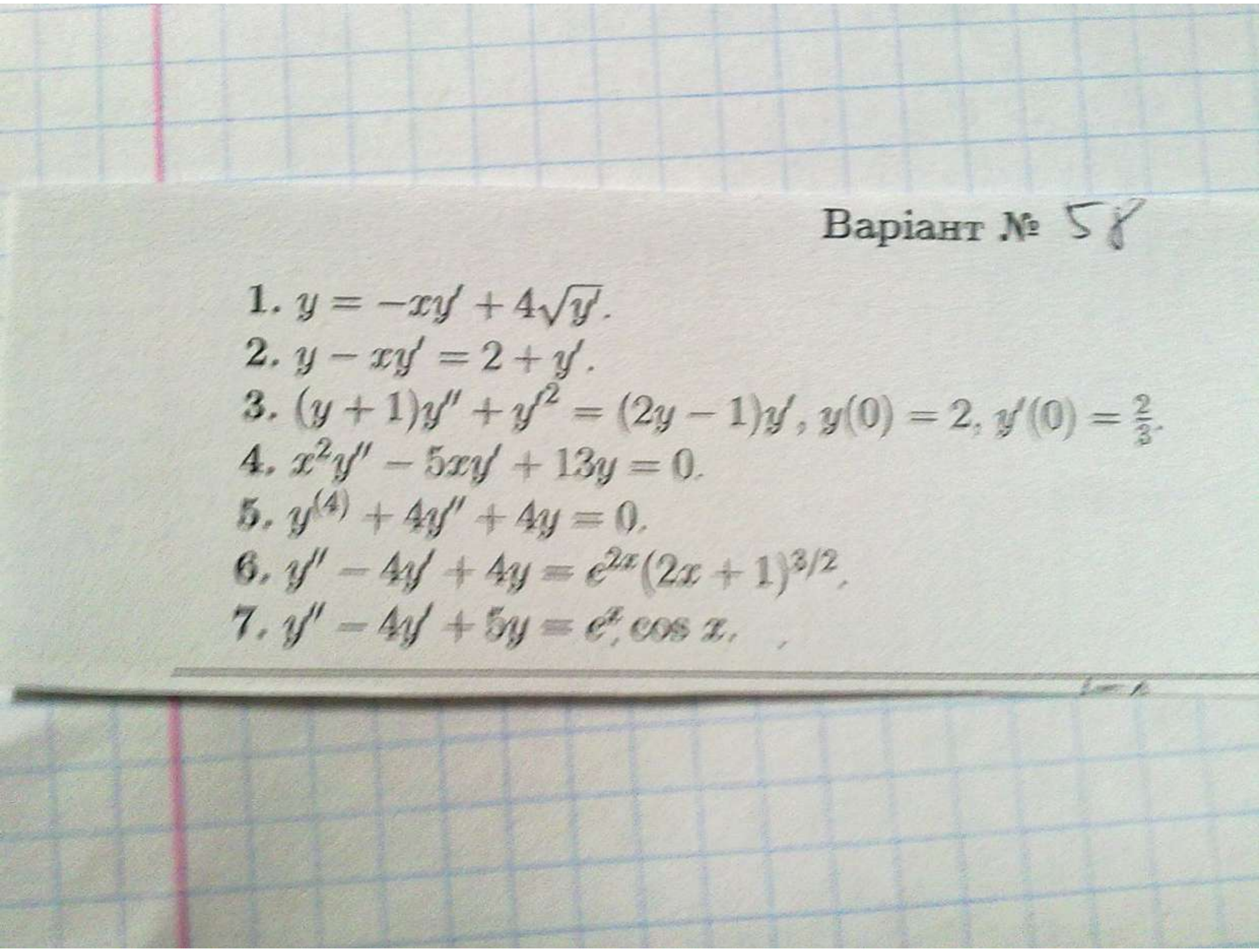


Варіант № 58

1. $y = -xy' + 4\sqrt{y'}$.
2. $y - xy' = 2 + y'$.
3. $(y + 1)y'' + y'^2 = (2y - 1)y'$, $y(0) = 2$, $y'(0) = \frac{2}{3}$.
4. $x^2y'' - 5xy' + 13y = 0$.
5. $y^{(4)} + 4y'' + 4y = 0$.
6. $y'' - 4y' + 4y = e^{2x}(2x + 1)^{3/2}$.
7. $y'' - 4y' + 5y = e^x \cos x$.

