Математичний аналіз

ПМО-11.

Папіж Вікторії

Екзаменаційний білет №1. (практична част.)

1. а) $\lim\limits_{x \to 5} \frac{x^2-3x-10}{2x^2-4x-30} = \frac{25-15-10}{50-20-30} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 5} \frac{x^2+2x-5x-10}{2x^2-4x-30} =$

$= \frac{x(x+2)-5(x+2)}{2(x^2-2x-15)} = \frac{(x+2)(x-5)}{2(x^2+3x-5x-15)} = \frac{(x+2)(x-5)}{2(x(x+3)-5(x+3)} =$

$\frac{(x+2)\cancel{(x-5)}}{2(x+3)\cdot\cancel{(x-5)}} = \frac{x+2}{2(x+3)} = \lim\limits_{x\to 5} \frac{5+2}{2(5+3)} = \boxed{\frac{7}{16}}$

б) $\lim\limits_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 7} \frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3}} =$

$= \frac{4-(x-3)}{(x-7)(x+7)(2+\sqrt{x-3}} = \frac{7-x}{(x-7)(x+7)(2+\sqrt{x-3}} = \frac{-\cancel{(x-7)}}{\cancel{(x-7)}(x+7)(2+\sqrt{x-3}} =$

$\lim\limits_{x\to 7} \frac{-1}{(x+7)(2+\sqrt{x-3}} = \frac{-1}{(7+7)(2+\sqrt{7-3}} = \boxed{-\frac{1}{56}}$

в) $\lim\limits_{x\to\infty} \left(\frac{2x-1}{2x+1}\right)^{2x-3} = \left[\cancel{\frac{\infty}{\infty}}\right]\left[\infty^{\infty}\right] = \lim\limits_{x\to\infty}\left(1+\left(-\frac{2}{2x+1}\right)\right)^{2x-3} =$

$= \left(1+\left(-\frac{2}{2x+1}\right)\right)^{-\left(\frac{2x+1}{2}\right)\left(\frac{(2x+3)\cdot 2}{2x+1}\right)} = e^{-\left(\frac{4x-6}{2x+1}\right)} = \boxed{e^{-2}}$

г) $\lim\limits_{x\to 0} \frac{\sin 8x \cdot \operatorname{tg} 5x}{2x^2} = \left[\frac{0}{0}\right] \stackrel{L}{=} \frac{(\sin 8x \cdot \operatorname{tg} 5x)'}{(2x^2)'} =$

$\frac{\cos 8x \cdot 8 \operatorname{tg} 5x + \sin 8x \cdot \frac{5}{\cos^2 5x}}{4x} =$

$$= \frac{4\sin 10x \cdot \cos 8x + 5\sin 8x}{\cos^2 5x \cdot 4x} = \frac{1}{} \lim_{x\to 0} \frac{(4\cos 10x \cdot 10\cos 8x + 4\sin 10x)}{4\cos^2 5x - 20x \cdot \sin 10x}$$

$$\frac{8(-\sin 8x) + 8(5\cos 8x)}{} = \frac{4(10\cos 10x \cdot \cos 8x - 8\sin 10x \cdot \sin 8x + 10\cos 8x}{4(\cos^2 5x - 5x \cdot \sin 10x)}$$

$$= \frac{10\cos 10x \cdot \cos 8x - 8\sin 10x \cdot \sin 8x + 10\cos 8x}{\cos^2 5x - 5x \cdot \sin 10x} = \frac{10+10}{1} =$$

$\boxed{20}$

② a) $y = (28x+12)^{2020}$

$$y' = ((28x+12)^{2020})' = 2020(28x+12)^{2019} \cdot (28+12)' =$$

$$2020(28x+12)^{2019} \cdot 28 = 56\,560\,(28x+12)^{2019}.$$

б) $y = e^{\cos(5x+2)}$

$$y' = (e^{\cos(5x+2)})' = e^{\cos(5x+2)} \cdot (-\sin(5x+2))(5x+2)' =$$

$$e^{\cos(5x+2)} \cdot (-\sin(5x+2)) \cdot 5 = -5e^{\cos(5x+2)} \cdot \sin(5x+2)$$

③ $y = \frac{x^2-6x+13}{x-3}$

1) $D(y) : \mathbb{R} \setminus \{3\}$

2) неперіодична, ні парна, ні непарна ⇒ загального типу.

$x = 3$ - т. розриву II роду.

3) a) $x = 3$ - вертикальна асимптота.

б) $\exists k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2-6x+13}{x(x-3)} = \frac{x^2-6x+13}{x^2-3x} = \boxed{1}$

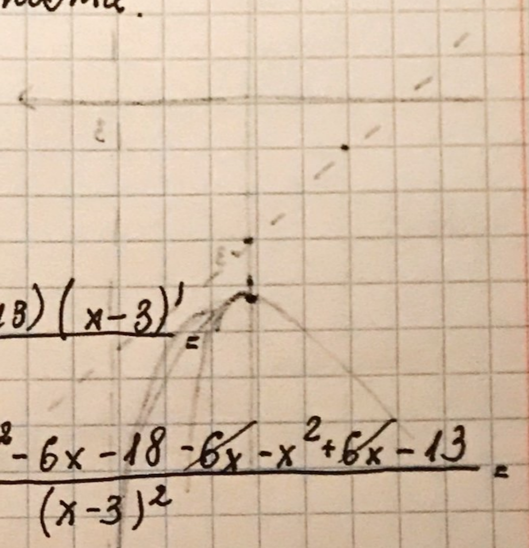
$$\exists b = \lim_{x\to\infty} f(x) - k(x) = \lim_{x\to\infty} \frac{x^2-6x+13}{x-3} - 1x = \frac{x^2-6x+13-x+3}{x-3} = \frac{x^2-7x+16}{x-3} =$$

$y = kx + b$

$y = x - 3$ – похила асимптота.

