Optimizing single-qubit control with Floquet theory

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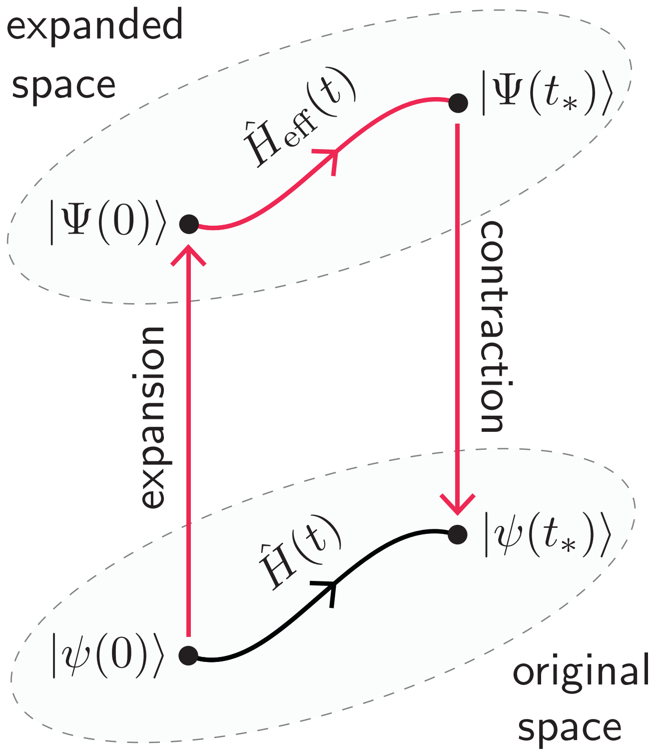
Outline

- **Quantum optimal control using Floquet theory in a nutshell**
- **1QB gates for the transmon qubit**
  - **Implementation and numerical details**
  - **Convergence of the Fourier Ansatz**
  - **Speed limit vs qubit anharmonicity**
  - **Spectral analysis: discovering the DRAG Ansatz**
- **1QB gates for the heavy-fluxonium qubit**
  - **Microwave gates via charge coupling**
  - **Single-cycle gates via flux coupling**
  - **Which gate is best for your fluxonium?**

Optimal control with Floquet theory in a nutshell

- **Floquet theory <u>is</u> an efficient way to solve the time-periodic Schrodinger equation**
- **We solve a simpler, time-independent Schrodinger equation in a bigger Hilbert space:**



Simpler time-independent problem in a bigger space $\Rightarrow$ Floquet

Differentiable

Original time-dependent problem $\Rightarrow$ QuTiP

Optimal control with Floquet theory in a nutshell

- **What do we mean by differentiable and why is it important?**

Model

Propagator

$$\boldsymbol{\hat{H}(\lambda) \Rightarrow U_t(\lambda)}$$

Model parameter (drive amplitude, coupling strength, etc.)

Propagator derivative

$$\boldsymbol{\partial_{\lambda_i}\hat{H}(\lambda) \Rightarrow \partial_{\lambda_i}U_t(\lambda)} = \underbrace{\frac{\boldsymbol{U_t}(\lambda_i + \Delta) - \boldsymbol{U_t}(\lambda_i)}{\Delta}}$$

Model derivative

How much does the propagator changes when we change the system or <u>drive parameters</u>

- **Optimal-control loop (i.e., gradient descent):**

Model $\boldsymbol{\hat{H}(\lambda)}$ ⇒ **Propagator** $\boldsymbol{U_t(\lambda)}$ ⇒ **Gate infidelity** $\boldsymbol{1 - F(\lambda)}$ ⇒ **Gradients** $\boldsymbol{\partial_\lambda[1 - F](\lambda)}$

$\lambda \to \lambda + \eta\, \nabla$, **with** η **being the learning rate**

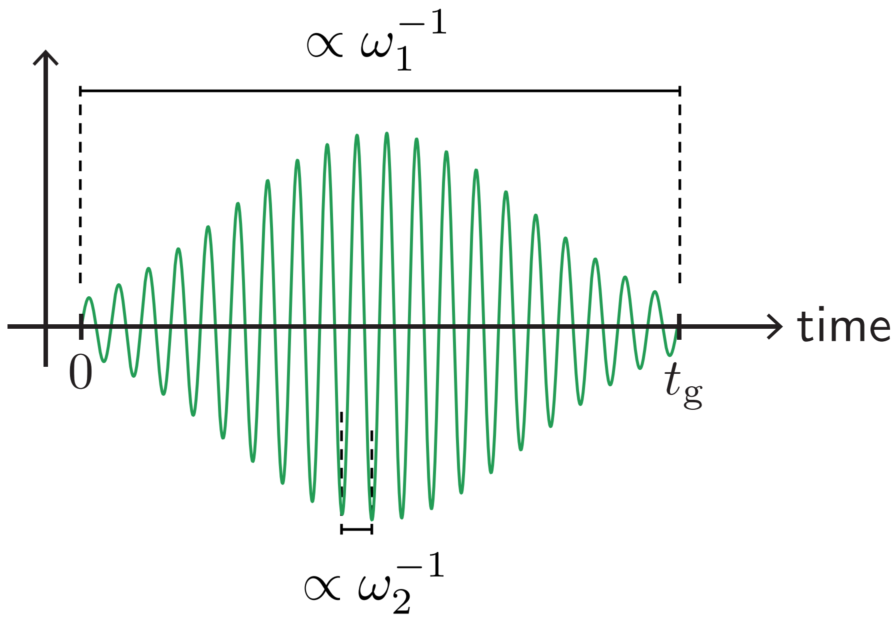
Unique properties of our optimal control method

✓	✗
Exact 1st and higher-order derivatives	Dimensionality / memory
Handles nonlinear controls (e.g., fast-flux)	Runtime scales with K
Avoids approximations such as RWA (well-suited for driven problems)	Implementation
Uses a physical basis: • Easier to integrate with experiment • Convergence is exponential with K	
Parallelizable	
Handles dissipation: • Deterministic (Liouvillian) • Stochastic (non-Hermitian Hamiltonian)	

<u>Other benefits of our implementation</u>:
- Full average-gate-fidelity metric as cost-function w/ 1QB-phase corrections
- Custom optimizer w/ adaptive learning rate → the cost never increases

Single-qubit microwave pulses

Typical microwave pulse as a two-tone problem



$a_k - d_k$ determine the spectral weight of the pulse at frequencies $\omega_2 \pm k\omega_1$

Two-quadrature driving Hamiltonian

$$\hat{H}_d = [\mathcal{E}_x(t)\cos(\omega_2 t) + \mathcal{E}_y(t)\sin(\omega_2 t)]\,\hat{n}$$

$$\mathcal{E}_x(t) = \sum_{k=0}^{K-1} a_k \sin(k\omega_1 t) + b_k \cos(k\omega_1 t) \qquad \mathcal{E}_y(t) = \sum_{k=0}^{K-1} c_k \sin(k\omega_1 t) + d_k \cos(k\omega_1 t)$$

Parameters + (Z correction, drive frequency ω_2, QB params)

Gradient descent

0. Initial condition

1. Make operator

$$\begin{cases} \theta_1(t) = \omega_1 t \to \hat{\theta}_1 \ \ (2\,\mathrm{M}_1 + 1\ \mathrm{dim}) \\ \theta_2(t) = \omega_2 t \to \hat{\theta}_2 \ \ (2\,\mathrm{M}_2 + 1\ \mathrm{dim}) \end{cases}$$

