Заняття 11

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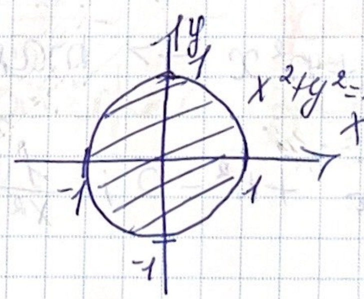
$\boxed{№3139}$

$u = \dfrac{1}{\sqrt{x^2+y^2-1}}$ $\qquad D(u)$ — ?



$u(x;y)$

$x^2+y^2-1>0$

$x^2+y^2>0$

$D(u) = \{(x;y) \mid x^2+y^2>1\}$

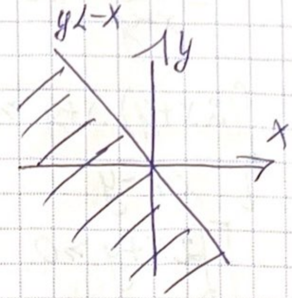
$\boxed{№3143}$

$U = \ln(-x-y)$; $D(U)$ — ?

$U(x;y)$

$-x-y > 0 \Leftrightarrow -x > y$

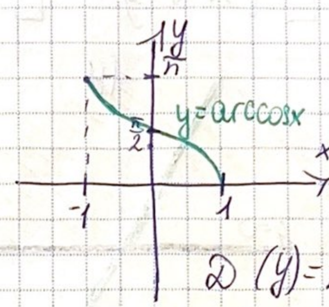
$x < -y \qquad y < -x$



$D(U) = \{(x;y) \mid y < -x\}$

$\boxed{№3145}$

$U = \arccos \frac{x}{x+y}$; $D(U)$ — ?



$D(y) = [-1;1]$

$E(y) = [\pi;0]$

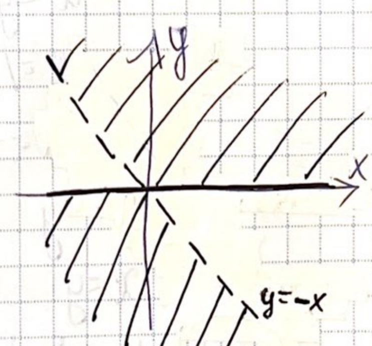
$-1 \le \frac{x}{x+y} \le 1$

$\begin{cases} \frac{x}{x+y} \le 1 \\ \frac{x}{x+y} \ge -1 \end{cases}$ $\quad\begin{cases} \frac{x}{x+y} - 1 \le 0 \\ \frac{x}{x+y} + 1 \ge 0 \end{cases}$

(1) (2) $\begin{cases} \frac{-y}{x+y} \le 0 \\ \frac{2x+y}{x+y} \ge 0 \end{cases}$

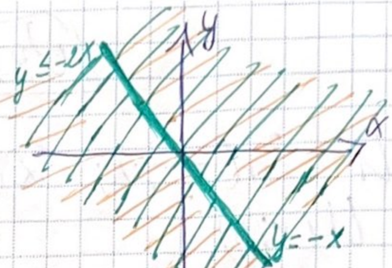
(1) $\frac{-y}{x+y} \le 0$

$\begin{cases} x \ne -y \\ -y(x+y) \le 0 \end{cases}$ $\quad\left[\begin{array}{l} \begin{cases} x > -y \\ -y \ge 0 \end{cases} \\ \begin{cases} x < -y \\ -y \le 0 \end{cases} \end{array}\right.$ $\quad\left[\begin{array}{l} \begin{cases} x > -y \\ y \le 0 \end{cases} \\ \begin{cases} x < -y \\ y \ge 0 \end{cases} \end{array}\right.$



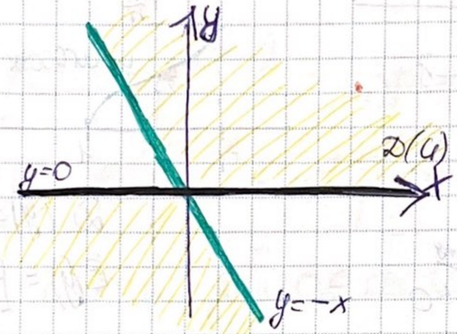
$\left[\begin{array}{l} \begin{cases} y < -x \\ y \le 0 \end{cases} \\ \begin{cases} y > -x \\ y \ge 0 \end{cases} \end{array}\right.$

