Домашнє завдання 1.
студентки групи ПМО-21
Кравець Ольги

① $\sqrt{y^2+1}\,dx = xy\,dy$
$y(e) = \sqrt{3}$

$\sqrt{y^2+1}\,dx = xy\,dy \quad \Big| \cdot \frac{1}{x} \cdot \frac{1}{\sqrt{y^2+1}}$

$$\int \frac{dx}{x} = \int \frac{y\,dy}{\sqrt{y^2+1}}$$

$$\int \frac{y\,dy}{\sqrt{y^2+1}} = \left\{ \begin{array}{l} \text{Заміна:} \\ y^2+1 = z \\ 2y\,dy = dz \end{array} \right\} = \int \frac{dz}{2\sqrt{z}}$$

$$\int \frac{dx}{x} = \int \frac{dz}{2\sqrt{z}} = \frac{1}{2}\int \frac{1}{\sqrt{z}}\,dz = \frac{1}{2}\cdot 2\sqrt{z} = \sqrt{z} = \sqrt{y^2+1} + c$$

$$\ln|x| = \sqrt{y^2+1} + c$$

$$\ln|x| - \sqrt{y^2+1} = c$$

В: $\ln|x| = \sqrt{y^2+1} + c$

② $(x^2-1)y' + 2xy^2 = 0$

$y(0) = 1$

$(x^2-1)\, y' = -2xy^2$

$(x^2-1)\,\frac{dy}{dx} = -2xy^2 \;\Big/ \cdot \frac{-(x^2-1)y^2}{dx}$

$\frac{-dy}{y^2} = \frac{2x\,dx}{x^2-1}$

$t = x^2-1,\ t' = 2x$

$-\int\frac{dy}{y^2} = \int\frac{2x\,dx}{x^2-1} = \int\frac{1}{x^2-1}\,dt = \int\frac{1}{t}\,dt = \ln|t| = \ln|x^2-1| + c$