2. Solve w/ Floquet

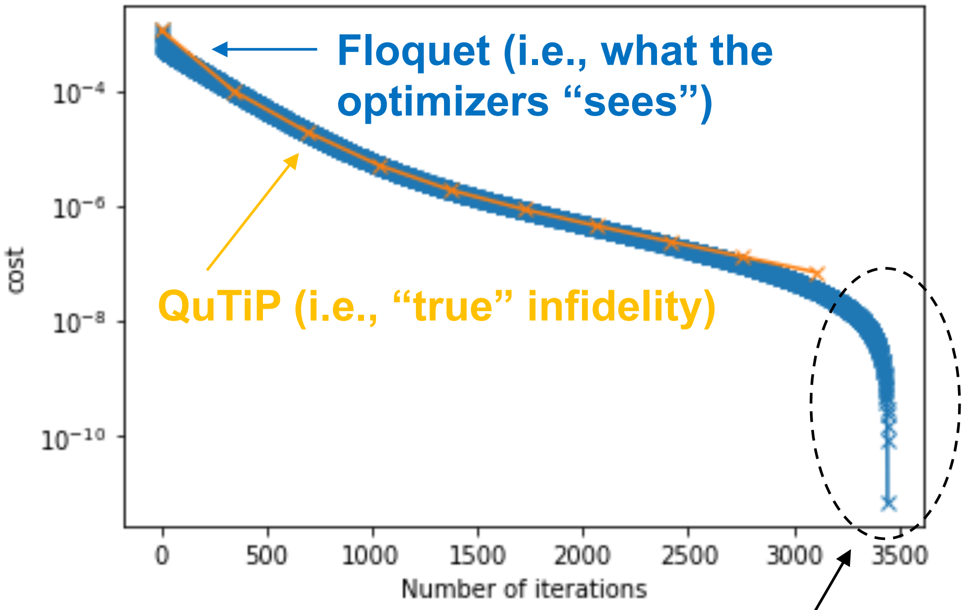
3. Compute fidelity

4. Compute gradients

K Progressive optimization $K+1$

Typical runs and consistency checks

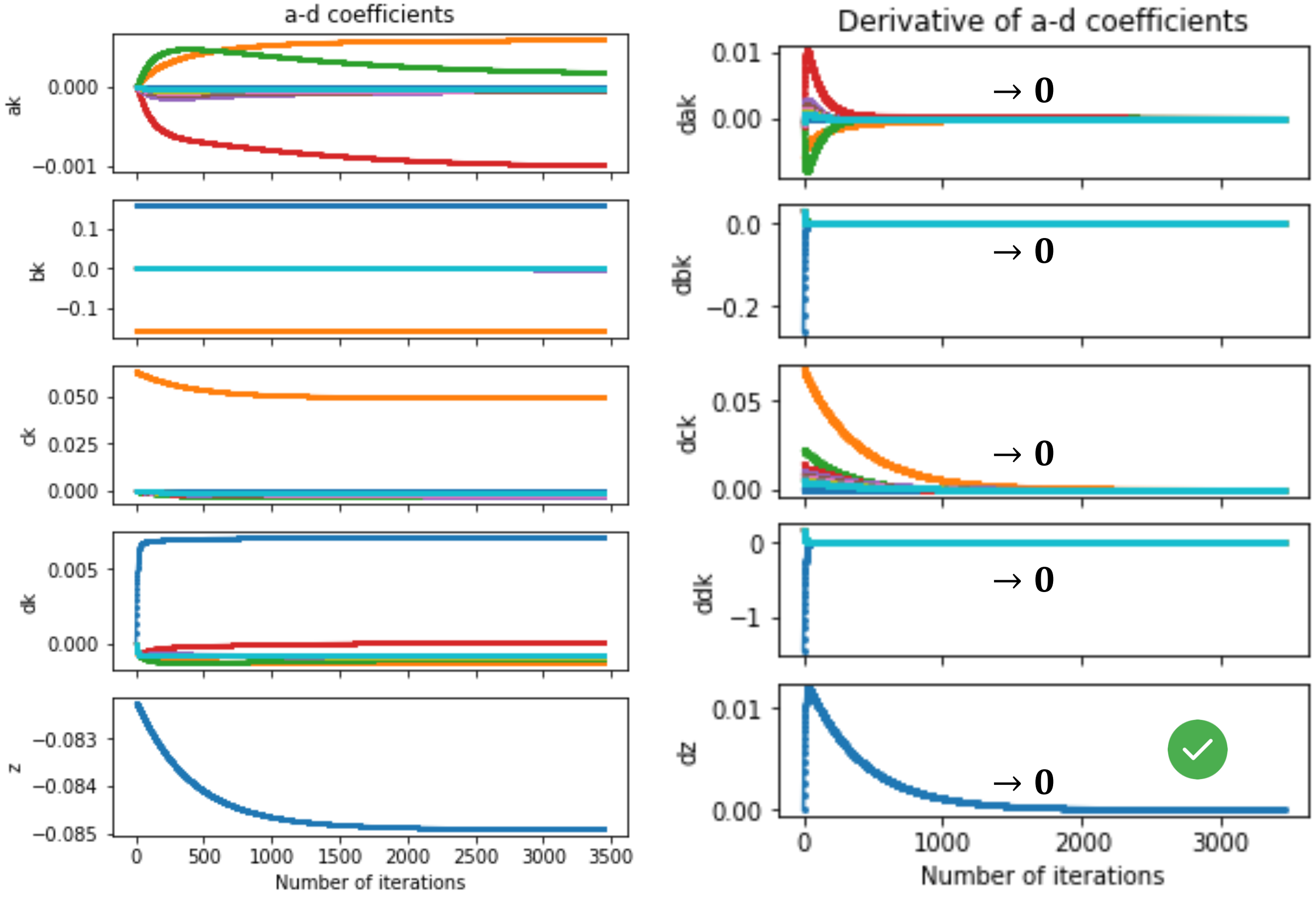
Cost function minimization



Floquet accuracy comparable to gate infidelity → failure

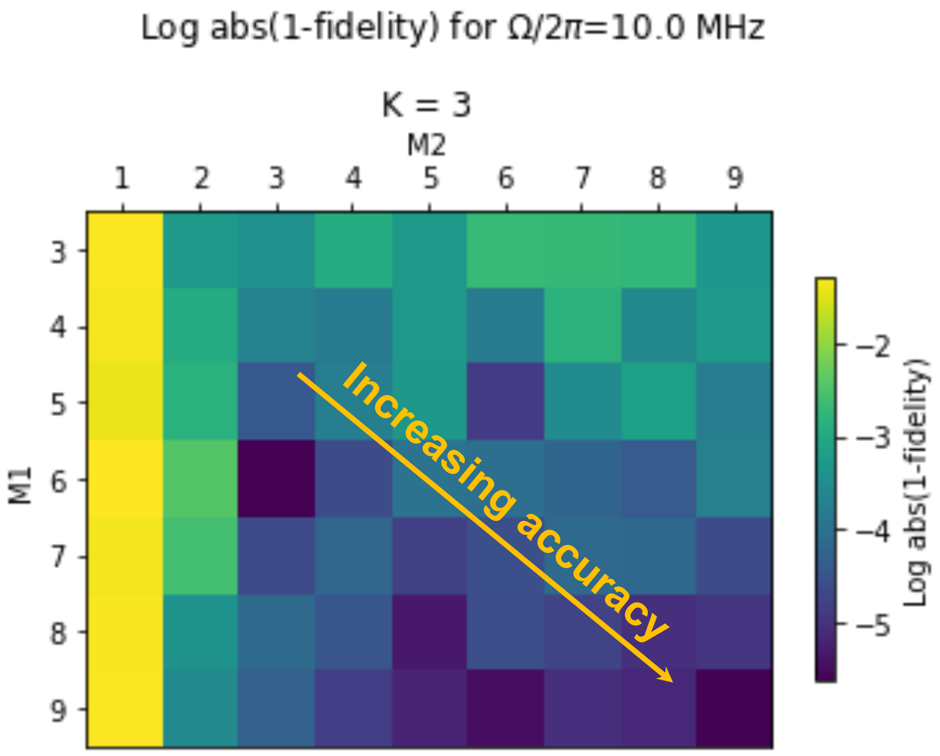
Re-adjust Floquet dims (M1 and M2)

