Quantified logic, p.1

Week 5 . Topic introduction.

Recall:

What's next: Quantified logic

The vocabulary of QL builds on the vocabulary of SL, which is what gives QL a lot more expressive power!

We will have:

- Connectives: ∧, v, →, ¬, ↔
- Parentheses: (,)
- Predicate/relation terms: capital letters like A, B …
- A special, logical relation: =
- Constants: lower case letters like a, b …
- Variables: x, y, z…
- Quantifiers: ∀, ∃

New pieces of vocab are indicated in purple!

We will learn these by using them in translations!

Properties

- We usually think of properties as something like “my mug is blue” and so the mug as the “blue” property.

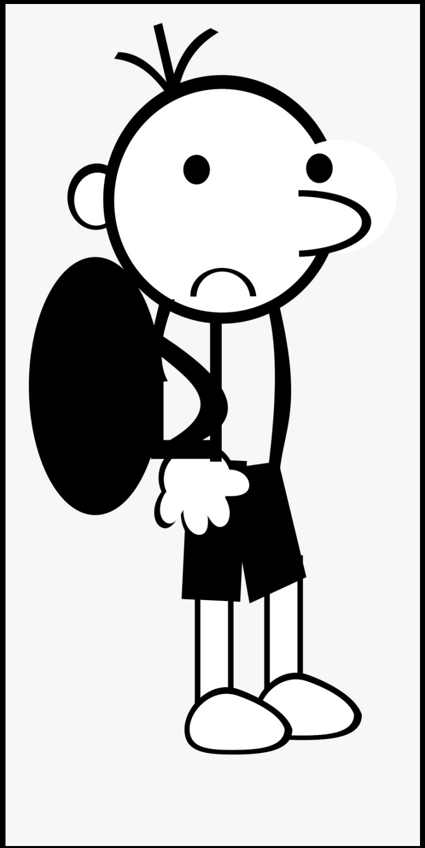
- “Fabio is a student” -> Fabio has the property “being a student”
- “Socrates is wise” -> Socrates has the property “being wise”

Properties

- In logic, we understand properties a bit more broadly.

- "My dog is running" → we understand the dog to have the property "is running"

How do we symbolize this?



- These examples all have one object who possesses a property.

- “Fabio is a student” involves the object “Fabio” and the property “is a student” that Fabio possesses
- In QL, we symbolize this sentence as Sf, where
  - S: is a student
  - f: Fabio

Predicates

- This way, we can see the parallel structure of sentences:
  - Fabio is a student = Sf
  - William is a student = Sw
  - Jingyi is a student = Sj

'S' is like a partial sentence:
____ is a student.

- We call partial sentences like 'S' a **predicate**.
- Note1: Capitalization now starts to matter!
- Note 2: There is only one blank in this partial sentence, so we also sometimes say that it's a 1-place predicate.

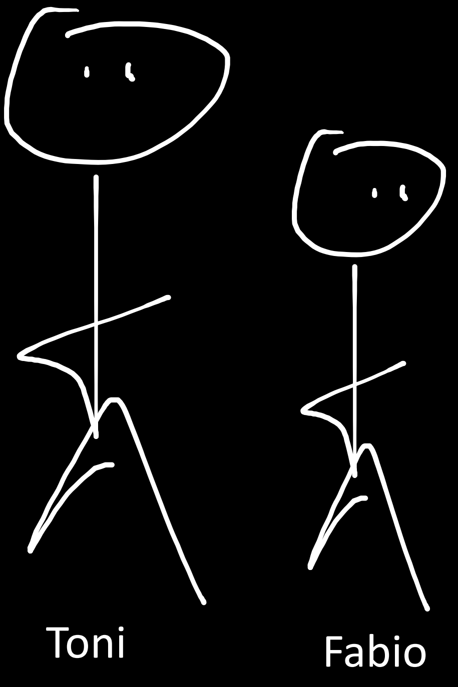
Relations

- In logic, we not only expand the notion of "properties" to include actions like "is running"; we also expand it to include expressions like comparisons or transitive verbs.

- So: "Toni is taller than Fabio" is considered a property that Toni and Fabio collectively hold.

