What are proofs?

Week 3 . Topic Introduction

Thus far, we have been checking the validity of arguments using truth tables.

This week, we begin to learn a new method: proofs.

Why do we need a new method when truth tables work just fine?

Recall:

TRUTH TABLE SHORTCOMINGS...

Suppose we're considering the argument:

$$P, Q, R, S, T \vDash P$$

We'd have to check all the rows on which the premises are true.

So we'd need to write out the whole truth table. That's $2^5 = 32$ rows!

Surely there's an easier way?

The argument is so simple after all!

Besides practical reasons, proofs tend to give a better understanding of the argument.

- Think back to the practice problems last week: once the arguments became more complex, did you really understand why the arguments were (in)valid?

Proofs: the basics

A proof is a sequence of wffs of a formal system, each member of which is an assumption or an axiom, or is derived from previous wffs by the inference rules of that system.

To start writing our own proofs, we need to know the inference rules of our system: sentential logic! The proof system we're learning is called "natural deduction."

Proofs: the basics

Proofs give us another notion of validity: deductive validity.

> An argument is deductively valid just in case we can *prove* its conclusion from its premise(s) .

Deductive validity gets a fancy symbol too: $\vdash$

If we want to say A, B, …, C ∴ D is deductively valid, we write:

$$A, B, \ldots, C \vdash D.$$

What are the inference rules...?