

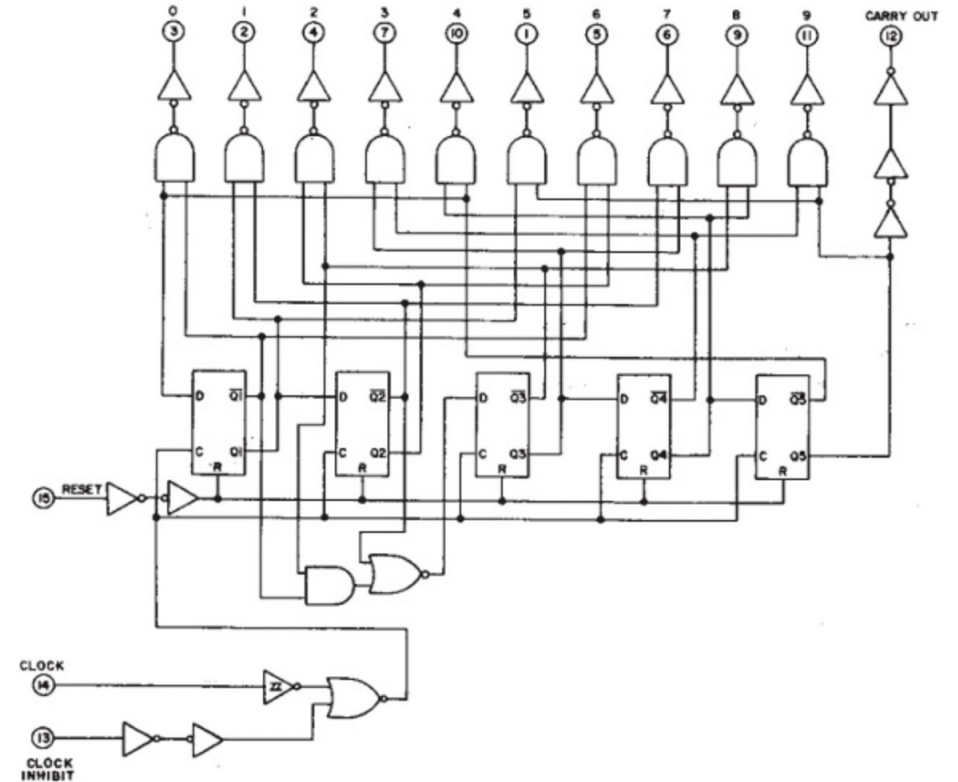
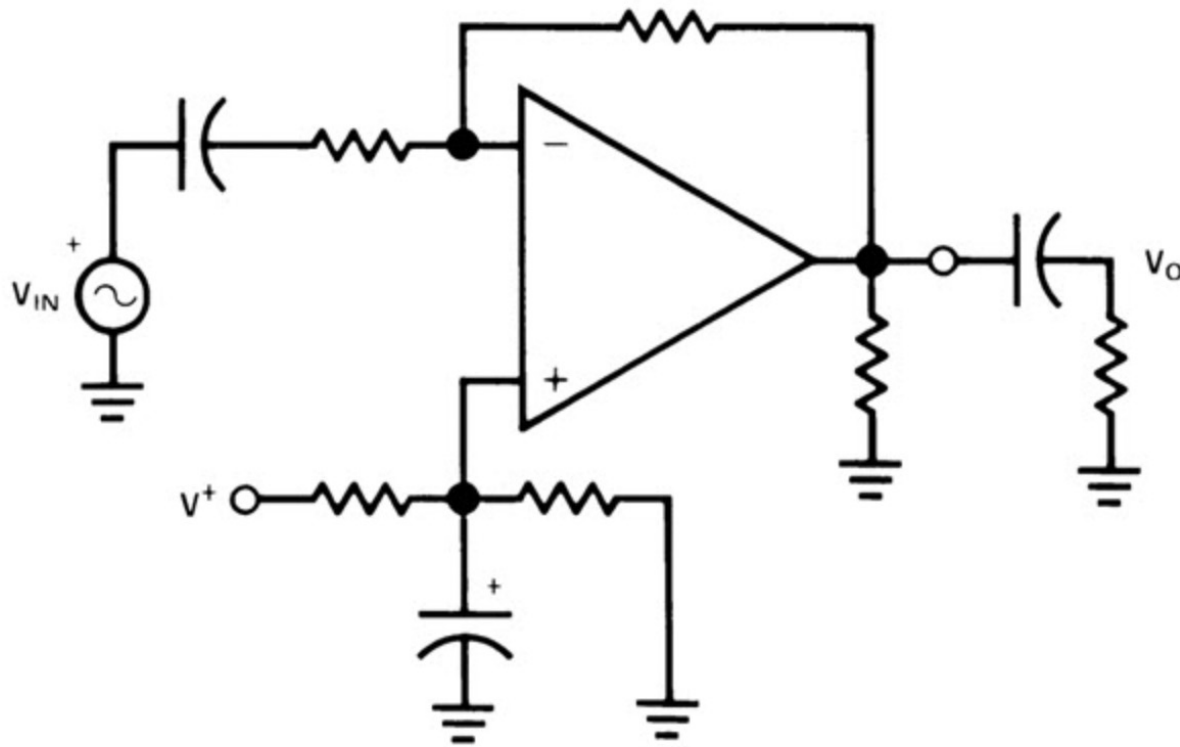
# Logic Gates

Supplemental module

# electronic circuits

analog

digital



digital circuits

consist of logic gates

constructed from transistors.

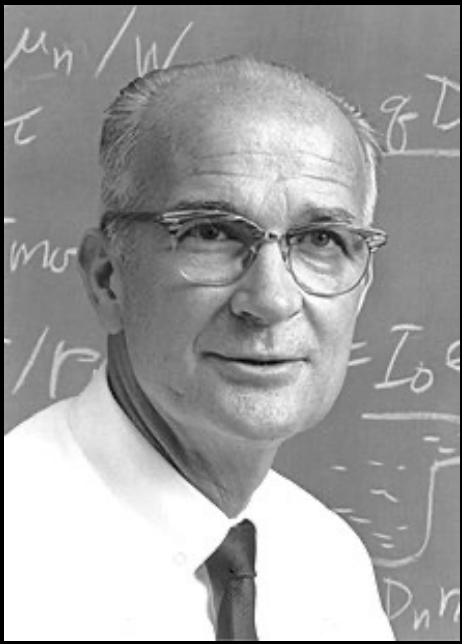
Learning objective: be able to translate between combinations of logic gates and wffs of SL.

digital circuits

consist of logic gates

constructed from transistors.





William  
Shockley

supervised



Walter  
Brattain

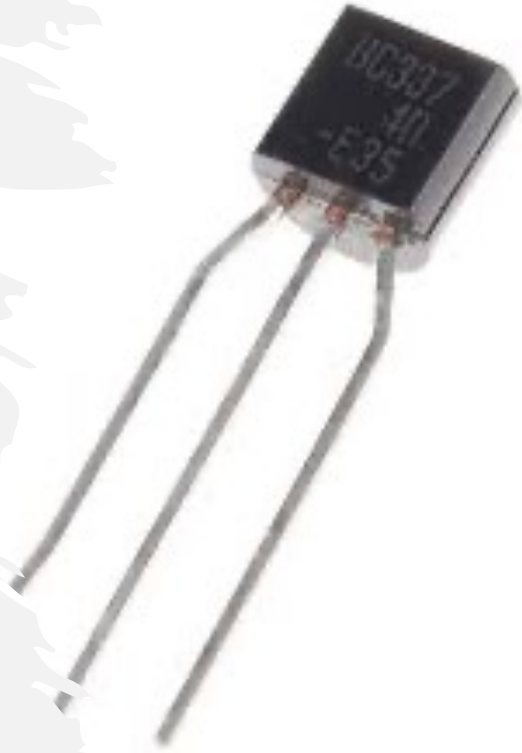
and



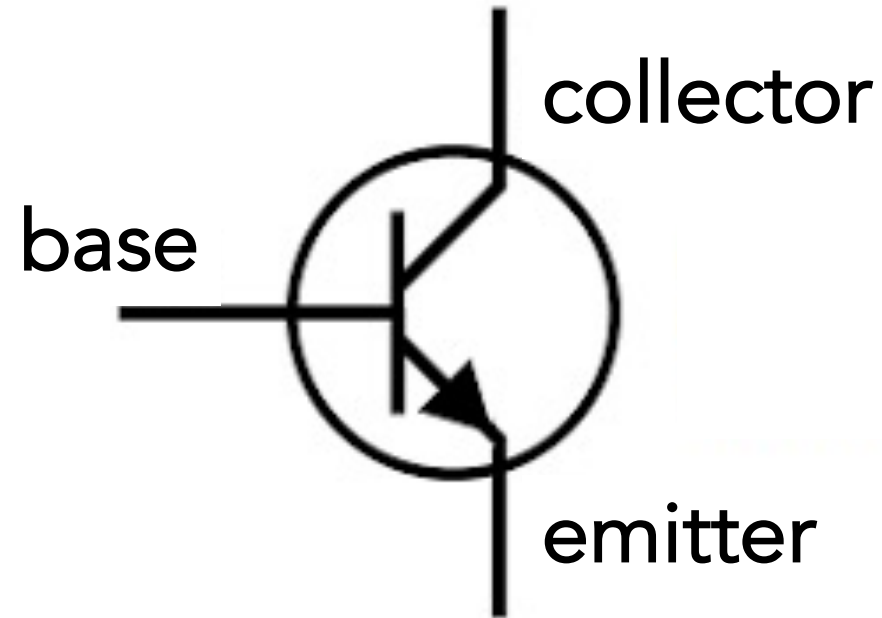
John  
Bardeen

who invented the transistor in 1948 at Bell labs  
and won the Nobel Prize in Physics in 1956.

