Домашнє завдання 2.
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З збірника Філіппова
№137.

$$(2x+1)y' = 4x + 2y$$

$$y' = \frac{4x+2y}{2x+1}$$

$$y' - \frac{2y}{2x+1} = \frac{4x}{2x+1}$$

$$y_0: y' - \frac{2y_0}{2x+1} = 0$$

$$y' = \frac{2y_0}{2x+1}$$

$$\ln|y_0| = \frac{\cancel{2}}{\cancel{2}} \ln|2x+1| + \ln|C|$$

$$y_0 = C(2x+1)$$

$$y_* = \alpha(x) \cdot 2x+1$$

$$(\alpha(x)(2x+1))' - \frac{2\alpha(x)\cancel{(2x+1)}}{\cancel{2x+1}} = \frac{4x}{2x+1}$$

$$\alpha'(x)(2x+1) + \cancel{2\alpha(x)} - \cancel{2\alpha(x)} = \frac{4x}{2x+1}$$

$$\alpha'(x) = \frac{4x}{(2x+1)^2}$$

$$\alpha(x) = \int \frac{4x}{(2x+1)^2}dx = 2\int \frac{((2x+1)-1)\,dx}{(2x+1)^2} =$$

$$= 2\left(\int \frac{dx}{2x+1} - \int \frac{dx}{(2x+1)^2}\right) =$$

$$= 2\left(\frac{1}{2}\ln|2x+1| + \frac{1}{2(2x+1)}\right) =$$

$$= \ln(2x+1) + \frac{1}{2x+1}$$

$$y_* = (2x+1)\ln(2x+1) + 1$$

$$y = y_0 + y_* = C(2x+1) + 1 + (2x+1)\cdot \ln(2x+1)$$

В: $y = C(2x+1) + 1 + (2x+1)\cdot \ln(2x+1)$

№140.

$x^2y' + xy + 1 = 0$

$x^2y' = -xy - 1 \;/\cdot \frac{1}{x^2}$

$y' = \frac{-xy-1}{x^2}$

$y' = -\frac{\cancel{x}y}{x^{\cancel{2}}} - \frac{1}{x^2}$

$y' = -\frac{y}{x} - \frac{1}{x^2}$

$y' + \frac{y}{x} = -\frac{1}{x^2}$

$y_0:\; y' + \frac{y_0}{x} = 0$

$y' = -\frac{y_0}{x}$

$\frac{dy_0}{dx} = \frac{-y_0}{x} \;/\cdot \frac{dx}{-y_0}$

$-\frac{dy_0}{y_0} = +\frac{dx}{x}$

$-\int\frac{dy_0}{y_0} = \int\frac{dx}{x}$

$-\ln y_0 = \ln x + \ln c$

$\ln y_0 = -\ln x + \ln c$

$y_0 = \frac{c}{x}$

$y_* = \alpha(x)\cdot\frac{1}{x}$

$\alpha(x) = \int\frac{-\frac{1}{x^2}}{\frac{1}{x}}dx = \int -\frac{1}{x^{\cancel{2}}}\cdot\cancel{x}\,dx =$

$= \int -\frac{1}{x}dx = -\ln|x|$

$y_* = -\ln x\cdot\frac{1}{x}$

$y = \frac{c}{x} - \frac{\ln x}{x} \;/\cdot x$

$xy = c - \ln x$

В: $\boxed{xy = c - \ln x}$

№143

$(xy'-1)\ln x = 2y$

$xy'\ln x - \ln x = 2y$

$xy'\ln x = 2y + \ln x$

$xy'\ln x - 2y = \ln x \,/\cdot \frac{1}{x\cdot \ln x}$

$y' - \frac{2y}{x\ln x} = \frac{1}{x}$

$y_0 : y' - \frac{2y_0}{x\ln x} = 0$

$y' = \frac{2y_0}{x\ln x}$

$\frac{dy_0}{dx} = \frac{2y_0}{x\ln(x)} \,/\cdot \frac{dx}{y_0}$

$\int \frac{dy_0}{y_0} = 2\int \frac{dx}{x\ln x} \to 2\int \frac{d(\ln(x))}{\ln(x)}$

$\ln(y_0) = 2\ln(\ln x) + \ln c$

$y_0 = c\cdot \ln^2(x)$

$y_* = \alpha(x)\cdot \ln^2(x)$

$\alpha(x) = \int \frac{\frac{1}{x}}{\ln^2 x}dx = \int \frac{1}{x\ln^2 x}dx =$

$= -\frac{1}{\ln x}$

$y_{(*)} = -\frac{1}{\cancel{\ln x}}\cdot \cancel{\ln^2 x} = -\ln x$

$y = -\ln x + c\ln^2(x) = c\cdot \ln^2 x - \ln x$

В: $\boxed{y = c\cdot \ln^2 x - \ln x}$

№146.

