Домашнє робота (за з. 12)

№ 2.101

$$\lim_{x\to\pi} \frac{\sin mx}{\sin nx},\ n, m \in N$$

$$\lim_{x\to\pi} \frac{\sin mx}{\sin nx} = \lim_{x\to\pi} \overbrace{\frac{\sin mx}{\sin nx}}^{\to 1} \cdot \frac{n}{m} = \boxed{\frac{n}{m}}$$

№ 2.103

$$\lim_{x\to 0} \frac{\overset{\to \text{обм.}}{\operatorname{tg} x}}{\underset{\to \infty}{x}} = \left[\frac{\text{обм.}}{\infty}\right] = \boxed{0}$$

№2.106

$$\lim_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}=\lim_{x\to 0}\frac{2\cos 4x\sin x}{\sin x}=\lim_{x\to 0}2\cos 4x=\boxed{2}$$

№2.108

$$\lim_{x\to \pi/4} tg\,2x\cdot tg\left(\frac{\pi}{4}-x\right)=\lim_{x\to\pi/4}\left(\frac{\sin 2x}{\cos 2x}\cdot\frac{\sin(\pi/4-x)}{\cos(\pi/4-x)}\right)=$$

$$=\lim_{x\to\pi/4}\frac{\sqrt{2}\cos 2x\cos x-\sqrt{2}\cos 2x\sin x-\frac{\sqrt{2}\sin 2x\cos x+\sqrt{2}\sin 2x\sin x}{2}}{-\sqrt{2}\sin 2x\cos x-\sqrt{2}\sin 2x\sin x+\frac{\sqrt{2}\cos 2x\cos x-\sqrt{2}\cos 2x\sin x}{2}}=$$

$$=\lim_{x\to\pi/4}\frac{\frac{2\sqrt{2}\cos 2x\cos x-2\sqrt{2}\cos 2x\sin x-(\sqrt{2}\sin 2x\cos x+\sqrt{2}\sin 2x\sin x)}{2}}{\frac{-2\sqrt{2}\sin 2x\cos x-2\sqrt{2}\sin 2x\sin x+\sqrt{2}\cos 2x\cos x-\sqrt{2}\cos 2x\sin x}{2}}=$$

$$=\lim_{x\to\pi/4}\frac{\sqrt{2}(2\cos 2x\cos x-2\cos 2x\sin x-\sin 2x\cos x-\sin 2x\sin x)}{\sqrt{2}(-2\sin 2x\cos x-2\sin 2x\sin x+\cos 2x\cos x-\cos 2x\sin x}=$$

$$=\lim_{x\to\pi/4}\frac{2\cos\frac{\pi}{2}\cdot\frac{\sqrt{2}}{2}-2\cos\pi/2\cdot\frac{\sqrt{2}}{2}-\sin\pi/2\,\frac{\sqrt{2}}{2}-\sin\pi/2\,\frac{\sqrt{2}}{2}}{-2\sin\pi/2\,\sqrt{2}/2-2\sin\pi/2\cdot\sqrt{2}/2+\cos\pi/2\cdot\sqrt{2}/2-\cos\pi/2\cdot\sqrt{2}/2}=$$

$$=\frac{0-0-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{-\sqrt{2}-\sqrt{2}+0-0}=\frac{-\sqrt{2}}{-2\sqrt{2}}=-\frac{1}{-2}=\boxed{\frac{1}{2}}$$

№2.110

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}\frac{2\cos\frac{x+a}{2}\sin\frac{x-a}{2}}{x-a}=$$

$$=\lim_{x\to a}\left(\cos\frac{x+a}{2}\right)\cdot\overbrace{\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}}^{\to 1}=\boxed{\cos a}$$

№2.112

$$\lim_{x\to a}\frac{tg x-tg a}{x-a}=\lim_{x\to a}\frac{\frac{\sin(x-a)}{\cos x\cos a}}{x-a}=$$

$$=\lim_{x\to a}\frac{\sin(x-a)}{(\cos x\cos a)(x-a)}=\lim_{x\to a}\overbrace{\left(\frac{\sin x-a}{x-a}\right)}^{\to 1}\cdot\frac{1}{\cos x\cos a}=\boxed{\cos^2 a}$$