$\varphi_1 = -2\int x^2\sqrt{x-3}\,dx =$ [redacted]

замена: $\sqrt{x-3} = t$
$x = t^2+3$
$dx = 2t\,dt$

$\int (t^2+3)^2\cdot t\cdot 2t\,dt = 2\int t^2(t^2+3)^2dt = 2\int t^2(t^4+6t^2+9)\,dt = 2\int t^6+6t^4+9t^2\,dt = 2\left(\frac{t^7}{7}+6\frac{t^5}{5}+9\frac{t^3}{3}+C_3\right)$

$\varphi_1 = -\frac{4}{7}(\sqrt{x-3})^7 - \frac{12}{5}(\sqrt{x-3})^5 - 6(\sqrt{x-3})^3 - 4C_3$

$\varphi_2 = 2\int x\sqrt{x-3}\,dx = 2\int(x-3)\sqrt{x-3}\,dx + 6\int\sqrt{x-3}\,dx = 2\int(x-3)^{3/2}dx + 6\int(x-3)^{1/2}dx =$
$= 2\cdot\frac{2(x-3)^{5/2}}{5} + 6\cdot\frac{(x-3)^{3/2}\cdot 2}{3} + C_2$

---

⑨ $y'' + 5y' + 4y = e^{-x}(x-1)$

1) $\lambda^2+5\lambda+4=0$
$D = 25-16 = 9$
$\lambda_1 = \frac{-5+3}{2} = -1$
$\lambda_2 = \frac{-5-3}{2} = -4$

$y_o = C_1\cdot e^{-x} + C_2\cdot e^{-4x}$

2) $f(x) = e^{-x}(x-1) = e^{\alpha x}P_m(x)$
$\alpha = -1$ - корінь $\Rightarrow$ $k = 1$
$m = 1$

$\tilde{y}$ [redacted] $= x^k e^{\alpha x}Q_m(x) = xe^{-x}(ax+b) =$ [redacted]

$$\begin{array}{r|l} 4 & \tilde{y} = xe^{-x}(ax+b) = e^{-x}(ax^2+bx) \\ 5 & \tilde{y}' = -e^{-x}(ax^2+bx) + e^{-x}(2ax+b) = e^{-x}(-ax^2-bx+2ax+b) \\ 1 & \tilde{y}'' = -e^{-x}(ax^2-bx+2ax+b) + e^{-x}(-2ax-b+2a) = e^{-x}(ax^2+bx-2ax-b-2ax-b+2a) \\ \hline ⊕ & e^{-x}(x-1) = 4e^{-x}(ax^2+bx) + 5e^{-x}(-ax^2-bx+2ax+b) + -e^{-x}(-ax^2-bx+2ax+b) + e^{-x}(-2ax-b+2a) \end{array}$$

$(x-1) = \underline{4ax^2}+\cancel{4bx} - \cancel{5ax^2} - \cancel{5bx} + \underline{10ax} + \underline{5b} + \cancel{ax^2} + \cancel{bx} - \underline{2ax} - \underline{b} - \underline{2ax} - \underline{b} + 2a$
$x-1 = 6ax+3b+2a$

$x^1: \begin{cases} 1 = 6a \\ -1 = 3b+2a \end{cases}$ $\quad a = 1/6 \quad \swarrow \quad -1 = 3b+\frac{1}{3} \qquad 3b = -1-\frac{1}{3} = -\frac{4}{3}\ \Big|\cdot 3$
$x^0:$

$b = -4$

$\tilde{y} = xe^{-x}\left(\frac{1}{6}x-4\right)$

$\boxed{y = y_o + \tilde{y}}$

④ $y^{(4)} + 3y'' + 2y = 0$

$\lambda^4 + 3\lambda^2 + 2 = 0$

Замена: $\lambda^2 = t$

$t^2 + 3t + 2 = 0$

$D = 9 - 8 = 1$

$t_1 = \frac{-3+1}{2} = -1$

$t_2 = \frac{-3-1}{2} = -2$

$\lambda^2 = -1 \qquad \lambda^2 = -2$

$\lambda = \pm i \qquad \lambda = \pm\sqrt{2}\, i$

$$\boxed{y = C_1 \cos x + C_2 \sin x + C_3 \cos\sqrt{2}x + C_4 \sin\sqrt{2}x}$$

⑤ $x^2y'' - 5xy' + 10y = 0$

$\lambda(\lambda-1) - 5\lambda + 10 = 0$

$\lambda^2 - \lambda - 5\lambda + 10 = 0$

$\lambda^2 - 6\lambda + 10 = 0$

$D = 36 - 40 = -4$

$\lambda_{1,2} = \frac{6 \pm 2i}{2} = 3 \pm i$

$x^\lambda = x^{3+i} = x^3 \cdot x^i = x^3 \cdot e^{\ln x^i} = x^3 \cdot e^{i\ln x} = x^3(\cos \ln x + i \sin \ln x)$

$y = C_1 \cdot x^3 \cos \ln x + C_2 \cdot x^3 \sin \ln x$

---

⑥ $y'' - 2y' + y = 2x\sqrt{x-3}\, e^x$

1) $y'' - 2y' + y = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$D = 4 - 4 = 0$

$\lambda_{1,2} = \frac{2}{2} = 1$

$(\lambda - 1)^2 = 0 \qquad x = 2$

$y = C_1 \cdot e^x + C_2 \cdot x \cdot e^x$

2) $\boxed{y = \varphi_1(x) \cdot e^x + \varphi_2(x) \cdot x \cdot e^x}$

3) $\begin{cases} \varphi_1'(x) \cdot e^x + \varphi_2'(x)\, x e^x = 0 \\ \varphi_1'(x) \cdot e^x + \varphi_2'(x)(e^x + xe^x) = 2x\sqrt{x-3}\, e^x \end{cases}$

