Quantifier rules: existential elimination

WEEK 7 . TOPIC INTRODUCTION

EXISTENTIAL ELIMINATION

1. $\exists X \Phi[x]$
2. | $\Phi[x/c]$:assumption
3. | χ
4. $\Phi[x/c] \rightarrow \chi$
5. χ E∃ 1, 4

The **Φ**[x/c] part means 'Take **Φ** and replace every instance of x with c'

Instead, we end us saying, suppose it is this particular object. Then χ is true. Then, by E∃, χ. This is like saying it doesn't matter what object instantiates the property.

EXISTENTIAL ELIMINATION

1. ∃XΦ[x]
2. | Φ[x/c] :assumption
3. | χ
4. Φ[x/c]→ χ
5. χ E∃ 1, 4

Restrictions:

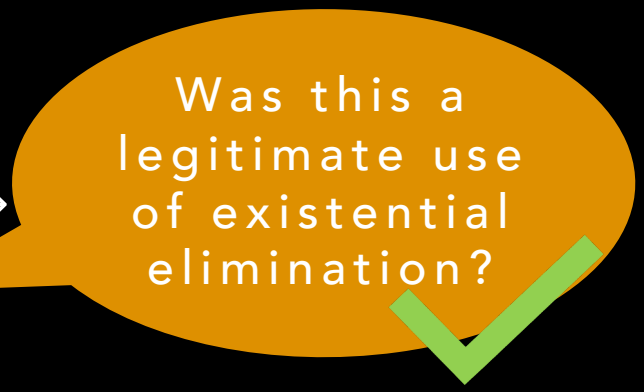
1. c can't occur in an open assumptions (premises or subproof assumptions).
2. The name c can't occur in Φ already.
3. The name c can't occur in the conclusion χ.

EXISTENTIAL ELIMINATION

1. $\exists X \Phi[x]$
2. | $\Phi[x/c]$:assumption
3. | χ
4. $\Phi[x/c] \rightarrow \chi$
5. χ E∃ 1, 4

Prove: $\exists x \forall y Lxy \vdash \forall x \exists y Lyx$

1. $\exists x \forall y Lxy$:assumption
2. | $\forall y Lay$:assumption
3. | Lab :E∀2
4. | $\exists y Lyb$:I∃3
5. | $\forall x \exists y Lyx$: I∀4
6. $\forall y Lay \rightarrow \forall x \exists y Lyx$:I→
7. $\forall x \exists y Lyx$:E∃ 1,6

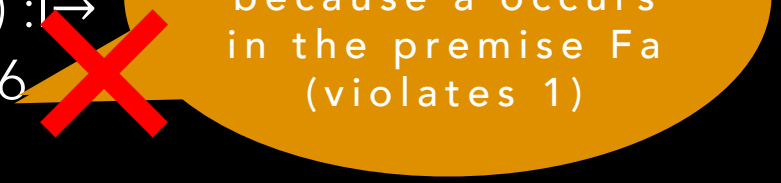


EXISTENTIAL ELIMINATION

1. $\exists X \Phi[x]$
2. | $\Phi[x/c]$:assumption
3. | χ
4. $\Phi[x/c] \rightarrow \chi$
5. χ E∃ 1, 4

Prove: $\exists x Gx, Fa \vdash \exists x(Gx \wedge Fx)$

1. $\exists x Gx$:assumption
2. Fa : assumption
3. | Ga :assumption
4. | Ga ∧ Fa : I ∧ 2,3
5. | ∃x(Gx ∧ Fx):I∃4
6. Ga → ∃x(Gx ∧ Fx) :I→
7. ∃x(Gx ∧ Fx): E∃ 1,6



EXISTENTIAL ELIMINATION

1. ∃X**Φ**[x]
2. | **Φ**[x/c] :assumption
3. | **χ**
4. **Φ**[x/c]→ **χ**
5. **χ** E∃ 1, 4

Prove: **∃xGxx** ⊢ ∃xGxa

1. ∃xGxx :assumption
2. | Gaa: assumption
3. | ∃xGxa :I∃2
4. Gaa → ∃xGxa : I→
5. ∃xGxa: E∃ 1,4

Not legitimate because we do ϕ[x/a] but a occurs in the conclusion (violates 3)