# transistor



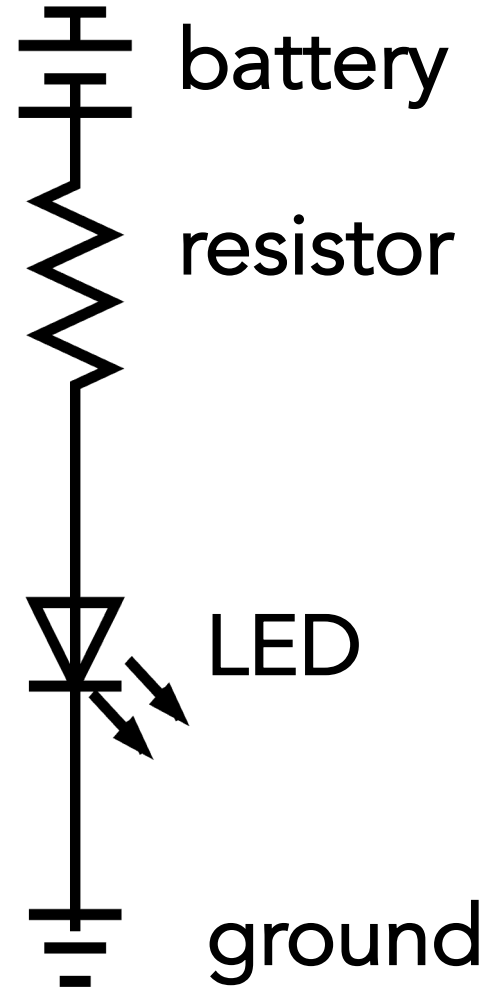
(Negative-Positive-Negative)



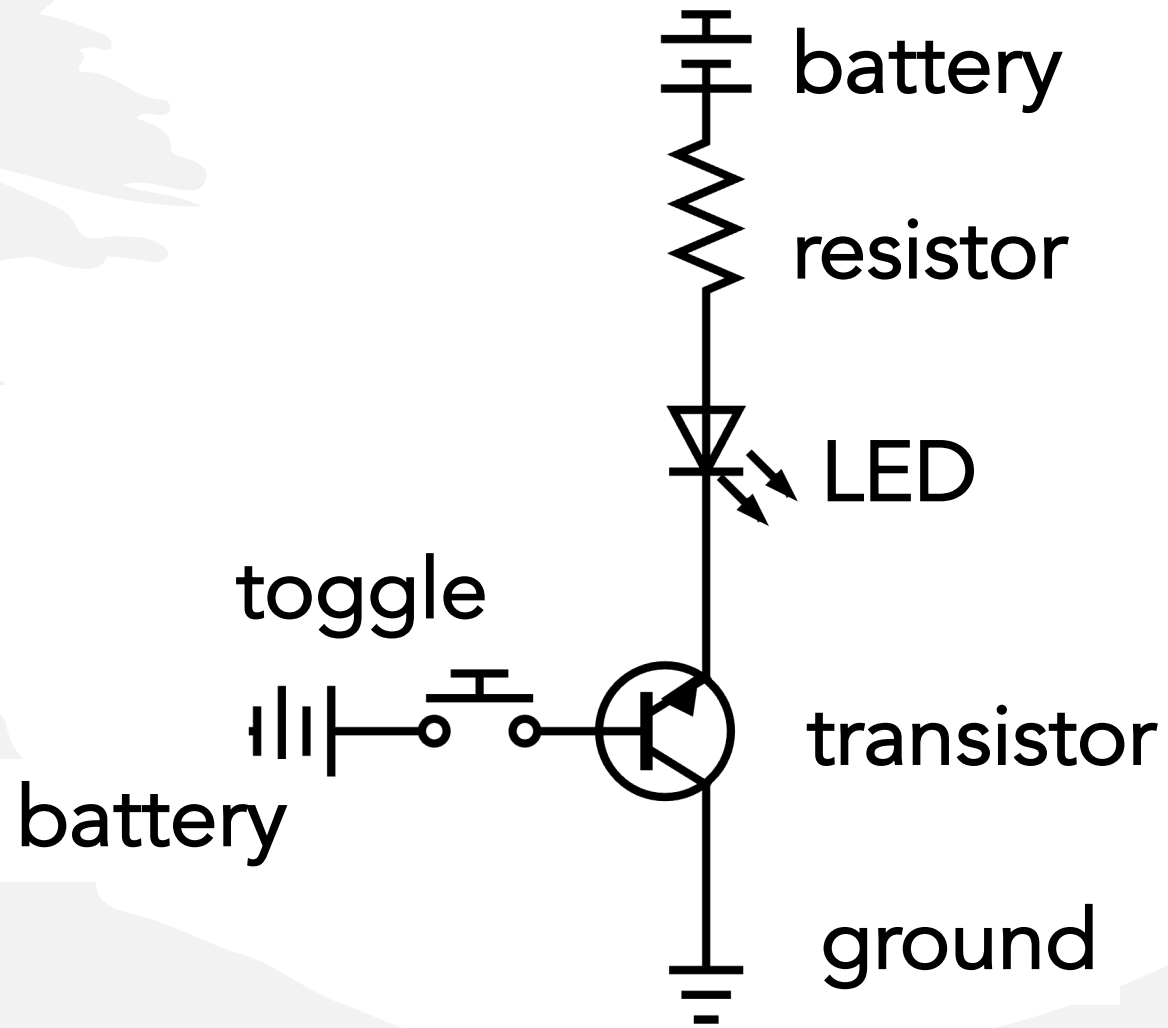
we can only go so far....  
for more on transistors,  
you can watch this video:







# transistor



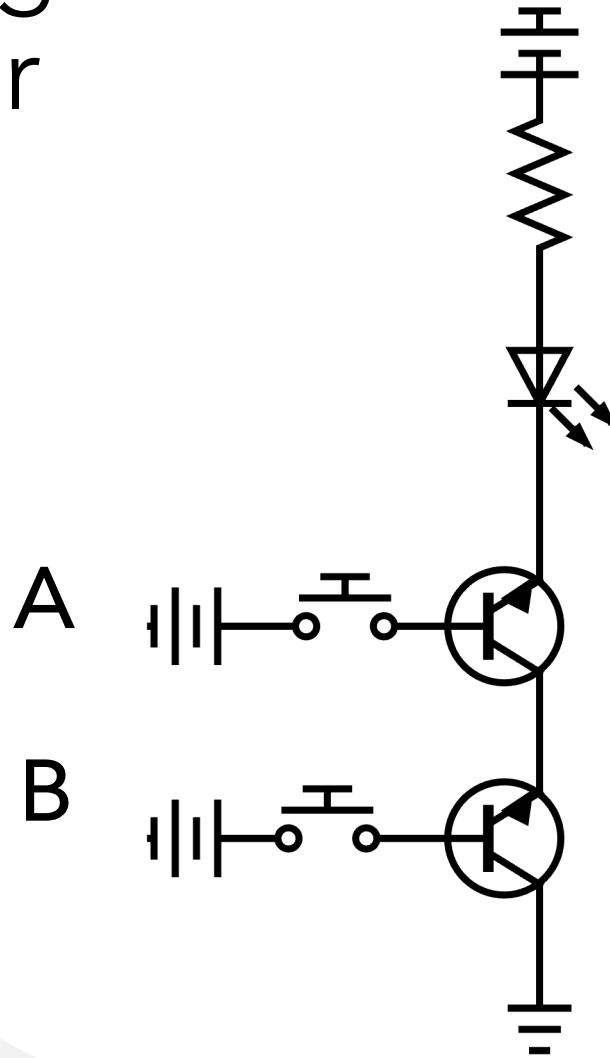
digital circuits

consist of logic gates

constructed from transistors.

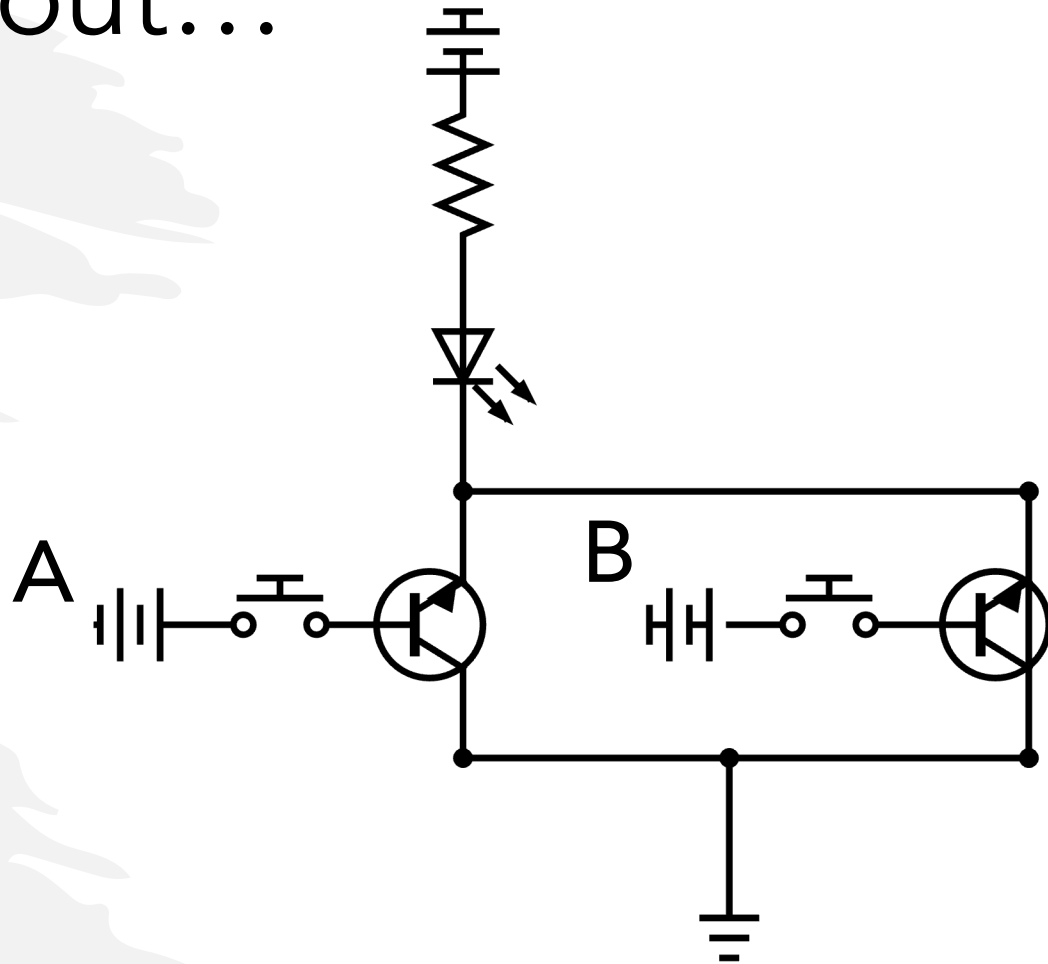


making a logic gate  
from a transistor

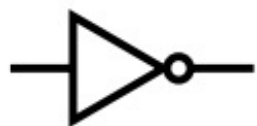


we've  
created an  
"and" gate!

what about...



it's an "or"  
gate



NOT



AND



OR



NAND

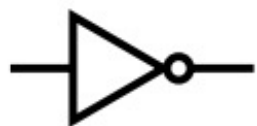


NOR



XOR

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$\neg(A \wedge B)$	$\neg(A \vee B)$	$(A \vee B) \wedge \neg(A \wedge B)$
T	T						
T	F						
F	T						
F	F						



NOT



AND



OR



NAND

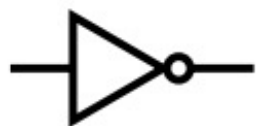


NOR



XOR

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$\neg(A \wedge B)$	$\neg(A \vee B)$	$(A \vee B) \wedge \neg(A \wedge B)$
T	T	F	T	T	F	F	F
T	F	F	F	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	F	F	T	T	F



NOT

AND

OR

NAND

NOR

XOR

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$\neg(A \wedge B)$	$\neg(A \vee B)$	$(A \vee B) \wedge \neg(A \wedge B)$
1	1	0	1	1	0	0	0
1	0	0	0	1	1	0	1
0	1	1	0	1	1	0	1
0	0	1	0	0	1	1	0



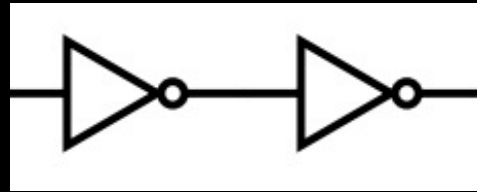
digital circuits



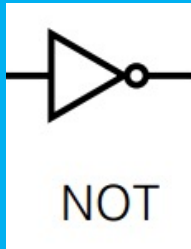
consist of logic gates

constructed from transistors.

can you translate this into a wff of SL?