② $\frac{2x+y}{x+y} \geq 0$



$$\begin{cases} (2x+y)\,x+y \geq 0 \\ x \neq -y \end{cases}$$

$$\left[\begin{array}{l} \begin{cases} 2x+y \geq 0 \\ x > -y \end{cases} \\ \begin{cases} 2x+y \leq 0 \\ x < -y \end{cases} \end{array}\right. \quad \left[\begin{array}{l} \begin{cases} 2x \geq -y \\ x > -y \end{cases} \\ \begin{cases} 2x \leq -y \\ x < -y \end{cases} \end{array}\right. \quad \left[\begin{array}{l} \begin{cases} x \geq -\frac{y}{2} \\ x > -y \end{cases} \\ \begin{cases} x \leq -\frac{y}{2} \\ x < -y \end{cases} \end{array}\right. \quad \left[\begin{array}{l} \begin{cases} y < -x \\ y \leq -2x \end{cases} \\ \begin{cases} y > -x \\ y \geq -2x \end{cases} \end{array}\right.$$



$D(4) = \{(x;y) \mid -1 \leq \frac{x}{x+y} \leq 1\}$

$\boxed{№3153}$

$z = x^2 - y^2$; лінії рівня — ?

$z \geq 0$

$z=0$: $0 = x^2 - y^2$

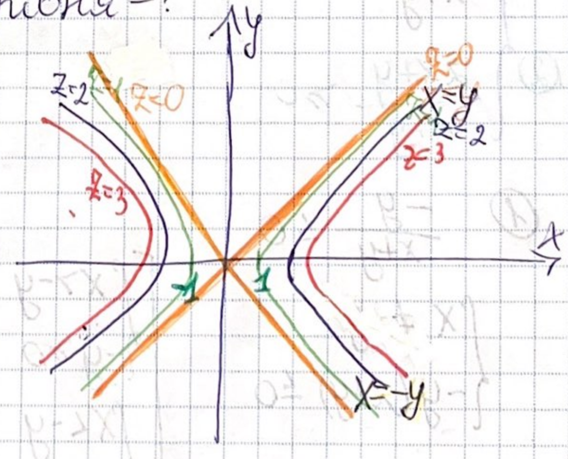
$x^2 = y^2$

$x = |y|$

$x = -|y|$

$x = -y$

$x = y$



$z=1$: $1 = x^2 - y^2$

$x^2 = 1 + y^2$

$x = \sqrt{1+y^2}$ $\quad x = -\sqrt{y^2+1}$

$z=2$: $x^2 - y^2 = 2$

$x = \sqrt{y^2+2}$

$x = -\sqrt{y^2+2}$

$z=3$: $x^2-y^2=3$

$x=\sqrt{y^2+3}$

$x=-\sqrt{y^2+3}$

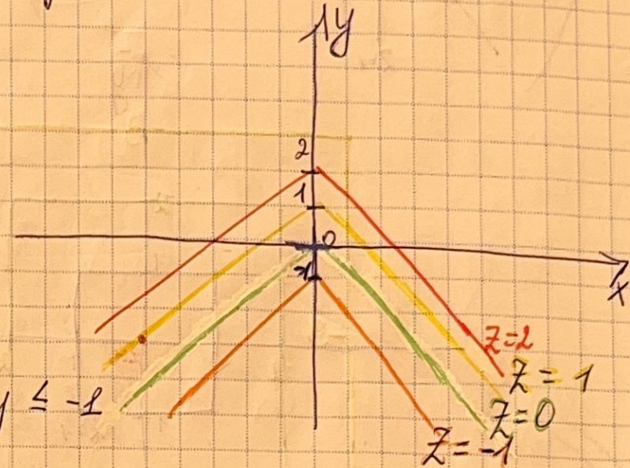
$\boxed{N3158}$

$z=|x|+y$; лінії рівня-?

