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Домашнее задание №3

1.5.2. $xy' - 2y = 2x^4 \mid : x$

$y' - 2y\frac{1}{x} = 2x^3$

а) $y' = 2\frac{1}{x}y + 2x^3$

1) $y' = 2\frac{1}{x}y$

$y' = \frac{dy}{dx}$

$\frac{dy}{dx} = 2\frac{1}{x}y \mid \cdot \frac{dx}{2y}$

$\int \frac{dy}{2y} = \int \frac{dx}{x}$

$\frac{1}{2}\ln|y| = \ln|x| + \ln|C|$

$y^{\frac{1}{2}} = xC$

$y = x^2C$

2) $y = x^2\varphi(x)$

3) $(x^2\varphi(x))' = 2x^3 + 2\frac{1}{x}x^2\varphi(x)$

$2x\varphi(x) + x^2\varphi'(x) = 2x\varphi(x) + 2x^3$

$x^2\varphi'(x) = 2x^3 \mid : x^2$

$\varphi'(x) = 2x$

$\varphi(x) = \int 2x\,dx = x^2 + C_1$

$y = x^2(x^2 + C_1)$

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Перевірка!

$x=0 \to$ не розв., то не існує y'

$x^2=0 \to$ не розв.

1.59. $y' = \frac{1}{2x-y^2} \mid \cdot 2x-y^2$

$(2x-y^2)y' = 1$

$(2x-y^2)\frac{dy}{dx} = 1 \mid \cdot \frac{dx}{dy}$

$2x-y^2 = \frac{dx}{dy}$

o) $x' = 2x - y^2$

1) $x' = 2x$

$\frac{dx}{dy} = 2x \mid \cdot \frac{dy}{2x}$

$\int \frac{dx}{2x} = \int dy$

$\frac{1}{2}\ln|x| = |y| + \ln|C|$

$x^{\frac{1}{2}} = e^y C$

$x = e^{2y} C$

2) $x = e^{2y}\varphi(y)$

3) $2e^{2y}\varphi(y) + e^{2y}\varphi'(y) = 2e^{2y}\varphi(y) - y^2$

$e^{2y}\varphi'(y) = -y^2 \mid : e^{2y}$

$\varphi'(y) = -\frac{y^2}{e^{2y}}$

$\varphi(y) = -\int \frac{y^2}{e^{2y}} dy = \frac{y^2+y}{2e^{2y}} + \frac{1}{4e^{2y}} + C_1$

$x = e^{2y}\left(\frac{y^2+y}{2e^{2y}} + \frac{1}{4e^{2y}} + C_1\right)$

перевірка:

$dx = 0 \Rightarrow x = C_2$ - не розв., бо y' не існує

$dy = 0 \Rightarrow y = C_3$, $y' = 0$ - не розв.

$2x - y^2 = 0$ - не розв.

1.5.13. $y' + 3x^2 y = e^{-x^3}$

а) $y' = -3x^2 y + e^{-x^3}$

1) $y' = -3x^2 y$

$\frac{dy}{dx} = -3x^2 y \mid \cdot \frac{dx}{y}$

$\int \frac{dy}{y} = -\int 3x^2 dx$

$\ln|y| = -x^3 + \ln|C|$

$y = e^{-x^3} C$

2) $y = e^{-x^3}\varphi(x)$

3) $-3x^2 e^{-x^3}\varphi(x) + e^{-x^3}\varphi'(x) = -3x^2 e^{-x^3}\varphi(x) + e^{-x^3}$

$e^{-x^3}\varphi'(x) = e^{-x^3} \mid : e^{-x^3}$

$\varphi'(x) = 1$

$\varphi(x) = \int dx = x + C_1$

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$y = e^{-x^2}(x + C_1)$

~~Перевірка:~~

$dx = 0$ - не розв.

1.5.12. $(x + xy^2 - y^2)dy = y(1+y^2)dx \mid : y(1+y^2)$
$\mid : dy$

$$\frac{dx}{dy} = \frac{x + xy^2 - y^2}{y(1+y^2)}$$

$$\frac{dx}{dy} = \frac{x(1+y^2)}{y(1+y^2)} - \frac{y^2}{y(1+y^2)}$$

$$\frac{dx}{dy} = \frac{x}{y} - \frac{y}{1+y^2}$$

o) $x' = \frac{x}{y} - \frac{y}{1+y^2}$

1) $x' = \frac{x}{y}$

$$\frac{dx}{dy} = \frac{x}{y} \mid \cdot \frac{dy}{x}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\ln|x| = \ln|y| + \ln|C|$$

$x = yC$

2) $x = y\varphi(y)$

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3) $\varphi(y) + y\varphi'(y) = \frac{y\varphi(y)}{y} - \frac{y}{1+y^2}$

$y\varphi'(y) = -\frac{y}{1+y^2} \quad | : y$

$\varphi'(y) = -\frac{1}{1+y^2}$

$\varphi(y) = -\int \frac{1}{1+y^2} dy = -\operatorname{arctg}(y) + C_1$

$x = y(-\operatorname{arctg} y + C_1)$

Перевірка:

$y(1+y^2) = 0$ $\qquad dy = 0$

$y = 0$ $\qquad y = 0$

$0 = 0$ - розв. $\qquad$ не тотожне.

