

Theorems:

Proving something from nothing

Week 4 . Deep dive.

Recall:

Consider $\models (p \vee \neg p)$. There aren't any premises!

p	$\models (p \vee \neg p)$
T	✓ T T F T
F	✓ F T T F

There are no premises. What row do we check?

Any argument with a tautology as a conclusion is valid.

Think about it as saying: " $(p \vee \neg p)$ will be valid no matter what the premises are."

Theorem: a wff of some formal system (e.g., sentential logic) which is the conclusion of some proof of that system that does not contain any non-hypothetical assumptions.

Informally, a theorem is a statement that's provable from nothing!

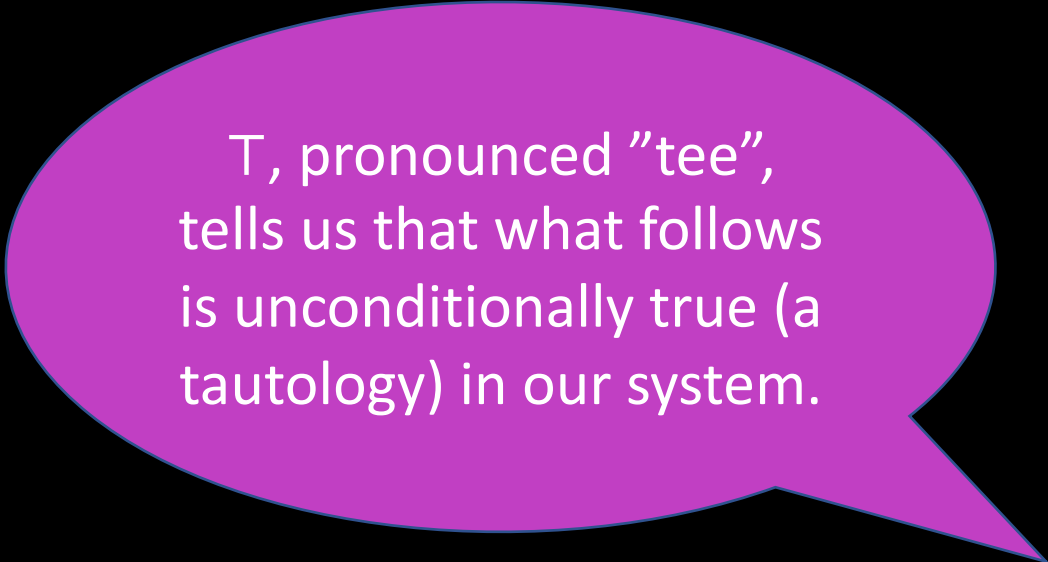
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Notation:

$$\vdash \sim(P \wedge \sim P)$$

or

$$\top \vdash \sim(P \wedge \sim P)$$



\top , pronounced "tee", tells us that what follows is unconditionally true (a tautology) in our system.

Example:

$\vdash (P \wedge Q) \rightarrow (P \vee Q)$

1.	$(P \wedge Q)$: assumption
2.	P	: $E \wedge 1$
3.	$(P \vee Q)$: $I \vee 2$
4.	$(P \wedge Q) \rightarrow (P \vee Q)$: $I \rightarrow$

Example:

$\top \vdash (P \rightarrow \sim \sim P)$

1. | P

: assumption

2. | | $\sim P$

: assumption

3. | | \perp

: E \sim 1,2

4. | $\sim \sim P$

: I \sim

5. $(P \rightarrow \sim \sim P)$

: I \rightarrow