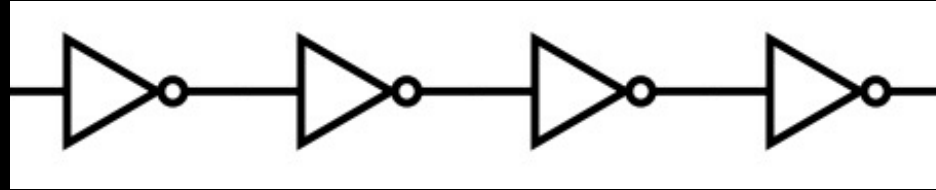


Recall:



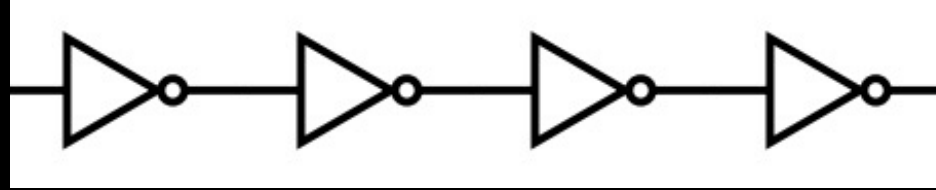
So we have  $\neg\neg a$

can you translate this into a wff of SL?



$\neg\neg\neg\neg a$

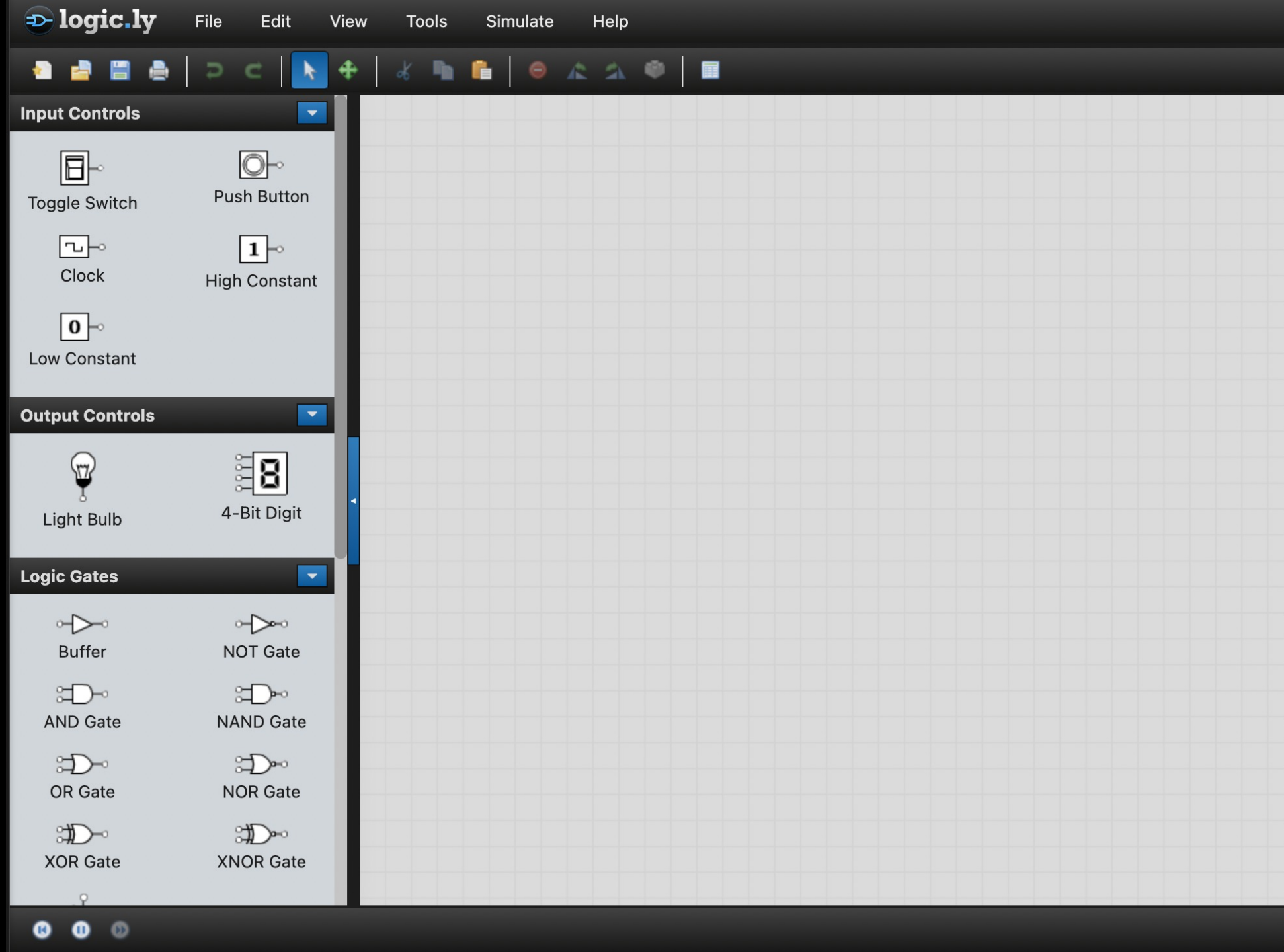
What is the output if the input at point (a) is a 0 bit?



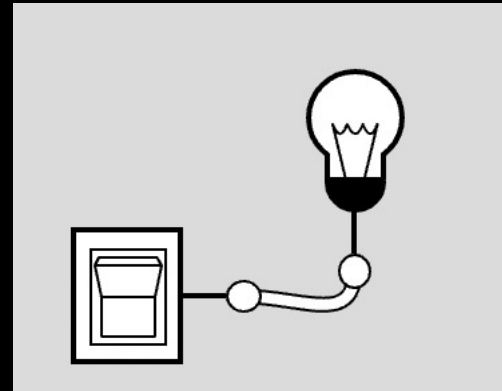
0 → 1 → 0 → 1 → 0 →

$a$	$\neg a$	$\neg \neg a$	$\neg \neg \neg a$	$\neg \neg \neg \neg a$
0	1	0	1	0

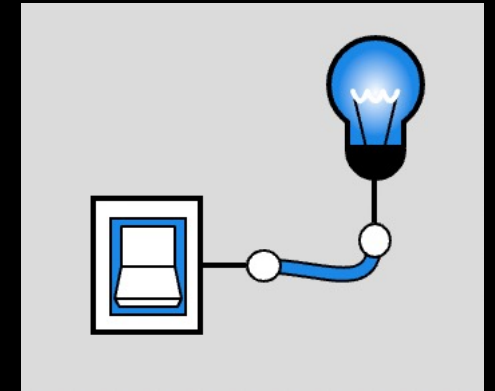
we'll use  
this tool to  
construct  
our logic  
gates.



we'll use  
this tool to  
construct  
our logic  
gates.

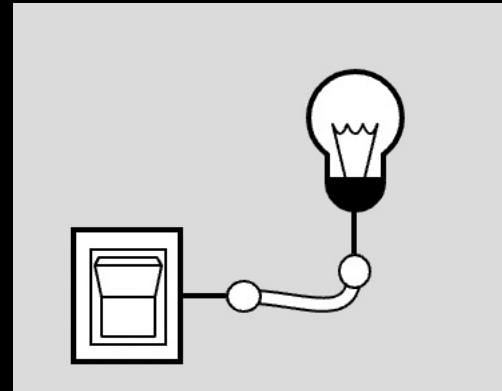


a is false, light is off

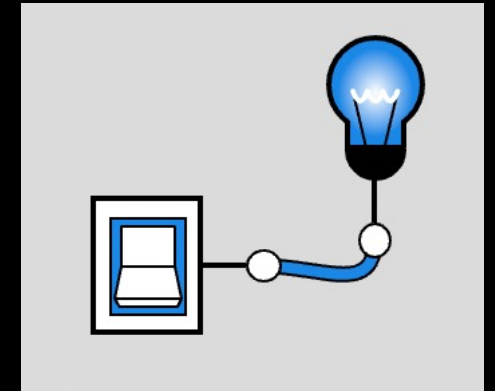


a is true, light is on

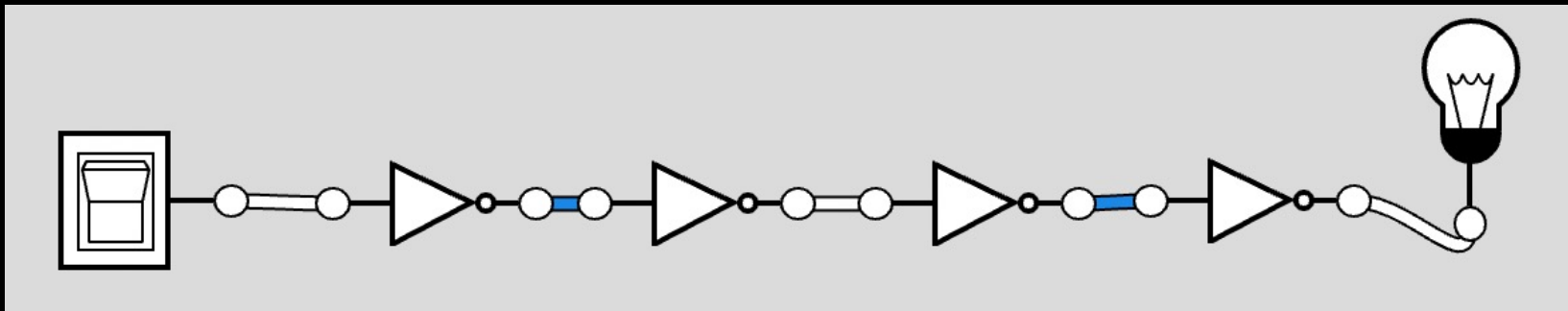
we'll use  
this tool to  
construct  
our logic  
gates.