$\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x(1+x) \end{vmatrix} = e^{2x}(1+x) - e^{2x} \cdot x = e^{2x}(1+x-x) = e^{2x}$

$\Delta_1 = \begin{vmatrix} 0 & xe^x \\ 2xe^x\sqrt{x-3} & e^x(1+x) \end{vmatrix} = -2x^2 e^{2x}\sqrt{x-3}$

$\Delta_2 = \begin{vmatrix} e^x & 0 \\ e^x & 2xe^x\sqrt{x-3} \end{vmatrix} = 2xe^{2x}\sqrt{x-3}$

$\varphi_1' = \frac{\Delta_1}{\Delta} = -2x^2\sqrt{x-3} \qquad \varphi_2' = \frac{\Delta_2}{\Delta} = 2x\sqrt{x-3}$

3) $\cancel{-2\frac{1}{p^3}\varphi(p)} + \frac{1}{p^2}\cdot\varphi'(p) = \cancel{-\frac{2}{p}\frac{1}{p^2}\cdot\varphi(p)} + 12p$

$\frac{1}{p^2}\cdot\varphi'(p) = 12p \quad |\cdot p^2$

$\varphi'(p) = 12p^3$

$\varphi(p) = 12\int p^3dp = 12\frac{p^4}{4} + A = 3p^4 + A$

$\underbrace{x = \frac{1}{p^2}(3p^4 + A)}$ – 2-га частина відповіді

$\boxed{\begin{cases} x = \frac{1}{p^2}(3p^4 + A) \\ y = 2xp - 4p^3 \end{cases}}$ – ~~[illegible]~~ розв.

[illegible]: $dp = 0$

$\underbrace{p = C}$ – 2-га част.

$\begin{cases} y = 2xp - 4p^3 \\ p = c \end{cases}$ $\boxed{y = 2xC - 4C^3}$

$p = 0$

$\begin{cases} y = 2xp - 4p^3 \\ p = 0 \end{cases}$ $\boxed{y = 0}$

---

③ $yy'' + (2y+1)y'^2 = 3yy'$ $\quad y(0) = 1 \quad y'(0) = \frac{3}{4}$

нема x !!!

заміна: $y(x) \rightsquigarrow z(y)$

$y' = z(y)$

$y'' = (y')'_x = (z(y))'_x = z'y' = z'z$

$yz'z + (2y+1)z^2 = 3yz \quad |:z$

$yz' + (2y+1)z = 3y$

$yz' = 3y - (2y+1)z \quad |:y$

0) $z' = -\frac{2y+1}{y}z + 3$ – ЛНОР

1) $z' = -\frac{2y+1}{y}z$ – ЛОД

$\frac{dz}{dy} = -\frac{2y+1}{y}z \quad |\cdot dy \; |:z$

$\int\frac{dz}{z} = -2\int dy - \int\frac{dy}{y}$

$\ln|z| = -2y - \ln|y| + \ln|C|$

$z = e^{-2y}\cdot y\cdot C$

2) ~~[illegible]~~ $z = e^{-2y}\cdot y\cdot\varphi(y)$

3) $-2e^{-2y}y\cdot\varphi(y) + e^{-2y}\varphi(y) + e^{-2y}y\cdot\varphi'(y) = \cancel{-\frac{2y+1}{y}e^{-2y}y\cdot\varphi(y)} + 3$

$\cancel{e^{-2y}\varphi(y)(-2y+1)} + \dots$

$e^{-2y}y\cdot\varphi'(y) = 3 \quad |:e^{-2y}\cdot y$

$\varphi'(y) = \frac{3}{e^{-2y}y} = \frac{3e^{2y}}{y}$

$\varphi(y) = 3\int e^{2y}\cdot\frac{1}{y}dy = \left\{\begin{matrix} u = \frac{1}{y} & dv = e^{2y}dy \\ du = -\frac{1}{y^2}dy & v = \frac{1}{2}e^{2y} \end{matrix}\right\} =$

Варіант 41

① $y'^3 = 3(xy' - y)$

$y'^3 = 3xy' - 3y$

$3y = 3xy' - y'^3 \;\Big|\,:3$

$y = xy' - \frac{y'^3}{3}$ – р-ня Клеро

Метод введення параметра: $y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = pdx$

$\underbrace{y = xp - \frac{p^3}{3}}$ – 1-ша частина відповіді

$dy = d(xp - \frac{1}{3}p^3)$

$\cancel{pdx} = \cancel{pdx} + xdp - p^2dp$

$(x - p^2)dp = 0$

$x - p^2 = 0$ або $dp = 0$

$\underbrace{x = p^2}$ – 2-га частина $p = C$

$\boxed{\begin{cases} x = p^2 \\ y = xp - \frac{p^3}{3} \end{cases}}$ – особливий розв'язок

$\begin{cases} y = xp - \frac{1}{3}p^3 \\ p = C \end{cases}$ $\boxed{y = xC - \frac{1}{3}C^3}$ – розв'язок

