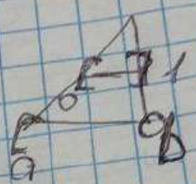
4) Дано:
$[0;1]$
$[a,b)$, $a<b$

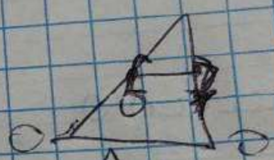
Довести:
$[0;1] \sim [a,b)$, $a<b$



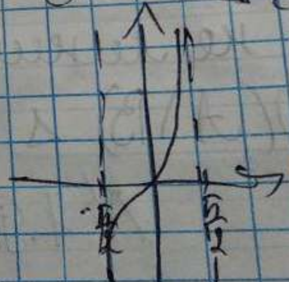
▽ f – біекція
$f: [0;1] \to [a;b]$, $a<b$
$0 \xrightarrow{f} a$
решта $f(x)=x$ △

5) Дано:
$[0;1]$
$\mathbb{R}$

Довести:
$[0;1] \sim \mathbb{R}$



▽ f – біекція
$f: \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \to \mathbb{R}$ $f = tg\,x$ – біекція



$\forall y \in \mathbb{R}$ $\exists x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

№ 1130

$X = \{x_n = \frac{(-1)^n}{n} + \frac{1+(-1)^n}{2} ; n \in \omega\}$

$\exists \max X - ?$ $\exists \min X - ?$ $\sup X - ?$ $\inf X - ?$

$x_1 = -1$

$x_2 = \frac{3}{2} = 1{,}5$

$x_3 = -\frac{1}{3}$

$x_4 = \frac{5}{4}$

$x_5 = -\frac{1}{5}$

$x_6 = \frac{7}{6}$

$x_7 = -\frac{1}{7}$

$\exists \min X = -1$

$\exists_{\max} X = 1{,}5$

$\inf X = -1$

$\sup X = 1{,}5$

№ 1132

$X = \{x_n = 1 + \sin \frac{n\pi}{2} ; n \in \mathbb{Z}\}$

$\exists \max X - ?$ $\exists \min X - ?$ $\sup X - ?$ $\inf X - ?$

$x_1 = 1 + \sin \frac{\pi}{2} = 1+1 = 2$

$x_2 = 1 + \sin \frac{2\pi}{2} = 1 + \sin \pi = 1 + 0 = 1$

$x_3 = 1 + \sin \frac{3\pi}{2} = 1 - 1 = 0$

$x_4 = 1 + \sin \frac{4\pi}{2} = 1 + 0 = 1$

$x_5 = 1 + \sin \frac{5\pi}{2} = 1 + 1 = 2$

$x_6 = 1 + \sin \frac{6\pi}{2} = 1 + 0 = 1$

$x_7 = 1 + \sin \frac{7\pi}{2} = 1 - 1 = 0$

$x_8 = 1 + \sin \frac{8\pi}{2} = 1 + 0 = 0$

$\exists \max X = 2$

$\exists \min X = 0$

$\sup X = 2$

$\inf X = 0$

№1.119.

Дано:
$A \subset \mathbb{R}$
$B \subset \mathbb{R}$
$A \neq \varnothing$
$M = \sup A$
$m = \inf A$

Знайти: $\sup B$
$\inf B$

Розв.:
$\sup B = (\max A)^3$
$\inf B = (\min A)^3$

№1.121.

Дано:
$A \subset \mathbb{R}$
$B \subset \mathbb{R}$
$A \neq 0$
$M = \sup A$
$m = \inf A$

Знайти: $\sup B$, $\inf B$

Розв.:
$\sup B = M + a,\ a \in \mathbb{R}$
$\inf B = m + a,\ a \in \mathbb{R}$

№1.126

$\inf\{x^2+3 : x \in \mathbb{R}\} = 3$

$\inf\{x^2+3 : x \in \mathbb{R}\} = 3 \overset{def}{\Leftrightarrow} \begin{cases} 1)\ \forall a \in \{x^2+3 : x \in \mathbb{R}\}\ a \geq 3 \\ 2)\ (\forall \varepsilon > 0)\ (\exists a^* \in \{x^2+3 : x \in \mathbb{R}\})\ \{a^* < 3 + \varepsilon\} \end{cases}$

№1.127.

$\sup\{\sin x : x \in \mathbb{R}\} = 1$

$\sup\{\sin x : x \in \mathbb{R}\} = 1 \Leftrightarrow \begin{cases} 1)\ \forall a \in \{\sin x : x \in \mathbb{R}\}\ \{a \leq 1\} \\ 2)\ (\forall \varepsilon > 0)(\exists a^* \in \{\sin x : x \in \mathbb{R}\}) : \{a^* > 1 - \varepsilon\} \end{cases}$

№1.128

$\sup A$ – ? $\inf A$ – ? $\exists \max A$ – ? $\exists \min A$ – ?