$-\int y^{-2}\,dy = \ln|x^2-1| + c$

$\frac{1}{y} = \ln(x^2-1) + c$

$y = \frac{1}{\ln(x^2-1)} \cancel{+c}$

В: $y = \frac{1}{\ln(x^2-1)+c}$

③ $xy' + y = y^2$

$y(1) = \frac{1}{2}$

$$x\frac{dy}{dx} = y^2 - y$$

$$x\frac{dy}{dx} = y(y-1) \Rightarrow y = 0 \text{ - задов.}$$

$$y = 1 \text{ - задов.}$$

$$\frac{x\,dy}{dx} = y(y-1) \Big/ \frac{dx}{xy(y-1)}$$

$$\frac{dy}{y(y-1)} = \frac{dx}{x}$$

$$\int \frac{dy}{y(y-1)} = \int \frac{dx}{x}$$

$$\frac{A}{y} + \frac{B}{y-1} = \frac{1}{y(y-1)}$$

$$A = \frac{1}{\cancel{y}(y-1)}\Big|_{y=0} = -1$$

$$B = \frac{1}{y\cancel{(y-1)}}\Big|_{y=1} = 1$$

$$\int \left(\frac{1}{y-1} - \frac{1}{y}\right) dy = \int \frac{dx}{x}$$

$$\ln|y-1| - \ln|y| = \ln|x| + c$$

$$\ln\left|\frac{y-1}{y}\right| = \ln|x| + c$$

$$e^{\ln\left|\frac{y-1}{y}\right|} = e^{\ln|x| + \ln|c|}$$

$$\frac{y-1}{y} = xc \Big/ \cdot y$$

$$y - 1 = yxc$$

$$y - yxc = 1$$

$$y(1 - xc) = 1$$

$$y = 1$$

$$y(1) = \frac{1}{2}$$

$$\frac{1}{2}(1 - c \cdot 1) = 1 \Big/ \cdot 2$$

$$1 - c = 2$$

$$-c = 1$$

$$c = -1$$

$$y(1 - x \cdot (-1)) = 1$$

$$y(1 + x) = 1$$

В: $y(1 - xc) = 1$; $c \in \mathbb{Z}$

$y = 0$

$y(1 + x) = 1$

④ $e^{-s}\left(1+\frac{ds}{dt}\right)=1$

$$e^{-s}+e^{-s}\cdot\frac{ds}{dt}=1$$

$$\frac{1}{e^{s}}+\frac{1}{e^{s}}\cdot\frac{ds}{dt}=1\ \Big/\cdot e^{s}$$

$$1+\frac{ds}{dt}=e^{s}$$

$$\frac{ds}{dt}=e^{s}-1 \Rightarrow \begin{matrix} e^{s}=1 \\ s=0 \end{matrix}$$

$$\frac{ds}{dt}=e^{s}-1\ \Big/\cdot\frac{dt}{e^{s}-1}$$

$$\frac{ds}{e^{s}-1}=dt$$

$$\int\frac{ds}{e^{s}-1}=\int dt$$

$$e^{s}-1=t$$

$$\int\frac{1}{e^{s}-1}\cdot\frac{1}{e^{s}}\,dt=\int\frac{1}{e^{s}t}\,dt=$$

$$=\int\frac{1}{(e^{s}+1-1)t}\,dt=\int\frac{1}{(t+1)t}\,dt\ \ominus$$

$$\frac{A}{t+1}+\frac{B}{t}=\frac{1}{(t+1)t}$$

$$A=\frac{1}{\cancel{(t+1)}t}\Big|_{t=-1}=-1$$

$$B=\frac{1}{(t+1)\cancel{t}}\Big|_{t=0}=1$$

$$\ominus\int-\frac{1}{t+1}+\frac{1}{t}\,dt=-\int\frac{1}{t+1}\,dt+\int\frac{1}{t}\,dt=$$

