№ 1.149

$$\lim_{n\to\infty} \frac{10000n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{10000n}{n^2}}{\frac{n^2}{n^2}+\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\frac{10000}{n}}{1+\frac{1}{n^2}} =$$

$$= \left[\frac{0}{1}\right] = 0$$

№ 1.151

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4+n-1}}{n+2} = \lim_{n\to\infty} \frac{\sqrt{\frac{n^4}{n^4}+\frac{n}{n^4}-\frac{1}{n^4}}}{\frac{n}{n^2}+\frac{2}{n^2}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+\overset{\to 0}{\frac{1}{n^3}}+\overset{\to 0}{\frac{1}{n^4}}}}{\underset{\to 0}{\frac{1}{n}}+\underset{\to 0}{\frac{2}{n^2}}} = \left[\frac{1}{0}\right] = \infty$$

№ 1.159.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{i(i+1)} = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{1\cdot 1}{i(i+1)}\right) =$$

$$= \lim_{n\to\infty} \left(\underset{i=1}{\frac{1}{1\cdot 2}} + \underset{i=2}{\frac{1}{2\cdot 3}} + \underset{i=3}{\frac{1}{3\cdot 4}} + \underset{i=4}{\frac{1}{4\cdot 5}} + \ldots + \underset{i=n}{\frac{1}{n\cdot(n+1)}}\right) =$$

$$= \lim_{n\to\infty} \left(\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \ldots + \cancel{\frac{1}{n}} - \frac{1}{n+1}\right) =$$

$$= \lim_{n\to\infty} \left(1 - \overset{\to 0}{\frac{1}{n+1}}\right) = 1 - 0 = 1$$

$Ax^2+Bx+C = A(x-x_1)(x-x_2)$

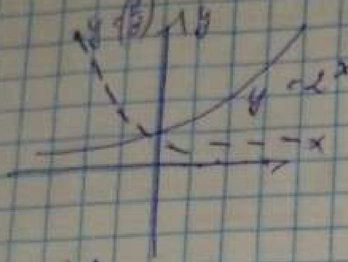
$9k^2+6k-8 = 9\left(k-\frac{2}{3}\right)\left(k-\left(-\frac{4}{3}\right)\right) = 3\left(k-\frac{2}{3}\right)\cdot 3\left(k+\frac{4}{3}\right) = (3k-2)(3k+4)$

$D = 36-4\cdot 9\cdot(-8) = 36+288 = 324$

$\sqrt{D} = 18$

$k_1 = \frac{-6+18}{18} = \frac{12}{18} = \frac{2}{3}$

$k_2 = \frac{-6-18}{18} = \frac{-24}{18} = -\frac{4}{3}$



$$\lim_{n\to\infty} q^n = \begin{cases} \infty & \text{при } q>1 \\ 1 & \text{при } q=1 \\ 0 & \text{при } q<1 \\ 0 & \text{при } -1<q<1 \\ \nexists & \text{при } q\le -1 \end{cases}$$

$\left\{\left(-\frac{1}{2}\right)^n\right\}$: $-\frac{1}{2}$; $\frac{1}{4}$; $-\frac{1}{8}$; $\frac{1}{16}$ ($n=1$, $n=2$, $n=3$, $n=4$) | $\{(-1)^n\}$: $-1; 1; -1; 1; \dots$

Домашня робота з(6)

I. $\boxed{2}$ $x_n = \frac{2n-1}{n+1}$; $a=2$; $\varepsilon=1$, $\varepsilon=0{,}1$, $\varepsilon=0{,}01$, $\varepsilon=0{,}001$; $\forall n \ge N$: $|x_n - a| < \varepsilon$

$\nabla \lim_{n\to\infty} \frac{2n-1}{n+1} = 2 \overset{\text{def}}{\Leftrightarrow} (\forall\varepsilon>0)(\exists N)(\forall n \ge N)\left\{\left|\frac{2n-1}{n+1} - 2\right| < \varepsilon\right\}$

$\left|\frac{2n-1}{n+1} - \frac{2}{1}\right| = \left|\frac{2n-1-2n-1}{n+1}\right| = \left|\frac{-2}{n+1}\right| = \frac{|-2|}{|n+1|} = \frac{2}{n+1} < \varepsilon$

$n+1 > \frac{2}{\varepsilon}$

$n > \frac{2}{\varepsilon} - 1$

$\exists N = \left[\frac{2}{\varepsilon} - 1\right] + 1$

при $\varepsilon = 0{,}1$: $N=20$

при $\varepsilon = 1$: $N=2$

при $\varepsilon = 0{,}01$: $N=200$

при $\varepsilon = 0{,}001$: $N=2000$ $\triangle$

II.