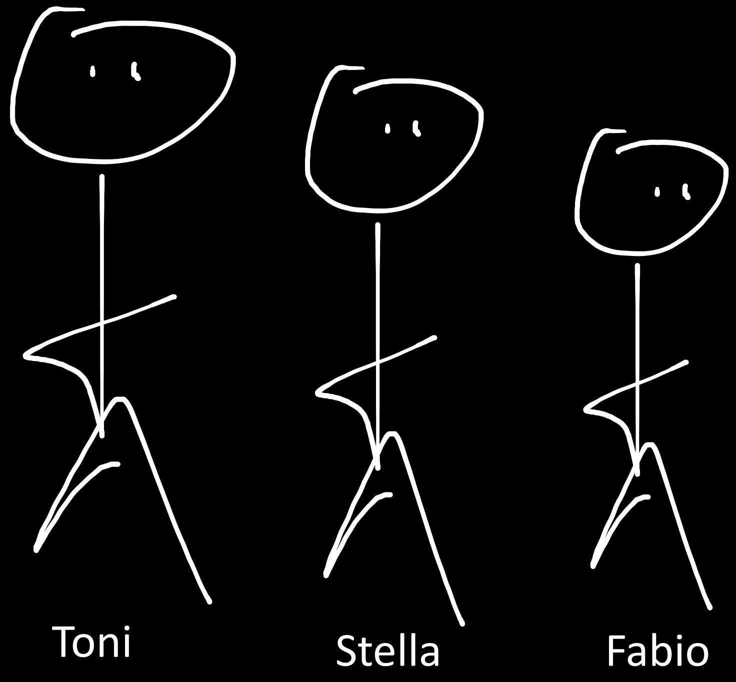
- "The dog is chasing the cat" is a property that the dog and the cat collectively hold.

We can translate
"Toni is taller than Fabio" as follows:
"t" means "Toni" and "f" means
"Fabio" (both are objects).
"T" means "___ is taller than ___"
is the partial sentence.
"Toni is taller than Fabio"
then translates to Ttf.



- We can have parallel sentences like:
  - "Toni is taller than William": Ttw
  - "Greg is taller than William": Tgw
- The partial sentence "___ is taller than ___" is called a ***relation***. Because it takes two blanks, it's a **2-place relation.**

Can we have more than 2-place relations?



- Sure! Here's an example of a 3-place relation:
  - "Stella is between Toni and Fabio" = Bstf

Just because a relation has 3 places it doesn't mean it requires 3 people! Consider: "Stella introduced herself to Fabio" = Issf

Relations & predicates: observations

- From a formal perspective, ***predicates*** **can be seen as 1-place relations**, and ***relations*** **are n-place predicates.**
  - There's no deep significance to this distinction though.

- Note: Once we have n-place predicates where n>1, order matters!
  - E.g., if I had written $Ifss$ accidently, this would translation to "Fabio introduced Stella to Stella" which wouldn't make sense…

Relations & predicates: observations

Also note: just like we had with SL, we have some flexibility in translation in QL.

- Recall: we had "Stella introduced herself to Tobi" translated as "Isst."
- We can also translate it into "I_2st". I_2 is "___ introduced oneself to ___".

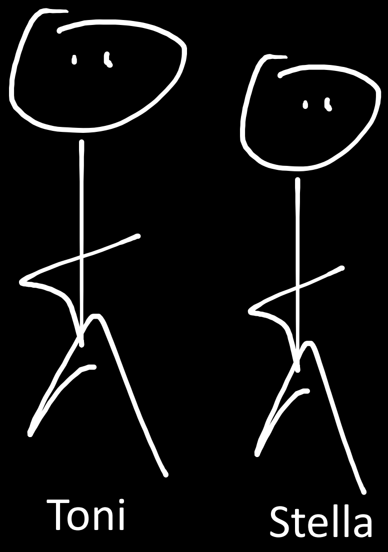
- This depends on how much internal structure we want to pick out when translating.

2-place relations can be…

- symmetric
- reflexive
- transitive

Properties of 2-place relations: symmetry

- Order *can* matter in a relation, but it doesn't always.
  - E.g., "Stella is next to Tobi" (Nst) and "Tobi is next to Stella" (Nts) *always have the same truth value*.
  - Other examples: "___ is same age as ___" "___ is as tall as ___"
- When this happens, we say that the relation is **symmetric**.
  - A relation that's not symmetric is *asymmetric*.

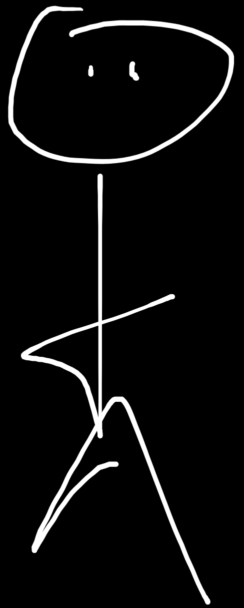


2-place relations can be…

A relation R is such that…

- symmetric: for all x and y, if Rxy, then Ryx.
- reflexive
- transitive

Properties of 2-place relations: reflexivity



Josiah

- Sometimes a relation *always holds between an object and itself.*
  - E.g., “Josiah is as tall as himself” (Tjj)
  - E.g., “Jessica is the same age as herself” (Ajj).