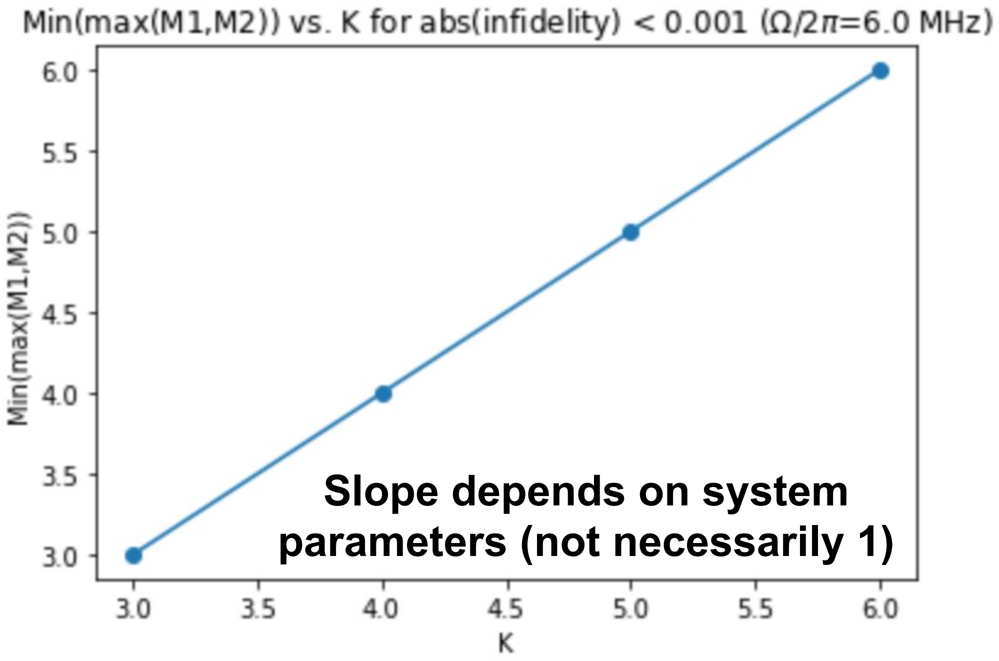
Do parameters converge w/ iteration number?



Adjusting the accuracy of the Floquet solver

Plot $F(U_{\text{Floquet}}, U_{\text{QuTiP}})$ averaged over randomly sampled drive parameters $a_k - d_k$

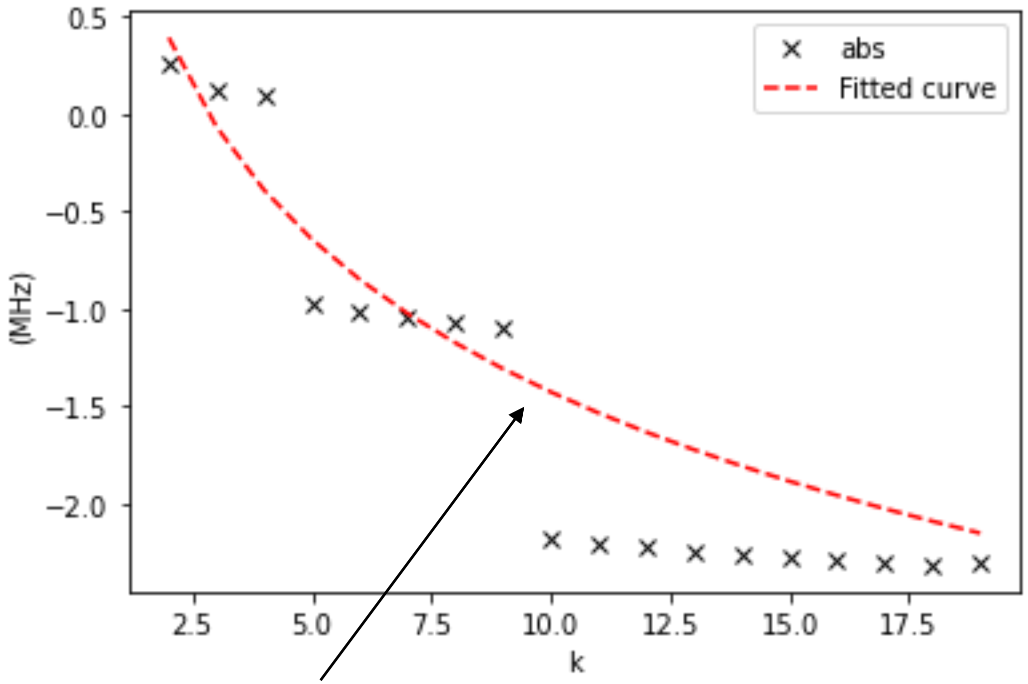




Minimum M scales linearly w/ K for same accuracy

How do we select M1 and M2?

- To benchmark, we create random pulses and compares the Floquet solver's propagator for that pulse against QuTiP's propagator.
- We increment M1 & M2 until we find (M1, M2) that produces an infidelity lower than a given target infidelity (e.g., 1.e-4)