$\boxed{2}$ $\lim\limits_{n\to\infty} \frac{4n+3}{2n+1} = 2 \} \underline{\underline{def}} \{ (\forall \varepsilon > 0)(\exists N)(\forall n > N)$

$\{ |\frac{4n+3}{2n+1} - 2| < \varepsilon \}$

∇ Розгл. $|\frac{4n+3}{2n+1} - 2|^{2n+1} = |\frac{4n+3-4n-2}{2n+1}| = |\frac{1}{2n+1}| =$

$= \frac{|1|}{|2n+1|} = \frac{1}{2n+1} < \frac{\varepsilon}{1}$

$2n+1 > \frac{1}{\varepsilon}$

$2n > \frac{1}{\varepsilon} - 1 \,/\cdot\frac{1}{2}$

$n > \frac{1}{2\varepsilon} - \frac{1}{2}$

$N = [\frac{1}{2\varepsilon} - \frac{1}{2}] + 1$ Δ

$\boxed{4}$ $\lim\limits_{n\to\infty} \frac{3-2n}{n+2} = -2 \} \underline{\underline{def}} \{ (\forall \varepsilon > 0)(\exists N)(\forall n > N)$

$\{ |\frac{3-2n}{n+2} + 2| < \varepsilon \}$

∇ Розгл. $|\frac{3-2n}{n+2} + 2|^{n+2} = |\frac{3-\cancel{2n}+\cancel{2n}+4}{n+2}| = |\frac{-1}{n+2}| =$

$= \frac{|-1|}{|n+2|} = \frac{1}{n+2} < \frac{\varepsilon}{1}$

$n+2 > \frac{1}{\varepsilon}$

$n > \frac{1}{\varepsilon} - 2$

$N = [\frac{1}{\varepsilon} - 2] + 1$ Δ

$\boxed{3}$ $\lim\limits_{n\to\infty} \frac{3n+5}{n+1} = 3 \} \underline{\underline{def}} \{ (\forall \varepsilon > 0)(\exists N)(\forall n > N)$

$\{ |\frac{3n+5}{n+1} - 3| < \varepsilon \}$

∇ Розгл. $|\frac{3n+5}{n+1} - 3|^{n+1} = |\frac{\cancel{3n}+5-\cancel{3n}-3}{n+1}| = |\frac{2}{n+1}| = \frac{|2|}{|n+1|} = \frac{2}{n+1} < \frac{\varepsilon}{1}$

$n+1 > \frac{2}{\varepsilon}$

$n > \frac{2}{\varepsilon} - 1$

$N = \left[\frac{2}{\varepsilon} - 1\right] + 1 \quad \Delta$

$\boxed{6}$ $\lim_{n\to\infty} \frac{6n^3+3n+1}{2n^3+n+1} = 3$

$\left\{ \left| \frac{6n^3+3n+1}{2n^3+n+1} - 3 \right| < \varepsilon \right\}$

∇ Розв. $\left| \frac{6n^3+3n+1}{2n^3+n+1} - 3^{(2n^3+n+1} \right| = \left| \frac{\cancel{6n^3}+\cancel{3n}+1-\cancel{6n^3}-\cancel{3n}-3}{2n^3+n+1} \right| =$

$= \left| \frac{-2}{2n^3+n+1} \right| = \frac{|-2|}{|2n^3+n+1|} = \frac{2}{2n^3+n+1} < \frac{\varepsilon}{1}$

$2n^3+n+1 > \frac{2}{\varepsilon}$

$2n^3+n > \frac{2}{\varepsilon} - 1$

$n(2n^2+1) > \frac{2}{\varepsilon} - 1 \quad \Big/ \cdot \frac{1}{2n^2+1}$

$n > \frac{1}{\varepsilon(2n^2+1)} - \frac{1}{2n^2+1}$

$n > \frac{1-\varepsilon}{(2n^2+1)\varepsilon}$

$N = \left[\frac{1-\varepsilon}{(2n^2+1)\varepsilon}\right] + 1 \quad \Delta$

III.

