HOW DO I CHECK FOR VALIDITY WITH TRUTH TABLES?

WEEK 2 . TOPIC INTRODUCTION

LAST TIME...

we constructed truth tables for well-formed formulas.

TODAY...

we construct truth tables for argument forms.

RECALL

$(C \rightarrow D), C \therefore D$

Translation manual:

C is "He's a white supremacist" and

D is "He's a racist."

RECALL

$(C \rightarrow D), C \therefore D$

Translation manual:

C is "He's a white supremacist" and

D is "He's a racist."

We will say that an argument is *valid* if for all possible combinations of truth-values of the basic propositional letters in the formulas,

if all the premises are true on a combination, then the conclusion is true on that combination

For short we say: an argument is *valid* if
whenever all the premises are true,
the conclusion is true.

We symbolize validity with $\vDash$ as in:

$$(p \wedge q) \vDash q$$

We write $\phi_1, \ldots, \phi_n \vDash \psi$ to indicate that the argument with premises $\phi_1 \ldots, \phi_n$ and conclusion ψ is valid.

As you can see, when we have multiple premises, we separate them with commas.

We read $(p \wedge q) \vDash q$ as:

- $(p \wedge q)$ has q as a consequence
- $(p \wedge q)$ models q
- q follows from the premise $(p \wedge q)$
- The argument with premise $(p \wedge q)$ and conclusion q is valid.

HOW DO WE CHECK THIS WITH TRUTH TABLES?

For short we say: an argument is *valid* if
whenever all the premises are true,
the conclusion is true.

HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\vDash$	q
T	T	T	T	T		T
T	F	T	F	F		F
F	T	F	F	T		T
F	F	F	F	F		F

HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\models$	q
T	T	T	T	T		T
T	F	T	F	F		F
F	T	F	F	T		T
F	F	F	F	F		F

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HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\vDash$	q
T	T	T	T	T		T
T	F	T	F	F		F
F	T	F	F	T		T
F	F	F	F	F		F

For short we say: an argument is *valid* if
whenever all the premises are true,
the conclusion is true.

HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\vDash$	q
T	T	T	T	T	✓	T
T	F	T	F	F		F
F	T	F	F	T		T
F	F	F	F	F		F

For short we say: an argument is *valid* if whenever all the premises are true, the conclusion is true.

HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\models$	q
T	T	T	T	T	✓	T
T	F	T	F	F	✓	F
F	T	F	F	T	✓	T
F	F	F	F	F	✓	F

HOW DO WE CHECK THIS WITH TRUTH TABLES?

p	q	$(p$	$\wedge$	$q)$	$\vDash$	q
T	T	T	T	T	✓	T
T	F	T	F	F	✓	F
F	T	F	F	T	✓	T
F	F	F	F	F	✓	F

What about the rest of the rows?

The premises aren't true in the other rows.
So it doesn't matter what the conclusion is, the argument will be valid.

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

AN ARGUMENT IS *VALID* IF WHENEVER ALL THE PREMISES ARE TRUE, THE CONCLUSION IS TRUE.

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

AN ARGUMENT IS *VALID* IF WHENEVER ALL THE PREMISES ARE TRUE, THE CONCLUSION IS TRUE.

THIS IS LIKE THE CONDITIONAL TRUTH TABLE.

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

AN ARGUMENT IS *VALID* IF WHENEVER ALL THE PREMISES ARE TRUE, THE CONCLUSION IS TRUE.

THIS IS LIKE THE CONDITIONAL TRUTH TABLE.

Conditional is only false if the premise is true and the conclusion doesn't follow. An argument is only invalid if there's a case when the premises are all true but the conclusion isn't.