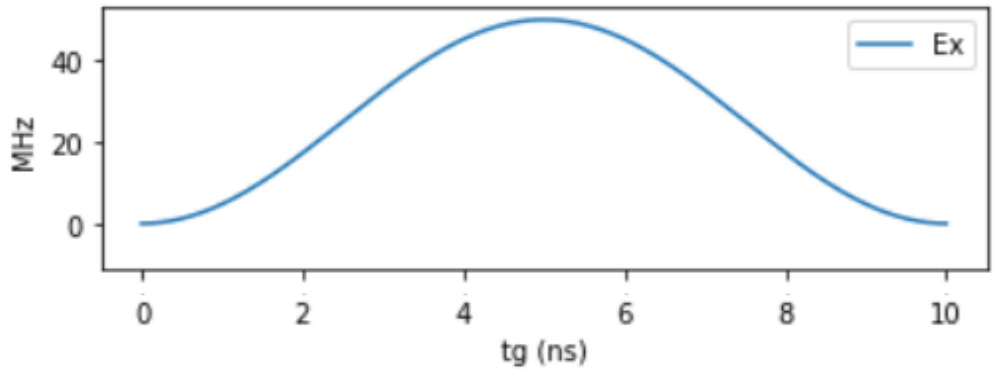


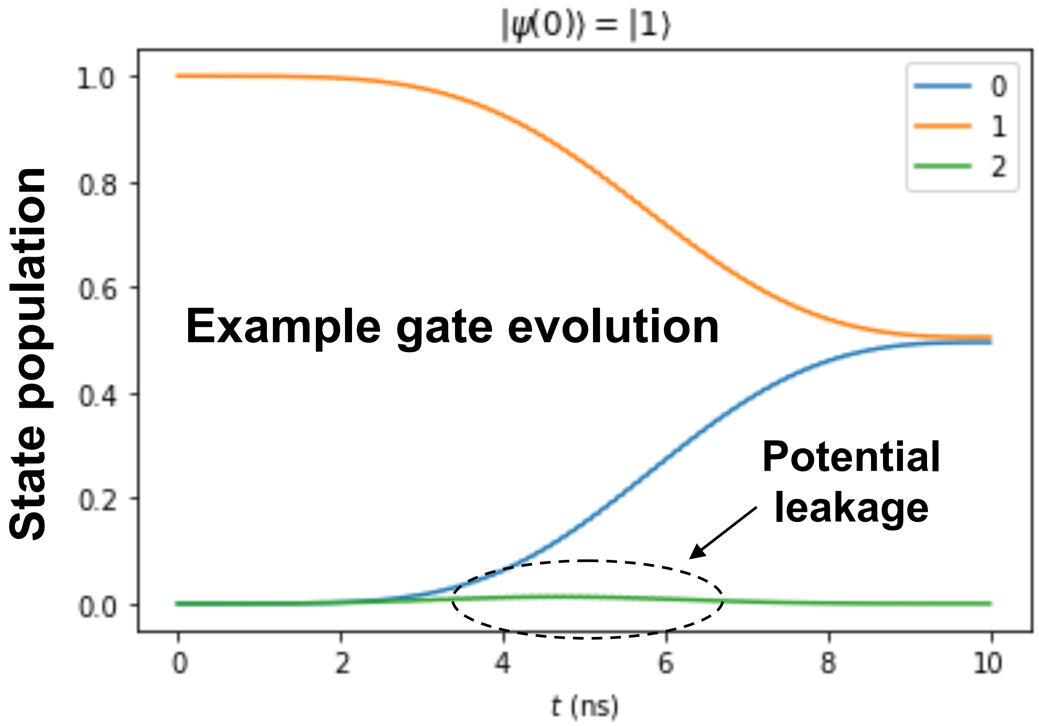
We fit the typical "decay" of pulse coefficients with *k* to a power law, and this information to generate realistic random pulses

Single-qubit gates for the transmon qubit

- Optimize gate fidelity of $\frac{\pi}{2}$-pulse as a function of:
  - Number of pulse parameters $\propto K$
  - Maximum drive amplitude ($|\Omega|/2\pi$). $\mathcal{E}_x(t) = \mathrm{Re}[\Omega(t)]$, $\mathcal{E}_y(t) = \mathrm{Im}[\Omega(t)]$,
  - Qubit anharmonicity ($\alpha/2\pi$)

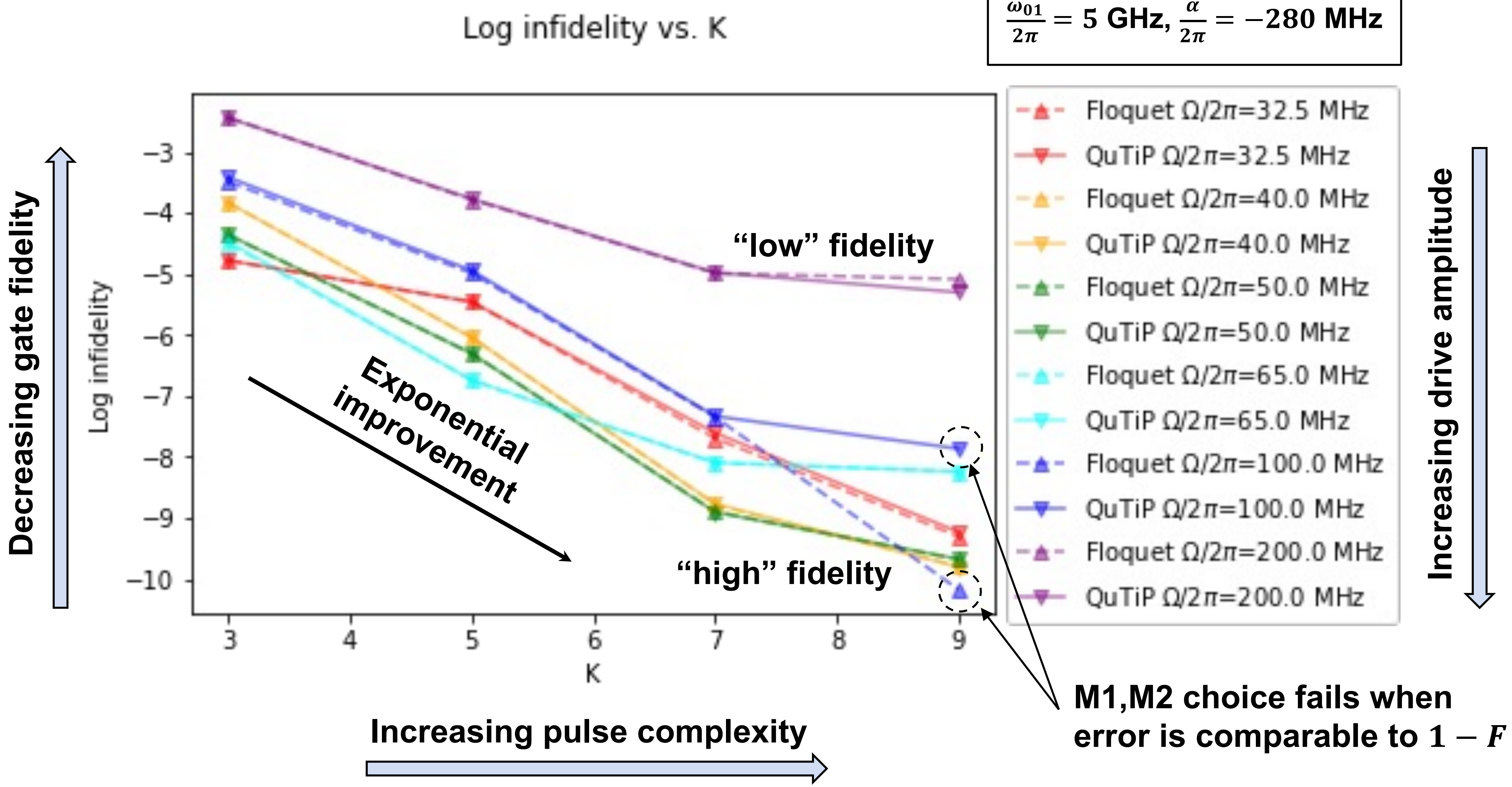
Initial condition: cosine pulse (single quad.)





