Контрольна робота №1.
Кравець Ольги
ПМО-21.

(5) $(\cos x - \frac{y}{x^2})\,dx + (\frac{1}{x} + 2y)\,dy = 0$

$$\underbrace{(\cos x - \frac{y}{x^2})}_{P}\,dx + \underbrace{(\frac{1}{x} + 2y)}_{Q}\,dy = 0$$

$P(x,y) = \cos x - \frac{y}{x^2}$ $\qquad Q(x,y) = \frac{1}{x} + 2y$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$\frac{\partial P}{\partial y} = -\frac{1}{x^2} \quad = \quad \frac{\partial Q}{\partial x} = -\frac{1}{x^2}$$

Рівняння в повних диференціалах.

Пошук потенціалу $U = U(x,y)$

$$U(x,y) = \int (\cos x - \frac{y}{x^2})\,dx = \sin x + \frac{y}{x} + \alpha(y)$$

$$\frac{\partial U}{\partial y} = (\sin x + \frac{y}{x} + \alpha(y))'_y = \cancel{\frac{1}{x}} + \alpha'(y) = \cancel{\frac{1}{x}} + 2y$$

$$\alpha'(y) = 2y$$

$$\alpha(y) = y^2$$

$$U(x,y) = \sin x + \frac{y}{x} + y^2$$

Запис. заг. розв. р-ня

$U(x,y) = c,\ c \in \mathbb{R};$

$\sin x + \frac{y}{x} + y^2 = c \ \Big|\ x$

Відповідь: $\boxed{x\sin x + y + xy^2 = cx,\ c \in \mathbb{R}}$

② Задача Коші

$(x+1)y' = y^2-4$

$y(0)=2$

$x=5$

$(x+1)y' = y^2-4 \quad / \cdot \frac{1}{x+1}$

$y' = \frac{y^2-4}{x+1}$

$\frac{dy}{dx} = \frac{y^2-4}{x+1} \quad / \cdot \frac{dx}{y^2-4}$

$\frac{dy}{y^2-4} = \frac{dx}{x+1}$

$\int \frac{dy}{y^2-4} = \int \frac{dx}{x+1}$

$\int \frac{dy}{(y-2)(y+2)} = \int \frac{dx}{x+1}$

$\quad \| $

$\| \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ln(x+1)+C$

$\int \frac{dy}{(y-2)(y+2)} = \frac{A}{y-1} + \frac{B}{y+2} = -\frac{1}{4}\int \frac{1}{y+2}dy + \frac{1}{4}\int\frac{1}{y-2}dy \ominus$

$A = \frac{1}{\cancel{(y-2)}(y+2)}\Big|_{y=2} = \frac{1}{4}$

$B = \frac{1}{(y-2)\cancel{(y+2)}}\Big|_{y=-2} = -\frac{1}{4}$

$\ominus -\frac{\ln(y+2)}{4} + \frac{\ln(y-2)}{4} + c$

$\ln(x+1) + C = \frac{\ln(y-2)}{4} - \frac{\ln(y+2)}{4}$

$\ln(x+1) + \ln(C) = \frac{\ln\left(\frac{y-2}{y+2}\right)}{4}$

$e^{\ln(x+1)+\ln(C)} = e^{\frac{\ln\left(\frac{y-2}{y+2}\right)}{4}}; \quad C(x+1) = \frac{\ln\left(\frac{y-2}{y+2}\right)}{4}; \; 4C(x+1) = \ln\left(\frac{y-2}{y+2}\right)$

В-дь: $\boxed{C(x+1) = \left(\ln\left(\frac{y-2}{y+2}\right)\right)/4}$

③ $y' + 2y = 6x^2 e^{-x}\sqrt{y}$

$y' + 2y = 6x^2e^{-x}\sqrt{y} \;/\cdot \frac{1}{\sqrt{y}}$

$\frac{y'}{\sqrt{y}} + 2\sqrt{y} = \frac{6x^2}{e^x}$

Рівняння Бернуллі

Заміна: $z = \sqrt{y} = y^{\frac{1}{2}}$;

