3.2.4. $3xy'' + 7y'' - 9 - y' = 0$ нема y

Заміна: $y(x) \rightsquigarrow z(x)$, $y' = z$, $y'' = z'$

$3xz' + 7z' - 9 - z = 0$

$z'(3x + 7) - 9 - z = 0 \mid : 3x+7$

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d) $z' = \frac{9}{3x+7} + \frac{z}{3x+7}$

1) $z' = \frac{z}{3x+7}$

$$\frac{dz}{dx} = \frac{z}{3x+7} \Big| \cdot \frac{dx}{z}$$

$$\int \frac{dz}{z} = \int \frac{dx}{3x+7}$$

$$\ln|z| = \frac{1}{3} \ln|3x+7| + \ln|C|$$

$$z = (3x+7)^{\frac{1}{3}} C$$

2) $z = (3x+7)^{\frac{1}{3}} \varphi(x)$

3) $\frac{1}{(3x+7)^{2/3}} \varphi(x) + \varphi'(x)(3x+7)^{\frac{1}{3}} = \cancel{\frac{(3x+7)^{\frac{1}{3}} \varphi(x)}{(3x+7)^{2/3}}} +$

$+ \frac{9}{3x+7}$

$$\varphi'(x)(3x+7)^{\frac{1}{3}} = \frac{9}{3x+7} \Big| : (3x+7)^{\frac{1}{3}}$$

$$\varphi'(x) = \frac{9}{(3x+7)^{4/3}}$$

$$\varphi(x) = \int \frac{9}{(3x+7)^{4/3}} dx = -\frac{9}{\sqrt[3]{3x+7}} + C_1$$

$$z = (3x+7)^{\frac{1}{3}} \left(-\frac{9}{\sqrt[3]{3x+7}} + C_1 \right) = -\frac{9\cancel{\sqrt[3]{3x+7}}}{\cancel{\sqrt[3]{3x+7}}} +$$

$+ (3x+7)^{\frac{1}{3}} C_1$

$$y' = -9 + (3x+7)^{\frac{1}{3}} C_1$$

$$y = \int (-9 + (3x+7)^{\frac{1}{3}} C_1) dx = \int -9dx + C_1 \int (3x+7)^{\frac{1}{3}} dx =$$

$= -9x + C_1 \frac{(3x+7)\sqrt[3]{3x+7}}{4} + C_2$

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Перевірка: $3x+7=0 \Rightarrow x \neq 2,3$ - не розв.

3.2.9 $x^6y''' + x^5y'' = 1$ нема y

Заміна: $y(x) \to z(x)$ $y' = z$, $y'' = z'$, $y''' = z''$

$x^6z'' + x^5z' = 1 \mid : x^6$

$z'' + \frac{z'}{x} = \frac{1}{x^6}$

$z'' = \frac{1}{x^6} - \frac{z'}{x}$

Заміна: $z' = t$, $z'' = t'$

0) $t' = \frac{1}{x^6} - \frac{t}{x}$

1) $t' = -\frac{t}{x}$

$\frac{dt}{dx} = -\frac{t}{x} \mid \cdot \frac{dx}{t}$

$\int\frac{dt}{t} = -\int\frac{dx}{x}$

$\ln|t| = -\ln|x| + \ln|C|$

$t = x^{-1}C$

2) $t = \frac{1}{x}\varphi(x)$

3) $\cancel{-\frac{1}{x^2}\varphi(x)} + \varphi'(x)\frac{1}{x} = \frac{1}{x^6} - \cancel{\frac{1}{x^2}\varphi(x)}$

$\varphi'(x)\frac{1}{x} = \frac{1}{x^6} \mid \cdot x$

$\varphi'(x) = \frac{1}{x^5}$

$\varphi(x) = \int \frac{1}{x^5} dx = -\frac{1}{4x^4} + C_1$

$t = \frac{1}{x}\left(-\frac{1}{4x^4} + C_1\right) = -\frac{1}{4x^5} + \frac{C_1}{x}$

$z' = -\frac{1}{4x^5} + \frac{C_1}{x}$

$y'' = -\frac{1}{4x^5} + \frac{C_1}{x}$ - диф р-ня на відшук. перв.

$y' = \frac{1}{16x^4} + C_1 \ln|x| + C_2$

$y = -\frac{1}{48x^3} + C_1 x \ln|x| - C_1 x + C_2 x + C_3$

Перевірка: $x^6 = 0 \to$ не розв., y' не існує

$dx = 0 \to x = C_4$

$t = 0 \to z' = 0 \to y'' = 0 \to y''' = 0$

$0 + 0 \neq 1$ не тотожне

не розв.