№2.114

$$\lim_{x\to a}\frac{\sec x-\sec a}{x-a}=\left[\frac{0}{0}\right]=\lim_{x\to a}\frac{\frac{1}{\cos x}-\frac{1}{\cos a}}{x-a}=\lim_{x\to a}\frac{\frac{\cos a-\cos x}{\cos x\cos a}}{x-a}=$$

$$=\lim_{x\to a}\frac{\cos a-\cos x}{\cos x\cos a\,(x-a)}=\lim_{x\to a}\underbrace{\frac{\cos a-\cos x}{x-a}}_{\to 1}\cdot\frac{1}{\cos x\cos a}=\boxed{\frac{1}{\cos^2 a}}$$

№2.117

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\operatorname{ctg}x\right)=[0-0]=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)=\lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\operatorname{tg}x/2=\boxed{0}$$

№2.119

$$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-1}{x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\left(\frac{\sqrt{1+x\sin x}-1}{x^2}\cdot\frac{\sqrt{1+x\sin x}+1}{\sqrt{1+x\sin x}+1}\right)=$$

$$=\lim_{x\to 0}\frac{\cancel{1}+\cancel{x}\cdot\sin x-\cancel{1}}{x^{\cancel{2}}(\sqrt{1+x\sin x}+1)}=\lim_{x\to 0}\frac{\sin x}{x(\sqrt{1+x\sin x}+1)}=$$

$$=\lim_{x\to 0}\left(\underbrace{\frac{\sin x}{x}}_{1}\cdot\frac{1}{\sqrt{1+x\sin x}+1}\right)=\frac{1}{\sqrt{1}+1}=\boxed{\frac{1}{2}}$$

№2.121

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{1+x}-1}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\cos x}{\frac{1}{2\sqrt{1+x}}}=\lim_{x\to 0}(2\sqrt{1+x}\cdot\cos x)=\boxed{2}$$

№2.123

$$\lim_{x\to 0}\frac{1-\cos x}{x\sin x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\left(\frac{\sin x}{\sin x+x\cos x}\right)=\lim_{x\to 0}\left(\frac{\cos x}{2\cos x-x\sin x}\right)=\boxed{\frac{1}{2}}$$

№2.125

$$\lim_{x\to 0}\frac{1-\cos 2x+x\sin x}{\sin^2 x}=\left[\frac{0}{0}\right]\lim_{x\to 0}\left(\frac{2\cdot\frac{1-\cos 2x}{2}+x\sin x}{\sin^2 x}\right)=$$

$$=\lim_{x\to 0}\left(\frac{2\sin^2 x+x\sin x}{\sin^2 x}\right)=\lim_{x\to 0}\left(\frac{\cancel{\sin x}(2\sin x+x)}{\sin^{\cancel{2}} x}\right)=$$

$$=\lim_{x\to 0}\left(\frac{2\sin x+x}{\sin x}\right)=\lim_{x\to 0}\left(\frac{2\cancel{\sin x}}{\cancel{\sin x}}+\frac{x}{\sin x}\right)=\lim_{x\to 0}\left(2+\frac{x}{\sin x}\right)=2+1^{-1}=\boxed{3}$$

№ 2.127

$$\lim_{x\to0}\frac{tg^2\frac{x}{2}}{x^2}=\left[\frac{0}{0}\right]=\left\{2\lim_{x\to0}\frac{\sin\frac{x}{2}}{\cos^3\frac{x}{2}\cdot 2x}\right\}=\lim_{x\to0}\frac{\frac{\cos x/2}{2}}{2\cos^3\frac{x}{2}-3x\cos^2\frac{x}{2}\sin\frac{x}{2}}=$$

$$=\lim_{x\to0}\frac{\cos x/2}{2\cos x/2\,\left(2\cos^2 x/2-3x\cos x/2\,\sin x/2\right)}=\lim_{x\to0}\frac{1}{2\cdot\frac{4\cos^2 x/2-3x\sin x}{2}}$$

$$=\lim_{x\to0}\frac{1}{4\cos^2\frac{x}{2}-3x\sin x}=\frac{1}{4\cos(\frac{1}{2}\cdot0)-3\cdot0\cdot\sin(0)}=\boxed{\frac{1}{4}}$$

№ 2.129

$$\lim_{x\to0}\frac{x^2 ctg 2x}{\sin 3x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{x^2\frac{\cos 2x}{\sin 2x}}{\sin 3x}=\lim_{x\to0}\frac{x^2}{\sin 3x\, tg 2x}=$$

$$=\lim_{x\to0}\frac{3x}{\sin 3x}\cdot\frac{2x}{tg 2x\cdot 6}=\boxed{\frac{1}{6}}$$

