Домашнє завдання №4

1.41. $\underbrace{(2x^3 - xy^2)}_{M}dx + \underbrace{(2y^2 - x^2y)}_{N}dy = 0$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(2x^3 - xy^2) - \frac{\partial}{\partial x}(2y^2 - x^2y) =$

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$= (0-2xy)-(0-2xy)=0$

$$\begin{cases} \frac{\partial u}{\partial x} = 2x^3 - xy^2 \\ \frac{\partial u}{\partial y} = 2y^3 - x^2y \end{cases} \quad (*)$$

$(*_1) \,\Big|\, \int dx \qquad u = \int 2x^3 - xy^2 dx = \int 2x^3 dx -$

$- \int xy^2 dx = \frac{x^4}{2} - \frac{x^2y^2}{2} + \varphi(y)$

$$\frac{\partial}{\partial y}\left(\frac{x^4}{2} - \frac{x^2y^2}{2} + \varphi(y)\right) = 2y^3 - x^2y$$

$0 - \cancel{x^2y} + \varphi'(y) = 2y^3 - \cancel{x^2y}$

$\varphi'(y) = 2y^3$

$\varphi(y) = \int 2y^3 dy = \frac{y^4}{2} + C_1$

$$\boxed{\frac{x^4 - x^2y^2 + y^4}{2} = C}$$

1.4.12. $\underbrace{\frac{y}{x}}_{M} dx + \underbrace{(y^3 + \ln x)}_{N} dy = 0$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}\left(\frac{y}{x}\right) - \frac{\partial}{\partial x}(y^3 + \ln x) =$

$= \frac{1}{x} - \frac{1}{x} = 0$

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{y}{x} \\ \frac{\partial u}{\partial y} = y^3 + \ln x \end{cases} \quad (*)$$

$(*_1) \mid \int dx \qquad u = \int \frac{y}{x} dx = y\ln|x| + \varphi(y)$

$\frac{\partial}{\partial y}(y\ln|x| + \varphi(y)) = y^3 + \ln x$

$\ln x + \varphi'(y) = y^3 + \ln x$

$\varphi'(y) = y^3$

$\varphi(y) = \int y^3 dy = \frac{y^4}{4} + C_1$

$$\boxed{y\ln x + \frac{y^4}{4} = C}$$

1.4.13. $\underbrace{\left(\frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y}\right)}_{M} dx + \underbrace{\left(\frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} - \frac{x}{y^2}\right)}_{N} dy = 0$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y}\right) -$$

$$- \frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} - \frac{x}{y^2}\right) = -\frac{xy}{(x^2+y^2)\sqrt{x^2+y^2}} -$$

$$- \frac{1}{y^2} - \left(-\frac{xy}{(x^2+y^2)\sqrt{x^2+y^2}} - \frac{1}{y^2}\right) = 0$$

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y} \\ \frac{\partial u}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} - \frac{x}{y^2} \end{cases} \quad (*)$$

$(*_1) \mid \int dx \qquad u = \int \frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y} dx =$

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$$= \sqrt{x^2+y^2} + \ln|x| + \frac{x}{y} + \varphi(y)$$

$$\frac{\partial}{\partial y}\left(\sqrt{x^2+y^2} + \ln|x| + \frac{x}{y} + \varphi(y)\right) = \frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} -$$

$$-\frac{x}{y^2}$$

$$\cancel{\frac{y}{\sqrt{x^2+y^2}}} - \cancel{\frac{x}{y^2}} + \varphi'(y) = \cancel{\frac{y}{\sqrt{x^2+y^2}}} + \frac{1}{y} - \cancel{\frac{x}{y^2}}$$

$$\varphi'(y) = \frac{1}{y}$$

$$\varphi(y) = \int \frac{1}{y}\,dy = \ln|y| + C_1$$

$$\boxed{\sqrt{x^2+y^2} + \ln|x| + \frac{x}{y} + \ln|y| = C}$$

1.4.22. $xy^2(xy'+y) = 1$

$$x^2y^2\frac{dy}{dx} + xy^3 = 1 \,\Big|\cdot dx$$

$$x^2y^2\,dy + xy^3\,dx = dx$$

$$\underbrace{(xy^3-1)}_{M}dx + \underbrace{x^2y^2}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(xy^3-1) - \frac{\partial}{\partial x}x^2y^2 = 3xy^2 - 2xy^2 =$$

$$= xy^2$$

Знайдемо інтег. м-ник: $(*)\,|\cdot\mu$

$$\underbrace{\mu(xy^3-1)}_{M}dx + \underbrace{\mu x^2y^2}_{N}dy = 0 \qquad (**)$$

