Домашня робота

I.

2) $x_n = (-1)^{n+2}(3+2n)$

1) $n=2k$:

$x_{2k} = (-1)^{2k+2}(3+2\cdot 2k) = 1\cdot(3+4k) = 3+4k$

$\lim\limits_{k\to\infty} x_{2k} = \lim\limits_{k\to\infty}(3+4k) = \boxed{3}$

2) $n=2k-1$:

$x_{2k-1} = -(3+2(2k-1)) = -(3+4k-2) = -(4k+1) = -4k-1$

$\lim\limits_{k\to\infty} x_{2k-1} = \lim\limits_{k\to\infty}(-4k-1) = \boxed{-1}$

мнж. част. гран.: $\{1; 3\}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = 3$; $\underline{\lim\limits_{n\to\infty}} x_n = -1$

3) $x_n = (-1)^{n+1}\cdot n+n$

1) $n=2k$: $x_{2k} = (-1)^{2k+1}\cdot 2k+2k = -2k+2k=0$

$\lim\limits_{k\to\infty} x_{2k} = \lim\limits_{k\to\infty} 0 = \boxed{0}$

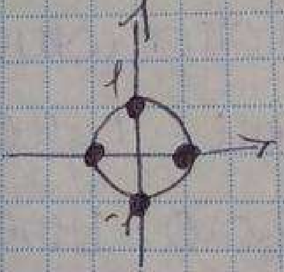
2) $n=2k-1$: $x_{2k-1} = -(2k-1)+(2k-1) = -2k+1+2k-1 = 0$

$\lim\limits_{k\to\infty} x_{2k-1} = \lim\limits_{k\to\infty} 0 = \boxed{0}$

мнж. част. гран. $\{0\}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = 0$; $\underline{\lim\limits_{n\to\infty}} x_n = 0$

4) $x_n = \sin \pi n + 2$



$n=1$: $x_1 = \sin\pi + 2 = 0+2 = 2$

$n=2$: $x_2 = \sin 2\pi + 2 = 0+2 = 2$

$n=3$: $x_3 = \sin 3\pi + 2 = 0+2 = 2$

$n=4$: $x_4 = \sin 4\pi + 2 = 0+2 = 2$

$x_{2k-1} = 2$; $x_{2k} = 2$

$\lim\limits_{k\to\infty} x_{2k-1} = \boxed{2}$ $\lim\limits_{k\to\infty} x_{2k} = \boxed{2}$

В: $\lim\limits_{n\to\infty} x_n = 2$

⑥ $x_n = (-1)^{n+1}\frac{1}{2}+3$

1) $n=2k$: $x_{2k} = -\frac{1}{2}+3=2,5$; $\lim\limits_{k\to\infty} x_{2k} = \lim\limits_{k\to\infty} 2,5 = \boxed{2,5}$ } частк. границі

2) $n=2k-1$: $x_{2k-1} = \frac{1}{2}+3=3,5$; $\lim\limits_{k\to\infty} x_{2k-1} = \lim\limits_{k\to\infty} 3,5 = \boxed{3,5}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = 3,5$; $\underline{\lim\limits_{n\to\infty}} x_n = 2,5$

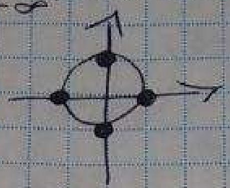
⑦ $x_n = ((-1)^n+1)\cdot 2n$

1) $n=2k$: $x_{2k} = (1+1)\cdot 2\cdot 2k = 2\cdot 2\cdot 2k = 8k$; $\lim\limits_{k\to\infty} x_{2k} = \lim\limits_{k\to\infty} 8k = \boxed{+\infty}$ } часткові границі

2) $n=2k-1$: $x_{2k-1} = \lim\limits_{k\to\infty} 0 = \boxed{0}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = +\infty$; $\underline{\lim\limits_{n\to\infty}} x_n = 0$

⑧ $x_n = 1+\sin\frac{\pi n}{2}$



$n=1$: $x_1 = 1+\sin\frac{\pi}{2} = 1+1=2$

$n=2$: $x_2 = 1+\sin\pi = 1+0=1$

$n=3$: $x_3 = 1+\sin\frac{3\pi}{2} = 1-1=0$

$n=4$: $x_4 = 1+\sin 2\pi = 1$

$n=5$: $x_5 = 1+\sin\frac{5\pi}{2} = 2$

$n=6$: $x_6 = 1+\sin 3\pi = 1+0=1$

$n=7$: $x_7 = 1+\sin\frac{7\pi}{2} = 1-1=0$

$n=8$: $x_8 = 1+\sin 4\pi = 1$

$n=9$: $x_9 = 1+\sin\frac{9\pi}{2} = 1+1=2$

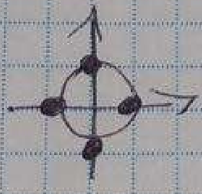
$n=4k-3$: $x_{4k-3} = 2$; $\lim\limits_{k\to\infty} x_{4k-3} = \boxed{2}$

$n=4k-1$: $x_{4k-1} = 0$; $\lim\limits_{k\to\infty} x_{4k-1} = \boxed{0}$

$n=2k$: $x_{2k} = 1$; $\lim\limits_{k\to\infty} x_{2k} = \boxed{1}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = 2$; $\underline{\lim\limits_{n\to\infty}} x_n = 0$