---

② $y = 2xy' - 4y'^3$ – р-ня Лагранжа

Метод вв. пар.: $y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = pdx$

$\underbrace{y = 2xp - 4p^3}$, – 1-ша частина

$dy = d(2xp - 4p^3)$

$pdx = 2xdp + 2pdx - 12p^2dp$

$pdx - 2xdp - 2pdx + 12p^2dp = 0$

$(p - 2p)dx + (-2x + 12p^2)dp = 0$

$(-p)dx + (12p^2 - 2x)dp = 0 \;\Big|\,:dp$

$(-p)\frac{dx}{dp} + 12p^2 - 2x = 0 \;\Big|\,:(-p)$

$\frac{dx}{dp} - 12p + 2x\frac{1}{p} = 0$

0) $x' = -\frac{2}{p}x + 12p$ – ЛНОР

1) $x' = -\frac{2}{p}x$ – ЛОР

$\frac{dx}{dp} = -\frac{2}{p}x \;\Big|\cdot dp \;\Big|\,:x$

$\int\frac{dx}{x} = -\int\frac{2}{p}dp$

$\ln|x| = -2\ln|p| + \ln|C|$

$x = p^{-2}\cdot C$

2) $x = \frac{1}{p^2}\cdot \varphi(p)$

Варіант № 41

1. $y'^3 = 3(xy' - y).$
2. $y = 2xy' - 4y'^3.$
3. $yy'' + (2y+1)y'^2 = 3yy',\ y(0) = 1,\ y'(0) = \frac{3}{4}.$
4. $y^{(4)} + 3y'' + 2y = 0.$
5. $x^2y'' - 5xy' + 10y = 0.$
6. $y'' - 2y' + y = 2x\sqrt{x-3}e^x.$
7. $y'' + 5y' + 4y = e^{-x}(x-1).$

[redacted] 15

$\varphi_1' = \frac{\Delta_1}{\Delta} = -\frac{x}{x^2-4}$ $\qquad$ $\varphi_2' = \frac{\Delta_2}{\Delta} = \frac{1}{x^2-4}$

$\varphi_1 = -\int \frac{x\,dx}{x^2-4} = \int \frac{x\,dx}{4-x^2} = -\frac{1}{2}\ln|4-x^2| + C_1$

$\varphi_2 = \int \frac{dx}{x^2-4} = -\int \frac{dx}{4-x^2} = \frac{1}{4}\ln\left|\frac{2+x}{2-x}\right| + C_2$

---

② $y'' + 2y' + y = e^{-x}\sin x$

1) $y'' + 2y' + y = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$D = 4 - 4 = 0$

$\lambda_{1,2} = \frac{-2}{2} = -1$

$(\lambda+1)^2 = 0 \qquad k = 2$

$y_0 = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x}$

2) $f(x) = e^{-x}\sin x = e^{\alpha x}(P_{m_1}(x)\cos\beta x + P_{m_2}(x)\sin\beta x)$

$\alpha = -1$

$\beta = 1 \qquad \alpha + i\beta = -1+i$ - не корінь $\Rightarrow k=0$

$m_1 = 0$

$m_2 = 0 \qquad m = \max\{m_1; m_2\} = 0$

$\tilde{y} = x^k e^{\alpha x}(Q_m(x)\cos\beta x + Q_m(x)\sin\beta x) = e^{-x}(a\cos x + b\sin x)$

1 | $\tilde{y} = e^{-x}(a\cos x + b\sin x)$

2 | $\tilde{y}' = -e^{-x}(a\cos x + b\sin x) + e^{-x}(-a\sin x + b\cos x) = e^{-x}(-a\cos x - b\sin x - a\sin x + b\cos x)$

1 | $\tilde{y}'' = -e^{-x}(-a\cos x - b\sin x - a\sin x + b\cos x) + e^{-x}(a\sin x - b\cos x - a\cos x - b\sin x)$

(+)

$e^{-x}\sin x = e^{-x}(a\cos x + b\sin x) + 2e^{-x}(-a\cos x - b\sin x - a\sin x + b\cos x) - e^{-x}(-a\cos x - b\sin x - a\sin x + b\cos x)$
$+ e^{-x}(a\sin x - b\cos x - a\cos x - b\sin x)$

$\sin x = \underline{a\cos x} + b\sin x - \underline{2a\cos x} - 2b\sin x - 2a\sin x + \underline{2b\cos x} + \underline{a\cos x} + b\sin x + a\sin x - \underline{b\cos x} + a\sin x - \underline{b\cos x} -$
$- \underline{a\cos x} - b\sin x$

$\sin x = \cos x(\cancel{a} - 2a + \cancel{2b} + a - \cancel{b} - \cancel{b} - \cancel{a}) + \sin x(\cancel{b} - \cancel{2b} - \cancel{2a} + \cancel{b} + \cancel{a} + \cancel{a} - b)$

$\sin x = \cos x(-a) + \sin x(-b)$

$\sin x: \quad \begin{cases} 1 = -b & b = 1 \\ 0 = -a & a = 0 \end{cases}$
$\cos x:$