1QB gates for transmons: $1 - F \propto e^{-\beta K}$

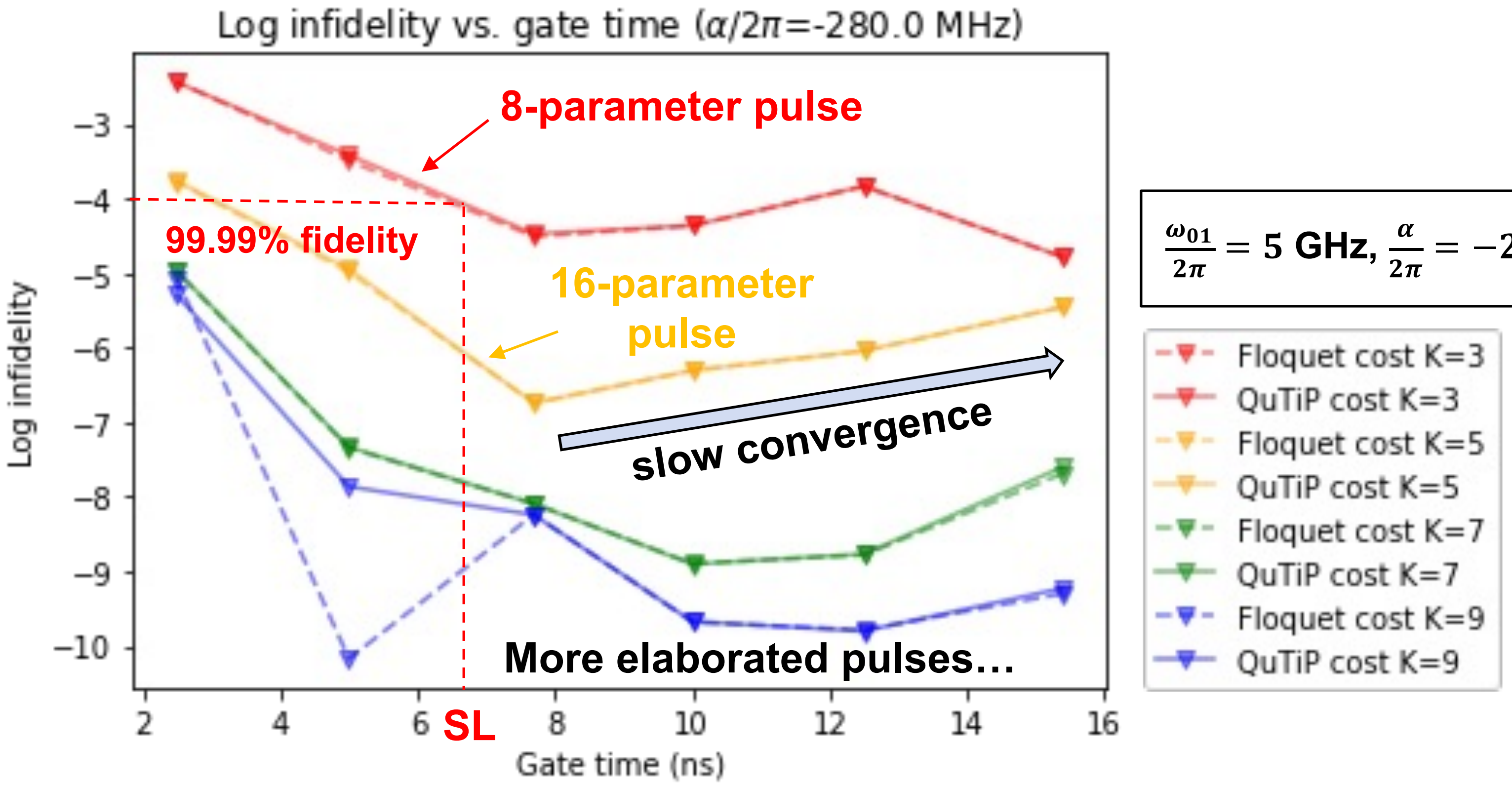
$\frac{\omega_{01}}{2\pi} = 5$ GHz, $\frac{\alpha}{2\pi} = -280$ MHz



Expected behavior according to S. Lloyd and S. Montangero, Phys. Rev. Lett. **113**, 010502 (2014)

Driving a typical transmon: speed limit (SL)

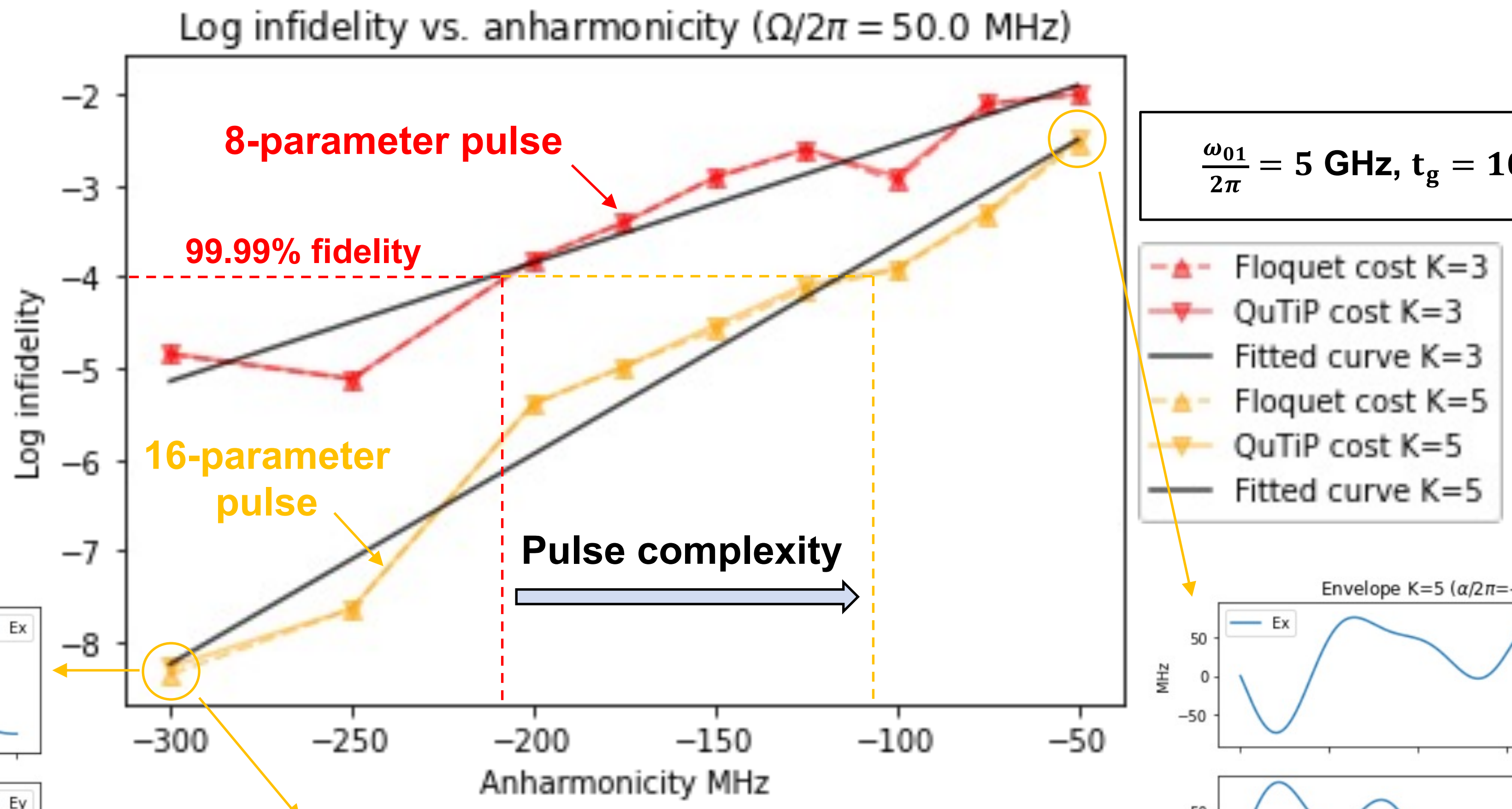
- **Speed limit: how fast can we drive a $\pi/2$ rotation? ~ 6 ns for a "nice" pulse**

