$= \lim\limits_{x \to \infty} \frac{(x-1)}{(5x-1)} \cdot \frac{(x-2)}{(5x-1)} \cdot \frac{(x-3)}{(5x-1)} \cdot \frac{(x-4)}{(5x-1)} \cdot \frac{(x-5)}{(5x-1)} = \boxed{\frac{1}{5^5}}$

$= \boxed{\left(\frac{3}{2}\right)^{10}}$

Домашня робота (з. 10)

№2.2.

$\lim\limits_{x \to 1} \frac{1}{(1-x)^2} = +\infty$

∇ Тр. дов. $\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x: \{|x-1| < \delta \Rightarrow \frac{1}{(1-x)^2} > \varepsilon\}$

$\frac{1}{(1-x)^2} = \frac{1}{(x-1)^2} = \frac{1}{\delta^2} > \varepsilon;\quad \delta^2 < \frac{1}{\varepsilon};\quad \delta < \frac{1}{\sqrt{\varepsilon}} \quad \Delta.$

№2.4

$\lim\limits_{x \to +\infty} \frac{1}{x} = 0$

∇ Тр. дов. $\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x: \{|x+\infty| < \delta \Rightarrow$

$\Rightarrow \frac{1}{x} > \varepsilon\};\quad \frac{1}{\delta} > \varepsilon;\quad \delta > \frac{1}{\varepsilon} \quad \Delta.$

№2.6

$\lim\limits_{x\to a+0} f(x) = b$ def $\forall \varepsilon > 0 \ \exists \delta = \delta(\varepsilon)$

$\forall x: \{x \in U_\delta(a+0) \Rightarrow f(x) \in U_\varepsilon(b)\}$

$\forall \varepsilon > 0 \quad \exists \delta = \delta(\varepsilon) > 0$

$\forall x: \{ a < x < a+\delta \Rightarrow f(x) < -\varepsilon \}$

№242

а) $\lim\limits_{x\to\infty} \dfrac{x^2-1}{2x^2-x-1} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{2x^2}{x^2} - \frac{x}{x^2} - \frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1-\frac{1}{x^2}}{2-\frac{1}{x}-\frac{1}{x^2}} = \boxed{\dfrac{1}{2}}$

б) $\lim\limits_{x\to 1} \dfrac{x^2-1}{2x^2-x-1} = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(2x+1)} = \boxed{\dfrac{2}{3}}$

$2x^2-x-1=0 \quad D=9 \quad x_1=1 \quad x_2=-1/2$

$2x^2-x-1 = 2(x-1)(x+\frac{1}{2}) = (x-1)(2x+1)$

в) $\lim\limits_{x\to 2} \dfrac{x^2-1}{2x^2-x-1} = \dfrac{2^2-1}{2\cdot4-2-1} = \dfrac{3}{5} = \boxed{0{,}6}$

№244

$\lim\limits_{x\to 0} \dfrac{(1+x)^5-(1+5x)}{x^2+x^5} = \lim\limits_{x\to 0} \dfrac{\cancel{1}+\cancel{5x}+10x^2+10x^3+5x^4+x^5-\cancel{1}-\cancel{5x}}{x^2(1+x^3)} =$

$= \lim\limits_{x\to 0} \dfrac{10x^2+10x^3+5x^4+x^5}{x^2(1+x^3)} = \lim\limits_{x\to 0} \dfrac{\cancel{x^2}(10+10x+5x^2+x^3)}{\cancel{x^2}(1+x^3)} =$

$= \lim\limits_{x\to 0} \dfrac{10+10x+5x^2+x^3}{1+x^3} = \dfrac{10+0+0+0}{1+0} = \boxed{10}$

№2.46

$\lim\limits_{x\to -2} \dfrac{x^3+3x^2+2x}{x^2-x-6} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to -2} \dfrac{x(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = \lim\limits_{x\to -2} \dfrac{(x+1)x}{x-3} =$

$= \dfrac{-2(-1)}{-5} = \boxed{-0{,}4}$

№2.50

$\lim\limits_{x\to 1} \dfrac{x^4-3x+2}{x^5-4x+3} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}(x^3+x^2+x-2)}{\cancel{(x-1)}(x^4+x^3+x^2+x-3)} =$

$= \lim\limits_{x\to 1} \dfrac{x^3+x^2+x-2}{x^4+x^3+x^2+x-3} = \dfrac{1+1+1-2}{1+1+1+1-3} = \boxed{1}$

$$\begin{array}{l|l} x^4-3x+2 & x-1 \\ \hline x^4-x^3 & x^3+x^2+x-2 \\ \quad x^3-3x & \\ \quad x^3-x^2 & \\ \quad\quad x^2-3x & \\ \quad\quad x^2-x & \\ \quad\quad -2x+2 & \\ \quad\quad -2x+2 & \\ \quad\quad\quad 0 & \end{array}$$

$$\begin{array}{l|l} x^5-4x+3 & x-1 \\ \hline x^5-x^4 & x^4+x^3+x^2+x-3 \\ \quad x^4-4x & \\ \quad x^4-x^3 & \\ \quad\quad x^3-4x & \\ \quad\quad x^3-x^2 & \\ \quad\quad\quad x^2-4x & \\ \quad\quad\quad x^2-x & \\ \quad\quad\quad -3x+3 & \\ \quad\quad\quad -3x+3 & \\ \quad\quad\quad\quad 0 & \end{array}$$