№ 2.131

$$\lim_{x\to0}\frac{x^3\, tg 5x}{\sin 6x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{x^3}{\sin 6x\, ctg 5x}=\lim_{x\to0}\frac{6x}{\sin 6x}\cdot\frac{5x}{ctg 5x\cdot 30}=\boxed{\frac{1}{30}}$$

№ 2.133

$$\lim_{x\to0}\frac{(1-\cos x)^2}{8x^4}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{2\sin x-\sin 2x}{32x^3}=\lim_{x\to0}\frac{2\cos x-2\cos 2x}{96x^2}=$$

$$=\lim_{x\to0}\frac{2(\cos x-\cos 2x)}{96x^2}=\lim_{x\to0}\frac{\cos x-\cos 2x}{48x^2}=\lim_{x\to0}\frac{-\cos x+4\cos 2x}{96}=$$

$$=\frac{-\cos 0+4\cos(0)}{96}=\frac{3}{96}=\boxed{\frac{1}{32}}$$

№ 2.135

$$\lim_{x\to0}\frac{x\, tg^2 4x}{\sin^3 11x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{x}{\sin^3 11x\, tg^2 4x}=\lim_{x\to0}\frac{(11x)^3}{\sin^3 11x}\cdot\frac{(4x)^2}{tg^2 4x\cdot 44}=$$

$$=\boxed{\frac{1}{44}}$$

№ 2.137

$$\lim_{x\to0}\frac{arctg x}{3x}=\lim_{x\to0}\left(\frac{\frac{1}{1+x^2}}{3}\right)=\lim_{x\to0}\left(\frac{1}{(1+x^2)3}\right)=\frac{1}{3(1+0^2)}=\boxed{\frac{1}{3}}$$

№2.139

$$\lim_{x\to0}\frac{x\sin4x}{\operatorname{arctg}^2 5x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\left(\frac{\sin4x+4x\cos4x}{\frac{10\operatorname{arctg}5x}{1+25x^2}}\right)=\lim_{x\to0}\frac{(\sin4x+4x\cos4x)(1+25x^2)}{10\operatorname{arctg}5x}=$$

$$=\lim_{x\to0}\frac{(5x)^2\sin4x\cdot4}{(\operatorname{arctg}^2 5x)\cdot4x\cdot25}=\boxed{\frac{4}{25}}$$

№2.141

$$\lim_{x\to0}\frac{1-\cos2x}{\sqrt{\operatorname{arctg}^2x+4}-4}=\lim_{x\to0}\frac{(\sqrt{\operatorname{arctg}^2x+4}+4)(1-\cos2x)}{\operatorname{arctg}^2x}=\frac{(2+4)\cdot(1-1)}{0}=\boxed{0}$$

№2.143

$$\lim_{x\to\pi/6}\frac{\sin(x-\pi/6)}{\frac{\sqrt3}{2}-\cos x}=\left[\frac{0}{0}\right]=\lim_{x\to\pi/6}\frac{\sqrt3\cos x+\sin x}{\frac{2}{\sin x}}=\lim_{x\to\pi/6}\frac{2\left(\frac{\sqrt3}{2}\cos x+\frac12\sin x\right)}{\frac{2}{\sin x}}=$$

$$=\lim_{x\to\pi/6}\left(\frac{\sin\pi/3\cos x+\cos\pi/3\sin x}{\sin x}\right)=\lim_{x\to\pi/6}\left(\frac{\sin(\pi/3+x)}{\sin x}\right)=\frac{\sin(\pi/3+\pi/6)}{\sin\pi/6}=$$

$$=\frac{\sin\pi/2}{1/2}=\frac{1}{1/2}=\boxed{2}$$

№2.146

$$\lim_{x\to0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}=$$

$$=\lim_{x\to0}\frac{(\sin(a+2x)-\sin(a+x))+(\sin a-\sin(a+x))}{x^2}=$$

$$=\lim_{x\to0}\frac{-2\cos\left(a+\frac{3x}{2}\right)\cdot\cos\frac{x}{2}+\cos\left(a+\frac{x}{2}\right)\cos\frac{x}{2}}{x^2}=$$

$$=\lim_{x\to0}\frac{2\cos\frac{x}{2}\left(\cos\left(a+\frac{x}{2}\right)-\cos\left(a+\frac{3x}{2}\right)\right)}{x^2}=$$

