3.4.25. $y'' + y = \frac{1}{\sin^2 x}$

1) $y'' + y = 0$

$\lambda^2 + 1 = 0 \quad \lambda^2 = -1 \quad \lambda_{1,2} = \pm i$

$y_0 = C_1 \cos x + C_2 \sin x$

2) $z = \varphi_1(x)\cos x + \varphi_2(x)\sin x$

3) $\begin{cases} \varphi_1'(x)\cos x + \varphi_2'(x)\sin x = 0 \\ \varphi_1'(x)(-\sin x) + \varphi_2'(x)\cos x = \frac{1}{\sin^2 x} \end{cases}$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & \sin x \\ \frac{1}{\sin^2 x} & \cos x \end{vmatrix} = 0 - \frac{\sin x}{\sin^2 x} = -\frac{1}{\sin x}$$

$$\Delta_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \frac{1}{\sin^2 x} \end{vmatrix} = \frac{\cos x}{\sin^2 x}$$

$\varphi_1'(x) = \frac{\Delta_1}{\Delta} = \frac{-\frac{1}{\sin x}}{1} = -\frac{1}{\sin x}$

$\varphi_1(x) = \int -\frac{1}{\sin x}\,dx = -\ln\left(\operatorname{tg}\frac{x}{2}\right) + C_3 = 0$

$\varphi_2'(x) = \frac{\cos x}{\sin^2 x} \qquad \varphi_2(x) = \int \frac{\cos x}{\sin^2 x}\,dx = -\frac{1}{\sin x} + C_4 = 0$

В-дь: $y = C_1\cos x + C_2 \sin x + \left(-\ln\left(\operatorname{tg}\frac{x}{2}\right)\right)\cos x + \left(-\frac{1}{\sin x}\right)\sin x$

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3.4.36. $x^2y'' - 2xy' + 2y = \frac{4x^4}{x^2+1}$, $x > 0$

1) $x^2y'' - 2xy' + 2y = 0$

$\lambda(\lambda-1) - 2\lambda + 2 = 0$

$\lambda^2 - 3\lambda + 2 = 0 \quad \lambda_1 = 1 \quad \lambda_2 = 2$

$y_0 = C_1 x + C_2 x^2$

2) $z = \varphi_1(x)x + \varphi_2(x)x^2$

3) $\begin{cases} \varphi_1'(x)x + \varphi_2'(x)x^2 = 0 \\ \varphi_1'(x) + \varphi_2'(x)2x = \frac{4x^2}{x^2+1} \end{cases}$

$\Delta = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2$

$\Delta_1 = \begin{vmatrix} 0 & x^2 \\ \frac{4x^2}{x^2+1} & 2x \end{vmatrix} = 0 - x^2\frac{4x^2}{x^2+1} = -\frac{4x^4}{x^2+1}$

$\Delta_2 = \begin{vmatrix} x & 0 \\ 1 & \frac{4x^2}{x^2+1} \end{vmatrix} = \frac{4x^3}{x^2+1}$

$\varphi_1'(x) = -\frac{4x^2}{x^2+1}$

$\varphi_1(x) = \int -\frac{4x^2}{x^2+1}dx = -4x + 4\operatorname{arctg}x + C_3 = 0$

$\varphi_2'(x) = \frac{4x}{x^2+1}$

$\varphi_2(x) = \int \frac{4x}{x^2+1}dx = 2\ln|x^2+1| + C_4 = 0$

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В-дь: $y = C_1 x + C_2 x^2 + (-4x + 4\arctg x)x + (2\ln|x^2+1|)x^2$

10.5. $x^2y'' - xy' - 3y = 5x^4$

1) $x^2y'' - xy' - 3y = 0$

$\lambda(\lambda-1) - \lambda - 3 = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$D = 4 + 12 = 16 \quad \lambda_1 = -1 \quad \lambda_2 = 3$