MORE EXAMPLES...

p	q	$(\neg$	p	$\vee$	$q)$	$\models q$
T	T	F	T	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	F	T	F	F

MORE EXAMPLES...

p	q	($\neg$	p	$\vee$	q)	$\models$ q
T	T	F	T	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	F	T	F	F

MORE EXAMPLES...

p	q	$(\neg$	p	$\lor$	$q)$	$\models$	q
T	T	F	T	T	T		T
T	F	F	T	F	F		F
F	T	T	F	T	T		T
F	F	T	F	T	F		F

MORE EXAMPLES...

p	q	$(\neg$	p	$\vee$	$q)$	$\models$	q
T	T	F	T	T	T	✓	T
T	F	F	T	F	F		F
F	T	T	F	T	T	✓	T
F	F	T	F	T	F		F

MORE EXAMPLES...

p	q	$(\neg$	p	$\vee$	$q)$	$\models$	q
T	T	F	T	T	T	✓	T
T	F	F	T	F	F	✓	F
F	T	T	F	T	T	✓	T
F	F	T	F	T	F		F

MORE EXAMPLES...

p	q	($\neg$	p	$\vee$	q)	$\models$	q
T	T	F	T	T	T	✓	T
T	F	F	T	F	F	✓	F
F	T	T	F	T	T	✓	T
F	F	T	F	T	F		F

MORE EXAMPLES...

p	q	$(\neg$	p	$\vee$	$q)$	$\models$	q
T	T	F	T	T	T	✓	T
T	F	F	T	F	F	✓	F
F	T	T	F	T	T	✓	T
F	F	T	F	T	F	✗	F

Here we have the premises as true, but the conclusion is false. Just one row like this is sufficient to make the argument invalid!

MORE EXAMPLES...

p	q	$(\neg$	p	$\vee$	$q)$	$\models$	q
T	T	F	T	T	T	✓	T
T	F	F	T	F	F	✓	F
F	T	T	F	T	T	✓	T
F	F	T	F	T	F	✗	F

- $(\neg p \vee q)$ does not have q as a consequence
- $(\neg p \vee q)$ does not model q
- q does not follow from the premise $(\neg p \vee q)$
- The argument with premise $(\neg p \vee q)$ and conclusion q is invalid.

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\rightarrow$	$q),$	$(p$	$\rightarrow$	$\neg$	$q)$	$\vDash$	$\neg$	p
T	T	T	T	T	T	F	F	T		F	T
T	F	T	F	F	T	T	T	F		F	T
F	T	F	T	T	F	T	F	T		T	F
F	F	F	T	F	F	T	T	F		T	F

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\to$	$q),$	$(p$	$\to$	$\neg$	$q)$	$\models$	$\neg$	p
T	T	T	T	T	T	F	F	T		F	T
T	F	T	F	F	T	T	T	F		F	T
F	T	F	T	T	F	T	F	T		T	F
F	F	F	T	F	F	T	T	F		T	F

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\rightarrow$	$q),$	$(p$	$\rightarrow$	$\neg$	$q)$	$\vDash$	$\neg$	p
T	T	T	T	T	T	F	F	T		F	T
T	F	T	F	F	T	T	T	F		F	T
F	T	F	T	T	F	T	F	T		T	F
F	F	F	T	F	F	T	T	F		T	F

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\rightarrow$	$q),$	$(p$	$\rightarrow$	$\neg$	$q)$	$\vDash$	$\neg$	p
T	T	T	T	T	T	F	F	T		F	T
T	F	T	F	F	T	T	T	F		F	T
F	T	F	T	T	F	T	F	T	✓	T	F
F	F	F	T	F	F	T	T	F	✓	T	F

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\rightarrow$	$q),$	$(p$	$\rightarrow$	$\neg$	$q)$	$\vDash$	$\neg$	p
T	T	T	T	T	T	F	F	T	✓	F	T
T	F	T	F	F	T	T	T	F	✓	F	T
F	T	F	T	T	F	T	F	T	✓	T	F
F	F	F	T	F	F	T	T	F	✓	T	F

AN EXAMPLE WITH MULTIPLE PREMISES

p	q	$(p$	$\rightarrow$	$q),$	$(p$	$\rightarrow$	$\neg$	$q)$	$\models$	$\neg$	p
T	T	T	T	T	T	F	F	T	✓	F	T
T	F	T	F	F	T	T	T	F	✓	F	T
F	T	F	T	T	F	T	F	T	✓	T	F
F	F	F	T	F	F	T	T	F	✓	T	F

The argument with premises $(p \rightarrow q), (p \rightarrow \neg q)$ and conclusion $\neg p$ is valid.