$$=\lim_{x\to0}\frac{-4\cos\frac{x}{2}\cdot\cos\frac{x}{2}\sin(a+x)}{x^2}=\lim_{x\to0}\frac{-4\overbrace{\cos^2\frac{x}{2}}^{\to1}\cdot\sin(a+x)}{4\left(\frac{x}{2}\right)^2}=$$

$$=-\sin a$$

№2.161

$$\lim_{x\to0}\frac{1-\cos x\sqrt{\cos2x}\sqrt[3]{\cos3x}}{x^2}=\left[\frac{0}{0}\right]=$$

$$= \lim_{x\to 0} \frac{1-\cos^{\frac{1}{2}} x \cos^{\frac{1}{2}} 2x \cos^{\frac{1}{3}} 3x}{x^2} = \lim_{x\to 0} \frac{1-\cos x \sqrt[5]{\cos^3 2x \cos^2 3x}}{x^2} =$$

$$\boxed{0}$$?

№2.160

$$\lim_{x\to 0} \frac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\frac{\sin x}{2\sqrt{\cos x}}}{\frac{\sin\sqrt{x}}{2\sqrt{x}}} = \lim_{x\to 0} \frac{\sin x \sqrt{x}}{\sqrt{\cos x}\sin\sqrt{x}} =$$

$$= \lim_{x\to 0} \frac{\frac{2x\cos x + \sin x}{2\sqrt{x}}}{-\frac{\sin x \sin\sqrt{x}}{2\sqrt{\cos x}} + \frac{\sqrt{\cos x}\cdot\cos\sqrt{x}}{2\sqrt{x}}} = \lim_{x\to 0} \frac{\frac{2x\cos x + \sin x}{2\sqrt{x}}}{\frac{-\sqrt{x}\sin x \sin\sqrt{x} + \cos x \cos\sqrt{x}}{2\sqrt{\cos x}\cdot(x)}} =$$

$$= \lim_{x\to 0} \frac{2x\sqrt{\cos x}\cdot\cos x + \sin x \sqrt{\cos x}}{-\sqrt{x}\sin x \sin\sqrt{x} + \cos x \cos\sqrt{x}} = \boxed{0}$$

Домашня робота (за з. 13)

№ 171

$$\lim_{x\to\infty}\left(\frac{x+2}{2x-1}\right)^{x^2} = \lim_{x\to\infty}\left(\frac{(2x-1)-x+3}{2x-1}\right)^{x^2} = \lim_{x\to\infty}\left(\underbrace{\left(1+\left(\frac{-x+3}{2x-1}\right)\right)^{\frac{2x-1}{-x+3}}}_{e}\right)^{\frac{-x+3}{2x-1}x^2} =$$

$$= \lim_{x\to\infty} e^{\left(\frac{-x+3}{2x-1}\right)x^2} = e^{\frac{-x^3+3x^2}{2x-1}} = \boxed{0}$$

№ 172

$$\lim_{x\to\infty}\left(\frac{3x^2-x+1}{2x^2+x+1}\right)^{\frac{x^2}{1+x}} = \lim_{x\to\infty}\frac{(2x^2+x+1)+x^2-2x}{2x^2+x+1} = \lim_{x\to\infty}\left(\underbrace{\left(1+\frac{x^2-2x}{2x^2+x+1}\right)^{\frac{2x^2+x+1}{x^2-2x}}}_{e}\right)^{\frac{x^2}{1+x}\left(\frac{2x^2+x+1}{x^2-2x}\right)} =$$

$$= \lim_{x\to\infty} e^{\frac{x^2}{1+x}\cdot\frac{2x^2+x+1}{x^2-2x}} = e^{\frac{2x^3+x^2+x}{-x^2+3x-2}} = \boxed{0}$$

№2.175

$$\lim_{n\to\infty} \sin^n \frac{2\pi n}{3n+1} = \boxed{0}$$

№2.183

$$\lim_{x\to0}(1+x^2)^{ctg^2 x} = [1^\infty] = \lim_{x\to0}\left((1+x^2)^{\frac{1}{x^2}}\right)^{ctg^2 x} = \lim_{x\to0} e^{ctg^2 x} = \boxed{e^{ctg^2 x}}$$

