Theorems:
Proving something from nothing

Week 4 . Deep dive.

Recall:

Consider $\vDash (p \vee \neg p)$. There aren't any premises!

p	$\vDash (p \vee \neg p)$
T	✓ T T F T
F	✓ F T T F

There are no premises. What row do we check?

Any argument with a tautology as a conclusion is valid.

Think about it as saying: "$(p \vee \neg p)$ will be valid no matter what the premises are."

Theorem: a wff of some formal system (e.g., sentential logic) which is the conclusion of some proof of that system that does not contain any non-hypothetical assumptions.

Informally, a theorem is a statement that's provable from nothing!

Theorem: a wff of some formal system (e.g., sentential logic) which is the conclusion of some proof of that system that deos not contain any non-hypothetical assumptions.

Notation:

$$\vdash \sim(P \wedge \sim P)$$

$$or$$

$$\top \vdash \sim(P \wedge \sim P)$$

T, pronounced ”tee”, tells us that what follows is unconditionally true (a tautology) in our system.

Example:

$T \vdash (P \land Q) \to (P \lor Q)$

1.	$\quad (P \land Q)$	: assumption
2.	$\quad P$	: E $\land$ 1
3.	$\quad (P \lor Q)$	: I $\lor$ 2
4.	$(P \land Q) \to (P \lor Q)$	: I $\to$

Example:

$T \vdash (P \to \sim\sim P)$

1.	P	: assumption
2.	$\sim P$	: assumption
3.	$\bot$	: E ~ 1,2
4.	$\sim\sim P$	: I ~
5.	$(p \to \sim\sim p)$	: I →