3.4.1. $y'' - 4y' + 3y = 0$

$\lambda^2 - 4\lambda + 3 = 0$ - хар. ф-ция

$\lambda_1 = 3 \quad \lambda_2 = 1$

$\lambda_1 = 3 \rightsquigarrow e^{3x} \quad \lambda_2 = 1 \rightsquigarrow e^{x}$

$y = C_1 e^{x} + C_2 e^{3x}$

3.4.4. $y'' - 4y' + 13y = 0$

$\lambda^2 - 4\lambda + 13 = 0$

$D = 16 - 52 = -36$

$\lambda_1 = \frac{4 + \sqrt{36}}{2} = \frac{4 + 6i}{2} = 2 + 3i \quad \lambda_2 = 2 - 3i$

$\lambda_1 = 2 + 3i \rightsquigarrow e^{2x} \cos 3x$

$\lambda_2 = 2 - 3i \rightsquigarrow e^{2x} \sin(-3x)$

$y = C_1 e^{2x} \cos 3x + C_2 e^{2x} \sin(-3x)$

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3.4.9. $3y'' - 6y' + 4y = 0 \quad y(0) = 0 \quad y'(0) = -\frac{1}{\sqrt{3}}$

$3\lambda^2 - 6\lambda + 4 = 0$

$D = 36 - 48 = -12$

$\lambda_1 = \frac{6 + \sqrt{12}}{6} = 1 + \frac{\sqrt{3}}{3}i \quad \lambda_2 = 1 - \frac{\sqrt{3}}{3}i$

$\lambda_1 = 1 + \frac{\sqrt{3}}{3}i = e^x \cos\frac{\sqrt{3}}{3}x$

$\lambda_2 = 1 - \frac{\sqrt{3}}{3}i = e^x \sin\left(-\frac{\sqrt{3}}{3}x\right)$

$y = C_1 e^x \cos\frac{\sqrt{3}}{3}x + C_2 e^x \sin\left(-\frac{\sqrt{3}}{3}x\right)$

$$\begin{cases} y(0) = 0, \\ y'(0) = -\frac{1}{\sqrt{3}} \end{cases} \begin{cases} C_1 e^x \cos\frac{\sqrt{3}}{3}x + C_2 e^x \sin\left(-\frac{\sqrt{3}}{3}x\right) = 0 \\ C_1\left(e^x \cos\frac{\sqrt{3}}{3}x - \frac{\sqrt{3}e^x \sin\frac{\sqrt{3}}{3}x}{3}\right) + \\ + C_2\left(-e^x \sin\frac{\sqrt{3}}{3}x\right) - \frac{\sqrt{3}e^x \cos\frac{\sqrt{3}}{3}x}{3} = -\frac{1}{\sqrt{3}} \end{cases}$$

$$\begin{cases} C_1 = 0 \\ C_1(1 - 0) + C_2\left(0 - \frac{\sqrt{3}}{3}\right) = -\frac{1}{\sqrt{3}} \end{cases}$$

$$\begin{cases} C_1 = 0 \\ C_1 - \frac{\sqrt{3}}{3}C_2 = -\frac{1}{\sqrt{3}} \end{cases} \quad \begin{cases} C_1 = 0 \\ C_2 = 1 \end{cases} \quad y = e^x \sin\left(-\frac{\sqrt{3}}{3}x\right)$$

3.4.10. $y^{(IV)} - y''' - y' + y = 0$

$\lambda^4 - \lambda^3 - \lambda + 1 = 0$

$\lambda^3(\lambda - 1) - \lambda + 1 = 0$

$\lambda^3(\lambda - 1) - (\lambda - 1) = 0$

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$(\lambda-1)(\lambda^3-1)=0$

