Emulation of Stock Exchange

Through C++ Programming and Telegram bot

Kayumov Yakov*, Pianzin Artem†

May 12, 2019

Abstract

The study introduce a mathematical model of stock exchange implemented through $C++$ programming. The code automotate all calculations, pricing algorithms and accounting issues. The interface is provided by Telegram bot. This is an unfinished study, so the pool of actions a participant can do is quite constrained - speculation on buying and selling shares of two companies. However, the model has key features of financial market: price dynamics, impact of investors' decisions and zero-sum game. Moreover, the participants can observe some statistics trough price plots.

* Moscow Institute of Physics and Technology, Bachelor Programme of Fundamental and Applied Physics, Institutskiy pereulok, 9, Dolgoprudny, 141701, Russia

† New Economic School, Bachelor Programme in Economics, Skolkovskoye Shosse, 45, Moscow, 121353, Russia

Model Set Up

Trading is implemented by two types of prices: a *bid price* - price at which an investor buys the stock, and *ask price* - price at which he/she sells the stock. In this model both prices are determined by the automated system of the model, what means that the system doesn't use the *limit order book* and investors can't quote their prices. These restrictions will be reliased in further extended versions of the study.

Before all, we introduce two measures constitute the basis of pricing algorithm. The number *Net Amount of Stock Purchased* = *NASP*, equals to the sum of all previously bought shares minus sum of all previously sold shares:

$$NASP = \sum_{i=0}^{I} q_i - \sum_{j=0}^{J} q_j$$

The *Net Stock Purchases* = *NSP*, equals to the sum of money spent on buying shares minus sum of money gained on selling shares:

$$\sum_{i=0}^{I} q_i \cdot p_i^b - \sum_{j=0}^{J} q_j \cdot p_j^a$$

Where q_i is the number of shares bought by the investor through transaction i at price p_i^b; q_j is the number of shares sold by the investor through transaction j at price p_j^a; $1, 2, ..., i, ..., I$ is a sequence of past buying transactions; $1, 2, ..., j, ...,J$ is a sequence of past selling transactions.

To compute these measures through programming it is more convenient to use the recurrent formula. So, if there is a buying transaction the NASP and NSP after the transaction will change in such way:

$$NASP_t = NASP_{t-1} + q^b,$$

$$NSP_t = NSP_{t-1} + q^b \cdot p^b$$

where q^b is the number of bought shares during the transaction for the price p^b

If there is a selling transaction:

$$NASP_t = NASP_{t-1} - q^a$$

$$NSP_t = NSP_{t-1} - q^a \cdot p^a,$$

where q^a is the number of sold shares during the transaction for the price p^a

Bid price

Creation of the price at which the security can be bought (bid price) occurs according to this formula:

$$p^b = a \cdot NASP^{\alpha} + \frac{P}{Q}, \quad a, \alpha \in \mathbb{R}_+$$

where Q is total number of emitted shares; P is a constant; together Q and P form a fraction $\frac{P}{Q}$ what is initial (IPO) price; $NASP$ same as discussed above, a

and α are constants. It's important to determine the change of p^b with respect to $NASP$:

$$(p^b)'_{NASP} = \alpha \cdot a \cdot \frac{1}{NASP^{1-\alpha}} > 0$$

Naturally, when NASP grows, the price increases too. However, depending on α the growth can increase, decrease or doesn't change:

$$(p^b)''_{NASP} = \alpha \cdot (\alpha - 1) \cdot a \cdot \frac{1}{NASP^{2-\alpha}}$$

So, if $\alpha \in (0;1)$ growth decreases, if $\alpha \in (1;+\infty)$ - growth increases, if $\alpha = 1$ - growth is constant. We chose the value of alpha the most suitable for our model.

Ask price

The formula of the current price for a transaction $t+1$, at which the security can be sold (ask price), is

$$p^a_{t+1} = min\left[\beta\frac{NSP_t}{NASP_t},\ p^b_t\right], \qquad \beta \in (1;2) \tag{1}$$

Initially, we consider the most appropriate values for beta lie in the interval $(1.001; 1.01)$ depending on the initial bid price. The *min* condition is necessary to eliminate the arbitrage possibility after an investor just bought a stock.

So, the general formula of ask price is:

$$p^a_{t+1} = \beta \cdot \frac{\sum_{i=0}^{I} q_i \cdot p^b_i - \sum_{j=0}^{J} q_j \cdot p^a_j}{\sum_{i=0}^{I} q_i - \sum_{j=0}^{J} q_j}$$

Therefore,

$$p^a_{t+1} = \beta\frac{NSP_{t-1} + q^b_t \cdot p^b_t}{NASP_{t-1} + q^b_t}\ \ or\ \ \beta\frac{NSP_{t-1} - q^a_t \cdot p^a_t}{NASP_{t-1} - q^a_t},$$ depending on the transaction purpose.