$\tilde{y} = -e^{-x}\sin x$

$\boxed{y = y_0 + \tilde{y}}$

$$\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi}$$

④ $x^2y'' - xy' + 2y = 0$

$\lambda(\lambda-1) - \lambda + 2 = 0$

$\lambda^2 - \lambda - \lambda + 2 = 0$

$\lambda^2 - 2\lambda + 2 = 0$

$D = 4 - 8 = -4$

$\lambda_{1,2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$x^\lambda = x^{1+i} = x^1 \cdot x^i = x^1 \cdot e^{\ln x^i} = x^1 \cdot e^{i\ln x} = x(\cos\ln x + i\sin\ln x)$

$$\boxed{y = C_1 \cdot x \cdot \cos\ln x + C_2 \cdot \sin\ln x \cdot x}$$

⑤ $y^{(4)} + 7y'' + 12y = 0$

$\lambda^4 + 7\lambda^2 + 12 = 0$

$\begin{array}{r} \times 12 \\ 4 \\ \hline 48 \end{array}$

Заміна: $\lambda^2 = t$

$t^2 + 7t + 12 = 0$

$D = 49 - 48 = 1$

$t_1 = \frac{-7+1}{2} = -3$

$t_2 = \frac{-7-1}{2} = -4$

$\lambda^2 = -3$ $\lambda^2 = -4$

$\lambda = \pm\sqrt{3}i$ $\lambda = \pm 2i$

$$\boxed{y = C_1 \cdot \cos\sqrt{3}x + C_2\sin\sqrt{3}x + C_3\cos 2x + C_4\sin 2x}$$

⑥ $y'' + 6y' + 9y = \frac{e^{-3x}}{x^2-4}$

1) $y'' + 6y' + 9y = 0$

$\lambda^2 + 6\lambda + 9 = 0$

$D = 36 - 36 = 0$

$\lambda_{1,2} = \frac{-6}{2} = -3$

$(\lambda+3)^2 = 0$ $k = 2$

$y = C_1 \cdot e^{-3x} + C_2 \cdot x \cdot e^{-3x}$

2) $\boxed{y = \varphi_1(x)\, e^{-3x} + \varphi_2(x) \cdot x \cdot e^{-3x}}$

3) $\begin{cases} \varphi_1'(x)\, e^{-3x} + \varphi_2'(x) \cdot x \cdot e^{-3x} = 0 \\ \varphi_1'(x)(-3e^{-3x}) + \varphi_2'(x)(e^{-3x} + -3xe^{-3x}) = \frac{e^{-3x}}{x^2-4} \end{cases}$

$\Delta = \begin{vmatrix} e^{-3x} & x \cdot e^{-3x} \\ -3e^{-3x} & e^{-3x} - 3xe^{-3x} \end{vmatrix} = e^{-3x} \cdot e^{-3x}(1-3x) + 3xe^{-6x} = e^{-6x}(1-3x) + 3xe^{-6x} = e^{-6x}(1-3x+3x) =$
$= e^{-6x}$

$\Delta_1 = \begin{vmatrix} 0 & x \cdot e^{-3x} \\ \frac{e^{-3x}}{x^2-4} & e^{-3x} - 3xe^{-3x} \end{vmatrix} = -\frac{x\, e^{-6x}}{x^2-4}$

$\Delta_2 = \begin{vmatrix} e^{-3x} & 0 \\ -3e^{-3x} & \frac{e^{-3x}}{x^2-4} \end{vmatrix} = \frac{e^{-6x}}{x^2-4}$

1) $x = e^y \cdot (y+1)^{-2} \cdot C$

2) $x = e^y (y+1)^{-2} \varphi(y)$

3) $e^y (y+1)^{-2} \varphi(y) - 2e^y (y+1)^{-3} \varphi(y) + e^y (y+1)^{-2} \cdot \varphi'(y) = \frac{y-1}{y+1} e^y \frac{1}{(y+1)^2} \varphi(y) + \frac{3}{y+1}$

$e^y \varphi(y) \left(\frac{1}{(y+1)^2} - 2\frac{1}{(y+1)^3}\right) + e^y \ldots = \ldots$

$e^y \varphi(y) \frac{y+1-2}{(y+1)^3} + \ldots = \ldots$

$\cancel{e^y \cdot \varphi(y) \frac{y-1}{(y+1)^3}} + \ldots = \cancel{\frac{y-1}{(y+1)^3} \cdot e^y \varphi(y)} + \frac{3}{y+1}$

$e^y \cdot \frac{1}{(y+1)^2} \cdot \varphi'(y) = \frac{3}{y+1} \quad \Big| (y+1)^2$

$e^y \cdot \varphi'(y) = 3(y+1) \quad \Big| : e^y$

$\varphi'(y) = \frac{3(y+1)}{e^y}$

$\varphi(y) =$ [redacted] $\left\{\begin{matrix} u = y+1 & dv = e^{-y} dy \\ du = dy & v = -e^{-y} \end{matrix}\right] =$

$= 3\left(-e^{-y}(y+1) + \int e^{-y} dy\right) = 3\left(-e^{-y}(y+1) - e^{-y}\right) = -3e^{-y}(y+1+1) = -3e^{-y}(y+2) + C_2$

$x = e^y \frac{1}{(y+1)^2} \left((-3) \cdot \frac{1}{e^y}(y+2) + C_2\right) = -\frac{3(y+2)}{(y+1)^2} + \frac{C_2 \cdot e^y}{(y+1)^2}$

