Екзаменаційна робота
з математичного аналізу
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В -4.

1. $\int \frac{(1-\ln\sqrt{x})^{2021}}{x}dx = \left[\begin{array}{l} 1-\ln\sqrt{x}=t \\ \frac{1}{\sqrt{x}}\cdot(\sqrt{x})'dx=dt = \frac{1}{2x}\cdot \end{array}\right] =$

$= \int t^{2021}\, 2dt = 2\cdot\frac{t^{2022}}{2022} = \frac{t^{2022}}{1011} + C =$

$= \frac{(1-\ln\sqrt{x})^{2022}}{1011} + C$

2. $\int \frac{x}{(x+2)(x-4)(x+3)}dx = \frac{A}{x+2} + \frac{B}{x-4} + \frac{C}{x+3}$ (=)

$A = \frac{x}{\cancel{(x+2)}(x-4)(x+3)}\Big|_{x=-2} = \frac{-2}{-6\cdot 1} = \frac{1}{3}$

$B = \frac{x}{(x+2)\cancel{(x-4)}(x+3)}\Big|_{x=4} = \frac{4}{6\cdot 7} = \frac{4}{42} = \frac{1}{21}$

$C = \frac{x}{(x+2)(x-4)\cancel{(x+3)}}\Big|_{x=-3} = \frac{-3}{-1\cdot(-7)} = -\frac{3}{7}$

(=) $\frac{1}{3}\int\frac{dx}{x+2} + \frac{2}{21}\int\frac{dx}{x-4} - \frac{3}{7}\int\frac{dx}{x-4} =$

$= \frac{1}{3}\ln|x+2| + \frac{2}{21}\ln|x-4| - \frac{3}{4}\ln|x-4| + C$

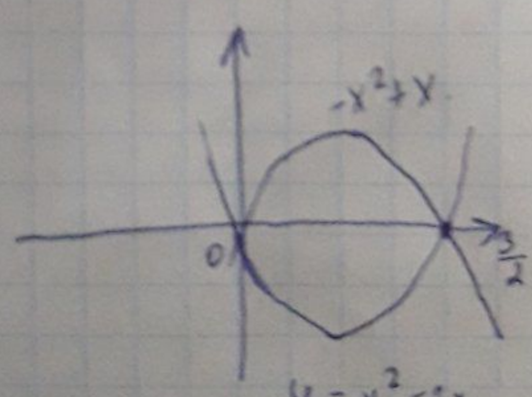
3. a) $y = -x^2 + x$ $\qquad y = x^2 - 2x$

$x^2 - 2x = -x^2 + x$

$x^2 - 2x + x^2 - x = 0$

$2x^2 - 3x = 0$

$x(2x-3) = 0$

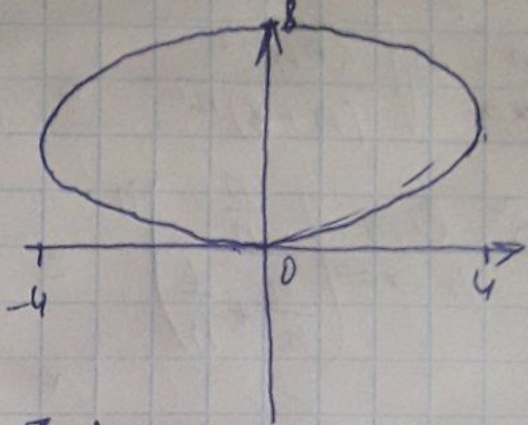


$x_1 = 0 \qquad x_2 = \frac{3}{2}$

$$\int_0^{\frac{3}{2}} (-2x^2+3x)\,dx = -\frac{2x^3}{3} + \frac{3x^2}{2}\Big|_0^{\frac{3}{2}} = -\frac{2}{3}\left(\frac{3}{2}\right)^3 + \frac{3}{2}\left(\frac{3}{2}\right)^2 - 0 =$$

$$= \frac{27}{8}\left(-\frac{2}{3}+1\right) = \frac{9}{8}$$

б) $r = 4(1+\sin\varphi)$



$\sin\varphi \geq 0$

$\sin 4\varphi \geq 0$

$0 + 2\pi n \leq \varphi \leq \pi + 2\pi n,\ n \in \mathbb{Z}$

$\cancel{\frac{\pi n}{2} \leq \varphi \leq \frac{\pi n}{2} + \frac{\pi}{4}}$

$\varphi = 0$	$r = 4$
$\varphi = \frac{\pi}{2}$	$r = 8$
$\varphi = \pi$	$r = 4$

$$S = \frac{1}{2}\int_0^{\pi} 4^2(1+\sin\varphi)^2 d\varphi = 8\int_0^{\pi}(1+\sin\varphi+\sin^2\varphi)\,d\varphi = 8\int (1+\sin\varphi +$$

$$= + \frac{1-\cos 2\varphi}{2})\,d\varphi = 8(\varphi - 2\cos\varphi + \frac{1}{2}\varphi - \frac{1}{2}\cdot\frac{1}{2}\sin 2\varphi)\Big|_0^{\pi} = 8\left(\frac{3}{2}\pi + \right.$$