$$=-\ln|t+1|+\ln|t|+c=$$

$$=-\ln|e^{s}\cancel{-1}+\cancel{1}|+\ln|e^{s}-1|=$$

$$=\ln|e^{s}-1|-\ln|e^{s}|=\ln\left|\frac{e^{s}-1}{e^{s}}\right|=$$

$$=\ln|1-e^{-s}|$$

$$\ln|1-e^{-s}|=t-\ln|c|$$

$$1-e^{-s}=-ce^{t}$$

$$e^{-s}=1+ce^{t}$$

$$s=0;\ c=0$$

В: $e^{-s}=1+ce^{t}$

⑤ $(x+2y)y'=1$

$y(0)=-1$

$z(x)=x+2y$

$z'=1+2y'$

$2y'=z'-1 \,/\cdot\frac{1}{2}$

$y'=\frac{z'-1}{2}$

$z\cdot\frac{z'-1}{2}=1$

$z(z'-1)=2 \,/\cdot\frac{1}{z}$

$(z'-1)=\frac{2}{z}$

$\frac{dz}{dx}-1=\frac{2}{z}$

$\frac{dz}{dx}=\frac{2}{z}+1$

$\frac{dz}{dx}=\frac{2+z}{z} \,/\cdot\frac{dx\cdot z}{(2+z)}$

$\frac{z}{2+z}dz=dx$

$\frac{z}{2+z}dz-dx=0$

$\int\frac{z}{2+z}dz-\int dx=\ln|c|$

$\int\frac{(2+z)-2}{2+z}dz=\int 1dz-\int\frac{2dz}{2+z}=$

$=\int 1dz-2\int\frac{dz}{2+z}$

$z-2\ln|2+z|-x=\ln|c|$

$\cancel{x}+2y-2\ln|2+x+2y|-\cancel{x}=\ln|c|$

$2y-2\ln|2+x+2y|=\ln|c|$

$2y=\ln|c|+2\ln|2+x+2y|$

$\ln(e^y)=\ln|c(2+x+2y)|$

$e^y=c(2+x+2y)$

$2+x+2y=0$

$2+z=0$

$z=-2$

$y(0)=-1$

$e^{-1}=c(\cancel{2+0-2})$

$e^{-1}=c(0)$

$c\in\emptyset$

B: $2+x+2y=0$

⑥ $(x-y)\,dx+(x+y)\,dy=0$

$(x-y)dx+(x+y)dy=0$

$(x+y)dy=-(x-y)dx \;\Big/ \cdot \frac{1}{dy\cdot dx\,(x+y)(x-y))}$

$\frac{dy}{dx}=\frac{-(x-y)}{x+y}$

$y'=\frac{dy}{dx}=\frac{\frac{y}{x}-1}{1+\frac{y}{x}}$

$\frac{y}{x}=v$

$y=xv$

$y'=v+xv'$

$\frac{v-1}{1+v}=v+xv'$

$\frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}=x\cdot\frac{dv}{dx}$

$\frac{dx}{x}=-\frac{(1+v)dv}{1+v^2}$

V $\int\frac{dx}{x}=-\int\frac{dv}{1+v^2}-\int\frac{1}{2}\frac{dv^2}{1+v^2}$

$\int-\frac{1+v}{1+v^2}dv=-\int\frac{1+v}{1+v^2}dv=$

$=-\int\frac{1}{1+v^2}+\frac{v}{1+v^2}dv=$

$=-\left(\int\frac{1}{1+v^2}dv+\int\frac{v}{1+v^2}dv\right)=$

$=-\left(\operatorname{arctg}v+\frac{1}{2}\ln(1+v^2)\right)$

V $\ln|x|=-\operatorname{arctg}v-\frac{1}{2}\ln|1+v^2| \,/\cdot 2$

$2\ln|x|=-2\operatorname{arctg}v-\ln|1+v^2|$

$2\ln|x|=-2\operatorname{arctg}\frac{y}{x}-$

$-\ln\left|1+\frac{y^2}{x^2}\right|$

$2\ln|x|=-2\operatorname{arctg}\frac{y}{x}-\ln\left|\frac{x^2+y^2}{x^2}\right|+c$

$2\ln|x|=-2\operatorname{arctg}\frac{y}{x}-\ln|x^2+y^2|+$

$+\ln|x^2|+c$

$\cancel{2\ln|x|}=-2\operatorname{arctg}\frac{y}{x}+\cancel{2\ln x}-$

$-\ln|x^2+y^2|$

$\ln|x^2+y^2|=C-2\operatorname{arctg}\frac{y}{x}$

B: $\ln|x^2+y^2|=C-2\operatorname{arctg}\frac{y}{x}$

(7) $2x^3y' = y(2x^2 - y^2)$

$$2x^3y' = y(2x^2 - y^2)$$

$$2x^3y' = 2x^2y - y^3 \Big/ \cdot \frac{1}{2x^3}$$

$$y' = \frac{\cancel{2x^2}y}{\cancel{2x^3}} - \frac{y^3}{2x^3}$$

$$y' = \frac{y}{x} - \frac{1}{2}\cdot\frac{y^3}{x^3}$$

$\frac{y}{x} = v;$ $\quad y = xv$

$$y' = v + xv'$$

$$\cancel{v} + xv' = \cancel{v} - \frac{1}{2}v^3$$

$$x\cdot\frac{dv}{dx} = -\frac{1}{2}v^3 \Big/ \cdot(-2)$$

$$-2x\frac{dv}{dx} = v^3 \Big/ \cdot \frac{dx}{x\cdot v^3}$$

$$\cancel{-2}\cdot\left(-\frac{1}{\cancel{2}v^2}\right) = \frac{1}{v^2}$$

$$-2\,\frac{dv}{v^3} = \frac{dx}{x}; \quad -2\int\frac{dv}{x^3} = \int\frac{dx}{x}$$

$$\frac{1}{v^2} = \ln|x| + \ln|c|$$

$$\frac{x^2}{y^2} = \ln(cx)$$

$$x^2 = y^2\cdot\ln(cx)$$

$$x = \pm y\sqrt{\ln cx}$$

В: $x = \pm y\sqrt{\ln cx}$

⑧ $(x^2+y^2)y' = 2xy$

$y(2) = -2$

$(x^2+y^2)y' = 2xy$

$y' = \frac{2xy}{x^2+y^2} \quad \begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$

$y' = \frac{2\frac{y}{x}}{1+\left(\frac{y}{x}\right)^2}$

$\frac{y}{x} = v$

$y = xv$

$y' = v + xv'$

$v + xv' = \frac{2v}{1+v^2}$

$x\frac{dv}{dx} = \frac{2v - v - v^3}{1+v^2} = \frac{v(1-v^2)}{1+v^2} \Big| \cdot \frac{dx}{v(1+v^2)}$