$A = \left\{\dfrac{n^2}{3n^2+2} ;\ n \in \mathbb{N}\right\}$

$\left\{ \cancel{n=-4\ \ \frac{8}{25}} \quad \cancel{n=-3\ \ \frac{9}{29}} \quad \cancel{n=-2\ \ \frac{2}{7}} \quad \cancel{n=-1\ \ \frac{1}{5}} \right\}$

$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
0	$\frac{1}{5}$	$\frac{2}{7}$	$\frac{9}{29}$	$\frac{8}{25}$

$n=5$	$n=6$	$n=7$	$n=8$
$\frac{25}{77}$	$\frac{18}{55}$	$\frac{49}{149}$	$\frac{32}{97}$

$\nexists \max A$
$\exists \min A = 0{,}2$

$\sup A = 1$
$\inf A = 0{,}2$

Домашня робота (з 4)

№1.93

Дано: $A \sim \infty$

Довести: $A \sim$ вдр. по$\{1\}$

▽ За лемою 1.95 $\exists B \subset A$: B — зліченне

B — вдр. по$\{1\}$

$A = B \cup (A \setminus B)$ △

№1.94

Дано: $A \subset \mathbb{R}$

Довести: A не більш як зліченна

▽ За лемою 1.95 $\exists B \subset A \{B \in$ зліченна$\} \Rightarrow$

$\exists A \subset \mathbb{R}: \{A \in$ зліченна$\}$

№1.98.

$[0;1]$

Дано:

1) $[0;1]$

$(0;1)$

Довести

$[0;1] \sim (0;1)$

▽ f — біекція $f: [0;1] \to (0;1)$

$0 \xmapsto{f} \frac{1}{2}$ $\frac{1}{n} \xmapsto{f} \frac{1}{n+2}$, $n \geq 2$, $n \in \mathbb{N}$

$1 \xmapsto{f} \frac{1}{3}$ інші числа $f(x) = x$

△

2) Дано:

$[0;1]$

$[0;1] \setminus \mathbb{Q}$

Довести

$[0;1] \sim [0;1] \mathbb{Q}$

▽

$\frac{m}{n} \xmapsto{f} a$ $\frac{m}{n} \in \mathbb{Z}$, $a \in \mathbb{N}$

3) Дано:

$[0;1]$

$[a;b]$, $a < b$

Довести:

$[0;1] \sim [a;b]$, $a < b$

▽ f — біекція $f: [0;1] \to [a;b]$, $a < b$

$0 \xmapsto{f} a$ $1 \xmapsto{f} b$ інші $f(x) = x$ △

Домашня робота (35)

№1.103

Дано: $A = \{a_n = n^2 2^{-n};\ n \in \mathbb{N}\}$

Довести: $\exists \min A$, $\exists \max A$

$A = \{\frac{1}{2};\ 1;\ \frac{9}{8};\ 1;\ \frac{25}{32};\ \frac{9}{16};\ \frac{49}{128};\ \frac{1}{4};\ \}$

$n=1:\ \frac{1}{2}$

$n=2:\ 2^2 \cdot 2^{-2} = 1$

$n=3:\ 3^2 \cdot 2^{-3} = \frac{9}{8}$

$n=4:\ 4^2 \cdot 2^{-4} = 1$

$n=5:\ 5^2 \cdot 2^{-5} = \frac{25}{32}$

$n=6:\ 6^2 6^{-2} = \frac{9}{16}$

$n=7:\ 7^2 \cdot 2^{-7} = \frac{49}{128}$

$n=8:\ 8^2 \cdot 2^{-8} = \frac{1}{4}$

$\exists \max A = 1$

$\nexists \min A$

№1.105

$A = \{(-1)^n (1 - \frac{1}{n}) \mid n \in \mathbb{N}\}$;

$\inf A$ - ? $\sup A$ - ?

$A = \{0;\ \frac{1}{2};\ -\frac{2}{3};\ \frac{3}{4};\ -\frac{4}{5};\ \frac{5}{6};\ \dots\}$

$\sup A = 1$ $\quad \inf A = -1$

№1.106

$A = \{\frac{n}{n+3} (2 + (-1)^n) \mid n \in \mathbb{N}\}$;

$A = \{0;\ \frac{1}{4};\ \frac{6}{5};\ \frac{1}{2};\ \frac{5}{8};\ 2 \dots\}$

$n=1:\ \frac{1}{4}(2-1) = \frac{1}{4}$

$n=2:\ \frac{2}{5} \cdot 3 = \frac{6}{5}$

$n=3:\ \frac{3!}{6^2} = \frac{1}{2}$

$n=4:\ \frac{4}{7}(2+1) = \frac{12}{7}$

$n=5:\ \frac{5}{8}$

$n=6:\ \frac{\cancel{6}^2}{\cancel{9}} \cdot \cancel{8} = 2$

$n=7:\ \frac{7}{10}$

$n=8:\ \frac{8}{11}$

$\sup A = +\infty$

$\inf A = \frac{1}{4}$