$a$  is false, light is off

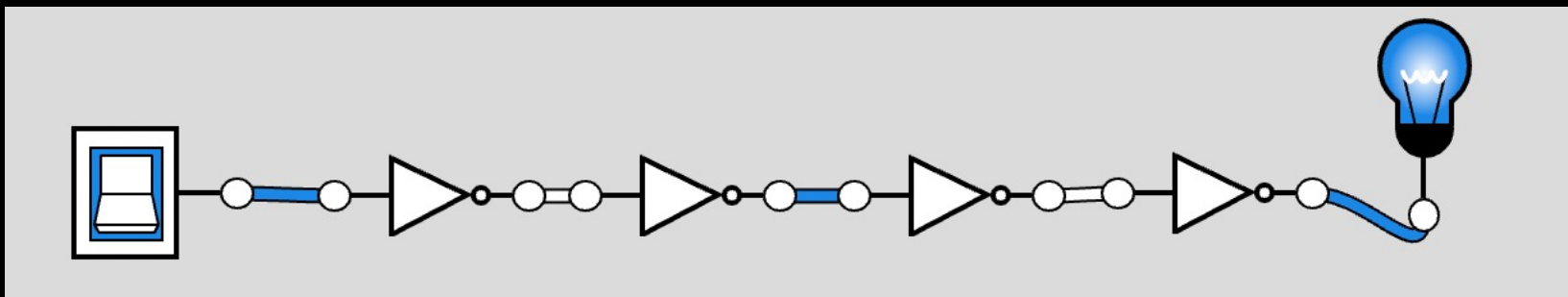


$a$  is true, light is on



$a$  is false

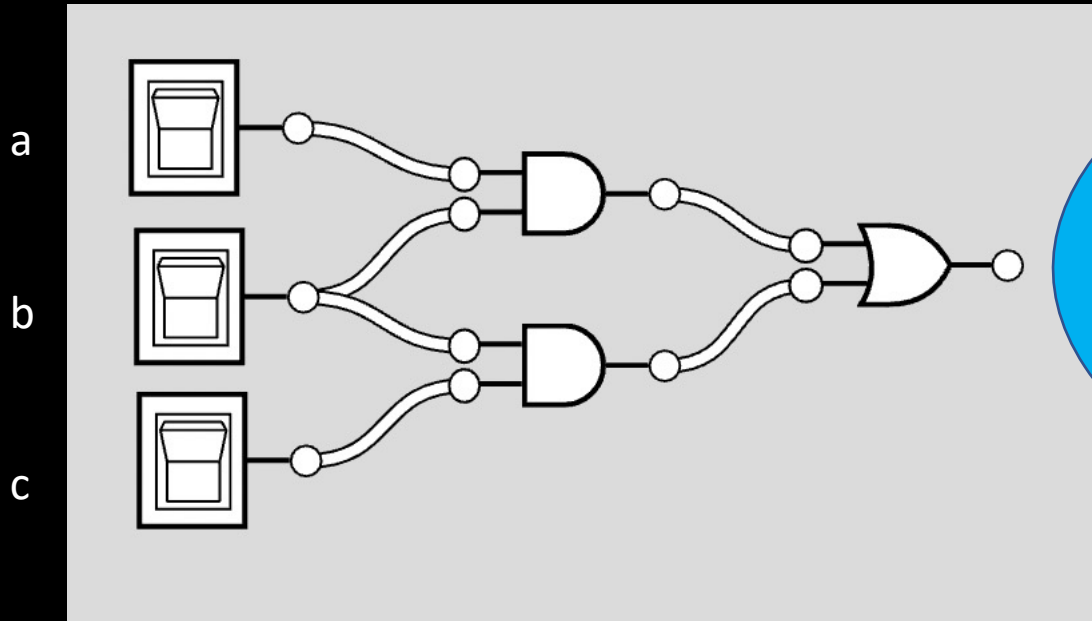
$\neg \neg \neg \neg a$  is false, light is off



$a$  is true

$\neg \neg \neg \neg a$  is true, light is on

can you translate this into a wff of SL?



Recall:



AND

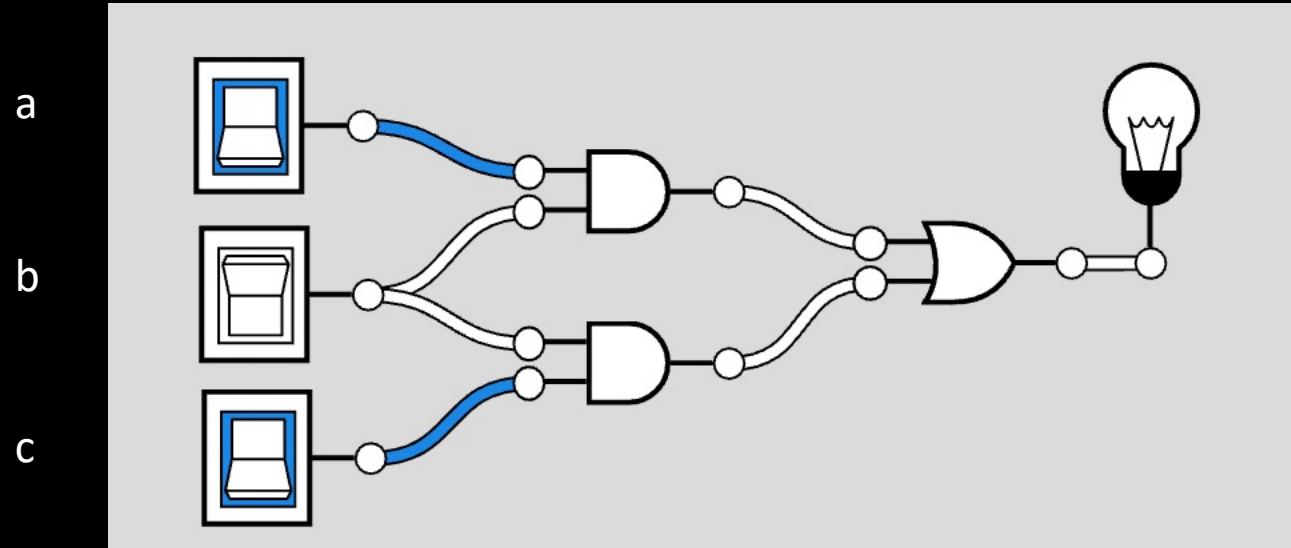


OR

$$(a \wedge b) \vee (b \wedge c)$$

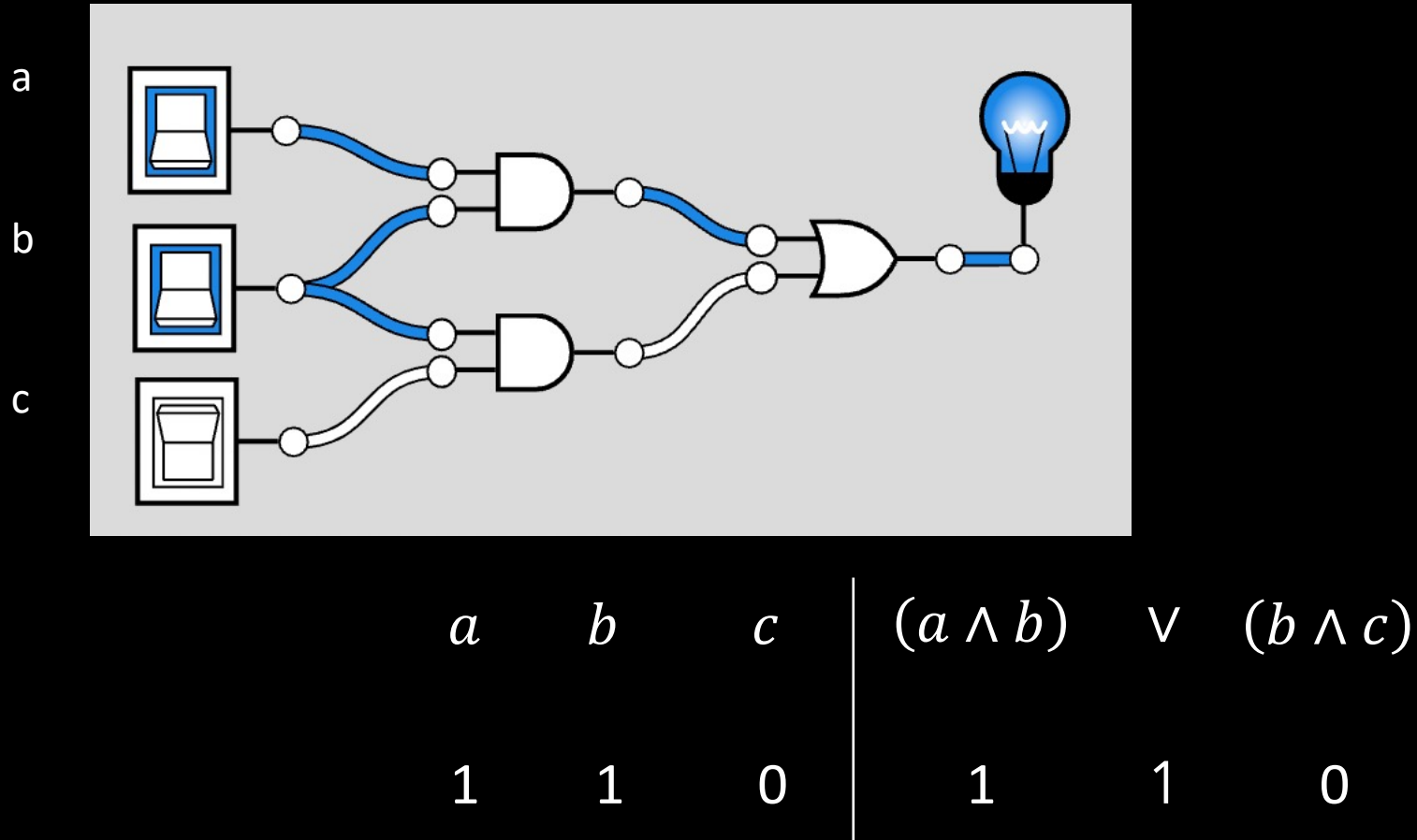


# If I connected a light, would it be on or off?

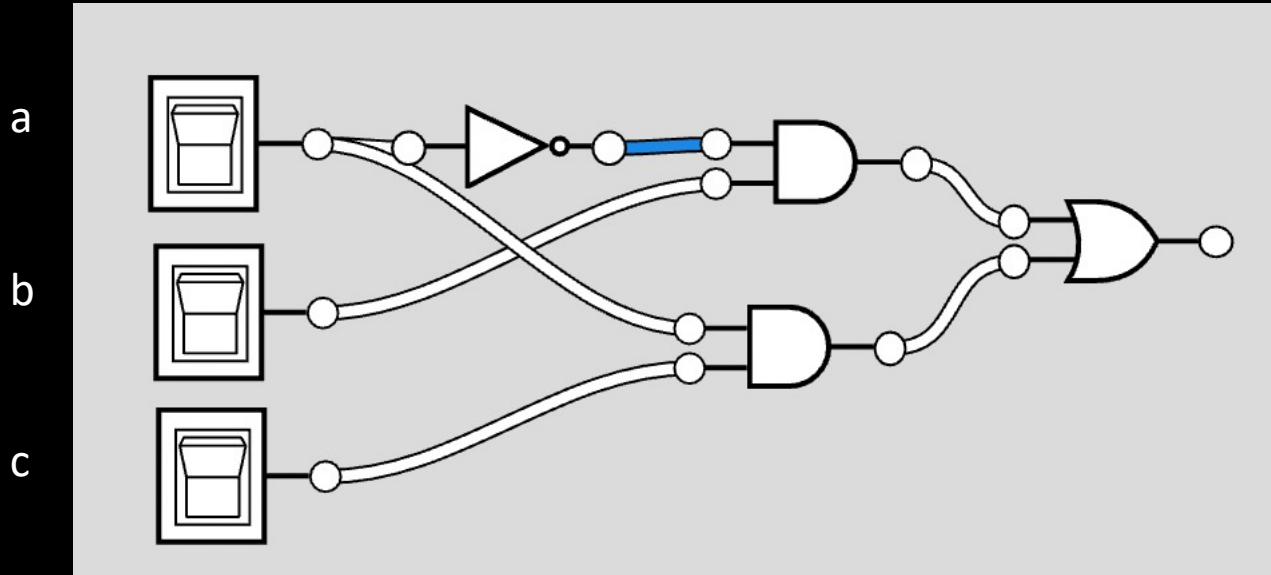


<i>a</i>	<i>b</i>	<i>c</i>	$(a \wedge b)$	$\vee$	$(b \wedge c)$
1	0	1	0	0	0

# If I connected a light, would it be on or off?

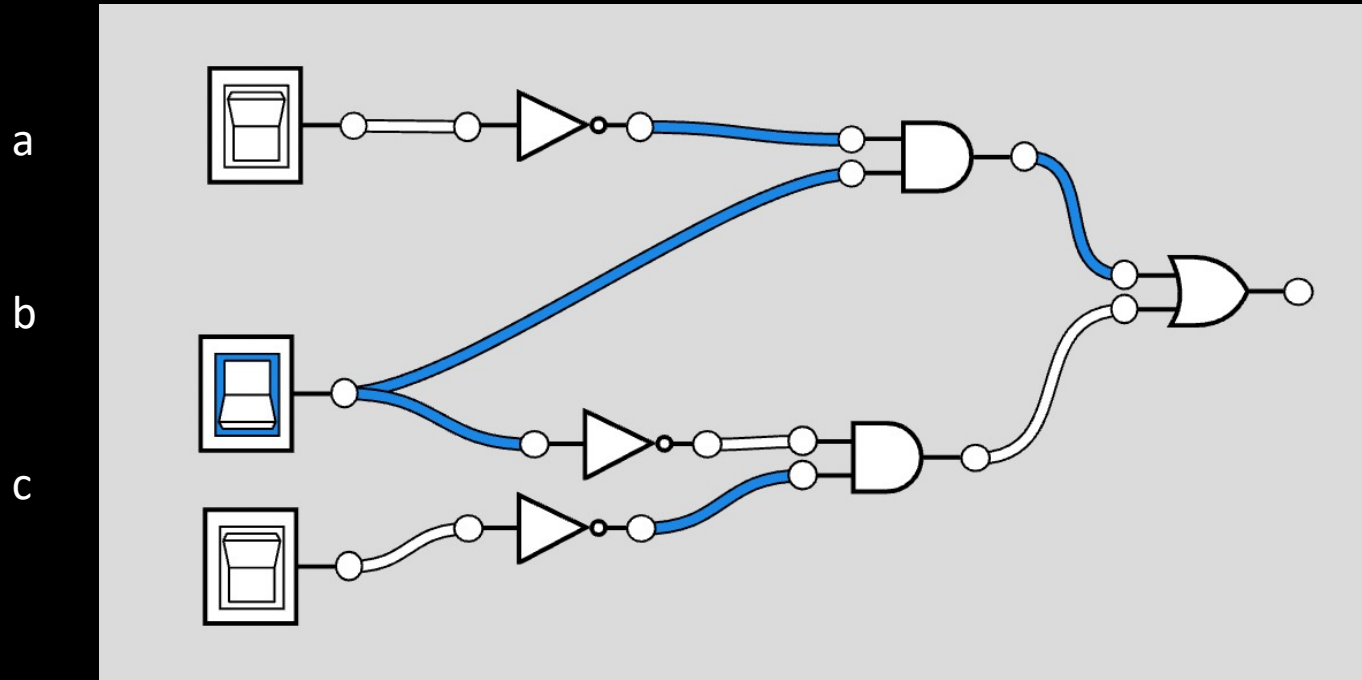


can you translate this into a wff of SL?

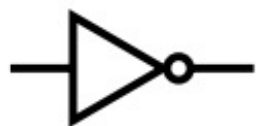


$$(\neg a \wedge b) \vee (a \wedge c)$$

can you translate this into a wff of SL?



$$(\neg a \wedge b) \vee (\neg b \wedge \neg c)$$



NOT



AND



OR



NAND



NOR



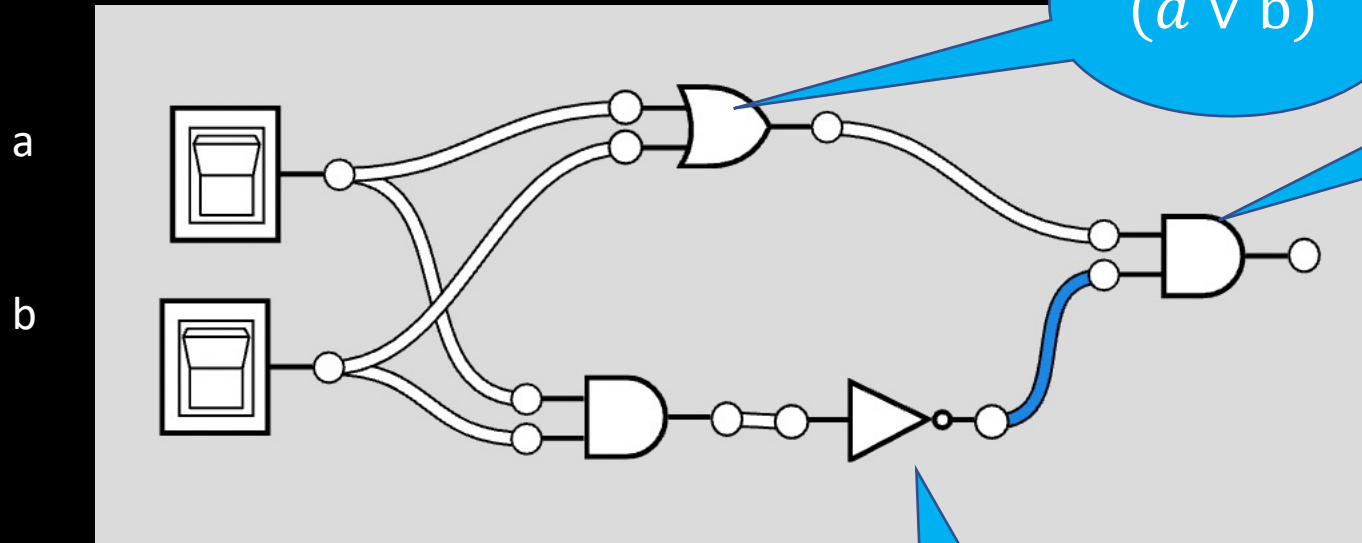
XOR

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$\neg(A \wedge B)$	$\neg(A \vee B)$	$(A \vee B) \wedge \neg(A \wedge B)$
1	1	0	1	1	0	0	0
1	0	0	0	1	1	0	1
0	1	1	0	1	1	0	1
0	0	1	0	0	1	1	0