3.2.14. $yy'' + 1 = y'^2$ нема x

Заміна: $y(x) \rightsquigarrow z(y)$ $y' = z(y)$, $y'' = z'z$

$yz'z + 1 = z^2 \mid : y$

$z'z = \frac{z^2}{y} - \frac{1}{y}$

Заміна: $z(y) \rightsquigarrow t(y)$

$t = z^2$ $\quad t' = 2zz'$ $\quad z'z = \frac{t'}{2}$

$\frac{t'}{2} = \frac{t}{y} - \frac{1}{y} \mid \cdot 2$

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0) $t' = \frac{2t}{y} - \frac{2}{y}$

1) $t' = \frac{2t}{y}$

$\frac{dt}{dy} = \frac{2t}{y} \Big| \cdot \frac{dy}{t}$

$\int \frac{dt}{t} = \int \frac{2}{y} dy$

$\ln|t| = 2\ln|y| + \ln|C|$

$t = y^2 C$

2) $t = y^2 \varphi(y)$

3) $2y\varphi(y) + \varphi'(y)y^2 = \frac{\cancel{2y^2\varphi(y)}}{y} - \frac{2}{y}$

$\varphi'(y)y^2 = -\frac{2}{y} \Big| : y^2$

$\varphi'(y) = -\frac{2}{y^3}$

$\varphi(y) = -\int \frac{2}{y^3} dy = \frac{1}{y^2} + C_1$

$t = y^2\left(\frac{1}{y^2} + C_1\right) = 1 + y^2 C_1$

$z^2 = 1 + y^2 C_1 \qquad z = \sqrt{1 + y^2 C_1}$

$y' = \sqrt{1 + y^2 C_1}$ – РВЗ

$\frac{dy}{dx} = \sqrt{1 + y^2 C_1} \Big| \cdot \frac{dx}{\sqrt{1 + y^2 C_1}}$

$\int \frac{dy}{\sqrt{1 + y^2 C_1}} = \int dx$

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$$\frac{\ln\left|y+\frac{\sqrt{C_1y^2+1}}{\sqrt{C_1}}\right|}{\sqrt{C_1}} = x + C_2$$

Перевірка: $y=0 \to y'=0 \to y''=0$

$0+1=0$

$1\neq 0$ - не розв, не точн

$dy=0 \to y=C_3 \to y'=0 \to y''=0$

$t=0 \to z=0 \to y'=0 \to y=C_4$

$dx=0 \to x=C_5$ - не розв, y' не існує

3.2.29. $(y+1)y''+y'^2=(2y-1)y'$, $y(0)=2$, $y'(0)=\frac{2}{3}$

немає x

Заміна: $y(x) \rightsquigarrow z(y)$ $y'=z(y)$ $y''=z'z$

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$(y+1)z'z + z^2 = (2y-1)z \mid : z \mid : y+1$

$z' + \frac{z}{y+1} = \frac{2y-1}{y+1}$

0) $z' = \frac{2y-1}{y+1} - \frac{z}{y+1}$

1) $z' = -\frac{z}{y+1}$

$\frac{dz}{dy} = -\frac{z}{y+1} \mid \frac{dy}{z}$

$\int \frac{dz}{z} = -\int \frac{dy}{y+1}$

$\ln|z| = -\ln|y+1| + \ln|C|$

$z = \frac{1}{y+1} C$

2) $z = \frac{1}{y+1} \varphi(y)$

3) $\cancel{-\frac{1}{(y+1)^2}\varphi(y)} + \varphi'(y)\frac{1}{y+1} = \frac{2y-1}{y+1} - \cancel{\frac{1}{(y+1)^2}\varphi(y)}$

$\varphi'(y)\frac{1}{y+1} = \frac{2y-1}{y+1} \mid \cdot y+1$

$\varphi'(y) = 2y-1$

$\varphi(y) = \int (2y-1)\,dy = y^2 - y + C_1$

$z = \frac{1}{y+1}(y^2 - y + C_1) = \frac{y^2 - y + C_1}{y+1}$

$y' = \frac{y^2 - y + C_1}{y+1}$

$y'(x) = \frac{(y(x))^2 - y(x) + C_1}{y(x)+1}$,

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$$y'(0) = \frac{(y(0))^2 - y(0) + C_1}{y(0) + 1}$$

$$\frac{2}{3} = \frac{4 - 2 + C_1}{3}; \quad \frac{2}{3} = \frac{2}{3} + \frac{C_1}{3}; \quad C_1 = 0$$

$$y' = \frac{y^2 - y}{y + 1}$$

$$\frac{dy}{dx} = \frac{y^2 - y}{y + 1} \Big| \cdot \frac{dx}{\frac{y^2 - y}{y+1}}$$

$$\int \frac{y+1}{y^2 - y}\, dy = \int dx$$

$$-\ln|y| + 2\ln|y-1| = x + C_2$$

$$\ln\left|\frac{y^2 - 2y + 1}{y}\right| = x + C_2$$

$$\ln\left|\frac{(y(x))^2 - 2(y(x)) + 1}{y(x)}\right| = x + C_2$$

$$\ln\left|\frac{4 - 4 + 1}{2}\right| = C_2$$

$$\ln\left|\frac{1}{2}\right| = C_2$$

$$\ln\left|\frac{y^2 - 2y + 1}{y}\right| = x - \ln|2|$$ – р-ок зад. Коші

Перевірка: $z = 0 \to y' = 0 \to y = C_3 \quad y' = 0 \quad y'' = 0$

$(C_3 + 1)0 + 0 = (2C_3 - 1)0$

$0 = 0$ – розв. р-ня $\qquad dy = 0 \to y = C_3$

$y = C_3$ не розв. зад. Коші

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$$\begin{cases} y(0)=2 \\ y'(0)=\frac{2}{3} \end{cases} \qquad \begin{cases} C_3=2 \\ 0\neq\frac{2}{3} \end{cases}$$

$y+1=0 \to y=-1$ розв. р-ня

не розв. зад. Коші

$dx=0 \to x=C_4$ - не розв., бо y' не існує

$$\frac{y^2-y}{y+1}=0$$

$y^2-y=0$

$y(y-1)=0$

$y=0 \quad y=1$