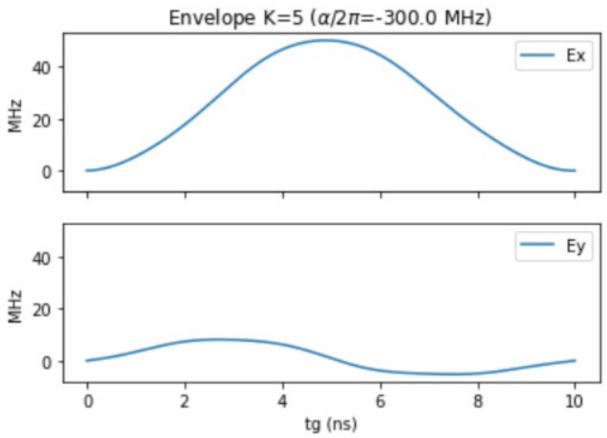


$$\frac{\omega_{01}}{2\pi} = 5\text{ GHz},\ \frac{\alpha}{2\pi} = -280\text{ MHz}$$

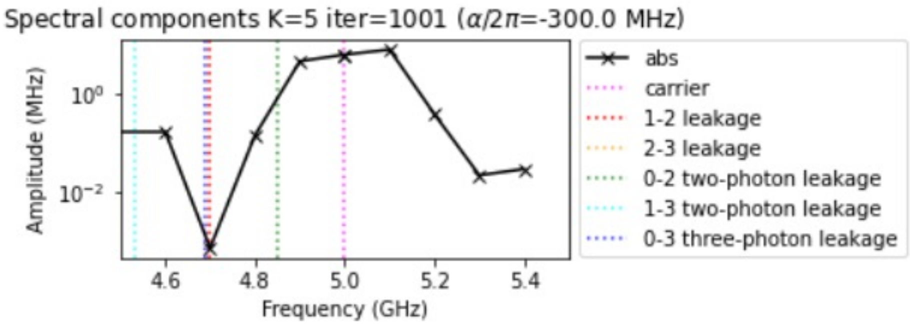
Driving weakly anharmonic transmons?

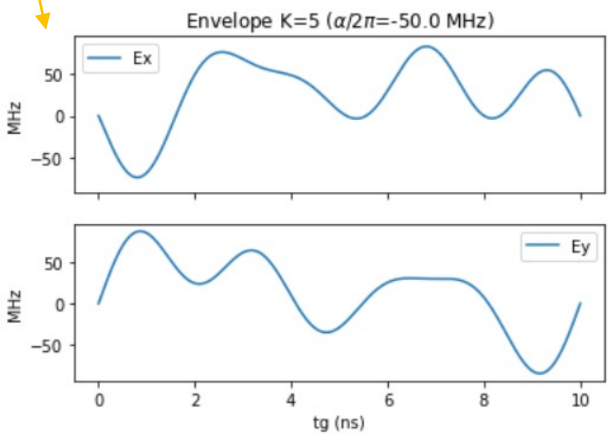
$$\frac{\omega_{01}}{2\pi} = 5\ \text{GHz},\ t_g = 10\ \text{ns}$$





