Домашнє завдання 3.
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№249.

$y'^3 + y^2 = yy'(y'+1)$

Введемо параметр

$y' = p;\ dy = p\,dx$

$p^3 + y^2 = yp(p+1)$

$p^3 + y^2 = yp^2 + yp$

$p^3 + y^2 = y(p^2+p)$

$p^3 + y^2 - y(p^2+p) = 0$

$y^2 - y(p^2+p) + p^3 = 0$

$D = (p^2+p)^2 - 4\cdot 1\cdot p^3 = (p^2+p)^2 - 4p^3 =$

$= p^4 + 2p^3 + p^2 - 4p^3 = p^4 - 2p^3 + p^2 =$

$= (p^2-p)^2$

$y_1 = \dfrac{-(p^2+p) - (p^2-p)^2}{2} = \dfrac{2p}{2} = p$

$y_2 = \dfrac{-(p+p) + (p^2-p)^2}{2} = p^2$

• $dx = \frac{1}{p}dy$

$dx = \frac{1}{p}dp$

$\left.\begin{array}{l} x = \ln p + c \\ y = p \end{array}\right\} \Rightarrow x = \ln y + \ln c$

$y = c\cdot e^x$

особливий

• $dx = \frac{1}{p}dy$

$dx = \frac{1}{p}\cdot 2p\,dp$

$dx = 2\,dp$

$\left.\begin{array}{l} x = 2p + c \\ y = p^2 \end{array}\right\} \Rightarrow x = 2\sqrt{y} - c$

$4y = (x+c)^2$

особливий

$y = 0$

В: $\begin{cases} y = c\cdot e^x \\ 4y = (x+c)^2 \\ y = 0 \end{cases}$

№259.

$y'^2 - 2yy' = y^2(e^x - 1)$

$y'^2 - 2yy' - y^2(e^x - 1) = 0$

$$\begin{cases} y' = \dfrac{2y + \sqrt{4y^2 + 4y^2(e^x - 1)}}{2} = y + y\sqrt{e^x} \\ y' = \dfrac{2y - \sqrt{4y^2 + 4y^2(e^x - 1)}}{2} = y - y\sqrt{e^x} \end{cases}$$

- $y' = y + y\sqrt{e^x}$

$\dfrac{dy}{dx} = y + y\sqrt{e^x} = y(1 + \sqrt{e^x}) \Big| \cdot \dfrac{dx}{y}$

$\int \dfrac{dy}{y} = \int (1 + \sqrt{e^x})\, dx \Rightarrow \int 1\, dx + \int e^{x/2}\, dx = x + 2e^{x/2}$

$\ln y = x + 2e^{x/2} + \ln c$

$\ln yc = x + 2e^{x/2}$

- $y' = y - y\sqrt{e^x}$

$\dfrac{dy}{dx} = y - y\sqrt{e^x} = y(1 - \sqrt{e^x}) \Big| \cdot \dfrac{dx}{y}$

$\int \dfrac{dy}{y} = \int 1 - \sqrt{e^x}\, dx$

$\ln y + \ln c = x - 2e^{x/2}$

$\ln yc = x - 2e^{x/2}$

B: $\begin{cases} \ln yc = x - 2e^{x/2} \\ \ln yc = x + 2e^{x/2} \\ y = 0 \end{cases}$

№270.

$y'(x - \ln y') = 1$

Введем параметр $p = y'$; $dy = p\,dx$

$p(x - \ln p) = 1$

$px - p \cdot \ln p = 1$

$px = 1 + p\ln p \;/\cdot \frac{1}{p}$

$x = \frac{1 + p\ln p}{p}$

$x = \frac{1}{p} + \frac{p\ln p}{p} = \frac{1}{p} + \ln p$

$dy = p\left(-\frac{1}{p^2} + \frac{1}{p}\right)dp$

$\int dy = -\int \frac{dp}{p} + \int dp$

$y = -\ln p + p + c$

$x = \frac{1}{p} + \ln p$

В: $\begin{cases} x = \frac{1}{p} + \ln p, \; p \in \mathbb{R} \\ y = -\ln p + p + c \end{cases}$

№272.

$y = \ln(1+y'^2)$

Введем параметр $y' = p$, $dy = p\,dx$

$y = \ln(1+p^2)$

$dy = \frac{1}{1+p^2} \cdot 2p\,dp$

$\int dx = \frac{dy}{p} = \int \frac{2p\,dp}{(1+p^2)p}$

$x = 2\int \frac{dp}{1+p^2} + c$

$x = 2 \cdot \operatorname{arctg} p + c$

В: $\begin{cases} y = \ln(1+p^2) \\ x = 2\operatorname{arctg} p + c \end{cases}$

№274.

$y = (y'-1)e^{y'}$

Введем параметр $y' = p$; $dy = p\,dx$

$dx = \frac{1}{p}\,dy$

$y = (p-1)e^p$; $dy = p \cdot e^p$; $p\,dx = p \cdot e^p\,dp \;/:p$; $dx = e^p\,dp$

$\{dx = \frac{1}{p}(e^p + pe^p - e^p)\,dp\}$

$dx = e^p\,dp$

$x = e^p + c$

В: $\begin{cases} y = (p-1)\cdot e^p \\ x = e^p + c \\ y = -1 - \text{особенная} \end{cases}$

№292.

$y = xy'^2 - 2y'^3$ — р-ция Клеро

$y' = p$

$dy = p\,dx$

$dx = \frac{1}{p}\,dy$

$y = xp^2 - 2p^3$

$p\,dx = d(xp^2 - 2p^3)$

$p = 1$

$p = 0$

$(p-1)\frac{dx}{dp} + 2x = 3p$

$x = 2p + 1 + \frac{C}{(p-1)^2}$

В:

$$\begin{cases} y = 0 \\ y = x - 2 \\ x = 2p + 1 + \frac{C}{(p-1)^2} \\ y = \frac{Cp^2}{(p-1)^2} + p^2 \end{cases}$$

№293.

$xy' - y = \ln y'$

$y = -xy' - \ln y'$ – ур-ние Клеро

$p' = p$

$dy = p\,dx$

$y = xp - \ln p; \quad dy = p\,dx + x\,dp - \frac{1}{p}\,dp$

$x - \frac{1}{p} = 0$

$x = \frac{1}{p} \Rightarrow \quad p = \frac{1}{x}$

$y = -\ln\frac{1}{x} + 1 = 1 + \ln x$

В: $\begin{cases} y = 1 + \ln x \\ y = px - \ln p \end{cases}$