- When this happens, the relation is **reflexive**.
  - A relation that’s not reflexive is *irreflexive*.

2-place relations can be…

A relation *R* is such that…

- symmetric: for all *x* and *y*, if *Rxy*, then *Ryx*.
- reflexive: for all all *x*, *Rxx*.
- transitive

2-place relations: symmetric and reflexive?

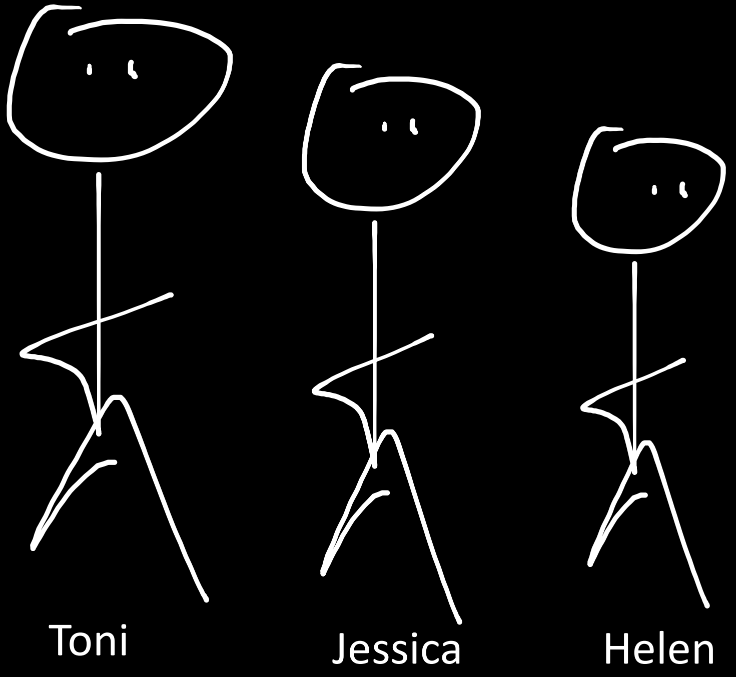
- Not all symmetric relations are reflexive
  - E.g., “___ stands next to ___” is symmetric but not reflexive since you can’t stand next to yourself.
- Not all reflexive relations are symmetric
  - E.g., “___ has heard of ___” you always have heard of yourself but not everyone you’ve heard of has heard of you.

2-place relations: transitivity

- A relation can also be **transitive**, which means that whenever Rab and Rbc, then Rac must be true.

- E.g., if "Toni is taller than Jessica" (Ttj) and "Jessica is taller than Helen" (Tjh) then "Toni is taller than Helen" (Tth) must be true.
  - Note: this relation is transitive but asymmetric and irreflexive.

- Example of a *non-transitive* but symmetric relation: "___ is holding hands with ___"
- ☹ example of an asymmetric, irreflexive, and non-transitive relation: ___ loves ___



2-place relations can be…

A relation *R* is such that…

- symmetric: for all *x* and *y*, if *Rxy*, then *Ryx*.
- reflexive: for all all *x*, *Rxx*.
- transitive: for all *x*, *y*, *z*, if *Rxy* and *Ryz* then *Rxz*.

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- ✓ $\boxed{\text{Constants: lower case letters like a, b …}}$
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Constants: general notes

- We've actually already seen them!
- Consider: "Fabio is a student" (Sf)
  - 'S' is a predicate
  - f is a constant.
- We use lower case letters that fall before "x" in the alphabet for constants.
- Constants denote objects that fill in the blanks of predicates.

Constants aren't the only one way to fill in those blanks! We'll learn other ways in the next lecture.

One type of constants: proper names

- No tricks here: names are names!
  - Mine is “Helen.”
  - Your TAs are “Jessica” and “Josiah.”
- Each name picks out a single individual:
  - “Helen” picks out me.

- When we say, e.g., Th for “Helen teaches”, we are assigning the property signalled by T (___teaches) to the object picked out by h (Helen).

But wait…what if two people have the same name?

This is how a “proper name” differs from a “natural language name”.

In logic, we assume that each proper name picks out exactly one object.

When two people share a natural language name, we assume that the names are logically differentiable. We can represent this with subscripts. E.g., we can say “$David_1$ is taller than $David_2$”.

But wait…what if I’m talking about something that doesn’t exist? What does it pick out?