We can make the rest  
from the basic logic gates



# can you make xor?

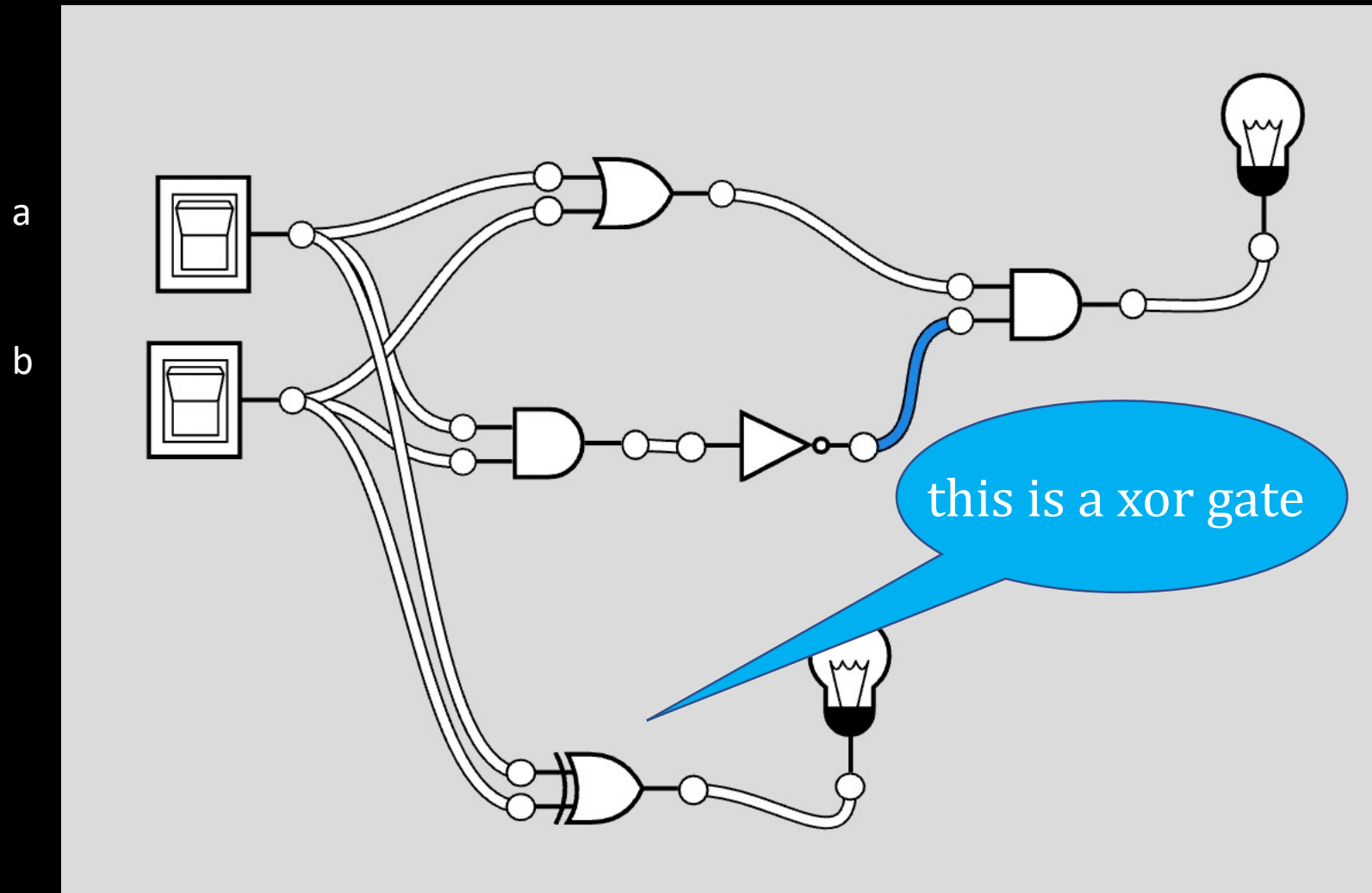


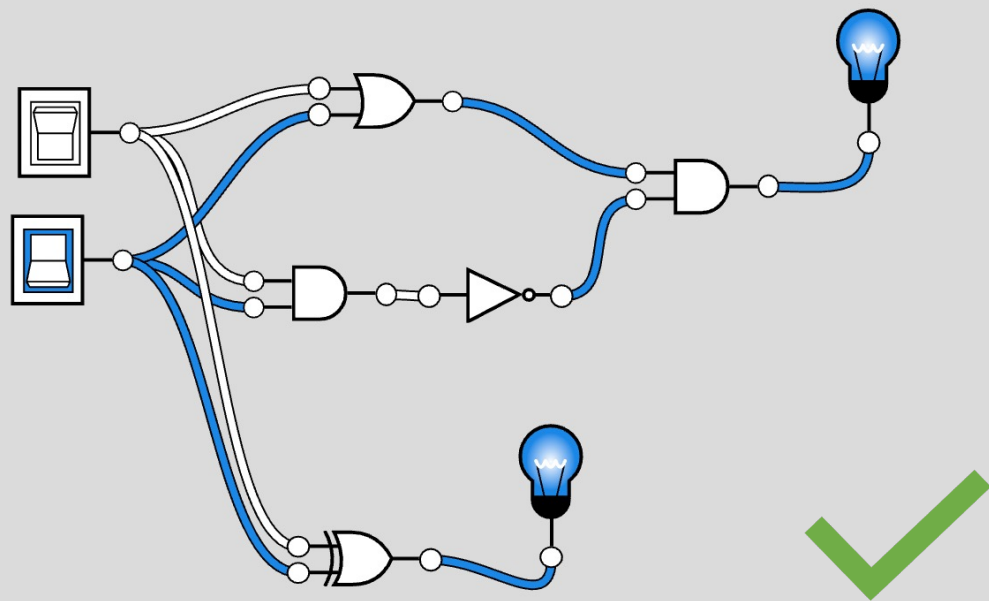
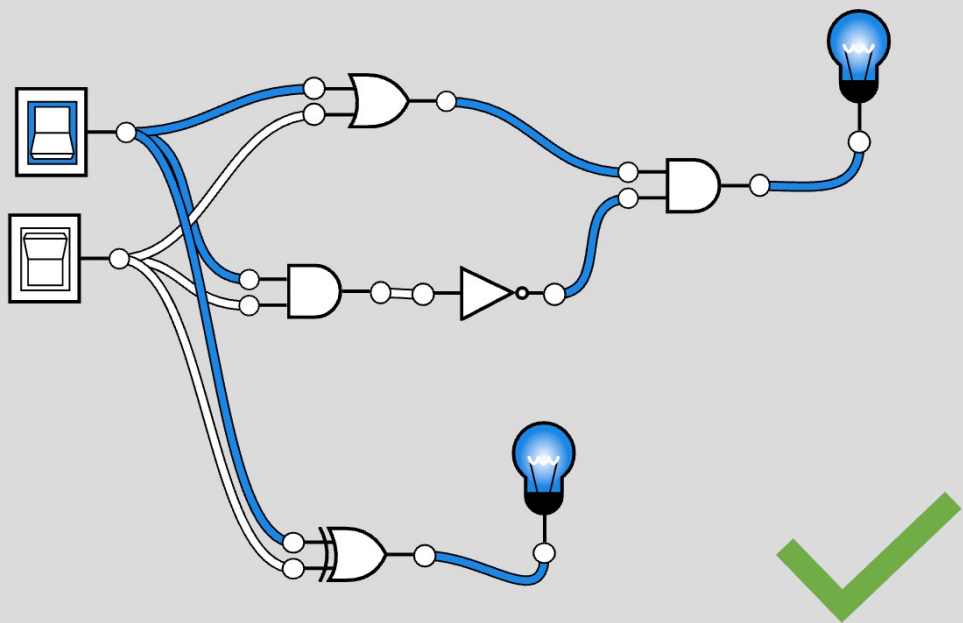
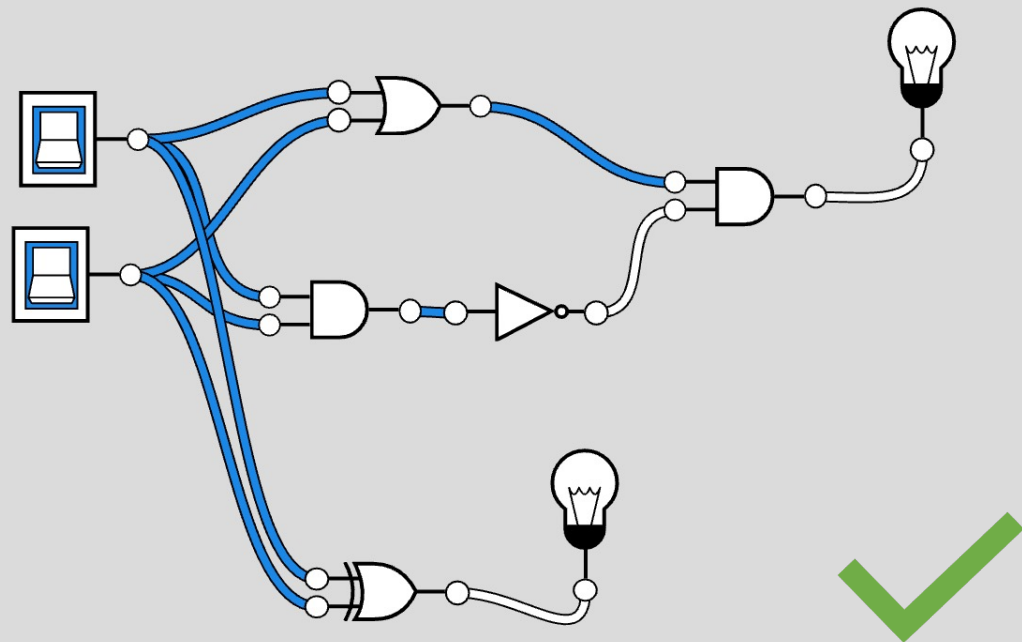
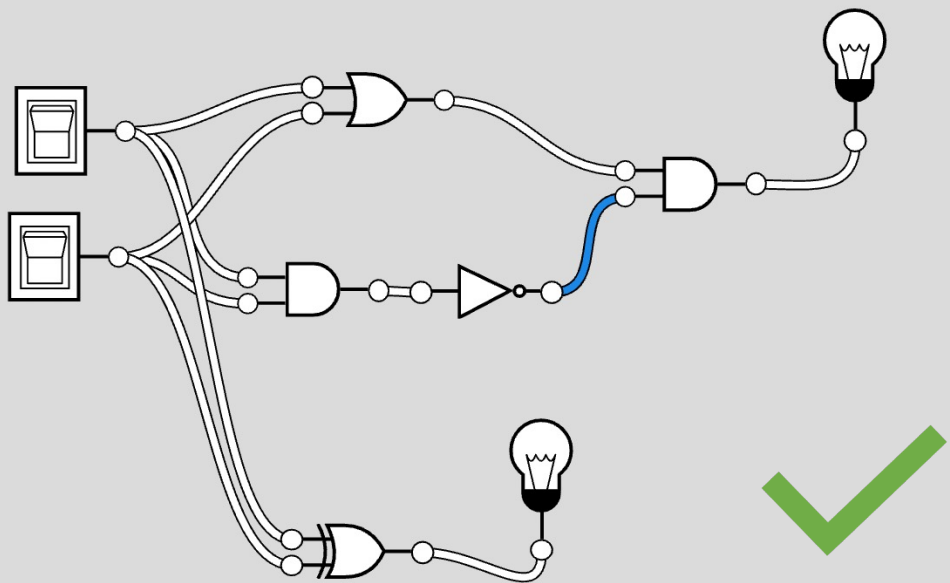
$$(a \vee b)$$

$$(a \vee b) \wedge \neg(a \wedge b)$$

$$\neg(a \wedge b)$$

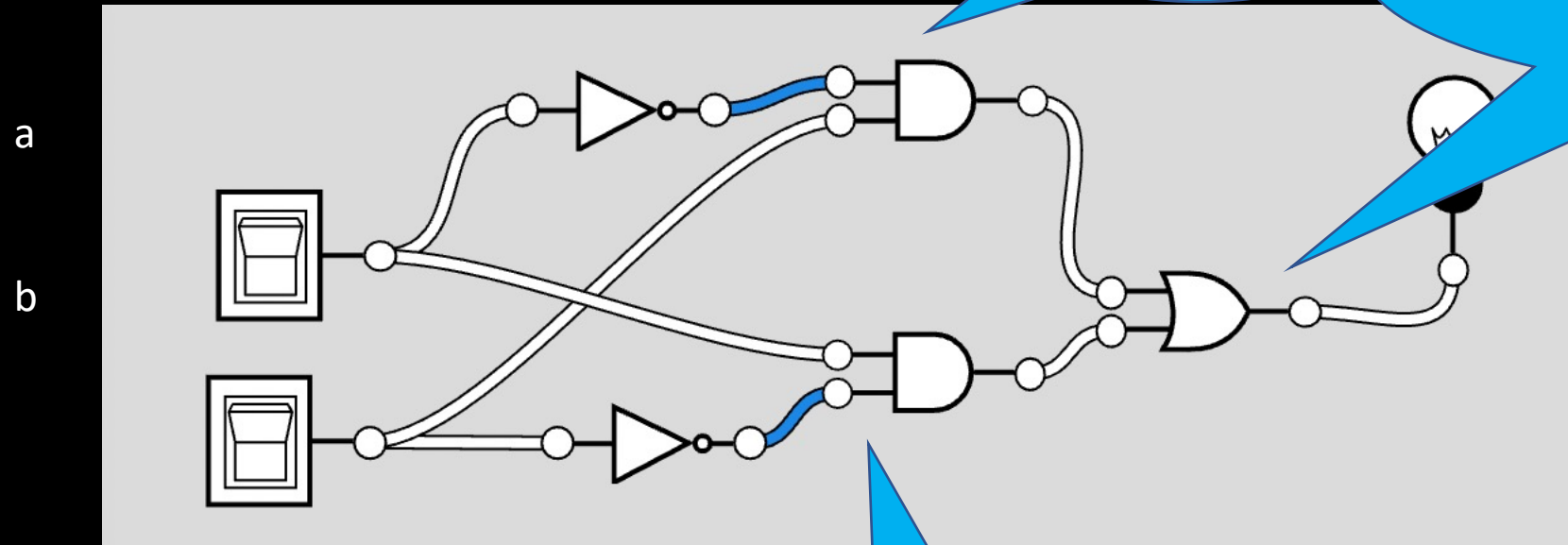
# does it work?







what about...