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Знайдемо $\mu(x)$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(\mu(x)(xy^3-1)) - \frac{\partial}{\partial x}(\mu(x)x^2y^2) =$$

$$= 0 + 3xy^2\mu(x) - \mu'(x)x^2y^2 - 2xy^2\mu(x) = xy^2\mu(x) - \mu'(x)x^2y^2 = 0$$

$$x^2y^2(\mu(x) - \mu'(x)x) \mid : xy^2$$

$$\mu(x) - \mu'(x)x = 0$$

$$\mu'(x) = \frac{\mu(x)}{x}$$ y тут нема $\Rightarrow$ $\mu(x)$ існує

$$\mu' = \frac{\mu}{x}$$

$$\frac{d\mu}{dx} = \frac{\mu}{x} \mid \cdot \frac{dx}{\mu}$$

$$\int \frac{d\mu}{\mu} = \int \frac{dx}{x}$$

$$\ln|\mu| = \ln|x| + \ln|C_1|$$

$\mu = xC_1$ $\quad$ $\mu = x$ – інтег. м-ник

$(*) \mid \cdot x$

$$x(xy^3-1)dx + x(x^2y^2)dy = 0$$

$$(x^2y^3 - x)dx + x^3y^2dy = 0$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(x^2y^3 - x) - \frac{\partial}{\partial x}(x^3y^2) =$$

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$= 3x^2y^2 - 3x^2y^2 = 0$

$$\begin{cases} \frac{\partial u}{\partial x} = x^2y^3 - x \\ \frac{\partial u}{\partial y} = x^3y^2 \end{cases} \quad (*)$$

$(*_1) \mid \int dx \qquad u = \int (x^2y^3 - x)dx = \frac{y^3x^3}{3} - \frac{x^2}{2} + \varphi(y)$

$$\frac{\partial}{\partial y}\left(\frac{y^3x^3}{3} - \frac{x^2}{2} + \varphi(y)\right) = x^3y^2$$

$\cancel{x^3y^2} + \varphi'(y) = \cancel{x^3y^2}$

$\varphi'(y) = 1$

$\varphi(y) = \int dy = y + C_1$

$$\boxed{\frac{y^3x^3}{3} - \frac{x^2}{2} + y = C}$$

Перевірка: $x=0$ – не розв., бо y' не існує

1.4.27. $\underbrace{(3x^2-1)}_{M}dx + \underbrace{\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)}_{N}dy = 0$

$y(1) = 1$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(3x^2-1) - \frac{\partial}{\partial x}\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right) =$$

$$= 0 - \left(0 - \frac{2}{y} + \frac{6x^2}{y}\right) = \frac{2-6x^2}{y}$$

Знайдемо інтег. м-ж: $(*) \mid \cdot \mu$

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$$\mu(3x^2-1)dx + \mu\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)dy = 0$$

Знайдемо $\mu(x)$:

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}\left(\mu(x)(3x^2-1)\right) - \frac{\partial}{\partial x}\left(\mu(x)\cdot\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)\right) = 0 - \left(\mu'(x)\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)\right) + \left(-\frac{2}{y} + \frac{6x^2}{y}\right)\mu(x) = 0$$

$$-\mu'(x)\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right) = -\mu(x)\left(-\frac{2}{y} + \frac{6x^2}{y}\right)$$

$$\mu'(x) = \frac{\frac{2-6x^2}{y}}{\frac{3y-2x+2x^3}{y}}\mu(x)$$

інтег. м-ника $\mu(x)$ не існує

Знайдемо $\mu(y)$:

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}\left(\mu(y)(3x^2-1)\right) - \frac{\partial}{\partial x}\left(\mu(y)\cdot\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)\right) = \mu'(y)(3x^2-1) - \left(-\frac{2}{y} + \frac{6x^2}{y}\right)\mu(y) = 0$$

$$\mu'(y)(3x^2-1) = \mu(y)\frac{-2+6x^2}{y} \quad \Big| : 3x^2-1$$

$$\mu'(y) = \mu(y)\frac{2\cancel{(-1+3x^2)}}{y\cancel{(3x^2-1)}}$$

x тут нема → $\mu(y)$ ~~не~~ існує

$$\mu' = \frac{2\mu}{y}$$

$$\frac{d\mu}{dy} = \frac{2\mu}{y} \quad \Big| \cdot \frac{dy}{2\mu}$$

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$$\int \frac{d\mu}{\mu} = \int \frac{2}{y}\,dy$$

$$\ln|\mu| = 2\ln|y| + \ln|C_1|$$

$\mu = y^2 C \qquad \mu = y^2$ – інтег. м-ник

$(*) \,|\cdot y^2$

$$y^2(3x^2-1)dx + y^2\left(3 - \frac{2x}{y} + \frac{2x^3}{y}\right)dy = 0$$

$$(3x^2y^2 - y^2)dx + (3y^2 - 2xy + 2x^3y)dy = 0$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(3x^2y^2 - y^2) - \frac{\partial}{\partial x}(3y^2 - 2xy + 2x^3y) = 6x^2y - 2y + 2y - 6x^2y = 0$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = 3x^2y^2 - y^2 \\ \dfrac{\partial u}{\partial y} = 3y^2 - 2xy + 2x^3y \end{cases} \qquad (*)$$