$(2e^y - x)y' = 1$

$2e^y - x = \frac{1}{y'}$

$\frac{1}{y'_x} = x'_y$

$2e^y - x = x'$

$2e^y = x' + x$

$x' + x = 0$

$x' = -x$

$\frac{dx}{dy} = -x \Big| \cdot \frac{dy}{x}$

$\frac{dx}{x} = -dy$

$\int \frac{dx}{x} = \int -dy$

$\ln x = -y + \ln c$

$e^{\ln x} = e^{-y} + e^{\ln c}$

$x = c \cdot e^{-y}$

$c'e^{-y} \cancel{- ce^{-y}} \cancel{+ ce^{-y}} = 2e^y$

$x = (e^{2y} + c)e^{-y}$

$x = e^y + c \cdot e^{-y}$

$\int dc = 2\int e^{2y} dy$

$c = 4e^{2y}$

В: $\boxed{x = e^y + c \cdot e^{-y}}$

№152.

$$(x+1)\ (y'+y^2)=-y \;/\cdot\frac{1}{x+1}$$

$$y'+y^2=\frac{-y}{x+1}$$

$$y'+\frac{y}{x+1}=-y^2$$

$$\frac{y'}{y^2}+\frac{y}{x+1}\cdot y^{-2}=-1$$

$$\frac{y'}{y^2}+\frac{y}{x+1}\cdot\frac{1}{y^2}=-1$$

$$z'=\left(\frac{1}{y}\right)'=-\frac{1}{y^2}\cdot y'$$

$$\frac{y'}{y^2}=-z'$$

$$\frac{1}{y^2}\cdot y'+\frac{1}{(x+1)y}=-1$$

$$-z'+\frac{1}{x+1}\cdot z=-1;\ \frac{1}{y}=z$$

$$z'-\frac{1}{x+1}\ z=1$$

$$z_0=\frac{dz}{dx}=\frac{1}{x+1}\ z$$

$$\frac{dz}{z}=\frac{dx}{x+1}$$

$$\ln|z|=\ln|x+1|+\ln|c|$$

$$z_0=c\cdot(x+1)$$

$$z_* =\lambda(x)\ (x+1)$$

$$\lambda(x)=\int\frac{1}{x+1}\,dx=\ln(x+1)$$

$$z_* =\ln(x+1)\cdot(x+1)$$

$$z=c(x+1)+\ln(x+1)\ (x+1)=$$

$$=(x+1)\cdot(c+\ln(x+1))$$

$$z=\frac{1}{y}$$

$$\frac{1}{y}=(x+1)\ (c+\ln(x+1))$$

$$y=\frac{1}{(x+1)(c+\ln(x+1))}$$

В:

$$\boxed{\left[\begin{array}{l} y=0 \\ y=\dfrac{1}{(x+1)(c+\ln(x+1))}\end{array}\right.}$$

№153.

$$y' = y^4\cos x + y\,tg x$$

$$y' = y^4\cos x + y\,tg x \;\Big/\cdot \frac{1}{y^4}$$

$$\frac{y'}{y^4} = \cos x + \frac{1}{y^3}tg x$$

$$\frac{y'}{y^4} - \frac{1}{y^3}tg x = \cos x$$

$$z = \frac{1}{y^3}; \quad z' = \left(\frac{1}{y^3}\right)'$$

$$z' = -\frac{3}{y^4}\cdot y'$$

$$-\frac{3y'}{y^4} = z'$$

$$\frac{y'}{y^4} = \frac{z'}{-3}$$

$$\frac{z'}{3} + z\,tg x = -\cos x \;\Big/\cdot 3$$

$$z' + 3z\,tg x = -3\cos x$$

$$z_0:\; z_0 + 3z_0\,tg x = 0$$

$$\frac{dz}{dx} = -3z_0\,tg x$$

$$\frac{dz}{z} = -3tg x\,dx$$

$$\ln(z) = 3\ln(\cos x) + \ln(c)$$

$$z = \cos^3 x\cdot c(x)$$

$$z_* = c(x)\cdot\cos^3 x$$

$$z'(x) = 3\cos^2 x\,(-\sin x)\,c(x) +$$

$$\cancel{-3\cos^2(x)\sin(x)\,c(x)} + c'(x)\cdot\cos^3(x) +$$

$$+\cancel{3\cos^3(x)\cdot c(x)\cdot tg(x)} = -3\cos(x)$$

$$c'(x)\cdot\cos^2(x) = -3\cos(x)$$

$$c'(x) = -\frac{3}{\cos^2(x)}$$

$$c(x) = -3tg x + C$$

$$z = \cos^3 x(-3tg x + c) =$$

$$= -3\sin x\cdot\cos^2 x + C\cdot\cos^3 x$$

$$\frac{1}{y^3} = C\cdot\cos^3 x - 3\sin x\cdot\cos^2 x$$

В.

