3.4.53. $y''' - 2y'' = x - 2 + \sin x$

1) $y''' - 2y'' = 0$

$\lambda^3 - 2\lambda^2 = 0 \quad \lambda^2(\lambda - 2) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = 2$ (кратн. = 2)

$$y_0 = C_1 + C_2 x + C_3 e^{2x}$$

2) $f(x) = x - 2 + \sin x = f_1(x) + f_2(x)$

2.1) $f_1(x) = x - 2 = e^{\alpha x} P_m(x)$

$\alpha = 0$ - корінь х. р. $\rightarrow k = 2$

$P_m(x) = x - 2, \quad m = 1$

$z_1 = x^k e^{\alpha x} Q_m(x) = x^2 Q_1(x) = x^2(ax + b)$

$z_1 = ax^3 + bx^2 \qquad z_1'' = 6ax + 2b$

$z_1' = 3ax^2 + 2bx \qquad z_1''' = 6a$

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$6a - 2(6ax + 2b) = x - 2$

$6a - 12ax - 4b = x - 2$

$x^1: \begin{cases} -12a = 1 \\ 6a - 4b = -2 \end{cases} \quad \begin{cases} a = -\frac{1}{12} \\ b = \frac{3}{8} \end{cases}$

$z_1 = -\frac{1}{12}x^3 + \frac{3}{8}x^2$

2.2) $f_2(x) = \sin x = e^{\alpha x}(P^1_{m_1}(x)\cos(\beta x) + P^2_{m_2}\sin(\beta x))$

$\alpha = 0 \quad P_{m_1} = 0, \; m_1 = 0$

$\beta = 1 \rightarrow \alpha + i\beta = 0 + i1 = i$ - кор. х. р $\rightarrow k = 0$

$P^2_{m_2}(x) = 1, \; m_2 = 0 \qquad \max\{m_1, m_2\} = 0$

$z_2 = x^k e^{\alpha x}(Q^1_m(x)\cos(\beta x) + Q^2_m(x)\sin(\beta x)) =$

$= c\cos x + d\sin x$

$z_2' = -c\sin x + d\cos x$

$z_2'' = -c\cos x - d\sin x$

$z_2''' = c\sin x - d\cos x$

$z_2 \rightarrow c\sin x - d\cos x - 2(-c\cos x - d\sin x) = \sin x$

$c\sin x - d\cos x + 2c\cos x + 2d\sin x = \sin x$

$\cos x: \; -d + 2c = 0 \quad \begin{cases} 2c - d = 0 \mid \cdot 2 \\ c + 2d = 1 \end{cases}$

$\sin x: \; c + 2d = 1$

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$$\begin{cases} 4c - 2d = 0 \\ c + 2d = 1 \end{cases}$$

$5c = 1$

$c = \frac{1}{5} \qquad d = \frac{2}{5}$

$z_2 = \frac{1}{5}\cos x + \frac{2}{5}\sin x$

$\boxed{y = y_0 + z_1 + z_2}$

3.4.54. $y''' - 4y' = x^2$

1) $y''' - 4y' = 0$

$\lambda^3 - 4\lambda = 0 \quad \lambda(\lambda^2 - 4) = 0 \quad \lambda(\lambda - 2)(\lambda + 2) = 0$

$\lambda_1 = 0 \quad \lambda_2 = 2 \quad \lambda_3 = -2$

$y_0 = C_1 + C_2 e^{2x} + C_3 e^{-2x}$

2) $f(x) = x^2 = e^{\alpha x} P_m(x)$

$\alpha = 0$ - корень х.р. $\rightarrow k = 1$

$P_m(x) = x^2, \quad m = 2$

$z = x^k e^{\alpha x} Q_m(x) = x(ax^2 + bx + c)$

$z = ax^3 + bx^2 + cx$

$z' = 3ax^2 + 2bx + c$

$z'' = 6ax + 2b$

$z''' = 6a$

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$6a - 12a ax^2 - 8bx - 4c = x^2$

x^2: $-12a = 1$ $\quad a = -\frac{1}{12}$

x: $-8b = 0$ $\quad b = 0$

x^0: $6a - 4c = 0$ $\quad c = -\frac{1}{8}$

$z_1 = -\frac{1}{12}x^3 - \frac{1}{8}x$

$\boxed{y = y_0 + z}$

3.4.57. $x^2y'' - 6y = 5x^3 + 8x^2$

1) $x^2y'' - 6y = 0$

$\lambda(\lambda-1) - 6 = 0 \quad \lambda^2 - \lambda - 6 = 0 \quad \lambda_1 = 3 \quad \lambda_2 = -2$

$y_0 = C_1x^3 + C_2x^{-2}$

2) $f(x) = 5x^3 + 8x^2 = \left\{ \begin{matrix} \text{замена} \\ x = e^t \end{matrix} \right\} = 5e^{3t} + 8e^{2t} =$

$= f_1(t) + f_2(t)$

2.1) $x^2y'' - 6y = 5x^3$

$f_1(x) = 5x^3 = f_1(t) = 5e^{3t} = e^{\alpha t} P_m(t)$

$\alpha = 3, \quad k = 1 \quad P_m(t) = 5, \quad m = 0$

$z_1(t) = t^k e^{\alpha t} Q_m(t) = te^{3t}a = \left\{ \begin{matrix} \text{замена} \\ x = e^t \\ t = \ln x \end{matrix} \right\} =$

$= \ln x \, x^3 a$

$z_1' = 3x^2 \ln x \, a + \frac{1}{x} x^3 a = 3x^2 a \ln x + ax^2$

$z_1'' = 6x\, a \ln x + \frac{1}{x} 3x^2 a + 2ax = 6x a \ln x + 3xa + 2ax = 5ax + 6x a \ln x$