DRAG-like

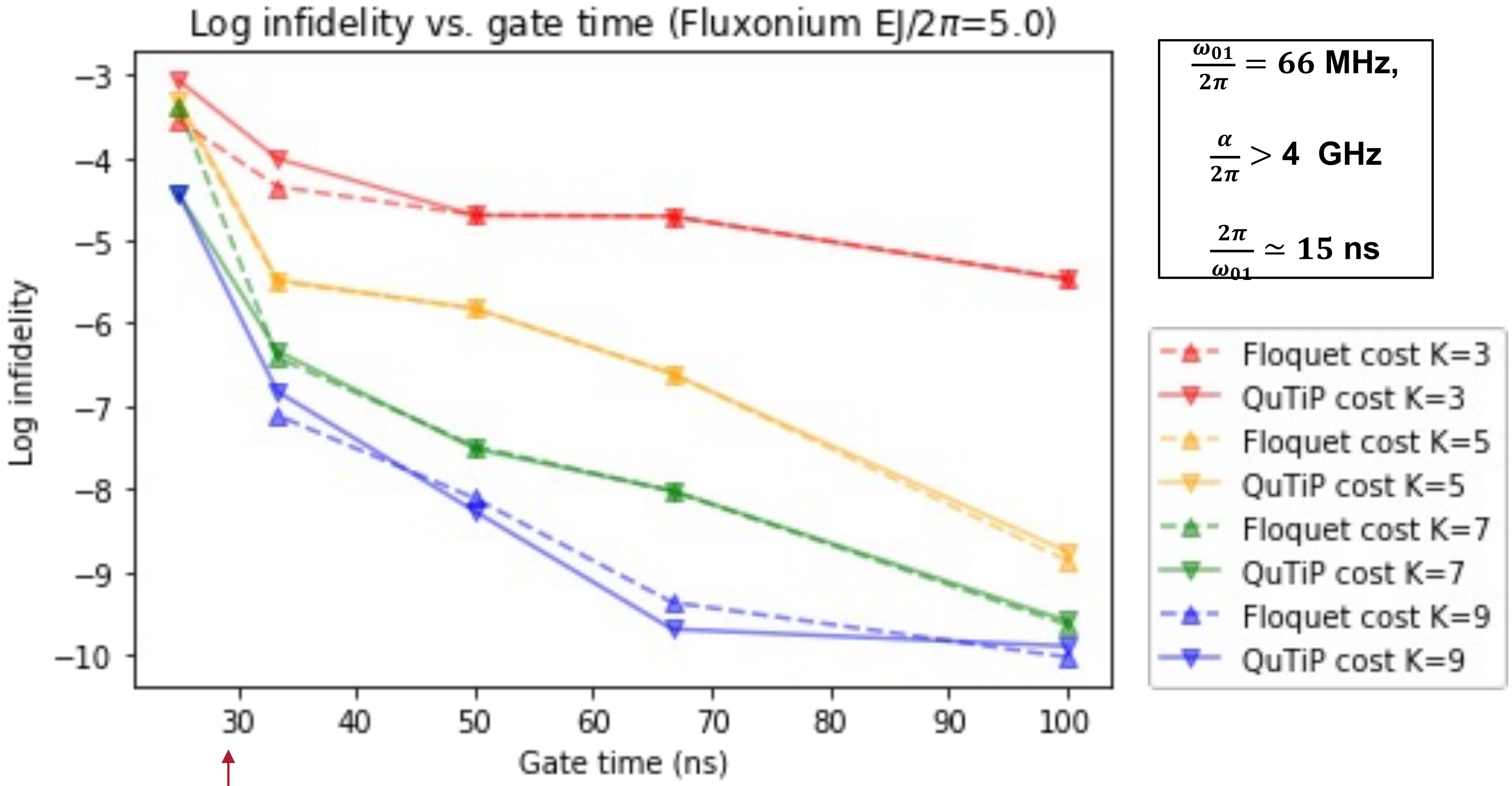




Complex pulse

Single-qubit gates for the fluxonium qubit

Gates via strong capacitive driving



$$\frac{\omega_{01}}{2\pi} = 66 \text{ MHz},$$

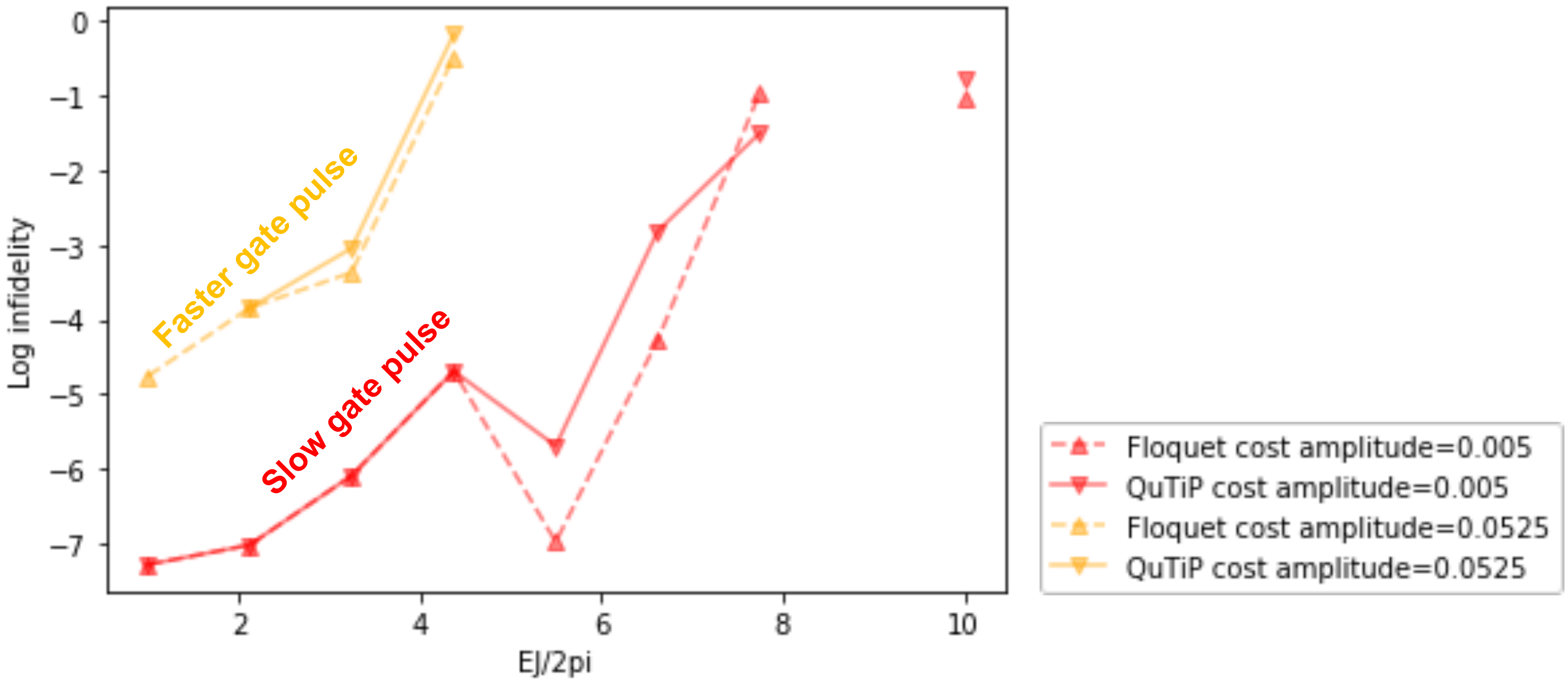
$$\frac{\alpha}{2\pi} > 4 \text{ GHz}$$

$$\frac{2\pi}{\omega_{01}} \simeq 15 \text{ ns}$$

2x Larmor frequency

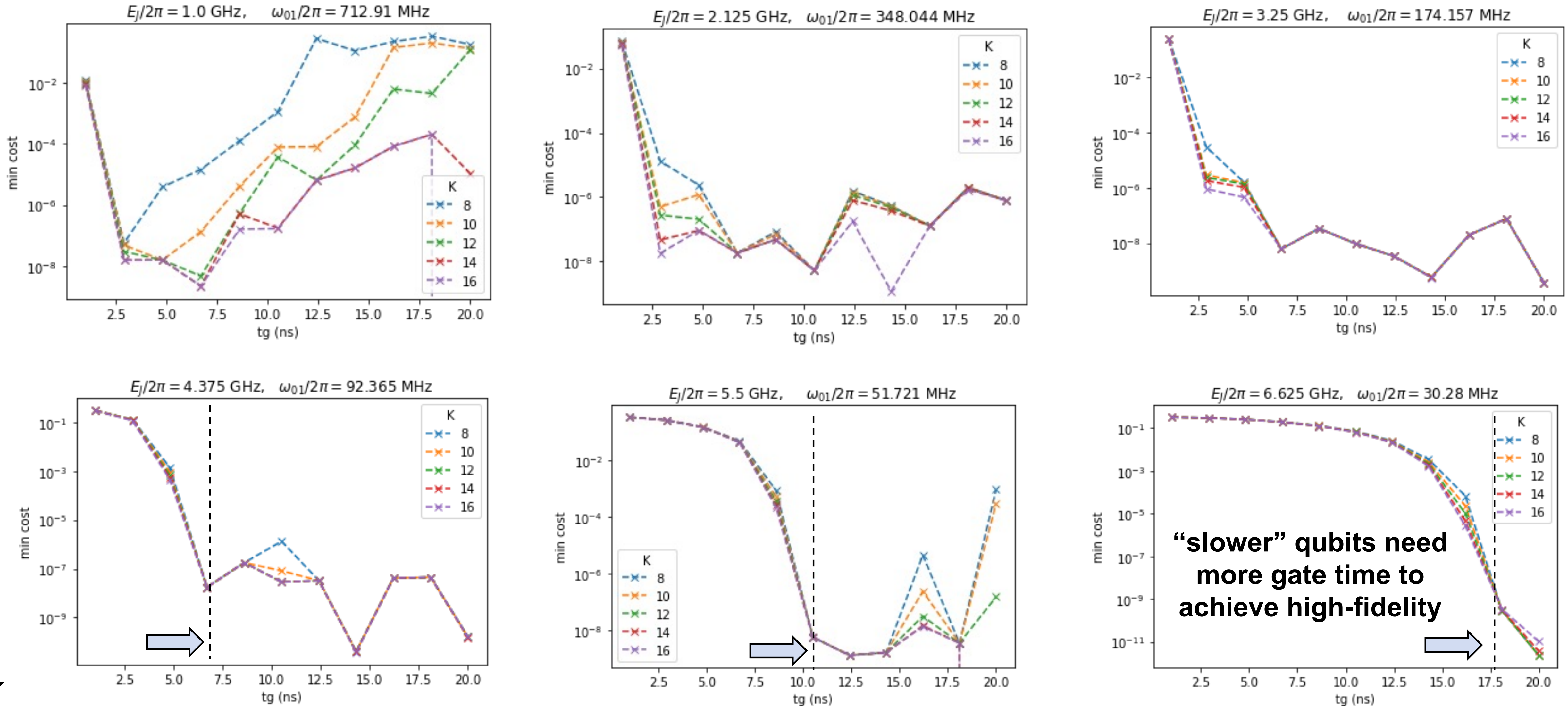
Optimizer fights mostly non-RWA terms

Microwave driving is harder at low frequency



Single-cycle 1QB fluxonium gates (fast-flux)

