Зошит з диференціальних рівнянь
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A) $x^2 y' = 16x^2 + xy + y^2$, $y(1)=0$

$y' = 16 + \frac{y}{x} + \frac{y^2}{x^2}$, $y(1)=0$

$y = xv$ $\quad v = \frac{y}{x}$ $\quad dy = xv' + v$

$xv' + v = 16 + v + v^2$

$xv' = 16 + v^2$

$v' = \frac{16+v^2}{x}$

$\frac{dv}{dx} = \frac{16+v^2}{x}$

$\int \frac{dv}{16+v^2} = \int \frac{dx}{x}$

$\frac{1}{4} \cdot \operatorname{arctg} \frac{v}{4} = \ln|x| + C$

$v = 4\,\mathrm{tg}(4(\ln|x| + C))$

$y = 4x\,\mathrm{tg}(4(\ln|x|+C))$

Задача Коші: $4\,\mathrm{tg}(4c) = 0$

$\mathrm{tg}(4c) = 0$

$c = 0$

$y = 4x\,\mathrm{tg}(4\ln x)$

D) $y'' - y' - 6y = 2(3-4x)e^{2x}, y(0)=0, y'(0)=2$

$\lambda^2 - \lambda - 6 = 0$

$D = 1 + 24 = 25$

$\lambda_1 = \frac{1+5}{2} = 3 \quad \lambda_2 = \frac{1-5}{2} = -2$

$y_0 = C_1 e^{3x} + C_2 e^{-2x}$

λ_1	λ_2	μ	ν	s
3	-2	2	0	1

$y_* = (ax+b)e^{2x}$

$y'_* = 2(ax+b)e^{2x} + ae^{2x}$

$y''_* = 2(2ax+2b+a)e^{2x} + 2ae^{2x}$

$4ax + 4b + 2a + 2a - 2ax - 2b - a - 6ax - 6b =$
$= 6 - 8x$

$4a - 2a - 6a = -8 \qquad 4b + 4a - 2b - a - 6b = 6$

$4a - 8a = -8 \qquad -4b + 3a = 6$

$-4a = -8 \qquad -4b + 6 = 6$

$a = 2 \qquad -4b = 0$

$y_* = 2xe^{2x} \qquad b = 0$

$y = C_1 e^{3x} + C_2 e^{-2x} + 2xe^{2x}$

$C_1 = -C_2$

$y(0) = 0 \quad \begin{cases} C_1 + C_2 = 0 \\ 3C_1 - 2C_2 = 0 \end{cases} \qquad -3C_2 - 2C_2 = 0$

$y'(0) = 2 \qquad -5C_2 = 0$

Відп: $\boxed{2xe^{2x}}$ $\qquad C_2 = 0 \quad C_1 = 0$

E)

$\boxed{E}$ $\begin{cases} \dot{x}_1 = 5x_1 - 2x_2 + 8 \\ \dot{x}_2 = 5x_1 - x_2 + 9 \end{cases}$

$A = \begin{pmatrix} 5 & -2 \\ 5 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$

$\det(A - E\lambda) = (5-\lambda)(-1-\lambda) + 10 =$

$= -5 - 5\lambda + \lambda + \lambda^2 + 10 =$ $\begin{pmatrix} 2\cos t \\ 3\cos t + \sin t \end{pmatrix} \begin{pmatrix} 2\sin t \\ 3\sin t - \cos t \end{pmatrix}$

$= \lambda^2 - 4\lambda + 5 = 0$

$D = 16 - 20 = -4$

$\lambda_1 = \frac{4+2i}{2} = 2+i$

$\lambda_2 = \frac{4-2i}{2} = 2-i$

$\lambda_1\ 2+i$ $\begin{pmatrix} 5-2+i & -2 \\ 5 & -1-2+i \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$h_1 = \begin{pmatrix} 2 \\ (3-i) \end{pmatrix}$

$3\alpha - i\alpha = 2\beta$

$5\alpha - 3\beta - i\beta = 0$

$\alpha(3-i) = 2\beta$

$\varphi(t) = e^{(2+i)t}\begin{pmatrix} 2 \\ 3-i \end{pmatrix} = e^{2t}(\cos t + i\sin t)\begin{pmatrix} 2 \\ 3-i \end{pmatrix} =$

$= e^{2t}\begin{pmatrix} 2\cos t + 2i\sin t \\ 3\cos t + 3i\sin t - i\cos t + \sin t \end{pmatrix}$

$x_{1o} = e^{2t}(2C_1\cos t + 2C_2\sin t)$

$x_{2o} = e^{2t}(C_1(3\cos t + \sin t) + C_2(3\sin t - \cos t))$

$X_* = (a_1\ a_2)$ $\qquad$ $b = (8, 9)$

$$\begin{cases} 5x_1 - 2x_2 + 8 = 0 \\ 5x_1 - x_2 + 9 = 0 \end{cases}$$

$5x_1 = -10$

$x_1 = -2$

$5x_1 - 2x_2 - 8$

$2x_2 - 8 - x_2 + 9 = 0$

$x_2 + 1 = 0$

$x_2 = -1$

$\boxed{8}$ $xy' + x^4 y^2 \cos x + 3y = 0 \quad y(\sqrt{\pi}) = \frac{1}{3\pi^3}$

$xy' + x^4 + y^2 \cos x = -3y$

$xy' + 3y = -x^4 y^2 \cos x$

$\frac{-y'}{y^2} - \frac{3}{xy} = x^3 \cos x$

$z = y^{1-2} \quad z' = (-1)y^{-2} y' \Rightarrow \frac{y'}{y^2} = \frac{1}{1-2} z'$

$z = \frac{1}{y}$

$\frac{1}{1-2} z' - \frac{3}{x} \cdot z = x^3 \cos x$

$z' - (1-2)\frac{3}{x} \cdot z = (1-2) x^3 \cos x$

$z' + \frac{3}{x} z = -x^3 \cos x$

$z^* = d(x) x^3$

$d'(x) = \frac{-x^3 \cos x}{x^3} = -\cos x$

$z' = \frac{3z}{x}$

$\frac{dz}{dx} = \frac{3z}{x}$

$\int \frac{dz}{z} = \int \frac{3}{x} dx$

$\ln|z| = 3 \ln|x| + \ln|c|$

$z = cx^3$

$d(x) = -\sin x$

$z^* = -\sin(x) x^3$

$z = cx^3 - \sin(x) x^3$

$y = \frac{1}{cx^3 - \sin(x) x^3}$

$x^3 y (c - \sin x) = 1$

$x^3 y = 1 - c + \sin x$

$x^3 y - c + \sin x$

Коші $\quad c = 3$

$\frac{1}{c\sqrt{\pi}^3} = \frac{1}{3\sqrt{\pi}^3}$

$y = \frac{1}{3x^3 - \sin(x) x^3} = \frac{1}{x^3(3 - \sin(x))}$