The beta plays an integral role: it allows the ask price to decrease when investors start to sell shares:

$$p^a_{t+1} = \beta\frac{NSP_{t-1} - q^a_t \cdot p^a_t}{NASP_{t-1} - q^a_t} \overset{(1)}{=} \beta\frac{NSP_{t-1} - q^a_t \cdot \beta\frac{NSP_{t-1}}{NASP_{t-1}}}{NASP_{t-1} - q^a_t}$$

$Average\ Price = AP_t = \dfrac{NSP_{t-1}}{NASP_{t-1}}$, intuitively, we divide the net money spent on shares over the net number of bought shares. This measure is close the current market price (CMP).

$$\text{Therefore, } p^a_{t+1} = \beta\frac{NASP_{t-1} \cdot AP_{t-1} - \beta \cdot AP_{t-1} \cdot q^a_t}{NASP_{t-1} - q^a_t}$$

$$(p^a_{t+1})'_{q^a_t} = \frac{AP_{t-1} \cdot NASP_{t-1} \cdot \beta(1-\beta)}{(NASP_{t-1} - q^a_t)^2} > 0$$

As we can see, the ask price decreases when investors sell the stocks.

To make the process of ask price creation more clear, we provide an *example*: Suppose $\beta = 1.005$, the market has just opened, and there is one investor on the market who buys one share of a company "K" for the bid price of 100\$. Therefore,

$NASP = 0{+}1 = 1;\ NSP = 1{\cdot}100 = 100$ and ask price is $p^a = min\left[\beta\frac{100}{1}, 100\right] = 100$

In other words, the investor now can sell his share at the price 100$. The *min* function eliminated the opportunity of arbitrage.

Second investor comes and also buys one share but now for the price 102$. Therefore,

$$NASP = 1 + 1 = 2;\ NSP = 100 + 1 \cdot 102 = 202 \text{ and ask price is}$$

$$p^a = min\left[1.005 \cdot \frac{202}{2}, 102\right] = 101.505$$

Both investors can sell their stocks at a price 101.505$.

Finally, assume the first investors decides to sell his security. He gains 101.505$. The ask price is:

$NASP = 2-1 = 1$; $NSP = 202-1\cdot 101.505 = 100.495$ and according to the formula

the ask price is $p^a = \beta\frac{100.495}{1}$. However, this is a special case.

If there is only 1 investor holding assets, we have to make the value of beta equal to 1. This is necessary to keep the amount of money constant. So, $p^a = 1 \cdot \frac{100.495}{1} = 100.495$.

As we can see, first investor has a profit of 1.505$, while second losses 1.505$. The pricing mechanism introduced above have a feature: it preserves the amount of money in the market constant and creates a zero-sum game.

Model extensions

The basic mechanism if formed. However, we provide some extensions to make the emulation more realistic and corresponding to investors' incentives.

Seasoned Equity Offering (SEO)

A firm might require more funding. To attract more liquidity it issues new shares. This release should drop the price of firm's shares. Within the model SEO is equivalent to increase of Q, from Q_1 to Q_2, where $Q_1 < Q_2$. This leads to the decrease of the bid price:

$$a \cdot NASP^{\alpha} + \frac{P}{Q_1} = p_t^b > p_{t+1}^b = a \cdot NASP^{\alpha} + \frac{P}{Q_2}$$

After the first selling transaction, after the SEO, the ask price also drops due to the *min* condition ($p^b \downdownarrows \Rightarrow p^b < p^a$).

Investors' expectations

Investors decisions of buying or not buying represent signal and their expectations which also affect the stock price. Thus, if no one buys or sells the shares and trading has stopped, it may imply that the price is too high or too low. Here, we introduce a new measure:

$$Share\ of\ Investments = SI = \frac{NSP}{M} = \frac{NSP}{NSP + C},$$

which reflects how much of the wealth is invested in the market. M is a total amount money in the economy; C is amount of money held by investors in cash.

Low SI means that investors don't want to hold securities because of negative expectations about low return. Therefore, return should be increased by lowering the price. Oppositely, high SI implies expectations abought good return, so the price increases. Assume, for instance, $SI_{low} = 0.35$ and $SI_{high} = 0.80$. If $IS < SI_{low}$ both ask and bid prices decrease. If $IS > SI_{high}-$ increase. Mathematically, we multiply both, old versions of bid and ask prices by the coefficient $\left(1+\frac{\gamma}{c}\right)$.

$$\gamma = ind \cdot (max[SI - SI_{high}, 0] + min[SI - IS_{low}, 0])$$

$$ind = \begin{cases} 0, & \Delta t < \Delta t^* \\ 1, & \Delta t \geq \Delta t^* \end{cases}$$

Δt is time passed since the last transaction of any type, Δt^* allowed time gap, for example, 6 hours, ind is indicator of time passed since the last transaction, c is a smoothing constant.

It is important to the total change of ask price to keep it always higher that than AP.

$$p_t^a = \left(1+\frac{\gamma}{c}\right)\beta \cdot AP_{t-1}$$

$$\text{Therefore, } \left(1+\frac{\gamma}{c}\right)\beta > 1 \overset{\beta>1}{\Rightarrow} c > \frac{\gamma\beta}{1-\beta}$$

$$\gamma \in [\gamma_{min}; \gamma_{max}] = [-SI_{min}; 1 - SI_{max}, \text{and can be negative and positive.}$$

$$\text{Therefore, } c > \frac{\gamma_{min}\beta}{1-\beta} = \frac{-SI_{min}\beta}{1-\beta}$$

We get the lower bound of c.

Conclusion

As was mentioned, this model is quite constrained, as investors can make only two types of decisions. However, we may improve the model by adding a possibility of taking short position, opening/closing orders, impacts of firms' financial statements, especially information about dividends, and new types of securities, such as bonds. Also, another pricing mechanism could be imbedded, which take into account the amount of shares an investor wants to buy/sell and differentiate the prices, which seems more plausible in small exchange.