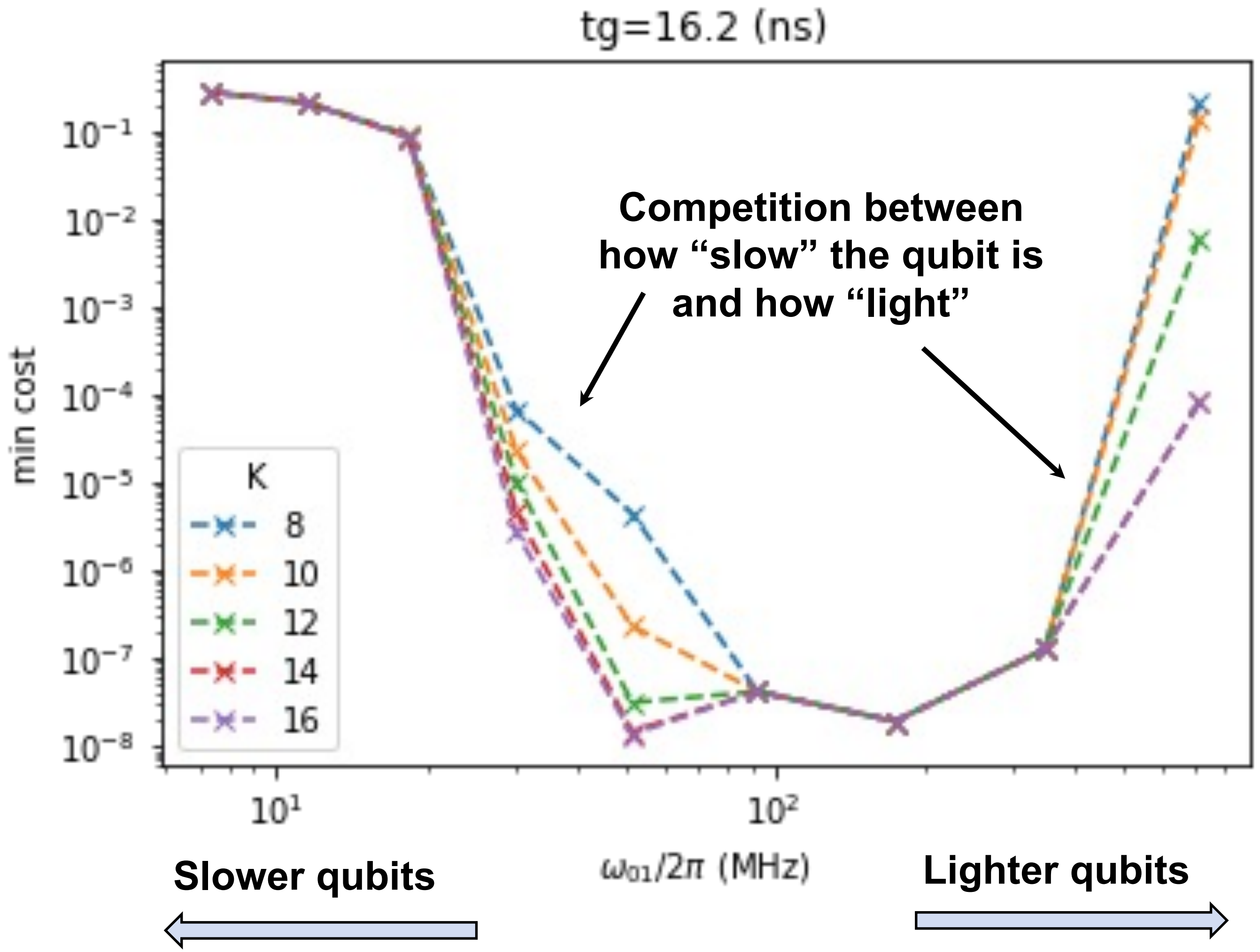
Heavier



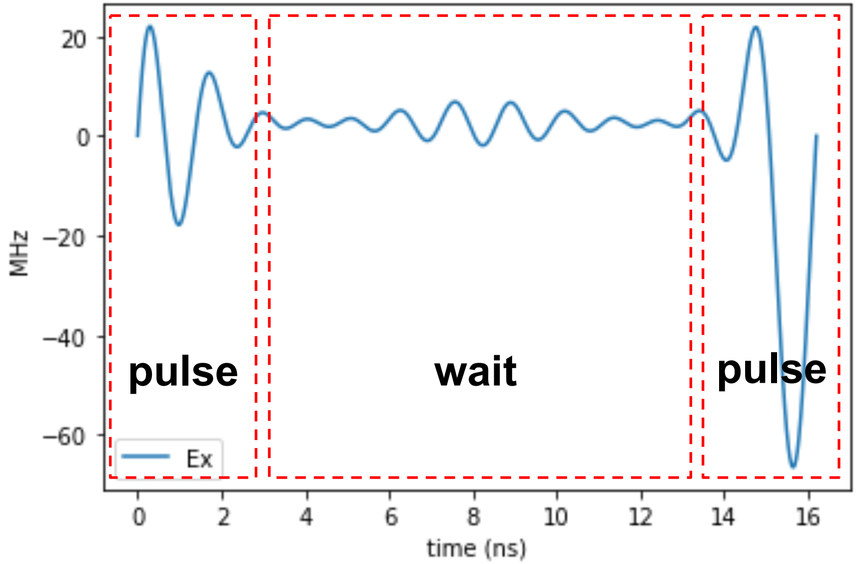
Heavier

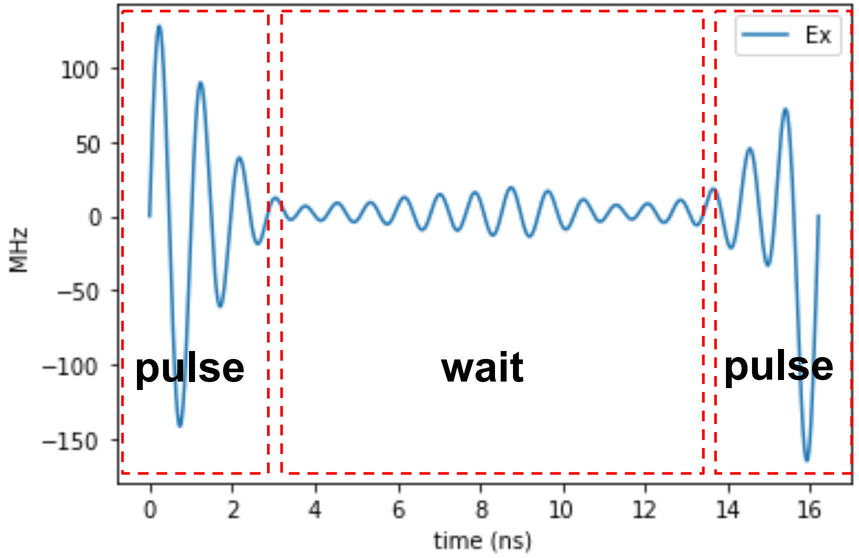
$$\frac{E_C}{h} = 1\ \text{GHz},\ \frac{E_L}{h} = 0.2\ \text{GHz}$$

Which kind of pulses are we "discovering"?



Typical pulses:





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