$\boxed{2}$ $\lim_{n\to\infty} \frac{14n^3+n^2+10}{5+n+3n^3} = \lim_{n\to\infty} \frac{\frac{14\cancel{n^3}}{\cancel{n^3}} + \frac{\cancel{n^2}}{n^{\cancel{3}}} + \frac{10}{n^3}}{\frac{5}{n^3} + \frac{\cancel{n}}{n^{\cancel{3}\,2}} + \frac{3\cancel{n^3}}{\cancel{n^3}}} =$

$= \lim_{n\to\infty} \frac{14 + \overset{\to 0}{\frac{1}{n}} + \overset{\to 0}{\frac{10}{n^3}}}{\overset{\to 0}{\frac{5}{n^3}} + \overset{\to 0}{\frac{1}{n^2}} + 3} = \boxed{\frac{14}{3}}$

$\boxed{4}$ $\lim\limits_{n\to\infty} \dfrac{1000n^3+n+2}{0{,}001n^5+n^3+1} = \lim\limits_{n\to\infty} \dfrac{\frac{1000n^3}{n^5}+\frac{n}{n^5}+\frac{2}{n^5}}{\frac{0{,}001n^5}{n^5}+\frac{n^3}{n^5}+\frac{1}{n^5}} =$

$= \lim\limits_{n\to\infty} \dfrac{\frac{1000}{n^2}+\frac{1}{n^4}+\frac{2}{n^5}}{0{,}001+\frac{1}{n^2}+\frac{1}{n^5}} = \left[\dfrac{0}{0{,}001}\right] = 0$

$\boxed{6}$ $\lim\limits_{n\to\infty} \dfrac{\sum\limits_{k=1}^{n}(5k+7)}{2n^2+1} = \lim\limits_{n\to\infty} \dfrac{2{,}5+9{,}5n}{2n^2+1} = \lim\limits_{n\to\infty} \dfrac{\frac{2{,}5}{n^2}+\frac{9{,}5n}{n^2}}{\frac{2n^2}{n^2}+\frac{1}{n^2}}$

$= \lim\limits_{n\to\infty} \dfrac{\frac{2{,}5}{n^2}+\frac{9{,}5}{n}}{2+\frac{1}{n^2}} = \left[\dfrac{0}{2}\right] = 0$

$\sum\limits_{k=1}^{n}(5k+7) = \sum\limits_{k=1}^{n}5k + \sum\limits_{k=1}^{n}7 = 5\left(\sum\limits_{k=1}^{n}k\right) + 7n =$

$= 5(1+2+3+\ldots+n)+7n = 5\cdot\dfrac{1+n}{2}+7n =$

$= 2{,}5+2{,}5n+7n = 2{,}5+9{,}5n$

$\boxed{8}$ $\lim\limits_{n\to\infty} \dfrac{n^2+7n+10}{199n+314} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{n\to\infty} \dfrac{\frac{n^2}{n^2}+\frac{7n}{n^2}+\frac{10}{n^2}}{\frac{199n}{n^2}+\frac{314}{n^2}} =$

$= \lim\limits_{n\to\infty} \dfrac{1+\frac{7}{n}+\frac{10}{n^2}}{\frac{199}{n}+\frac{314}{n^2}} = \left[\dfrac{1}{0}\right] = \infty$

$\boxed{10}$ $\lim\limits_{n\to\infty} \left(\sqrt{1+n^2}-\sqrt{n^2+7n+5}\right) = [\infty-\infty] =$

$= \lim\limits_{n\to\infty} \dfrac{\left(\sqrt{1+n^2}-\sqrt{n^2+7n+5}\right)\cdot\left(\sqrt{1+n^2}+\sqrt{n^2+7n+5}\right)}{\sqrt{1+n^2}+\sqrt{n^2+7n+5}} =$

$= \lim\limits_{n\to\infty} \dfrac{\left(\sqrt{1+n^2}\right)^2-\left(\sqrt{n^2+7n+5}\right)^2}{\sqrt{1+n^2}+\sqrt{n^2+7n+5}} =$

$= \lim\limits_{n\to\infty} \dfrac{1+\cancel{n^2}-\cancel{n^2}-7n-5}{\sqrt{1+n^2}+\sqrt{n^2+7n+5}} = \lim\limits_{n\to\infty} \dfrac{-7n-4}{\sqrt{1+n^2}+\sqrt{n^2+7n+5}} =$

$= \left[\dfrac{-4}{\infty+\infty}\right] = \left[\dfrac{-4}{\infty}\right] = 0$

II

$\boxed{2}$ $\lim\limits_{n\to\infty}\left(\frac{1}{3\cdot 7}+\frac{1}{7\cdot 10}+\dots+\frac{1}{(4n-1)(4n+3)}\right)=$

$=\lim\limits_{n\to\infty}\left(\frac{1}{3}-\cancel{\frac{1}{7}}+\cancel{\frac{1}{7}}-\cancel{\frac{1}{10}}+\dots+\cancel{\frac{1}{4n-1}}-\frac{1}{4n+3}\right)=$