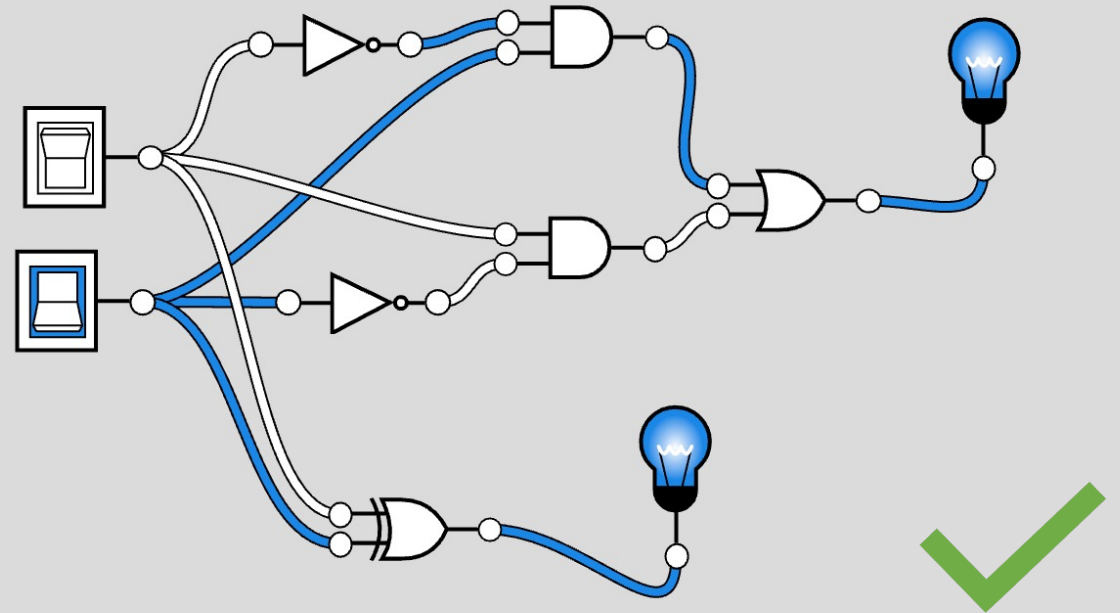
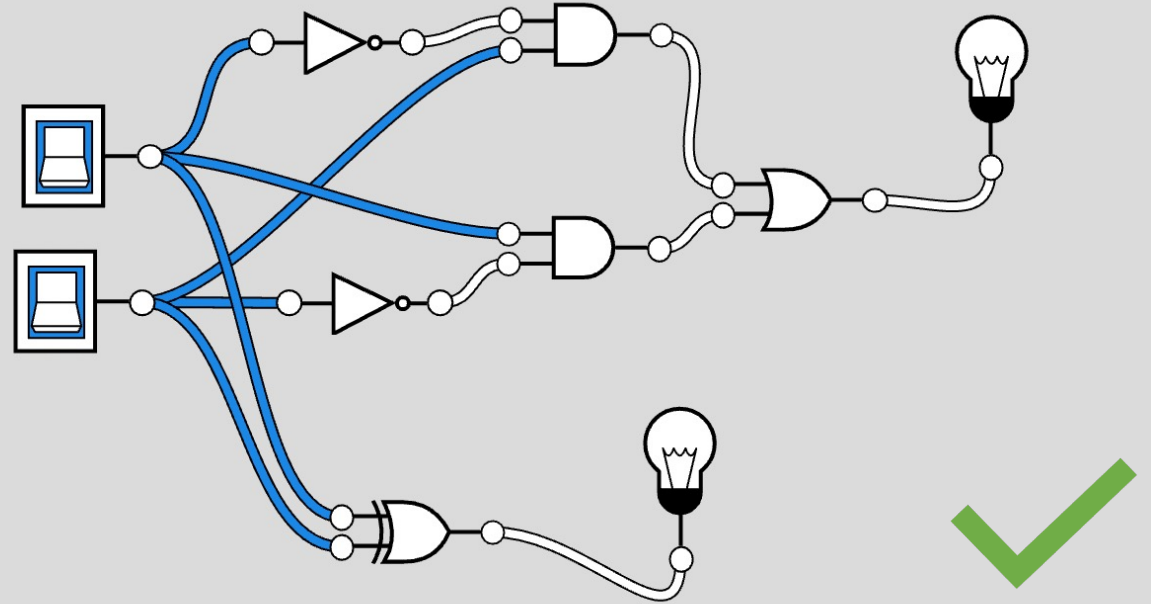
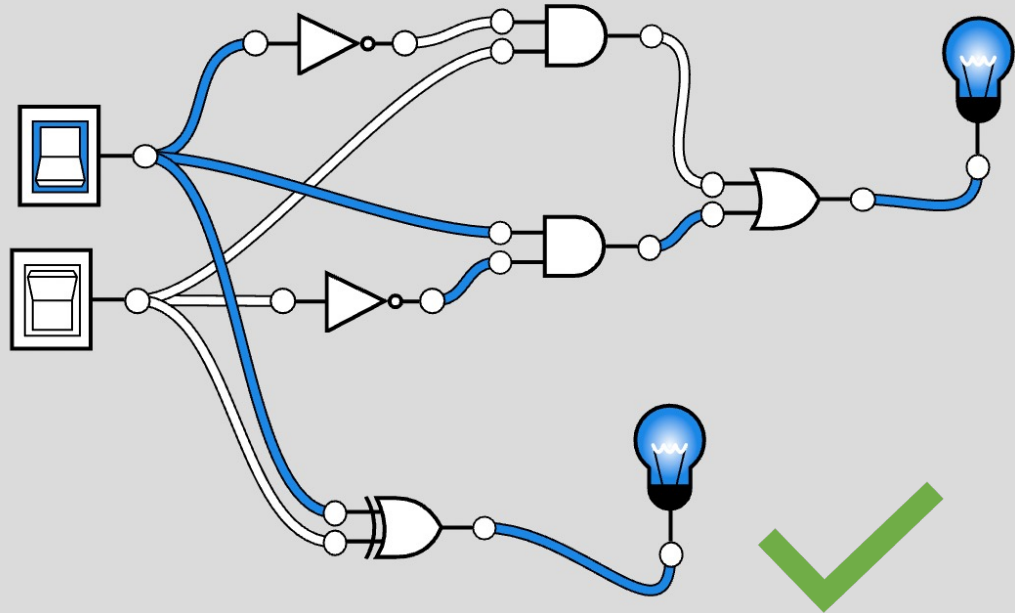
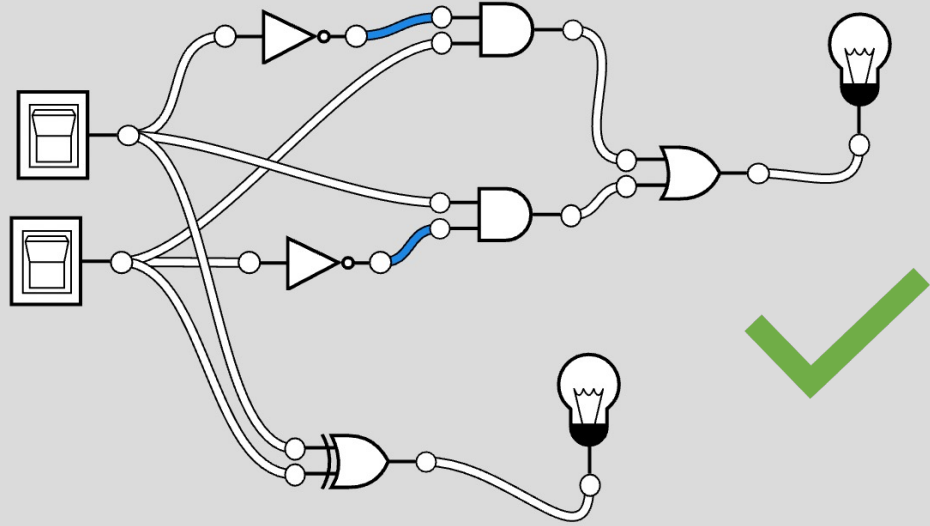


$$(\neg a \wedge b)$$

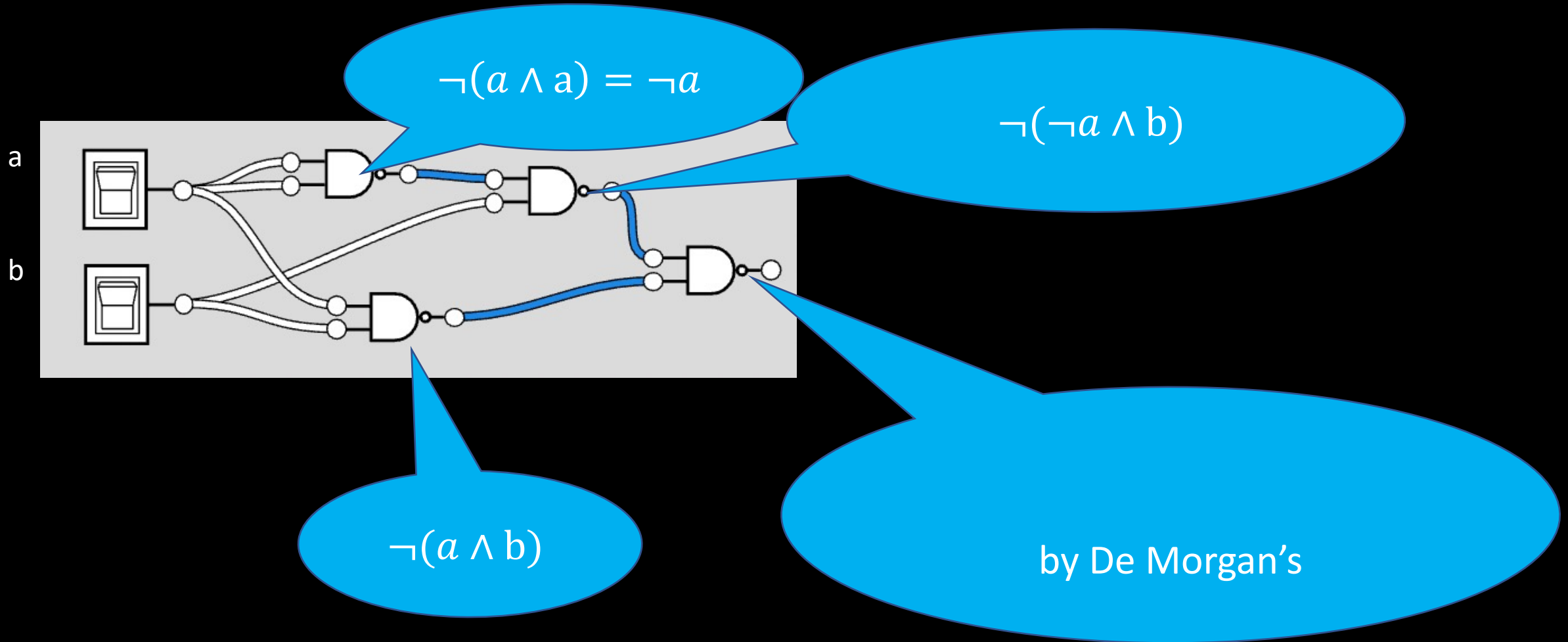
$$(\neg a \wedge b) \vee (a \wedge \neg b)$$

$$(a \wedge \neg b)$$

does it work?