$y' = -\frac{3(y+2)}{(y+1)^2} + \frac{C_2 \cdot e^y}{(y+1)^2}$ $\qquad y(0) = 0$, $y'(0) = -6$

$-6 = -\frac{3(0+2)}{(0+1)^2} + \frac{C_2 \cdot e^0}{(0+1)^2}$

$\cancel{-6} = \cancel{-6} + C_2$

$C_2 = 0$

$y' = -3 \frac{y+2}{(y+1)^2}$

$\frac{dy}{dx} = -3 \frac{y+2}{(y+1)^2} \quad \Big| \cdot dx \quad \Big| : \frac{y+2}{(y+1)^2}$

$\int \frac{(y+1)^2 dy}{y+2} = \int -3 dx$

[redacted]

$\frac{(y+1)^2}{y+2} = \frac{(y+2-1)^2}{y+2} = \frac{(y+2)^2 - 2(y+2) + 1}{y+2} =$

[redacted]

$= y+2-2+\frac{1}{y+2} = y + \frac{1}{y+2}$

$\int y dy + \int \frac{1}{y+2} dy = -3 \int dx$

[redacted] $\frac{y^2}{2} + \ln|y+2| = -3x + C_3$ $\qquad y(0) = 0$, $y'(0) = -6$

$0 + \ln 2 = -3 \cdot 0 + C_3$

$C_3 = \ln 2$

$\boxed{\frac{y^2}{2} + \ln|y+2| = -3x + \ln 2}$ – розв. задачі Коші

② $2y'^2(y-xy')=1$

$2y'^2y-2xy'^3=1$

$2y'^2y=1+2xy'^3 \mid : 2y'^2$

$y=\frac{1}{2y'^2}+xy'$

$y=xy'+\frac{1}{2}\cdot\frac{1}{y'^2}$ – р-ня Клеро

парам: $y'=p \Rightarrow \frac{dy}{dx}=p \Rightarrow dy=pdx$

$\underbrace{y=xp+\frac{1}{2}\cdot\frac{1}{p^2}}$ – 1-ша частина

$dy=d(xp+\frac{1}{2}\cdot\frac{1}{p^2})$

$\cancel{pdx}=\cancel{pdx}+xdp-\frac{1}{p^3}dp$

$(x-\frac{1}{p^3})dp=0$

$\underbrace{x-\frac{1}{p^3}=0}$ – 2-га частина

$\boxed{\begin{cases}y=xp+\frac{1}{2}\cdot\frac{1}{p^2}\\x=\frac{1}{p^3}\end{cases}}$ – особливий розв'язок

$dp=0$

$\underbrace{p=C}$ – 2-га частина

$\begin{cases}y=xp+\frac{1}{2}\cdot\frac{1}{p^2}\\p=C\end{cases}$ $\boxed{y=xC+\frac{1}{C^2}\cdot\frac{1}{2}}$, $C\neq0$

розв'язок

перевірка: $y'^2=0$

$y'=0$

$\cancel{y=C_2}$

$0\neq1$

---

③ $(y+1)y''-(y-1)y'^2=3y'$ $y(0)=0$ $y'(0)=-6$

немає x

заміна: $y(x)\to z(y)$

$y'=z(y)$

$y''=(y')'_x=(z(y))'_x=z'y'=z'z$

$(y+1)z'z-(y-1)z^2=3z \mid : z$

$(y+1)z'-(y-1)z=3$

$(y+1)z'=3+(y-1)z \mid : y+1$

0) $z'=\frac{y-1}{y+1}z+\frac{3}{y+1}$ – ЛНОР

1) $z'=\frac{y-1}{y+1}z$ – ЛОР

$\frac{dz}{dy}=\frac{y-1}{y+1}z \mid \cdot dy \mid : z$

$\int\frac{dz}{z}=\int\frac{y-1}{y+1}dy$

$\ln|z|=y-2\ln|y+1|+\ln|C|$

$\int\frac{y\,dy}{y+1}-\int\frac{dy}{y+1}=\int\frac{y+1}{y+1}dy-\int\frac{dy}{y+1}-\int\frac{dy}{y+1}=$

$=y-\ln|y+1|-\ln|y+1|+C$

②

Варіант 40

①

$y = 2xy' - \ln y'$ – р-ня ~~Клеро~~ Лагранжа

Метод вв. парам.: $y' = p \Rightarrow \frac{dy}{dx} = p \Rightarrow dy = p\,dx$

$\underbrace{y = 2xp - \ln p}$, – 1-ша частина відп.