$\frac{dx}{x} = \frac{(1+v^2)dv}{v(1-v^2)}$

$\frac{dx}{x} = \frac{dv}{v(1-v^2)} + \frac{1}{2}\frac{dv^2}{1-v^2}$

$\frac{dv}{v^3\left(\frac{1}{v^2}-1\right)} = -\frac{1}{2}\frac{d\left(\frac{1}{v^2}\right)}{\frac{1}{v^2}-1}$

$\int\frac{dx}{x} = -\frac{1}{2}\int\frac{d\left(\frac{1}{v^2}\right)}{\frac{1}{v^2}-1} + \frac{1}{2}\int\frac{dv^2}{1-v^2}$

$\ln|x| = -\frac{1}{2}\ln\left|\frac{1}{v^2}-1\right| + \frac{1}{2}\ln|1-v^2| + C$

$\ln x = -\frac{1}{2}\ln\frac{x^2-y^2}{x^2} - \frac{1}{2}\ln\frac{x^2-y^2}{y^2} + \ln c$

$\cancel{\ln x} = -\frac{1}{2}\ln(x^2-y^2) + \cancel{\ln x} - \frac{1}{2}\ln(x^2-y^2) + \ln y + \ln c$

$yc = x^2 - y^2; \; y = 0$

$y(2) = -2$

$-2c = 4 - 4$

$c = 0$

В: $cy = x^2 - y^2; \; y = 0$

⑨ $xy' - y = x\,tg\frac{y}{x}$

$xy' - y = x\,tg\frac{y}{x}$

$xy' = x\,tg\frac{y}{x} + y$

$x\,dy = (x\,tg\frac{y}{x} + y)\,dx \,/ \cdot \frac{1}{x\,dx}$

$y' = \frac{dy}{dx} = tg\frac{y}{x} + \frac{y}{x}$

$\frac{y}{x} = v;\quad y = xv$

$y' = v + xv'$

$\cancel{v} + xv' = tg\,v + \cancel{v}$

$\frac{dv}{dx} \cdot x = tg\,v$

$\frac{dv}{tg\,v} = \frac{dx}{x}$

$\frac{dv}{\frac{\sin v}{\cos v}} = \frac{dx}{x}$

$\frac{\cos v \cdot dv}{\sin v} = \frac{dx}{x}$

$\frac{\int \cos v\,dv}{\sin v} = \int\frac{dx}{x}$

$\int\frac{d(\sin v)}{\sin v} = \int\frac{dx}{x}$

$\ln \sin v = \ln|x| + c$

$\sin v = cx$

$\sin\frac{y}{x} = cx$

В: $\sin\frac{y}{x} = cx$

(10) $xy' = \sqrt{x^2 - y^2} + y$

$$xy' = \sqrt{x^2 - y^2} + y$$

$$x \cdot \frac{dy}{dx} = \sqrt{x^2 - y^2} + y$$

$$\frac{y}{x} = v; \quad y = xv$$

$$y' = v + xv'$$

$$x \cdot (v + xv') = xv + \sqrt{x^2 - (xv)^2}$$

$$x \cdot (v + xv') = xv + \sqrt{x^2 - x^2 v^2}$$

$$x \cdot (v + xv') = xv + \sqrt{x^2 (1 - v^2)}$$

$$x (v + xv') = xv + \sqrt{x^2} \cdot \sqrt{1 - v^2}$$

$$x (v + xv') = xv + x\sqrt{1 - v^2}$$

$$x (v + xv') = x(v + \sqrt{1 - v^2}) \Big/ \cdot \frac{1}{x}$$

$$\cancel{v} + xv' = \cancel{v} + \sqrt{1 - v^2}$$

$$x\frac{dv}{dx} = \sqrt{1 - v^2} \quad \Big/ \cdot \frac{dx}{x(\sqrt{1 - v^2})}$$

$$\frac{1}{\sqrt{1 - v^2}}\, dv = \frac{1}{x}\, dx$$

$$\int \frac{1}{\sqrt{1 - v^2}}\, dv = \int \frac{1}{x}\, dx$$

$$\pm \arcsin v = \ln x + c$$

$$\pm \arcsin \frac{y}{x} = \ln cx$$

В: $\pm \arcsin \frac{y}{x} = \ln cx$