$\lambda=1 \quad \lambda^3=1 \quad k=2$

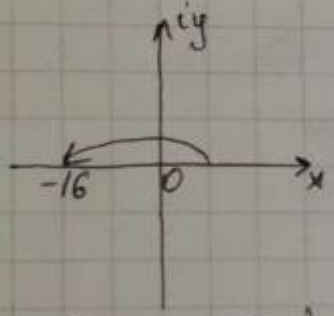
$\lambda=1 \rightsquigarrow e^x, xe^x$

$y=C_1e^x+C_2xe^x$

9.4. $y^{(4)}+16y=0$

$\lambda^4+16=0 \quad \lambda^4=-16 \quad \lambda=\sqrt[4]{16}$

$z=-16$



$r=16 \quad \varphi=\pi$

$(\sqrt[4]{-16})=\sqrt[4]{16}\left(\cos\frac{\pi+2\pi k}{4}+i\sin\frac{\pi+2\pi k}{4}\right) \quad k=0-3$

$\lambda_0=2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=\sqrt{2}+i\sqrt{2}$

$\lambda_1=-\sqrt{2}+i\sqrt{2} \quad \lambda_2=-\sqrt{2}-i\sqrt{2} \quad \lambda_3=\sqrt{2}-i\sqrt{2}$

$y=C_1e^{\sqrt{2}x}\cos(\sqrt{2}x)+C_2e^{-\sqrt{2}x}\sin(\sqrt{2}x)+C_3e^{-\sqrt{2}x}\cos(-\sqrt{2}x)+$

$+C_4e^{\sqrt{2}x}\sin(-\sqrt{2}x)$

3.4.15. $x^2y''-2y'=0, \quad x>0$

$\lambda(\lambda-1)(\lambda-2)-2\lambda=0$

$\lambda((\lambda-1)(\lambda-2)-2)=0$

$\lambda(\lambda^2-2\lambda-\lambda+2-2)=0$

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$\lambda(\lambda^2-3\lambda)=0$

$\lambda^2(\lambda-3)=0$

$\lambda=0, k=2 \rightsquigarrow x^0, x^0 \ln x$

$\lambda=3, k=1 \rightsquigarrow x^3$

$y=C_1 \cdot 1 + C_2 \ln x + C_3 x^3$

3.4.16. $x^2y''+3xy'+y=0, \quad x>0$

$\lambda(\lambda-1)+3\lambda+1=0$

$\lambda^2-\lambda+3\lambda+1=0$

$\lambda^2+2\lambda+1=0$

$(\lambda+1)^2=0$

$\lambda=-1, k=2 \rightsquigarrow \frac{1}{x}, \frac{1}{x}\ln x$

$y=C_1\frac{1}{x}+C_2\frac{1}{x}\ln x$

3.4.18. $x^3y'''+xy'-y=0, \quad x>0$

$\lambda(\lambda-1)(\lambda-2)+\lambda-1=0$

$(\lambda-1)(\lambda(\lambda-2)+1)=0$

$(\lambda-1)(\lambda^2-2\lambda+1)=0$

$(\lambda-1)(\lambda-1)^2=0 \qquad (\lambda-1)^3=0$

$\lambda=1, k=3 \rightsquigarrow x, x\ln x, x\ln x^2$

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$$y = C_1 x + C_2 x \ln x + C_3 x \ln x^2$$

3.4.20. $(2x+3)^3 y''' + 3(2x+3)y' - 6y = 0, \; x > -\frac{3}{2}$

$$8\left(x+\frac{3}{2}\right)^3 y''' + 6\left(x+\frac{3}{2}\right)y' - 6y = 0$$

$$8\lambda(\lambda-1)(\lambda-2) + 6\lambda - 6 = 0$$

$$8\lambda(\lambda-1)(\lambda-2) + 6(\lambda-1) = 0$$

$$(\lambda-1)(8\lambda(\lambda-2)+6) = 0$$

$$(\lambda-1)(8\lambda^2 - 16\lambda + 6) = 0$$

$\lambda_1 = 1 \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = \frac{3}{2}$

$\lambda_1 = 1 \rightsquigarrow (x+\frac{3}{2})^1$

$\lambda_2 = \frac{1}{2} \rightsquigarrow (x+\frac{3}{2})^{\frac{1}{2}}$

$\lambda_3 = \frac{3}{2} \rightsquigarrow (x+\frac{3}{2})^{3/2}$

$$y = C_1(x+\tfrac{3}{2}) + C_2(x+\tfrac{3}{2})^{1/2} + C_3(x+\tfrac{3}{2})^{3/2}$$