WHAT IF THE TRUTH TABLE IS MORE COMPLEX?

WEEK 2 . DEEPER DIVE

AN EXAMPLE FROM LAST TIME...

A	B	$A \land B$
T	T	T
T	F	F
F	T	F
F	F	F

What about for an expression like: $((p \to q) \wedge r)$?

How do we set up a truth table for this expression?

What about for an expression like: $((p \rightarrow q) \wedge r)$?

How do we set up a truth table for this expression?

Consider: how many rows did we need for $(a \wedge b)$?

4: two for each of the sentence letters a and b to represent their possible truth values

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General formula for truth tables:

We need *n* columns and 2^n rows where *n* is the number of sentence letters.

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We need 2^n rows where n is the number of sentence letters.

So for $((p \to q) \wedge r)$, we need 3 columns and $2^3 = 8$ rows

p	q	r	$((p \rightarrow q) \wedge r)$

How do we fill the columns?

1. We have *n* columns left of the vertical line, one for each of the sentence letters in our final expression.

p	q	r	$((p \to q) \wedge r)$

How do we fill the columns?

1. We have *n* columns left of the vertical line, one for each of the sentence letters in our final expression.
2. We write the sentence letters in alphabetical order above the horizontal line.

p	q	r	$((p \to q) \wedge r)$

How do we fill the columns?

1. We have *n* columns left of the vertical line, one for each of the sentence letters in our final expression.
2. We write the sentence letters in alphabetical order above the horizontal line.
3. We write the full expression we're considering the right of the vertical line.

p	q	r	$((p \to q) \land r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How do we fill the rows?

1. Starting from the left-most column: The first half (i.e., 2^{n-1}) are true and the remainder false.

p	q	r	$((p \to q) \wedge r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How do we fill the rows?

1. Starting from the left-most column: The first half (i.e., 2^{n-1}) are true and the remainder false.
2. Then, for the next column, 2^{n-2} are true and 2^{n-2} are false and we repeat this pattern all the way down.

p	q	r	$((p \to q) \land r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How do we fill the rows?

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2. Then, for the next column, 2^{n-2} are true and 2^{n-2} are false and we repeat this pattern all the way down.
3. Then, for the next column, 2^{n-3} are true and 2^{n-3} are false and we repeat this pattern all the way down again.

p	q	r	$((p \to q) \land r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How do we fill in the right side of the vertical line?

p	q	r	$((p \to q) \land r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How do we fill in the right side of the vertical line?

When we fill in the columns, we first copy over the truth values for the atomics.

p	q	r	$((p$	$\rightarrow q)$	$\wedge r)$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	F	T
F	F	F	F	F	F

Finally, we fill in the truth values for the connectives.

p	q	r	$((p$	$\to$	$q)$	$\land$	$r)$
T	T	T	T	T	T		T
T	T	F	T	T	T		F
T	F	T	T	F	F		T
T	F	F	T	F	F		F
F	T	T	F	T	T		T
F	T	F	F	T	T		F
F	F	T	F	T	F		T
F	F	F	F	T	F		F

We work our way out leaving the main connective (∧) for last.

p	q	r	$((p$	$\boxed{\to}$	$q)$	$\wedge$	$\boxed{r})$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	F	F	F	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	F

When we consider the main connective, we consider when $(p \wedge q)$ and r are both true.

Complete truth table:

p	q	r	$((p$	$\to$	$q)$	$\wedge$	$r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	F	F	F	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	F

We'll indicate the column that represents the truth values of the whole formula by circling it.

Another example

p	q	r	$((p$	$\land$	$q)$	$\lor$	$(p$	$\rightarrow$	$r))$
T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F
T	F	T	T	F	F	T	T	T	T
T	F	F	T	F	F	F	T	F	F
F	T	T	F	F	T	T	F	T	T
F	T	F	F	F	T	T	F	T	F
F	F	T	F	F	F	T	F	T	T
F	F	F	F	F	F	T	F	T	F

What if the expression is always true?

p	$(p$	$\vee$	$\neg p)$
T	T	T	F
F	T	T	T

What if the expression is always true?

p	$(p$	$\vee$	$\neg p)$
T	T	T	F
F	T	T	T

Or always false?

p	$(p$	$\wedge$	$\neg p)$
T	T	F	F
F	T	F	T

We call an expression that is always true a *tautology*.

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An expression that is always false is called a *contradiction*.

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An expression that is always false is called a *contradiction*.

An expression that is sometimes true sometimes false is called *contingent*.

Tautology:

p	$(p$	$\lor$	$\neg p)$
T	T	$\boxed{\text{T}}$	F
F	T	$\boxed{\text{T}}$	T

Contradiction:

p	$(p$	$\land$	$\neg p)$
T	T	$\boxed{\text{F}}$	F
F	T	$\boxed{\text{F}}$	T