$z' = \frac{z'}{2\sqrt{y}}$

$2z' + 2z = \frac{6x^2}{e^x} \;/\cdot \frac{1}{2}$

$z' + z = \frac{3x^2}{e^x}$

Однорідне рівняння $z' + z = 0$

$z' = -z$

$\frac{dz}{dx} = -z \;/\cdot \frac{dx}{z}$

$\frac{dz}{z} = -dx$

$\int \frac{1}{z} \cdot dz = \int -1 \cdot dx$

$\ln z = c - x; \quad e^{\ln z} = e^{c-x}$

$z_0 = e^{c-x}$

$z_0 = \frac{c}{e^x} = c \cdot \frac{1}{e^x}$

$z_x = \alpha(x) \cdot \frac{1}{e^x}; \quad \alpha(x) = \int \frac{\frac{3x^2}{e^x}}{\frac{1}{e^x}} dx = \int \frac{3x^2}{\cancel{e^x}} \cdot \frac{\cancel{e^x}}{1} dx = \int 3x^2 dx =$

$= \frac{\cancel{3}x^3}{\cancel{3}} = x^3; \quad z = x^3 \cdot \frac{1}{e^x} + c \cdot \frac{1}{e^x} = \frac{x^3 + c}{e^x}$

$\sqrt{y} = \frac{x^3 + c}{e^x} = (x^3 + c)e^{-x}; \quad y = 0$

В-дь: $\boxed{\begin{cases} \sqrt{y} = (x^3 + c)e^{-x} \\ y = 0 \end{cases}}$

④ $y' \cos x + y \sin x = 2x \cdot \cos^2 x$

$y(0) = 1$

$y' \cos x + y \sin x = 2x \cos^2 x \quad / \cdot \frac{1}{\cos x}$

$y' + y \cdot \frac{\sin x}{\cos x} = 2x \cdot \cos x$

Одн. диф. р-ня: $y' + y \cdot \frac{\sin x}{\cos x} = 0$

$y' = -y \cdot \frac{\sin x}{\cos x}$

$\frac{dy}{dx} = -y \cdot \frac{\sin x}{\cos x} \quad / \cdot \frac{dx}{y}$

$\frac{dy}{y} = -\frac{\sin x}{\cos x} dx$

$\int \frac{dy}{y} = \int -\frac{\sin x}{\cos x} dx = -\int \frac{\sin x}{\cos x} dx = -\int \frac{d(\cos x)}{\cos x} = \ln(\cos x) + c$

$\ln y = \ln(\cos x) + c$

$e^{\ln y} = e^{\ln \cos x + \ln c}$

$y_0 = c \cdot \cos x$

$y_* = \alpha(x) \cdot \cos x$

$\alpha(x) = \int \frac{2x \cdot \cancel{\cos x}}{\cancel{\cos x}} dx = \int 2x dx = \frac{\cancel{2}x^2}{\cancel{2}} = x^2 + c$

$y_* = x^2 \cdot \cos x$

$y = y_0 + y_* = c \cdot \cos x + x^2 \cos x = \cos x(c + x^2)$

Відповідь: $\boxed{y = \cos x(x^2 + c)}$

① $x^2y' = x^2 + xy + y^2 \Big/ \cdot \frac{1}{x^2}$

$$y' = 1 + \frac{y}{x} + \frac{y^2}{x^2}$$

Заміна: $\frac{y}{x} = v$; $y = xv$; $y' = v + xv'$

$$\cancel{v} + xv' = 1 + \cancel{v} + v^2$$

$$xv' = 1 + v^2$$

$$x \cdot \frac{dv}{dx} = 1 + v^2 \Big/ \cdot dx$$

$$x \cdot dv = (1 + v^2)\,dx \Big/ \cdot \frac{1}{x(1+v^2)}$$

$$\frac{dv}{1+v^2} = \frac{dx}{x}$$

$$\int \frac{dv}{1+v^2} = \int \frac{dx}{x}$$

$$\operatorname{arctg} v = \ln(x) + c$$

$$\operatorname{arctg}\left(\frac{y}{x}\right) = (\ln(x) + \ln(c)) \quad = e^{\ln x + \ln c} = x \cdot c$$

$$y = x \cdot tg(\ln|x \cdot c|)$$

Відповідь: $\boxed{y = x \cdot tg(\ln|x \cdot c|)}$