4. $$\int_0^{+\infty} \frac{x^2+1}{\sqrt[4]{x^9+x^{13}}}\,dx = \underbrace{\int_0^1 \frac{x^2+1}{\sqrt[4]{x^9+x^{13}}}}_{I_1} + \underbrace{\int_0^{+\infty} \frac{x^2+1}{\sqrt[4]{x^9+x^{13}}}}_{I_2}$$

$$I_1 = \int_0^1 \frac{x^2+1}{\sqrt[4]{x^9+x^{13}}} \underset{x\to 0}{\sim} \frac{x^2+1}{x^{9/4}} = \frac{1}{x^{\frac{1}{4}}}$$

збіжний при $x \to 0$, бо $\frac{1}{4} < 1$.

$$I_2 \int_0^{+\infty} \frac{x^2+1}{\sqrt[4]{x^9+x^{13}}} \underset{x\to\infty}{\sim} \frac{x^2+1}{x^{13/4}} = \frac{1}{x^{5/4}}$$

збіжний при $x \to \infty$, бо $\frac{5}{4} > 1$.

Отже, невласний інтеграл збіжний

5. $\sum_{n=1}^{\infty} \frac{5^n n^{n^2}}{(n+1)^{n^2}}$

За ознакою Коші

$$\lim_{n\to\infty} \sqrt[n]{\frac{5^n n^{n^2}}{(n+1)^{n^2}}} = \lim_{n\to\infty} \frac{5n^n}{(n+1)^{n^2}} = 5 \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n =$$

$$= 5 \lim_{n\to\infty} \left(\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}}\right)^n = 5 \lim_{n\to\infty} \left(\frac{1}{1+\frac{1}{n}}\right)^n = 5$$

ряд збіжний, бо $5 > 1$.

6. $\sum_{n=1}^{\infty} \frac{arctg(\sqrt{nx})}{x^n}$, $E = [1; +\infty)$

1) $u_n(x) = \left| \frac{arctg(\sqrt{nx})}{x^n} \right| = \frac{n^2}{1} = C_n$

2) $\sum_{n=1}^{\infty} C_n = \sum_{n=1}^{\infty} \frac{n^2}{1}$ збіжний, бо $2 > 1$.

отже, ряд рівномірно збігається

7. $\sum_{n=1}^{\infty} \frac{3^n x^n}{\sqrt{n^2+1}}$

$$R = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n\to\infty} \frac{3^n}{\sqrt{n^2+1}} \cdot \frac{\sqrt{(n+1)^2+1}}{3^{n+1}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+2n+2}}{\sqrt{n^2+1} \cdot 3} = \frac{1}{3} \lim_{n\to\infty} \sqrt{\frac{1+\frac{2}{n}+\frac{2}{n^2}}{1+\frac{1}{n^2}}} = \frac{1}{3}$$

$\left(x - \frac{1}{3};\ x + \frac{1}{3} \right)$ - інтервал збіжності

8. $f(x,y)=\frac{2x-y}{x+y}$

$$\lim_{x\to 0}\left(\lim_{y\to 0}\left(\frac{2x-y}{x+y}\right)\right)=\lim_{x\to 0}\left(\frac{2x-0}{x+0}\right)=0$$

$$\lim_{y\to 0}\left(\lim_{x\to 0}\left(\frac{2x-y}{x+y}\right)\right)=\lim_{y\to 0}\left(\frac{0-y}{0+y}\right)=0$$

Розглянемо $P_n\left(\frac{1}{n};0\right)\xrightarrow[n\to\infty]{} O(0;0)$

$$\lim_{n\to\infty}\left(\frac{2\frac{1}{n}-0}{\frac{1}{n}+0}\right)=2$$

Розглянемо $M_n\left(\frac{1}{n};\frac{1}{n}\right)\xrightarrow[n\to\infty]{} O(0;0)$

$$\lim_{n\to\infty}\frac{2\left(\frac{1}{n}\right)-\frac{1}{n}}{\frac{1}{n}+\frac{1}{n}}=\lim_{n\to\infty}\frac{\frac{2}{n}-\frac{1}{n}}{\frac{1}{n}+\frac{1}{n}}=\frac{1}{2}.$$

$2\neq\frac{1}{2}$, отже $\lim\limits_{\substack{x\to 0\\ y\to 0}}$ не існує.

9. $u=x^2+y^2+z^2+4x-8y-2z+6$

$x'=2x+4$

$y'=2y-8$

$z'=2z-2$

$$\begin{cases}2x+4=0\\2y-8=0\\2z-2=0\end{cases}\quad\begin{cases}x=-2.\\y=4\\z=1\end{cases}$$

$M(-2;4;1)$

$u''_{x^2}=2$

$u''_{y^2}=2$

$u''_{z^2} = 2.$

$u''_{xy} = 0$

$u''_{xz} = 0$

$u''_{yz} = 0$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$\Delta_1 = 2.$

$$\Delta_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$\Delta_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 8 + 0 + 0 - 0 - 0 - 0 = 8$$

Оскільки всі мінори > 0, то точка M є точкою min.

$$u_{min} = (-2)^2 + 4^2 + 1^2 + 4(-2) \cdot - 8 \cdot 4 - 2 - 1 + 6 =$$

$= -15.$