$=\lim\limits_{n\to\infty}\left(\frac{1}{3}-\frac{1}{4n+3}\right)=\frac{1}{3}-0=\frac{1}{3}$

$\boxed{4}$ $\lim\limits_{n\to\infty}\left(\sum\limits_{k=2}^{n}\frac{1}{k^2+k-2}\right)=\lim\limits_{n\to\infty}\left(\sum\limits_{k=2}^{n}\frac{1\cdot 1}{(k+2)(k-1)}\right)=$

$=\lim\limits_{n\to\infty}\left(\frac{1}{2\cdot(-1)}+\frac{1}{4\cdot 1}+\frac{1}{5\cdot 2}+\frac{1}{6\cdot 3}+\dots+\frac{1}{(n+2)(n-1)}\right)$

$=\lim\limits_{n\to\infty}\left(\frac{1}{3}\left(\frac{1}{2}+\frac{1}{1}\right)\right)+\frac{1}{3}\left(\frac{1}{4}-\frac{1}{1}\right)+\frac{1}{3}\left(\frac{1}{5}-\frac{1}{2}\right)+\frac{1}{3}\left(\frac{1}{6}-\frac{1}{3}\right)+\dots+\frac{1}{3}\left(\frac{1}{n+2}-\frac{1}{n-1}\right)$

$=-\frac{1}{3}\lim\limits_{n\to\infty}\left(\cancel{\frac{1}{2}}+\cancel{\frac{1}{1}}\right)+\left(\frac{1}{4}-\cancel{\frac{1}{1}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{3}}\right)+\dots+\left(\cancel{\frac{1}{n+2}}-\cancel{\frac{1}{n-1}}\right)$

$=-\frac{1}{3}\lim\limits_{n\to\infty}\left(\frac{1}{4}-\frac{1}{n-1}\right)=-\frac{1}{3}\cdot\frac{1}{4}=-\frac{1}{12}$

$\boxed{6}$ $\lim\limits_{n\to\infty}\left(\sum\limits_{k=2}^{n}\frac{12}{9k^2-12k-5}\right)=\lim\limits_{n\to\infty}\left(\frac{12\cdot 1}{(3k+1)(3k-5)}\right)=$

$=12\lim\limits_{n\to\infty}\left(\frac{1}{4\cdot(-2)}+\frac{1}{7\cdot 1}+\frac{1}{10\cdot 4}+\frac{1}{13\cdot 7}+\dots+\frac{1}{(3n+1)(3n-5)}\right)=$

$=12\lim\limits_{n\to\infty}\frac{1}{6}\left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{6}\left(\frac{1}{7}-\frac{1}{1}\right)+\frac{1}{6}\left(\frac{1}{10}-\frac{1}{4}\right)+\frac{1}{6}\left(\frac{1}{13}-\frac{1}{7}\right)+\dots+\frac{1}{6}\left(\frac{1}{3n+1}-\frac{1}{3n-5}\right)$

$=-\frac{1}{6}\cdot 12\lim\left(\left(\cancel{\frac{1}{4}}+\frac{1}{2}\right)+\left(\cancel{\frac{1}{7}}-\frac{1}{1}\right)+\left(\cancel{\frac{1}{10}}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{13}}-\cancel{\frac{1}{7}}\right)+\dots+\left(\cancel{\frac{1}{3n-1}}-\frac{1}{3n-5}\right)\right)=$

$=-2\lim\limits_{n\to\infty}\left(\frac{1}{2}-1-\frac{1}{3n-5}\right)=-2\cdot\left(-\frac{1}{2}\right)=1$

№ 1.149

$$\lim_{n\to\infty} \frac{10000n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{10000n}{n^2}}{\frac{n^2}{n^2}+\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\frac{10000}{n}}{1+\frac{1}{n^2}} =$$

$$= \left[\frac{0}{1}\right] = 0$$

№ 1.151

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4+n-1}}{n+2} = \lim_{n\to\infty}$$

№ 1.159.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{i(i+1)} = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{1\cdot 1}{i(i+1)} \right) =$$

$$= \lim_{n\to\infty} \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \dots + \frac{1}{n\cdot(n+1)} \right) =$$

$$= \lim_{n\to\infty} \left(\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \dots + \cancel{\frac{1}{n}} - \frac{1}{n+1} \right) =$$

$$= \lim_{n\to\infty} \left(1 - \frac{1}{n+1} \right) = 1 - 0 = 1$$