$(*_1)\,|\int dx \qquad u = \int (3x^2y^2 - y^2)dx = x^3y^2 - y^2x + \varphi(y)$

$$\frac{\partial}{\partial y}(x^3y^2 - y^2x + \varphi(y)) = 3y^2 - 2xy + 2x^3y$$

$$\cancel{2x^3}y - \cancel{2y}x + \varphi'(y) = 3y^2 - \cancel{2xy} + \cancel{2x^3}y$$

$$\varphi'(y) = 3y^2$$

$$\varphi(y) = \int 3y^2\,dy = y^3 + C_1$$

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$\boxed{x^3y^2 - xy^2 + y^3 = C}$

Перевірка: $y^2 = 0$ $y = 0$ – не розв

$y(1) = 1$

$1^3 \cdot 1^2 - 1 \cdot 1^2 + 1^3 = C$

$C = 1$

1.4.28. $\underbrace{y^2}_{M}dx - \underbrace{(xy + x^3)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(y^2) - \frac{\partial}{\partial x}(-xy - x^3) =$$

$$= 2y + y + 3x^2 = 3y + 3x^2$$

Знайдемо інтег. м-ка: $(*) | \cdot \mu$

$$\mu y^2 dx - \mu(xy + x^3)dy = 0$$

Знайдемо $\mu(x)$:

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{\partial}{\partial y}(\mu(x)y^2) - \frac{\partial}{\partial \cancel{x}}(\mu(x)(-xy - x^3)) =$$

$$= 2y\mu(x) - (\mu'(x)(-xy - x^3) + (-y - 3x^2)\mu(x)) =$$

$$= 2y\mu(x) - \mu'(x)(-xy - x^3) - (-y - 3x^2)\mu(x) =$$

$$= -\mu'(x)(-xy - x^3) + (2y + y + 3x^2)\mu(x) =$$

$$= -\mu'(x)(-xy - x^3) + (3y + 3x^2)\mu(x) = 0$$

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$\mu'(-xy-x^3)=\mu(x)(3y+3x^2) \mid : -xy-x^3$

$\mu' = +\dfrac{3y+3x^2}{-xy-x^3}\mu(x)$

$\mu' = \dfrac{3(y+x^2)}{-x(y+x^2)}\mu(x)$

$\mu' = -\dfrac{3\mu}{x}$ y тут нема $\to \mu(x)$ існує

$\dfrac{d\mu}{dx} = -\dfrac{3\mu}{x} \mid \cdot \dfrac{dx}{\mu}$

$\int\dfrac{d\mu}{\mu} = -\int\dfrac{3}{x}dx$

$\ln|\mu| = -3\ln|x| + \ln|C|$

$\mu = x^{-3}C$ $\mu = x^{-3}$ - інтег. м-ик

$(*) \mid \cdot x^{-3}$

$x^{-3}y^2dx - (x^{-2}y+1)dy = 0$

$\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x} = \dfrac{\partial}{\partial y}(x^{-3}y^2) - \dfrac{\partial}{\partial x}(-x^{-2}y-1) =$

$= 2x^{-3}y - 2x^{-3}y = 0$

$$\begin{cases} \dfrac{\partial u}{\partial x} = x^{-3}y^2 \\ \dfrac{\partial u}{\partial y} = -x^{-2}y-1 \end{cases} \quad (*)$$

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$(*_1)$ $|\int dx$ $\quad u=\int x^{-3}y^2dx=-\frac{y^2}{2x^2}+\varphi(y)$

$$\frac{\partial}{\partial y}\left(-\frac{y^2}{2x^2}+\varphi(y)\right)=-x^{-2}y-1$$

$$-\cancel{\frac{y}{x^2}}+\varphi'(y)=-\cancel{\frac{y}{x^2}}-1$$

$$\varphi'(y)=-1$$

$$\varphi(y)=-\int dy=-y+C_1$$

$$\boxed{-\frac{y^2}{2x^2}-y=C}$$

Перевірка: $x^{-3}=0 \Rightarrow x=0$ - розв.