This is a major difficulty of QL. In the standard QL, we cannot express “Nessy (the Loch Ness monster) does not exist”. This difficulty is called *the problem of non-referring terms*. It has been a major motivator for logicians to look beyond standard QL and develop more expressively powerful logics.

But wait…what if something has multiple names?

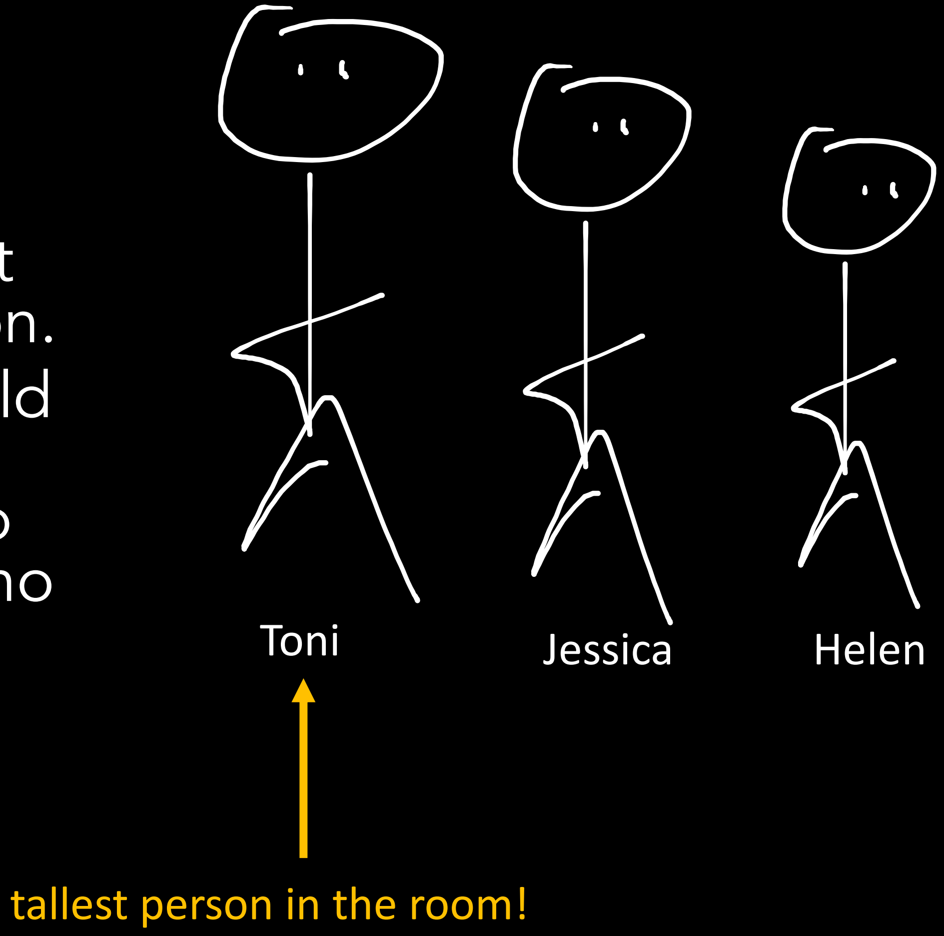
UCI goes by i: "University of California, Irvine" or say, a: "University of the Anteaters."

When this happens, we use a special relation "=" and write "i=a."

This identity relation is a lot more powerful than it looks. We'll revisit it again and again in the future.

The other type of constants: definite descriptions

- Another kind of constants are called **definite descriptions**.
  - They are "descriptions" in the usual sense of the term, and they are "definite" in that exactly one object would fit the description.
  - E.g., "the tallest person in the room" would pick out the tallest person in the room. It would not pick out anyone if there are two people who are both tallest, or if there is no one in the room.
  - E.g., "the instructor of this course" picks out me, but "the student in the course" wouldn't pick out any specific person.



Constants: definite descriptions

- We can use a definite description only when it picks out exactly one individual. In that case, we use it the same way we use a name.
- E.g., "the tallest person in the room is taller than William" can be translated as Ttw, where
  - T stands for the relation "is taller than"
  - t stands for the definite description "the tallest person in the room"
  - w stands for the proper name William

Properties vs. names

- In natural language, whether a description is definite (i.e. picks out a single object) seems not very important.
  - E.g., “Jeff is the tallest person in the room” seems to be not that different from “Jeff is one of the tallest people in the room”.

- However, in logic, we usually treat definite descriptions as name-like (i.e. constants), and indefinite descriptions as property-like (i.e. predicates).
  - E.g., we would translate “Jeff is the tallest person in the room” as j=t, and “Jeff is one of the tallest people in the room” as Tj.

Still to come!

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