$dy = d(2xp - \ln p)$

$p\,dx = 2x\,dp + 2p\,dx - \frac{1}{p}\,dp$

$p\,dx + (2x - \frac{1}{p})\,dp = 0 \quad |:dp$

$p\frac{dx}{dp} + (2x - \frac{1}{p}) = 0 \quad |:p$

$x' + \frac{2x}{p} - \frac{1}{p^2} = 0$

0) $x' = -\frac{2}{p}\cdot x + \frac{1}{p^2}$ – ЛНОР

1) $x' = -\frac{2}{p}x$ – ЛОР

$\frac{dx}{dp} = -\frac{2}{p}x \quad |:x \quad |\cdot dp$

$\int\frac{dx}{x} = \int -\frac{2}{p}\,dp$

$\ln|x| = -2\ln|p| + \ln|C|$

$x = p^{-2}\cdot C$

2) $x = p^{-2}\varphi(p)$

3) $\cancel{-2\cdot\frac{1}{p^3}\varphi(p)} + \frac{1}{p^2}\varphi'(p) = \cancel{-\frac{2}{p}\cdot\frac{1}{p^2}\varphi(p)} + \frac{1}{p^2}$

$\frac{1}{p^2}\varphi'(p) = +\frac{1}{p^2} \quad |\cdot p^2$

$\varphi'(p) = +1$

$\varphi(p) = +p + A$

$x = \frac{1}{p^2}(+p + A) = \frac{1}{p} + \frac{A}{p^2}$

$\underbrace{x = \frac{1}{p} + \frac{A}{p^2}}$ – друга частина відповіді

$$\boxed{\begin{cases} x = \frac{1}{p} + \frac{A}{p^2} \\ y = 2xp - \ln p \end{cases}}$$ – особливий р-ок

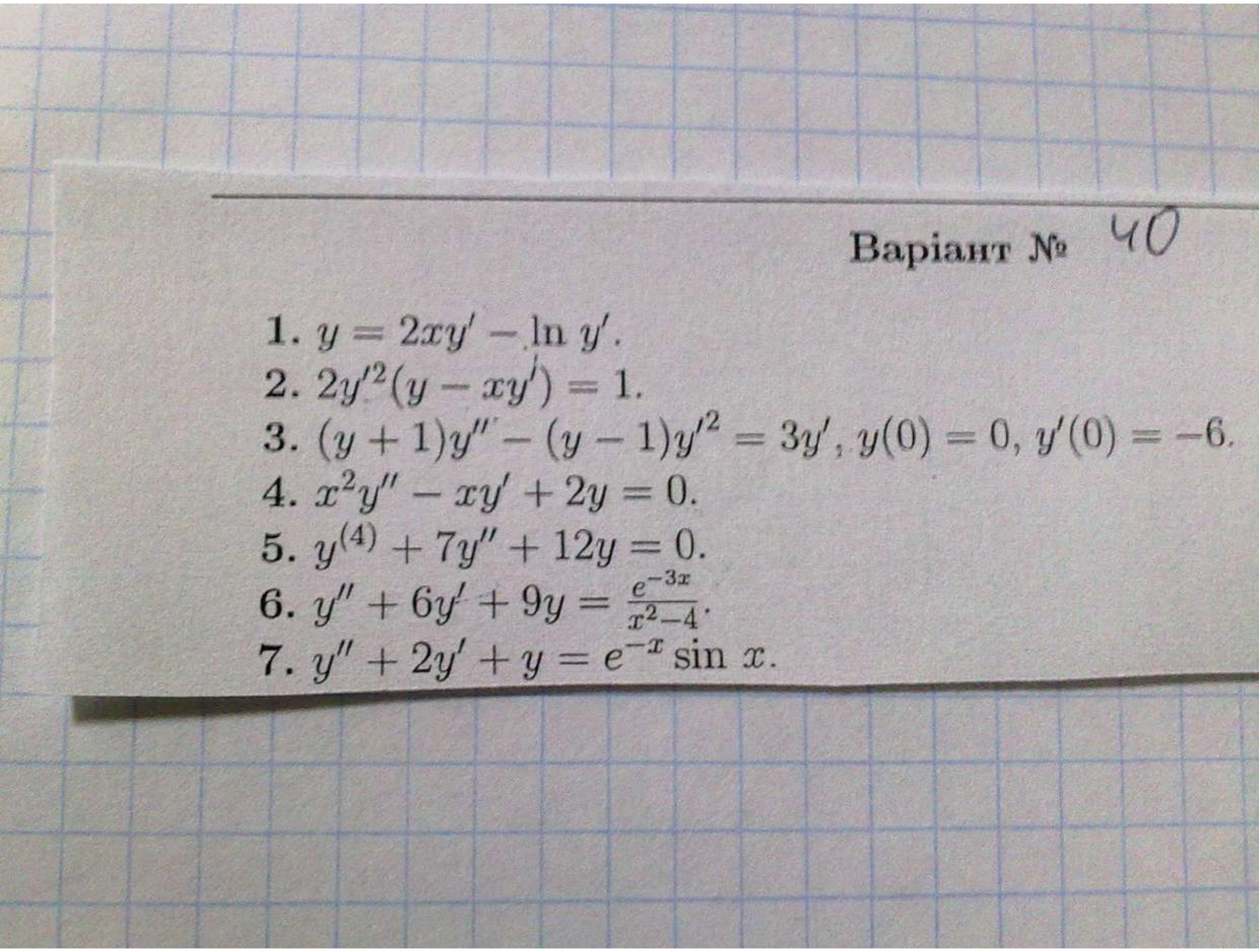
перевірка: $dp = 0$

$\underbrace{p = C_1}$ – 2-га частина відп.