④ $x = 0 \Rightarrow \left(0; -\frac{13}{3}\right)$

$y = 0 \Rightarrow x \notin \mathbb{R}$



⑤ т. extr, монотонність

$$f'(x) = \frac{(x^2-6x+13)'(x-3) - (x^2-6x+13)(x-3)'}{(x-3)^2} =$$

$$= \frac{(2x-6)(x-3) - (x^2-6x+13)}{(x-3)^2} = \frac{2x^2-6x-18\cancel{-6x}-x^2\cancel{+6x}-13}{(x-3)^2} =$$

$$\frac{x^2-6x+5}{(x-3)^2}$$

$f'(x) = 0$

$$\frac{x^2-6x+5}{(x-3)^2} = 0$$

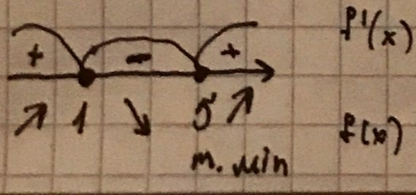
$$x^2 - 6x + 5 = 0$$

$6 \quad +5$

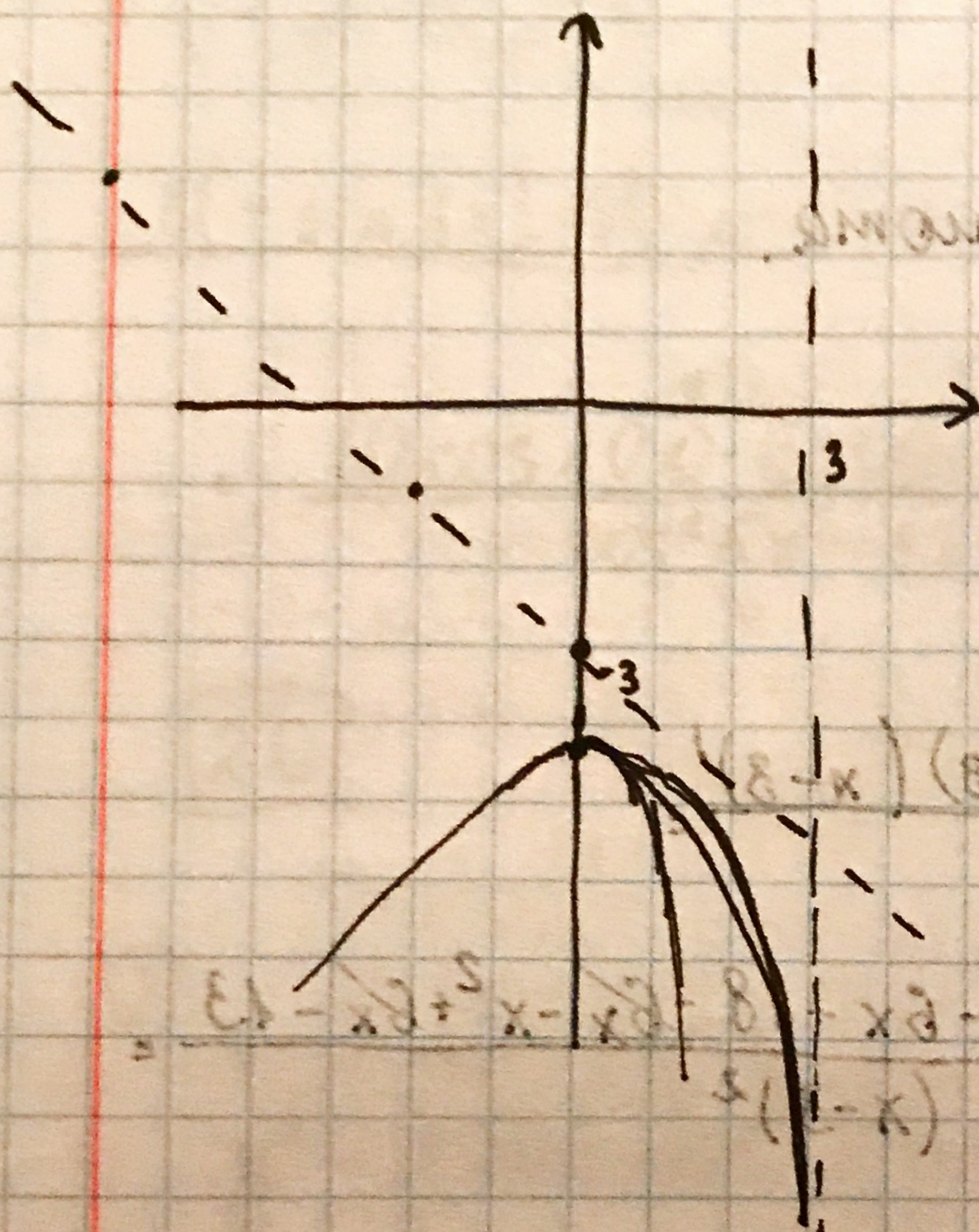
За т. Вієта: $x_1 = 5$, $x_2 = 1$ – критичні точки

$f'(x) > 0 \qquad \frac{x^2-6x+6}{(x-1)^3} > 0$

$$\begin{cases} \frac{x^2-6x+5}{(x-3)^2} > 0 \\ x \neq 3 \end{cases}$$



$f_{min} = f(5) = \frac{25-30+5}{4^3} = 0$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