№2.185

$$\lim_{x\to0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin x}} = \left\{\lim_{x\to0}\left(\frac{\cos x+\sin x}{\cos x+\cos x\sin x}\right)^{\frac{1}{\sin x}}\right\} =$$

$$= \lim_{x\to0}\left(1+\left(\frac{1+tgx}{1+\sin x}-1\right)\right)^{\frac{1}{\sin x}} = \lim_{x\to0}\left(1+\left(\frac{1+tgx-1-\sin x}{1+\sin x}\right)\right)^{\frac{1}{\sin x}} =$$

$$= \lim_{x\to0}\left(\underbrace{\left(1+\left(\frac{tgx-\sin x}{1+\sin x}\right)\right)^{\frac{1+\sin x}{tgx-\sin x}}}_{e}\right)^{\frac{tgx-\sin x}{1+\sin x}\cdot\frac{1}{\sin x}} = \lim_{x\to0} e^{\frac{tgx-\sin x}{\sin x+\sin^2 x}} =$$

$$= e^{\frac{1-\cos x}{\cos x(1+\sin x)}} = e^0 = \boxed{1}$$

№2.186

$$\lim_{x\to0}\left(\frac{1+tgx}{1+\sin x}\right)^{\frac{1}{\sin^3 x}} = \lim_{x\to0}\left(1+\left(\frac{1+tgx}{1+\sin x}-1\right)\right)^{\frac{1}{\sin^3 x}} =$$

$$= \lim_{x\to0}\left(\underbrace{1+\left(\frac{tgx-\sin x}{1+\sin x}\right)^{\frac{1+\sin x}{tgx-\sin x}}}_{e}\right)^{\frac{tgx-\sin x}{1+\sin x}\cdot\frac{1}{\sin^3 x}} =$$

$$= \lim_{x\to0} e^{\frac{tgx-\sin x}{\sin^3 x(1+\sin x)}} = e^{\frac{tgx-\sin x}{\sin^3 x+\sin^4 x}} = e^{\frac{1}{\cos x+\cos x\sin x+\cos^2 x+\cos^2 x\sin x}} =$$

$$= e^{\frac{1}{1+0+1+0}} = e^{\frac{1}{2}} = \boxed{\sqrt{e}}$$

№2.190

$$\lim_{x\to\pi/2}(\sin x)^{tgx} = [1^\infty] = \lim_{x\to\pi/2}\left(\underbrace{\left(1+(\sin x-1)\right)^{\frac{1}{\sin x-1}}}_{e}\right)^{(\sin x-1)\,tgx} =$$

$$= \lim_{x\to\pi/2} e^{(\sin x-1)\,tgx} = e^{\sin x\cdot\frac{\sin x}{\cos x}-tgx} = e^{\frac{\sin^2 x}{\cos x}-\frac{\sin x}{\cos x}} = e^{\frac{\sin x(\sin x-1)}{\cos x}}$$

$$= e^{\frac{1(1-1)}{0}} = e^0 = \boxed{1}$$

№ 2.194

$$\lim_{n\to\infty}\left(\frac{n+1}{n-1}\right)^n = \lim_{n\to\infty}\left(\frac{(n+1)-2}{n-1}\right)^n = \lim_{n\to\infty}\left(\underbrace{\left(1+\left(-\frac{2}{n-1}\right)\right)^{\frac{n-1}{2}}}_{e}\right)^{-\frac{2}{n-1}\cdot n} =$$

$$= e^{\frac{2n}{-n-1}} = \boxed{\frac{1}{e^{\frac{2n}{n+1}}}}$$

№ 2.203

$$\lim_{x\to+\infty}\frac{\ln(1+2^x)}{\ln(1+5^{2x})} = \lim_{x\to+\infty}\frac{\ln\left(2^x\left(1+\frac{1}{2^x}\right)\right)}{\ln\left(5^{2x}\left(1+\frac{1}{5^{2x}}\right)\right)} =$$

$$= \lim_{x\to+\infty}\frac{\ln 2^x + \ln\left(1+\frac{1}{2^x}\right)}{\ln 5^{2x} + \ln\left(1+\frac{1}{5^{2x}}\right)} = \lim_{x\to+\infty}\frac{x}{2x} = \boxed{\frac{1}{2}}$$

№ 2.208

$$\lim_{x\to 0}(x+e^x)^{\frac{1}{x}} = \left\{\text{Замена } \frac{1}{x} = t\right\} = \lim_{t\to 0}\left(\frac{1}{t}+e^{\frac{1}{t}}\right)^t =$$

$$= \lim_{x\to 0}\left(\frac{1+t\cdot e^{\frac{1}{t}}}{t}\right)^t = \boxed{1}$$