$$\boxed{\left[\begin{array}{l} y = 0 \\ y^3 = \dfrac{1}{C\cdot\cos^3 x - 3\sin x\cdot\cos^2 x}\end{array}\right.}$$

N 188.

$$\underset{P}{e^{-y}}\,dx - (\underbrace{2y + xe^{-y}}_{Q})\,dy = 0$$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

$$\frac{\partial P}{\partial y} = -e^{-y}$$

$$\frac{\partial Q}{\partial x} = e^{-y}$$

$$dU = P(x,y)\,dx + Q(x,y)\,dy$$

$$P(x,y) = \frac{du}{dx}$$

$$P(x,y)\,dx = U$$

$$\int -e^{-y}\,dx = -e^{-y}\cdot x = \frac{-x}{e^{y}} + \alpha(y) = U$$

$$Q(x,y) = \frac{\partial U}{\partial y}$$

$$\frac{\partial U}{\partial y} = \cancel{-xe^{-y}} + \alpha'(y) = -2y - \cancel{xe^{-y}}$$

$$\alpha'(y) = -2y$$

$$\int -2y\,dy = \frac{-\cancel{2}y^2}{\cancel{2}} = -y^2$$

$$C = xe^{-y} - y^2$$

В: $\boxed{C = xe^{-y} - y^2}$

№189.

$$\underbrace{\frac{y}{x}}_{P} dx + \underbrace{(y^3 + \ln x)}_{Q} dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{1}{x}$$

$$\frac{\partial Q}{\partial x} = \frac{1}{x}$$

$$P(x, y) = \frac{du}{dx}$$

$$\int \frac{y}{x} dx = \boxed{y \cdot \ln(x) + \lambda(y)}$$

$$\frac{\partial u}{\partial y} = \ln x + \lambda'(y) = \ln x + y^3$$

$$\lambda'(y) = y^3$$

$$\int y^3 dy = y^4 \cdot \frac{1}{4}$$

$$y \ln x + y^4 \frac{1}{4} = C \quad / \cdot 4$$

$$4y \ln x + y^4 = C$$

В: $\boxed{C = 4y \ln x + y^4}$

№191.

$$\underbrace{2x\left(1+\sqrt{x^2-y}\right)}_{P}dx-\underbrace{\sqrt{x^2-y}}_{Q}dy=0$$

$$\frac{\partial P}{\partial y}=-\frac{x}{\sqrt{x^2-y}}$$

$$\frac{\partial Q}{\partial x}=\frac{-2x}{2\sqrt{x^2-y}}=-\frac{x}{\sqrt{x^2-y}}$$

$$P(x,y)=\frac{du}{dx}$$

$$-\int\sqrt{x^2-y}\,dy=\boxed{\frac{2}{3}(x^2-y)^{1,5}+\alpha(x)}$$

$$\frac{du}{\partial x}=2x\left(\sqrt{x^2-y}\right)+\alpha'(x)=\underline{2x}+2x\sqrt{x^2-y}$$

$$\alpha'(x)=2x$$

$$\int 2x\,dx=\frac{\cancel{2}x^2}{\cancel{2}}=x^2$$

$$\frac{2}{3}(x^2-y)^{1,5}+x^2=C$$

В: $\boxed{C=\frac{2}{3}\left(x^2-y\right)^{1,5}+x^2}$

№193.

$$\underbrace{3x^2(1+\ln y)}_{P}\, dx = \underbrace{\left(2y - \frac{x^3}{y}\right)}_{Q} dy$$

$$3x^2(1+\ln y)\, dx - \left(2y - \frac{x^3}{y}\right) dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{3x^2}{y}$$

$$\frac{\partial Q}{\partial x} = \frac{3x^2}{y}$$

$$P(x,y) = \frac{du}{dx}$$

$$\int 3x^2(1+\ln y)\, dx = \boxed{x^3(1+\ln y) + \alpha(y)}$$

$$\frac{du}{dy} = \cancel{\frac{x^3}{y}} + \alpha'(y) = -2y + \cancel{\frac{x^3}{y}}$$

$$\alpha'(y) = -2y$$

$$-2\int y\, dy = -\frac{2y^2}{2} = -y^2$$

$$x^3(1+\ln y) - y^2 = C$$

В: $\boxed{C = x^3(1+\ln y) - y^2}$