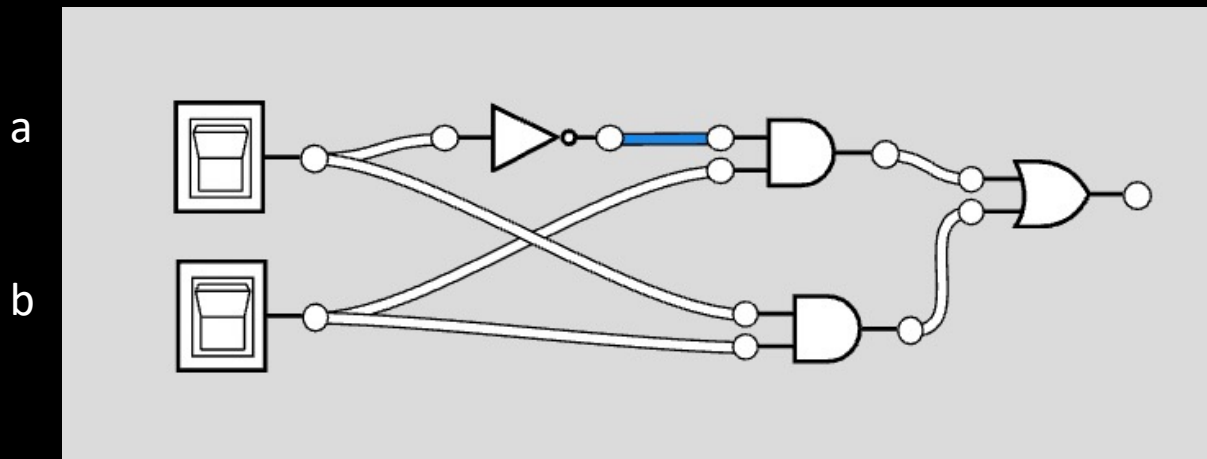
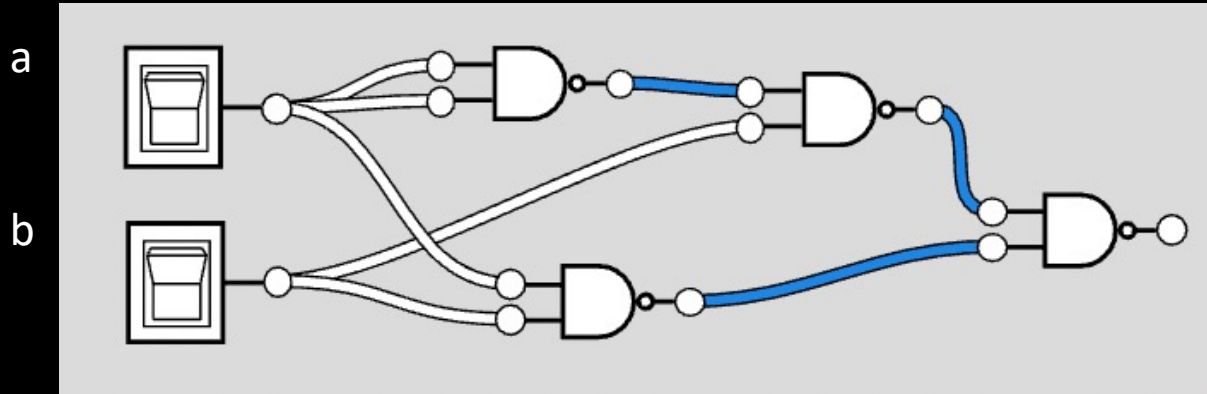


# what's going on here?



# why make it so complex?

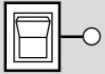
Both represent  
 $(\neg a \wedge b) \vee (a \wedge b)$   
but the top one uses a single  
gate (NAND) whereas the  
bottom one uses 3 different  
gates (NOT, OR, and AND).



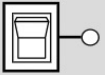
can you construct a logic gate diagram  
representing this wff of SL?:

$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$

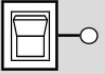
a



b



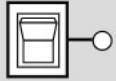
c



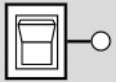
can you construct a logic gate diagram  
representing this wff of SL?:

$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$

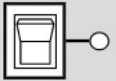
a



b



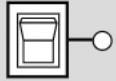
c



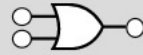
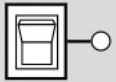
can you construct a logic gate diagram  
representing this wff of SL?:

$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$

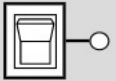
a



b

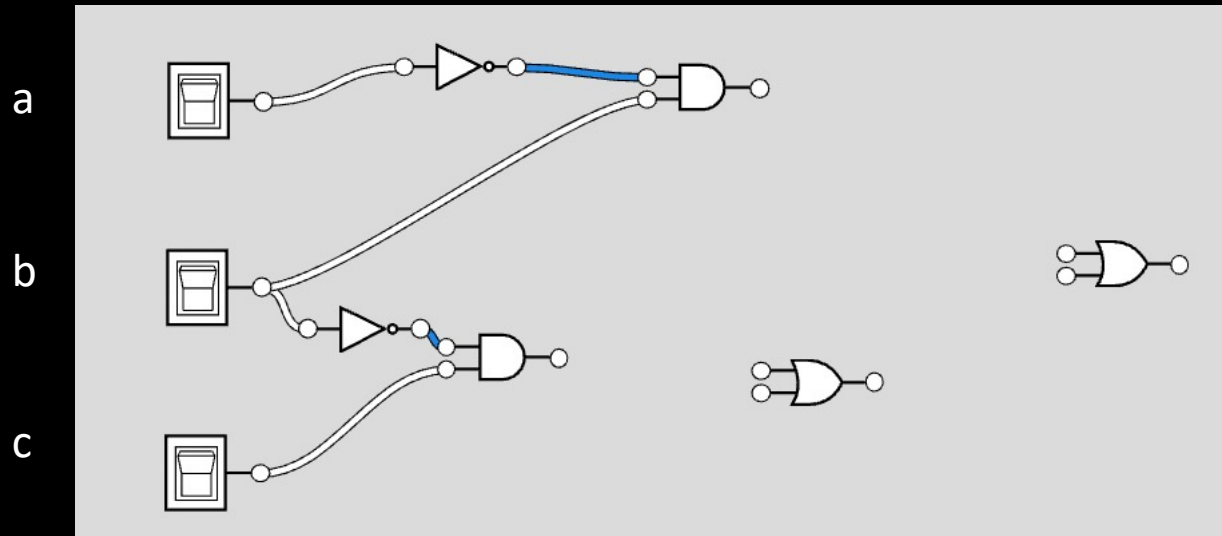


c



can you construct a logic gate diagram  
representing this wff of SL?:

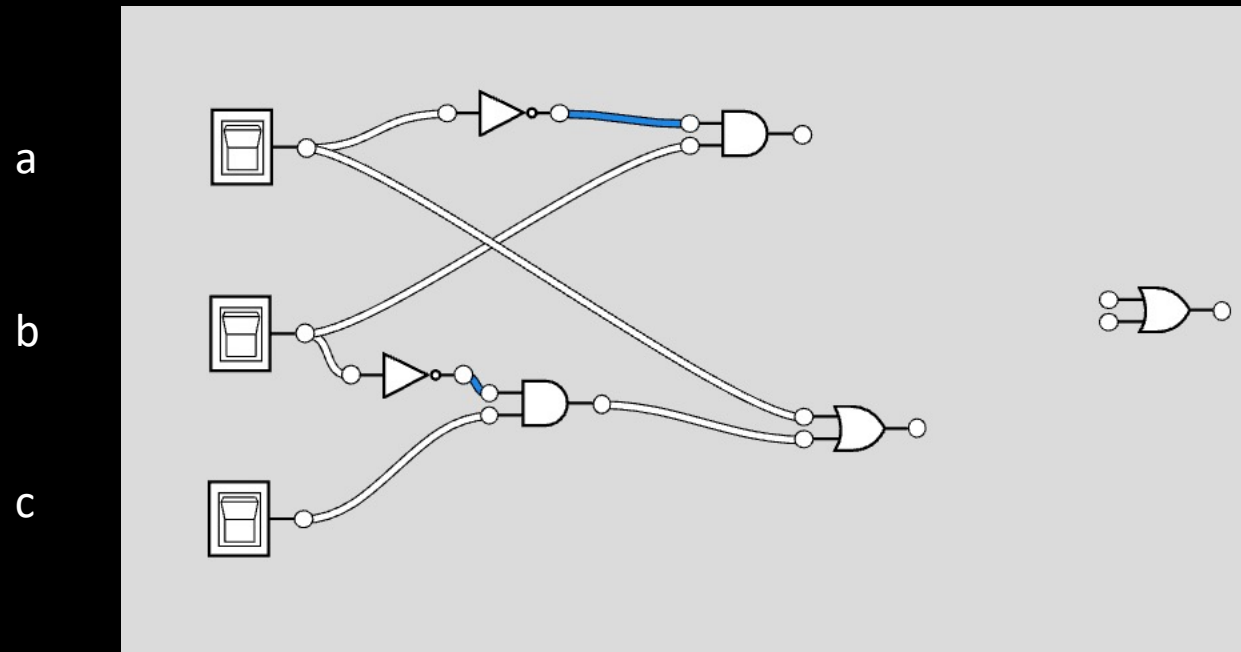
$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$





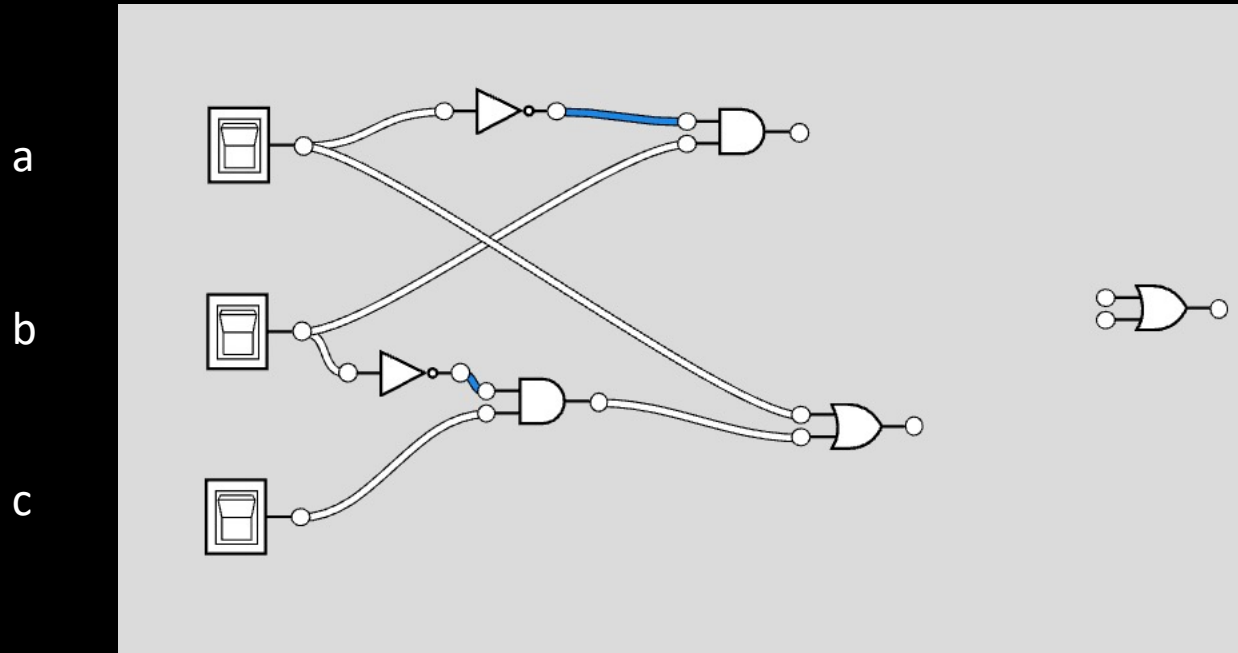
can you construct a logic gate diagram  
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$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$



can you construct a logic gate diagram  
representing this wff of SL?:

$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$



can you construct a logic gate diagram  
representing this wff of SL?:

$$(\neg a \wedge b) \vee (a \vee (\neg b \wedge c))$$