№2.53

$$\lim_{x\to-1}\frac{x^3-2x-1}{x^5-2x-1}=\left[\frac{0}{0}\right]=\lim_{x\to-1}\frac{(x+1)(x^2-x-1)}{(x^4-x^3+x^2-x-1)(x+1)}=$$

$$=\lim_{x\to-1}\left(\frac{x^2-x-1}{x^4-x^3+x^2-x-1}\right)=\frac{1+1-1}{1+1+1+1-1}=\boxed{\frac{1}{3}}$$

$$\begin{array}{l|l} x^3-2x-1 & x+1 \\ \hline x^3+x^2 & x^2-x-1 \\ \quad -x^2-2x & \\ \quad -x^2-x & \\ \quad\quad -x-1 & \\ \quad\quad -x-1 & \\ \quad\quad\quad 0 & \end{array}$$

$$\begin{array}{l|l} x^5-2x-1 & x+1 \\ \hline x^5+x^4 & x^4-x^3+x^2-x-1 \\ \quad -x^4-2x & \\ \quad -x^4-x^3 & \\ \quad\quad x^3-2x & \\ \quad\quad x^3+x^2 & \\ \quad\quad -x^2-2x & \\ \quad\quad -x^2-x & \\ \quad\quad\quad -x-1 & \\ \quad\quad\quad -x-1 & \\ \quad\quad\quad\quad 0 & \end{array}$$

№2.57

$$\lim_{x\to\infty}\frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(5x-1)^5}=\lim_{x\to\infty}\frac{(x-1)}{(5x-1)}\cdot\frac{(x-2)}{(5x-1)}\cdot\frac{(x-3)}{(5x-1)}\cdot\frac{(x-4)}{(5x-1)}\cdot\frac{(x-5)}{(5x-1)}$$

$$=\boxed{\frac{1}{5^5}}$$

№2.32

$y\to b-0$, якщо $x\to-\infty$

$$\Leftrightarrow \lim_{x\to-\infty} f(x)=b-0 \overset{def}{\Leftrightarrow} (\forall\varepsilon>0)(\exists\delta>0)\{\forall x: |x|>\delta\}:$$

$$\{b-\varepsilon<f(x)<b\}$$

№2.34

$y\to b+0$, якщо $x\to a \Leftrightarrow \lim_{x\to a} y(x)=b+0$

$\underline{def}$ $(\forall\varepsilon>0)\ \exists\delta=\delta(\varepsilon)>0$

$\forall x: \{x\in U_\delta(a) \Rightarrow y(x)\in U_\varepsilon(b+0)\}$

$\{a-\delta<x<a+\delta \Rightarrow b<y<b+\varepsilon\}$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(a) \Rightarrow f(x) \in U_{\varepsilon_1}(+\infty)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{a-\delta < x < a+\delta \Rightarrow f(x) > \varepsilon_1\}$

№2.8

$\lim\limits_{x \to -\infty} f(x) = b$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(-\infty) \Rightarrow f(x) \in U_{\varepsilon_1}(b)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{x < -\delta \Rightarrow g(x) \in U_{\varepsilon_1}(b)\}$

№2.16

$\lim\limits_{x \to a+0} f(x) = \infty$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(a+0) \Rightarrow f(x) \in U_{\varepsilon_1}(\infty)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{a < x < a+\delta \Rightarrow f(x) > \varepsilon_1\}$

№2.18

$\lim\limits_{x \to a+0} f(x) = -\infty$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(a+0) \Rightarrow f(x) < -\varepsilon_1\}$

$(\forall \varepsilon_1 > 0) \ (\exists \delta = \delta(\varepsilon_1) > 0) \ \forall x: \{a < x < a+\delta \Rightarrow f(x) < -\varepsilon_1\}$

№2.20

$\lim\limits_{x \to \infty} f(x) = -\infty$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(-\infty) \Rightarrow f(x) \in U_{\varepsilon_1}(\infty)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{x < -\delta \Rightarrow a - \varepsilon_1 < x < a + \varepsilon_1\}$

№2.22

$\lim\limits_{x \to -\infty} f(x) = \infty$

$\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(\infty) \Rightarrow f(x) \in U_{\varepsilon_1}(-\infty)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{a - \delta < x < a+\delta) \Rightarrow x < -\varepsilon_1\}$

№2.24

$\lim\limits_{x \to -\infty} f(x) = +\infty$ $\underline{def}$ $\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) \ \forall x: \{x \in U_\delta(+\infty) \Rightarrow f(x) \in U_{\varepsilon_1}(-\infty)\}$

$\forall \varepsilon_1 > 0 \ \exists \delta = \delta(\varepsilon_1) > 0 \ \forall x: \{x > \delta \Rightarrow x < -\varepsilon_1\}$

№ 2.26

$$\lim_{x \to +\infty} f(x) = -\infty$$

def $\forall \varepsilon_\varepsilon > 0 \ \exists \delta = \delta(\varepsilon_\varepsilon) \ \forall x : \{ x \in U_\delta(-\infty) \Rightarrow f(x) \in U_{\varepsilon_\varepsilon}(+\infty) \}$

$\forall \varepsilon_\varepsilon > 0 \ \exists \delta = \delta(\varepsilon_\varepsilon) > 0 \ \forall x : \{ x < -\delta \Rightarrow x > \varepsilon_\varepsilon \}$