


Quantifier rules: universal introduction

WEEK 7 . TOPIC INTRODUCTION

UNIVERSAL INTRODUCTION



This is like saying
everything in our domain
satisfies such and such.
This sounds like it'll be
hard to claim!

The $\Phi[a/x]$ part means 'Take Φ and replace every instance of a with x '

UNIVERSAL INTRODUCTION

1. Φa
2. $\forall x \Phi[a/x] \quad I\forall 1$

Restrictions:

1. a cannot occur in any open assumption(s)
 - cannot occur in the premises (which are open for the whole proof)
 - if within a subproof, cannot occur in the hypothesis for the subproof
2. you must replace *all* instances of the constant with the variable

UNIVERSAL INTRODUCTION

1. Φa
2. $\forall x \Phi[a/x] \text{ I}\forall 1$

Prove: $\forall z(Fz \rightarrow Gz) \vdash \forall x(Fx \rightarrow Gx)$

1. $\forall z(Fz \rightarrow Gz) : \text{assumption}$
2. $(Fa \rightarrow Ga) : E\forall 1$
3. $\forall x(Fx \rightarrow Gx) : I\forall 2$

Was this a
legitimate use
of universal
introduction?



UNIVERSAL INTRODUCTION

1. Φa
2. $\forall x \Phi[a/x] \text{ I}\forall 1$

Prove: $\forall x Fax, \forall x \forall y (Fxy \rightarrow Gyx) \vdash \forall x Gxa$

1. $\forall x Fax$:assumption
2. $\forall x \forall y (Fxy \rightarrow Gyx)$:assumption
3. Fab : $E\forall 1$
4. $\forall y (Fay \rightarrow Gya)$: $E\forall 2$
5. $Fab \rightarrow Gba$: $E\forall 4$
6. Gba : $E\rightarrow 3, 5$
7. $\forall x Gxa$: $I\forall 6$

Was this a
legitimate use
of universal
introduction?



UNIVERSAL INTRODUCTION

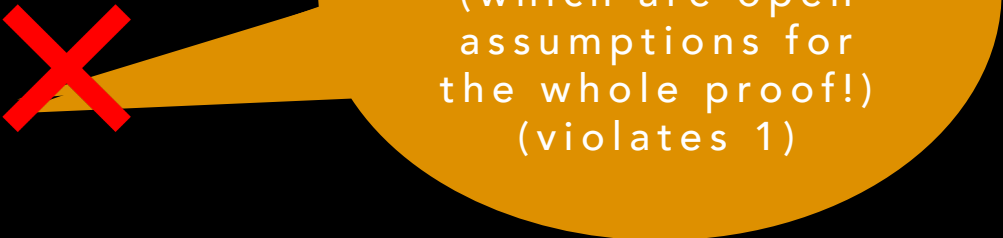
1. Φa
2. $\forall x \Phi[a/x] \quad I\forall 1$

Restrictions:

1. a cannot occur in any open assumption(s)
 - cannot occur in the premises (which are open for the whole proof)
 - if within a subproof, cannot occur in the hypothesis for the subproof
2. you must replace *all* instances of the constant with the variable

Prove: $\forall x(Fx \rightarrow Gx), Fb \vdash \forall xGx$

1. $\forall x(Fx \rightarrow Gx)$:assumption
2. Fb :assumption
3. $Fb \rightarrow Gb$:E $\forall 1$
4. Gb :E $\rightarrow 3, 2$
5. $\forall xGx$:I $\forall 4$



Not legitimate
because b occurs
in the premises
(which are open
assumptions for
the whole proof!)
(violates 1)