$x=y-z$;

$x=z-y,\ y\le z$

$z\in\mathbb{R}$



$z=-1$: $|x|+y=-1$

$x=y+1$

$x=-(y+1),\ y\le -1$

$z=0$: $|x|+y=0$

$x=y$

$x=-y,\ y\le 0$

$z=1$: $|x|+y=1$

$x=y-1$

$x=1-y,\ y\le 1$

$z=2$: $|x|+y=2$

$x=y-2$

$x=2-y,\ y\le 2$

$\boxed{N3159.2}$

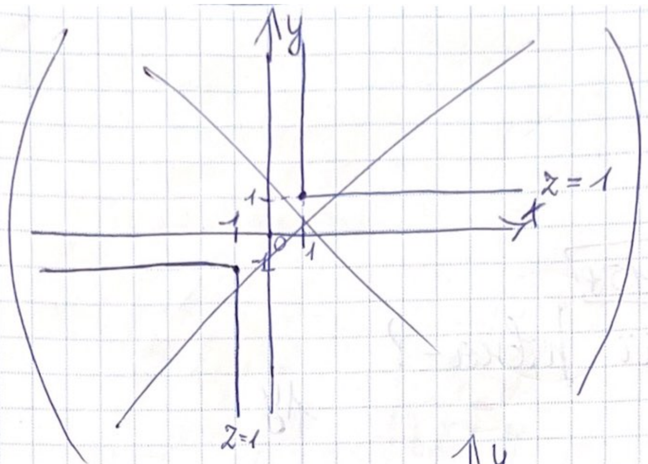
$z=\max(|x|,|y|)$; лінії рівня-?

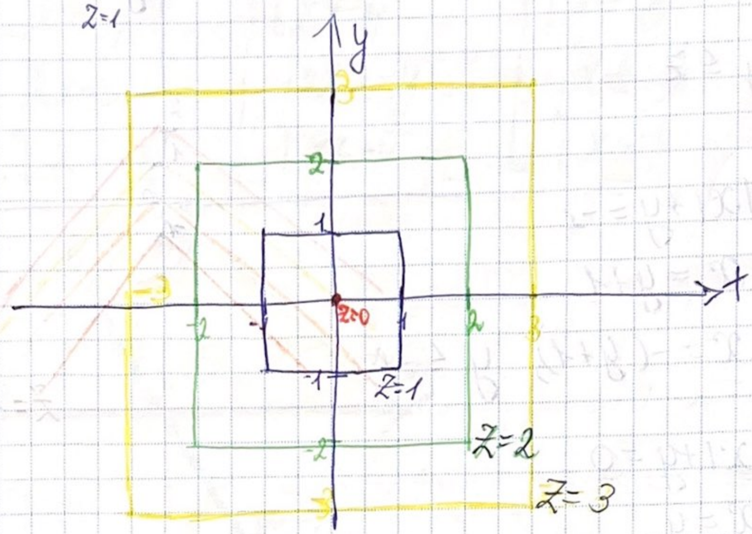
$z\in[0;+\infty)$

$z=1$: $\max(|x|,|y|)=1$

$$\begin{cases} x=\pm 1;\ -1\le y\le 1 \\ y=\pm 1;\ -1\le x\le 1 \end{cases}$$

$$\begin{cases} x=\pm 1 \\ y=\pm 1 \end{cases}$$





z=2 : $\max(|y|, |x|) = 2$; $\begin{cases} x = \pm 2 \\ y = \pm 2 \end{cases}$

z=3 : $\max(|x|, |y|) = 3$;

$$\begin{cases} x = \pm 3 \\ y = \pm 3 \end{cases}$$

z=0 : $\max(|x|, |y|) = 0$

$$\begin{cases} x = 0 \\ y = 0 \end{cases}$$

$\boxed{№3182}$

$$f(x,y) = \frac{x^2y^2}{x^2y^2 + (x-y)^2}$$

Показати, що

1) $\exists \lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\dfrac{\overset{=0}{x^2y^2}}{x^2\underset{0}{y^2}+(x-\underset{0}{y})^2}\right)=\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\dfrac{0}{x^2}\right)=0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ||$