$\begin{cases} y = 2xp - \ln p \\ p = C_1 \end{cases}$ $\boxed{y = 2xC_1 - \ln C_1}$

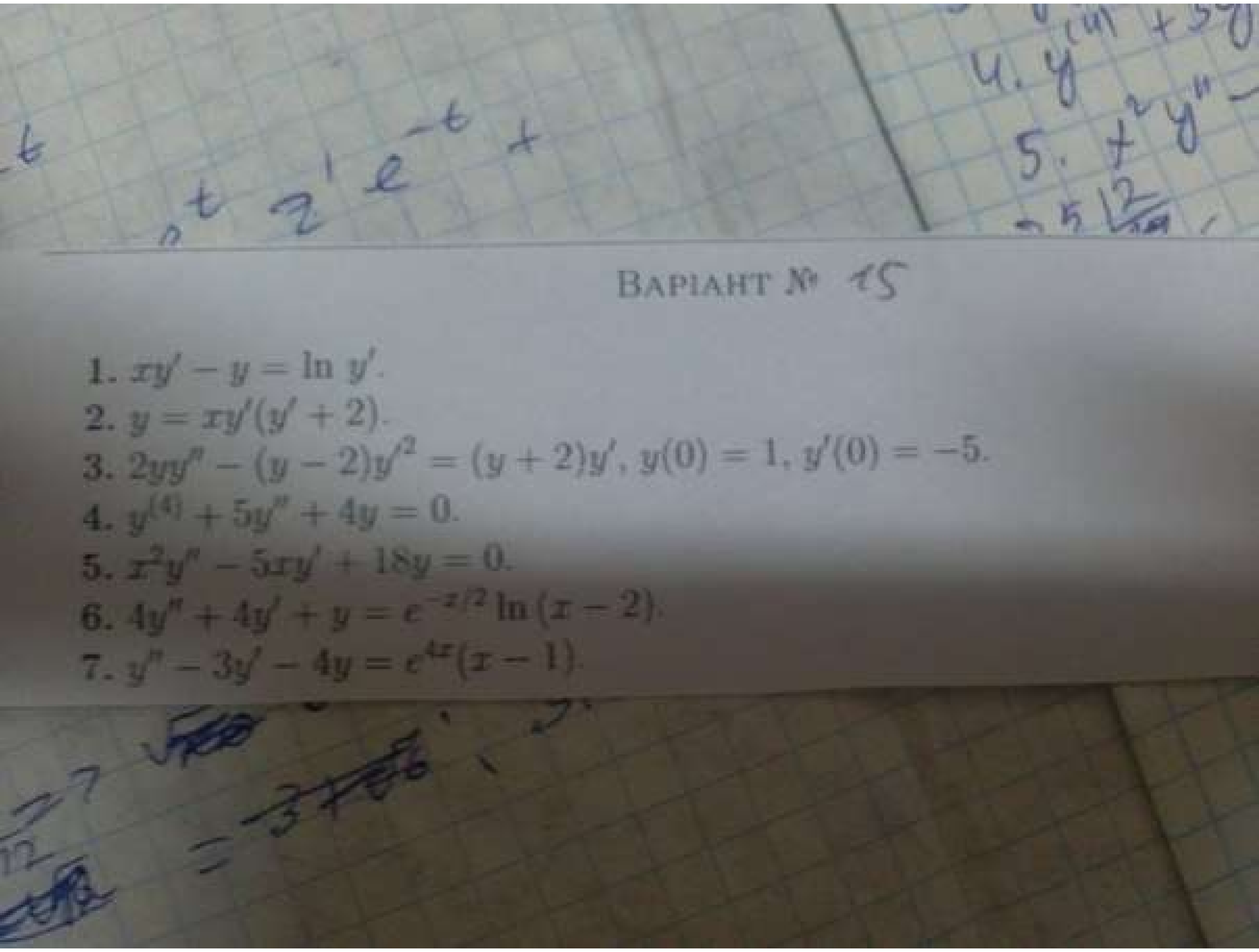
$\cancel{p = 0}$

$\cancel{\begin{cases} y = 2xp - \ln p \\ p = 0 \end{cases}}$



Варіант № 40

1. $y = 2xy' - \ln y'$.
2. $2y'^2(y - xy') = 1$.
3. $(y + 1)y'' - (y - 1)y'^2 = 3y'$, $y(0) = 0$, $y'(0) = -6$.
4. $x^2y'' - xy' + 2y = 0$.
5. $y^{(4)} + 7y'' + 12y = 0$.
6. $y'' + 6y' + 9y = \frac{e^{-3x}}{x^2-4}$.
7. $y'' + 2y' + y = e^{-x} \sin x$.



Варіант № 15

1. $xy' - y = \ln y'$.
2. $y = xy'(y' + 2)$.
3. $2yy'' - (y - 2)y'^2 = (y + 2)y'$, $y(0) = 1$, $y'(0) = -5$.
4. $y^{(4)} + 5y'' + 4y = 0$.
5. $x^2y'' - 5xy' + 18y = 0$.
6. $4y'' + 4y' + y = e^{-x/2}\ln(x - 2)$.
7. $y'' - 3y' - 4y = e^{4x}(x - 1)$.