$x^2(5ax + 6x a \ln x) - 6 \ln x \, x^3 a = 5x^3$

$5ax^3 + 6x^3 a \ln x - 6x^3 a \ln x = 5x^3$

$5ax^3 = 5x^3 \quad a = 1 \quad z = x^3 \ln x$

2.2) $f_2(x) = 8x^2 = f_2(t) = 8e^{2t} = e^{\alpha t} P_m(t)$

$\alpha = 2 \quad k = 0 \quad P_m(t) = 8, \; m = 0$

$z_2(t) = t^k e^{\alpha t} Q_m(t) = e^{2t} a = ax^2$

$z_2' = 2ax \qquad z_2'' = 2a$

$2ax^2 - 6ax^2 = 8x^2$

$-4ax^2 = 8x^2$

$a = -2 \qquad z_2 = -2x^2$

$\boxed{y = y_0 + z_1 + z_2}$

11.4. $y'' + y = 7\cos x$

1) $y'' + y = 0 \quad \lambda^2 + 1 = 0 \quad \lambda = \pm i$

$y_0 = C_1 \cos x + C_2 \sin x$

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2) $f(x) = 7\cos x = e^{\alpha x}(P^1_{m_1}(x)\cos(\beta x) + P^2_{m_2}(x)\sin(\beta x))$

$\alpha = 0 \quad P^1_{m_1}(x) = 7, \; m_1 = 0$

$\beta = 1 \to \alpha + i\beta = 0 + i \cdot 1 = i$ - кор. $\times$ р $\to L = 1$

$P^2_{m_2}(x) = 0, \; m_2 = 0 \qquad \max\{0; 0\} = 0$

$z = x^L e^{\alpha x}(Q^1_m(x)\cos(\beta x) + Q^2_m(x)\sin(\beta x)) = xe^{0x}$

$\cdot (Q^1_0(x)\cos x + Q^2_0(x)\sin x) = x(a\cos x + b\sin x)$

$z = ax\cos x + bx\sin x$

$z' = a\cos x - ax\sin x + b\sin x + bx\cos x =$

$= a\cos x + b\sin x + x(-a\sin x + b\cos x)$

$z'' = -a\sin x + b\cos x + (-a\sin x + b\cos x) +$

$+ x(-a\cos x - b\sin x)$

$-2a\sin x + 2b\cos x - ax\cos x - bx\sin x +$

$+ ax\cos x + bx\sin x = 7\cos x$

$\cos x: \; 2b = 7 \qquad b = 3{,}5$

$\sin x: \; -2a = 0 \qquad a = 0$

$z = 3{,}5x\sin x$

$y = y_0 + z$

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№6. $y'' + y' = 4x^3 + 5e^{-x} + 7\sin 2x$

1) $y'' + y' = 0$

$\lambda^2 + \lambda = 0 \quad \lambda(\lambda+1) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = -1$

$y_0 = C_1 + C_2 e^{-x}$

2) $f(x) = 4x^3 + 5e^{-x} + 7\sin 2x = f_1(x) + f_2(x) + f_3(x)$

2.1) $f_1(x) = 4x^3 = e^{\alpha x} P_m(x)$

$\alpha = 0$ - корінь х.р $\rightarrow k = 1$

$P_m(x) = 4x^3, \quad m = 3$

$z_1 = x^k e^{\alpha x} Q_m(x) = x(ax^3 + bx^2 + cx + d)$

2.2) $f_2(x) = 5e^{-x} = e^{\alpha x} P_m(x)$

$\alpha = -1$ - корінь х.р. $k = 1$

$P_m(x) = 5, \quad m = 0$

$z_2 = xe^{-x} a$

2.3) $f_3(x) = 7\sin 2x = e^{\alpha x}(P^1_{m_1}(x)\cos(\beta x) + P^2_{m_2}(x)\sin(\beta x))$

$\alpha = 0 \quad P^1_{m_1}(x) = 0, \quad m = 0$

$\beta = 2 \rightarrow \alpha + i\beta = 2i, \quad k = 0$

$P^2_{m_2}(x) = 7, \quad m = 0$

$z_3 = a\cos 2x + b\sin 2x$

$y = y_0 + z_1 + z_2 + z_3$

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11.6 $y'' - 2y' = 1 + 6e^{2x} + 5x\sin x$

1) $y'' - 2y' = 0$

$\lambda^2 - 2\lambda = 0 \quad \lambda(\lambda - 2) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = 2$

$y_0 = C_1 + C_2 e^{2x}$

2) $f(x) = 1 + 6e^{2x} + 5x\sin x = f_1(x) + f_2(x) + f_3(x)$

2.1) $f_1(x) = 1 = e^{\alpha x} P_m(x)$

$\alpha = 0, \; k = 1, \; P_m(x) = 1, \; m = 0$

$z_1 = x a$

2.2) $f_2(x) = 6e^{2x} = e^{\alpha x} P_m(x)$

$\alpha = 2, \; k = 1, \; P_m(x) = 6, \; m = 0$

$z_2 = x e^{2x} a$

2.3) $f_3(x) = 5x\sin x = e^{\alpha x}(P^1_{m_1}(x)\cos(\beta x) + P^2_{m_2}(x)\sin(\beta$

$\alpha = 0, \; P^1_{m_1}(x) = 0, \; m_1 = 0$

$\beta = 1 \rightarrow \alpha + i\beta = i, \; k = 0$

$P^2_{m_2}(x) = 5x, \; m_2 = 1 \qquad \max\{0, 1\} = 1$

$z_3 = (ax + b)\cos x + (cx + d)\sin x$

$\boxed{y = y_0 + z_1 + z_2 + z_3}$