2) $\exists \lim\limits_{y\to 0}\left(\lim\limits_{x\to 0}\dfrac{\overset{=0}{x^2y^2}}{\underset{0}{x^2}y^2+(\underset{0}{x}-y)^2}\right)=\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0}\dfrac{0}{y^2}\right)=0$

3) $\nexists \lim\limits_{\substack{x\to 0\\ y\to 0}}\left(\dfrac{x^2y^2}{x^2y^2+(x-y)^2}\right)$:

$(x_n, y_n)=P\left(\frac{1}{n};\frac{1}{n}\right)\xrightarrow[n\to\infty]{} O(0;0)$

$\lim\limits_{n\to\infty} f(x_n,y_n)=\lim\limits_{n\to y}\dfrac{\frac{1}{n^4}}{\frac{1}{n^4}}=1$

$(x'_n, y'_n)=\left(\frac{1}{n};\frac{1}{n}\right)\to(0;0)$

$\lim\limits_{n\to\infty} f(x'_n,y'_n)=\lim\limits_{n\to\infty}\dfrac{\frac{1}{n^4}}{\frac{1}{n^4}+\frac{4}{n^2}}=\lim\limits_{n\to\infty}\dfrac{\cancel{n^4}}{\cancel{n^4}(n^2+4)}=0$

Отже, $\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)$ не існує

$\boxed{№3184}$

(б) Знайти 1) $\lim\limits_{x\to a}\left(\lim\limits_{y\to b} f(x,y)\right)$

2) $\lim\limits_{y\to b}\left(\lim\limits_{x\to a} f(x,y)\right)$

$f(x,y)=\dfrac{x^y}{1+x^y}$; $a=\infty$; $b=+0$

1) $\lim\limits_{x\to\infty}\left(\lim\limits_{y\to +0}\left(\dfrac{x^y}{1+x^y}\right)\right)=\lim\limits_{x\to\infty}\left(\lim\limits_{y\to +0}\underbrace{\dfrac{x^0}{1+x^0}}_{=\frac{1}{2}}\right)=$

$=\lim\limits_{x\to\infty}\left(\lim\limits_{y\to +0}\dfrac{1}{2}\right)=\lim\limits_{x\to\infty}\dfrac{1}{2}=\boxed{\dfrac{1}{2}}$

2) $\lim\limits_{y\to +0}\left(\lim\limits_{x\to\infty}\left(\dfrac{x^y}{1+x^y}\right)\right)=\lim\limits_{y\to +0} 1=\boxed{1}$

(в) $f(x,y)=\sin\frac{\pi x}{2x+y}$, $a=\infty$, $b=\infty$

1) $\lim\limits_{x\to\infty}\left(\lim\limits_{y\to\infty}\sin\frac{\pi x}{2x+y}\right)=\lim\limits_{x\to\infty}(\sin 0)=\boxed{0}$

$\frac{\pi x}{2x+y}\to 0$

2) $\lim\limits_{y\to\infty}\left(\lim\limits_{x\to\infty}\sin\frac{\pi x}{2x+y}\right)=\lim\limits_{y\to\infty}\left(\sin\frac{\pi}{2}\right)=\boxed{1}$

$\frac{\pi x}{2x+y}\to\frac{\pi}{2}$

(д) $f(x,y)=\log_x(x+y)$, $a=1$, $b=0$

1) $\lim\limits_{x\to 1}\left(\lim\limits_{y\to 0}\left(\log_x(x+y)\right)\right)=\lim\limits_{x\to 1}\underbrace{\left(\log_x x\right)}_{=1}=\lim\limits_{x\to 1}1=\boxed{1}$

2) $\lim\limits_{y\to 0}\left(\lim\limits_{x\to 1}\left(\log_x(x+y)\right)\right)=\lim\limits_{y\to 0}\left(\log_1(1+y)\right)=$

$=\lim\limits_{y\to 0}\frac{\ln 1+y}{\ln 1}=\lim\limits_{y\to 0}\frac{\overset{\to 1}{\ln(1+y)}\cdot\overset{\to 0}{y}}{\underset{=\ln 1}{\ln 1}}=\frac{0}{\ln 1}=\boxed{\infty}$