1.5.19. $y\,tg\,x = \frac{y^3}{\cos^3 x} + 2y'$

$2y' = y\,tg\,x - \frac{y^3}{\cos^3 x}$

$y' = \frac{tg\,x\,y}{2} - \frac{y^3}{2\cos^3 x} \quad | : y^3$

$\frac{y'}{y^3} = \frac{tg\,x}{2y^2} - \frac{1}{2\cos^3 x}$

Заміна: $y(x) \rightsquigarrow z(x)$

$z = \frac{1}{y^2}, \quad z' = \left(\frac{1}{y^2}\right)' = -\frac{2y'}{y^3} = -2\frac{y'}{y^3}$

$\frac{y'}{y^3} = -\frac{z'}{2}$

$-\frac{z'}{2} = \frac{tg\,x}{2} z - \frac{1}{2\cos^3 x} \quad | \cdot (-2)$

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0) $z' = -\operatorname{tg} x \cdot z + \frac{1}{\cos^3 x}$

1) $z' = -\operatorname{tg} x \cdot z$

$\frac{dz}{dx} = -\operatorname{tg} x \cdot z \mid \cdot \frac{dx}{z}$

$\int \frac{dz}{z} = -\int \operatorname{tg} x \, dx$

$\ln|z| = \ln|\cos x| + \ln|C|$

$z = \cos x \cdot C$

2) $z = \cos x \cdot \varphi(x)$

3) $-\sin x \cdot \varphi(x) + \cos x \cdot \varphi'(x) = -\frac{\sin x}{\cos x} \cos x \cdot \varphi(x) + \frac{1}{\cos^3 x}$

$\cos x \cdot \varphi'(x) = \frac{1}{\cos^3 x} \mid : \cos x$

$\varphi'(x) = \frac{1}{\cos^2 x}$

$\varphi(x) = \int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C_1$

$z = \cos x (\operatorname{tg} x + C_1)$

Перевірка:

$y^3 = 0 \rightarrow y = 0$

$0 = 0$ - розв.

$\cos x = 0$ - не розв.

1.5.27 $y^2 dy + y^2 dx - 2xy\,dx = 0 \qquad y(-1) = 2$

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$x^2dy + y^2dx = 2xy\,dx \mid :dx$

$x^2\frac{dy}{dx} + y^2 = 2xy$

$x^2\frac{dy}{dx} = 2xy - y^2$

$\frac{dy}{dx} = \frac{2xy}{x^2} - \frac{y^2}{x^2}$

$y' = \frac{2}{x}y - y^2\frac{1}{x^2} \mid :y^2$

$\frac{y'}{y^2} = \frac{2}{x}\cdot\frac{1}{y} - \frac{1}{x^2}$

Zamena: $y(x) \rightsquigarrow z(x)$

$z = \frac{1}{y} \quad z' = \left(\frac{1}{y}\right)' = -\frac{y'}{y^2}$

$\frac{y'}{y^2} = -z'$

$-z' = \frac{1}{x}z - \frac{1}{x^2} \mid \cdot(-1)$

o) $z' = -\frac{2}{x}z + \frac{1}{x^2}$

1) $z' = -\frac{2}{x}z$

$\frac{dz}{dx} = -\frac{2}{x}z \mid \cdot\frac{dx}{z}$

$\int\frac{dz}{z} = -\int\frac{2}{x}dx$

$\ln|z| = -2\ln|x| + \ln|C|$

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$z = x^{-2} C$

2) $z = x^{-2} \varphi(x)$

3) $-2x^{-3}\varphi(x) + x^{-2}\varphi'(x) = -\frac{2}{x} x^{-2}\varphi(x) + \frac{1}{x^2}$

$\frac{1}{x^2}\varphi'(x) = \frac{1}{x^2} \mid : \frac{1}{x^2}$

$\varphi'(x) = 1$

$\varphi(x) = \int dx = x + C_1$

$z = x^{-2}(x + C_1)$

Перевірка:

$dx = 0$ - не розв.

$y^2 - 0 \to y = 0 \quad 0 = 0$ - розв

$\frac{1}{x^2} = 0$ - не розв

$y(-1) = 2$

$y = \frac{1}{z} = \frac{1}{x^{-2}(x + C_1)}$

$\frac{1}{(-1)^{-2}(-1 + C_1)} = 2$

$\frac{1}{-1 + C_1} = 2$

$C_1 = \frac{3}{2}$

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1.5.29 $xy^3y' = x^4 + y^4$

$y' = \frac{x^4}{xy^3} + \frac{y^4}{xy^3}$

$y' = x \cdot \frac{1}{y^3} + y \cdot \frac{1}{x}$

$y' = \frac{1}{x} y + x \frac{1}{y^3} \mid \cdot y^3$

$y'y^3 = \frac{1}{x} y^4 + x$

Замена: $y(x) \rightsquigarrow z(x)$

$z = y^4 \quad z' = 4y^3 y'$

$y'y^3 = \frac{z'}{4}$

$\frac{z'}{4} = \frac{1}{x} z + x \mid \cdot 4$

a) $z' = \frac{4}{x} z + 4x$

1) $z' = \frac{4}{x} z$

$\frac{dz}{dx} = \frac{4}{x} z \mid \cdot \frac{dx}{z}$

$\int \frac{dz}{z} = \int \frac{4}{x} dx$

$\ln|z| = 4 \ln|x| + \ln|C|$

$z = x^4 C$

2) $z = x^4 \varphi(x)$

3) $4x^3 \varphi(x) + x^4 \varphi'(x) = \frac{4}{x} \cdot x^4 \varphi(x) + 4x$

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$x^4\varphi'(x) = 4x \quad | : x^6$

$\varphi'(x) = \frac{4x}{x^4}$

$\varphi(x) = \int \frac{4}{x^3}\,dx = 4\int \frac{1}{x^3}\,dx = -\frac{2}{x^2} + C_1$

$z = x^4\left(-\frac{2}{x^2} + C_1\right)$

Перевірка.

$x = 0$ - не розв, бо не існує y'

$y^3 \neq 0 \rightarrow y = 0$ не тотожне.

$0 = x^2$