$y_0 = C_1 x^{-1} + C_2 x^3$

2) $f(x) = 5x^4 = e^{\alpha x} P_m(x)$

$\alpha = 0$ - не корінь x р. $\Rightarrow k = 0$

$P_m(x) = 5x^4, \quad m = 4$

$z = x^k e^{\alpha x} Q_m(x) = x^0 Q_4(x) = ax^4 + bx^3 + cx^2 + dx + e$

$z' = 4ax^3 + 3bx^2 + 2cx + d$

$z'' = 12ax^2 + 6bx + 2c$

$x^2(12ax^2 + 6bx + 2c) - x(4ax^3 + 3bx^2 + 2cx + d) - 3(ax^4 + bx^3 + cx^2 + dx + e) = 5x^4$

$12ax^4 + 6bx^3 + 2cx^2 - 4ax^4 - 3bx^3 - 2cx^2 - dx - 3ax^4 - 3bx^3 - 3cx^2 - 3dx - 3e = 5x^4$

$5ax^4 - 3cx^2 - 4dx - 3e = 5x^4$

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x^4: $5a = 5 \Rightarrow a = 1$

x^2: $-3c = 0 \Rightarrow c = 0$

x: $-4d = 0 \Rightarrow d = 0$

x^0: $-3e = 0 \Rightarrow e = 0$

$z = x^4$

О-ро: $y = C_1 x^{-1} + C_2 x^3 + x^4$

10.4. $y'' - 9y = 2e^{3x}\cos x$

1) $y'' - 9y = 0 \quad \lambda^2 - 9 = 0 \quad \lambda_{1,2} \neq 3$

$y_0 = C_1 e^{3x} + C_2 e^{-3x}$

2) $f(x) = 2e^{3x}\cos x = e^{\alpha x}(P^1_{m_1}(x)\cos\beta x + P^2_{m_2}(x)\sin\beta x)$

$\alpha = 3; \beta = 1 \quad \alpha + i\beta = 3 + i$ - не сор. $\Rightarrow k = 0$

$P^1_{m_1}(x) = 2 \rightarrow m_1 = 0$

$P^2_{m_2}(x) = 0 \rightarrow m_2 = 0$

$\Rightarrow m = \max\{m_1, m_2\} = 0$

$z = x^0 e^{3x}(Q^1_0(x)\cos x + Q^2_0(x)\sin x) = e^{3x}(a\cos x + b\sin x)$

$e^{3x}(a\cos x + b\sin x) = 2e^{3x}\cos x \mid : e^{3x}$

$a\cos x + b\sin x = 2\cos x \quad a = 2 \quad b = 0$

$z = e^{3x}(2\cos x + 0)$

О-ро: $y = C_1 e^{3x} + C_2 e^{-3x} + e^{3x} 2\cos x$

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3.4.39 $3y'' - 2y' - y = x + 1$

1) $3y'' - 2y' - y = 0$

$3\lambda^2 - 2\lambda - 1 = 0$

$D = 4 + 12 = 16 \quad \lambda_1 = 1 \quad \lambda_2 = -\frac{1}{3}$

$y_0 = C_1 x + C_2 x^{-\frac{1}{3}}$

2) $f(x) = x + 1 = e^{\alpha x} P_m(x)$

$\alpha = 0$ - нехарів х р. $\Rightarrow k = 0$

$P_m(x) = x + 1, \quad m = 1$

$z = x^k e^{\alpha x} Q_m(x) = ax + b$

$z' = a \quad z'' = 0$

$3 \cdot 0 - 2a - ax - b = x + 1$

$-2a - ax - b = x + 1$

$x: \begin{cases} -a = 1 \\ -2a - b = 1 \end{cases} \quad \begin{cases} a = -1 \\ b = 1 \end{cases}$

$z = x + 1$

З-ль: $y = C_1 x + C_2 x^{-\frac{1}{3}} + x + 1$