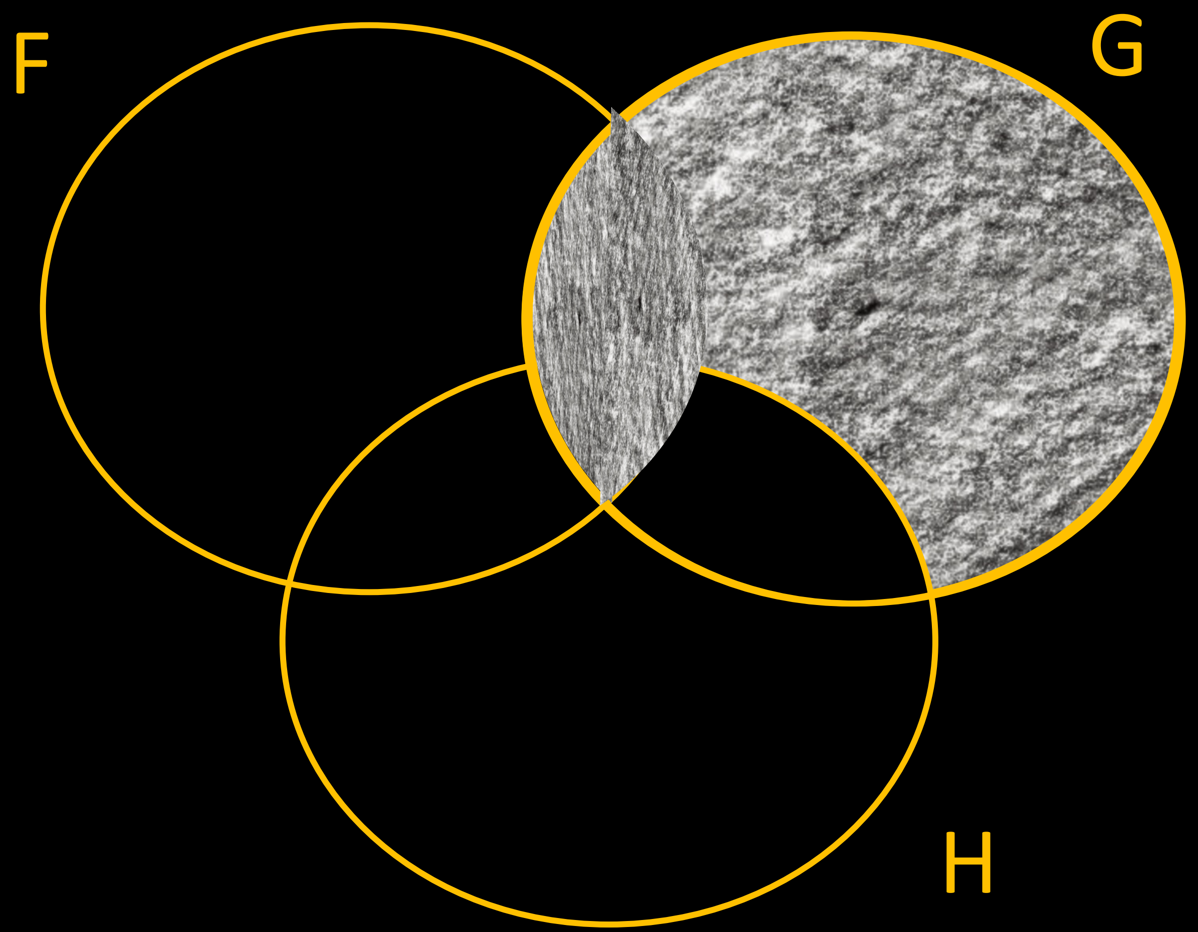


Consider the argument :

All P are Q.
No Q are S.
∴ No P are S.

valid

Using Venn diagram models for arguments

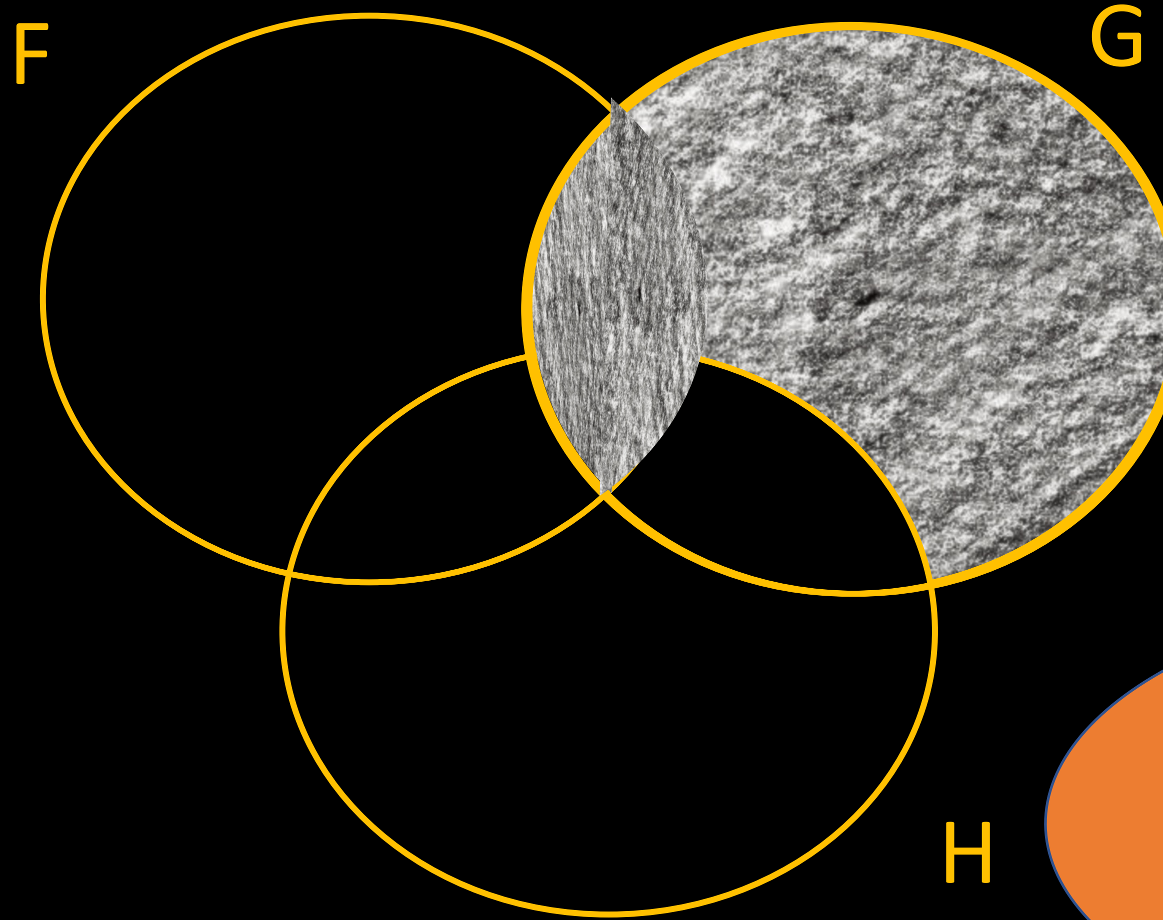


Consider the argument :

No F are G
All G are H.
∴ Some F are H

invalid

Using Venn diagram models for arguments



Consider the argument :

No F are G
All G are H.
∴ Some F are H

invalid

Why?
Recall, for the argument to be valid, the conclusion MUST follow. Here, we don't **have to have** any F that are H!