$\boxed{№3186}$

$\lim\limits_{\substack{x\to\infty\\y\to\infty}}\left(\frac{x^2+y^2}{x^4+y^4}\right)=\lim\limits_{\substack{x\to\infty\\y\to\infty}}\left(\frac{\cancel{x^4}\left(\frac{1}{x^2}+\frac{y^2}{x^4}\right)}{\cancel{x^4}\left(1+\frac{y^4}{x^4}\right)}\right)=$

$=\lim\limits_{\substack{x\to\infty\\y\to\infty}}\left(\frac{\overset{\to 0}{\frac{1}{x^2}}+\overset{\to\infty}{\frac{y^2}{x^4}}}{\underset{\to 1}{1}+\underset{\to\infty}{\frac{y^4}{x^4}}}\right)=\boxed{0}$

$\boxed{№3187}$

$\lim\limits_{\substack{x\to 0\\y\to a}}\frac{\sin xy}{x}=\lim\limits_{\substack{x\to 0\\y\to a}}\overset{\to 1}{\frac{\sin \overset{\to 0}{xy}}{\underset{\to 0}{xy}}}\cdot\overset{\to a}{y}=1\cdot a=a$

$\boxed{№3192}$

$$\lim_{\substack{x\to1\\y\to0}} \frac{\ln(x+e^{y})}{\sqrt{x^2+y^2}} = \lim_{\substack{x\to1\\y\to0}} (\ln 2) = \boxed{\ln 2}$$

$\boxed{N 31.94}$

$$u = \frac{1}{\sqrt{x^2+y^2}}$$

$D(u)$: $x^2+y^2>0$

$D(u) = \{(x;y) \in \mathbb{R}^2 \mid x^2+y^2>0\}$

$$\lim_{x^2+y^2\to0} \left(\frac{1}{\sqrt{x^2+y^2}}\right) = \lim_{x^2+y^2\to0} 1 = 1$$

Ф-я має розрив I роду в точці кола $x^2+y^2=0$

$\boxed{N 1.30}$

$f(x,y) = \frac{1}{xy} tg \frac{xy}{1+xy}$, $a=0$, $b=\infty$

1) $\lim_{x\to0}\left(\lim_{y\to\infty}\left(\frac{1}{xy} tg\frac{xy}{1+xy}\right)\right) =$

2) $\lim_{y\to\infty}\left(\lim_{x\to0}\left(\frac{1}{xy} tg \frac{xy}{1+xy}\right)\right) = \lim_{y\to\infty} 1 = 1$

$\boxed{N 1.34}$

$$1 \le \lim_{\substack{x\to0\\y\to0}} (x^2+y^2)^{x^2y^2} \le \lim_{\substack{x\to0\\y\to0}} (x^2+y^2)^{\frac{1}{4}(x^2+y^2)^2} = \left| \begin{array}{l} \text{Зам:} \\ x^2+y^2=t \\ t\to0 \end{array} \right| = \lim_{t\to0} t^{\frac{1}{4}t^2} = \lim_{t\to0} e^{\ln t^{\frac{1}{4}t^2}} =$$

$$= \lim_{t\to0} e^{\frac{1}{4}t^2\ln t} = e^{\frac{1}{4}\lim_{t\to0} t^2\ln t} = e^{\frac{1}{4}\lim_{t\to0}\frac{\ln t}{t^{-2}}} = e^{\frac{1}{4}\lim_{t\to0}\frac{\frac{1}{t}}{-2t^{-3}}} = \text{лопіталя} =$$

$$= e^{\frac{1}{4}\lim_{t\to0}\frac{\frac{1}{t}}{-2t^{-3}}} = e^{-\frac{1}{8}\lim_{t\to0} t^2} = e^0 = \boxed{1}$$

$0 \le (x-y)^2$

$0 \le x^2 - 2xy + y^2$

$2xy \le x^2+y^2 \mid :2$

$xy \le \frac{1}{2}(x^2+y^2)$

$x^2y^2 \le \frac{1}{4}(x^2+y^2)^2$