⑨ $x_n = \cos\pi n + 2$



$n=1$: $x_1 = \cos\pi + 2 = -1+2=1$

$n=2$: $x_2 = \cos 2\pi + 2 = 1+2=3$

$n=3$: $x_3 = \cos 3\pi + 2 = 1$

$n=4$: $x_4 = \cos 4\pi + 2 = 3$

$x_{2k-1} = 1$; $\lim\limits_{k\to\infty} x_{2k-1} = \boxed{1}$ } частк. границі

$x_{2k} = 3$; $\lim\limits_{k\to\infty} x_{2k} = \boxed{3}$

В: $\overline{\lim\limits_{n\to\infty}} x_n = 3$; $\underline{\lim\limits_{n\to\infty}} x_n = 1$

(10) $x_n = \frac{1}{2n}\left((-1)^n + \sin \pi n\right)$

$n=1: x_1 = -\frac{1}{2}$

$n=2: x_2 = 1/4$

$n=3: x_3 = -1/6$

$n=4: x_4 = 1/8$

1) $n = \frac{1}{2k}: x_{\frac{1}{2k}} = \frac{1}{4}; \lim\limits_{k\to\infty} x_{\frac{1}{2k}} = \boxed{\frac{1}{4}}$

2) $n = \frac{1}{2k}: x_{\frac{1}{2k}} = -\frac{1}{2}; \lim\limits_{k\to\infty} x_{\frac{1}{2k}} = \boxed{-\frac{1}{2}}$

} часткові границі

В: $\underline{\lim\limits_{n\to\infty}} x_n = \frac{1}{4}$; $\overline{\lim\limits_{n\to\infty}} x_n = -\frac{1}{2}$

(1) $x_n = (-1)^n \cdot \frac{2}{3} + 1$

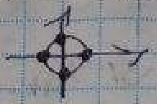
1) $n=2k: x_{2k} = \frac{2}{3} + 1 \overset{\text{В}}{=} \frac{5}{3}; \lim\limits_{k\to\infty} x_{2k} = \lim\limits_{k\to\infty} \frac{5}{3} = \boxed{\frac{5}{3}}$

2) $n=2k-1: x_{2k-1} = -\frac{2}{3} + 1 \overset{\text{В}}{=} \frac{1}{3}; \lim\limits_{k\to\infty} x_{2k-1} = \lim\limits_{k\to\infty} \frac{1}{3} = \boxed{\frac{1}{3}}$

} часткові границі

В: $\underline{\lim\limits_{n\to\infty}} x_n = 1/3$; $\overline{\lim\limits_{n\to\infty}} x_n = 5/3$

(5) $x_n = \dfrac{\cos \frac{n^2}{2} + 1}{n+1}$



$n=1: x_1 = \frac{\cos\frac{1}{2} + 1}{1+1} = \frac{\cos 0{,}5 + 1}{2} = 0{,}93 = \frac{\cos\frac{1}{2}+1}{2}$

$n=2: x_2 = \frac{\cos 2 + 1}{3} = 0{,}29 = \frac{\cos 2 + 1}{3}$

$n=3: x_3 = \frac{\cos 4{,}5 + 1}{4} = 0{,}19 = \frac{\cos\frac{9}{2}+1}{4}$

$n=4: x_4 = \frac{\cos 8 + 1}{5} = 0{,}17 = \frac{\cos 8 + 1}{5}$

$n=5: x_5 = \frac{\cos 12{,}5 + 1}{6} = 0{,}33 = \frac{\cos\frac{25}{2}+1}{6}$

$n=6: x_6 = \frac{\cos 18 + 1}{7} = \frac{\cos 18 + 1}{7}$

$n=7: x_7 = \frac{\cos 24{,}5 + 1}{8} = \frac{\cos\frac{49}{2}+1}{8}$

$n=8: x_8 = \frac{\cos 32 + 1}{9} = \frac{\cos 32 + 1}{9}$

$n = \dfrac{\cos\frac{k^2}{2}}{k+2}: x_{\frac{\cos\frac{k^2}{2}}{k+2}} = \dfrac{\cos\frac{1}{2}+1}{2};$

$\lim\limits_{k\to\infty} x_{\frac{\cos\frac{k^2}{2}}{k+2}} = \lim\limits_{k\to\infty} \dfrac{\cos\frac{1}{2}+1}{2} = \dfrac{\cos\frac{1}{2}+1}{2}$

$n = \dfrac{\cos 6\frac{k+4}{2} + 1}{k+2}:$

$x_{\frac{\cos\frac{k+4}{2}+1}{k+2}} = \dfrac{\cos 2 + 1}{3};$

$\lim\limits_{k\to\infty} x_{\frac{\cos\frac{k+4}{2}+1}{k+2}} = \lim\limits_{k\to\infty} \dfrac{\cos 2 + 1}{3} = \dfrac{\cos 2 + 1}{3}$

II. (11) $\lim\limits_{n\to\infty} \left(\underbrace{4 + \frac{1}{n}}_{4}\right)^n = \lim\limits_{n\to\infty} 4^n = \boxed{+\infty}$

(12) $\lim\limits_{n\to\infty} \left(\frac{2n-3}{n+1}\right)^n = \left[\frac{2^n}{1}\right] = \boxed{+\infty}$

(13) $\lim\limits_{n\to\infty} \left(\frac{2n-1}{3n+4}\right)^{n/3} = \left[\frac{2}{3}^{+\infty}\right] = \boxed{0}$

(14) $\lim\limits_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n+2} = \lim\limits_{n\to\infty} \left(\underbrace{\left(1 + \frac{1}{n}\right)^n}_{e}\right)^{\frac{n+2}{n}} = e^1 = \boxed{e}$

Вчасткові границі

(15) $\lim\limits_{n\to\infty}\left(\frac{n+1}{n+3}\right)^n = \lim\limits_{n\to\infty}\left(\frac{(n+3)-2}{n+3}\right)^n = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{-2}{n+3}\right)\right)^{\frac{-n+3}{2}}}_{e}\right)^{\frac{-2n}{n+3}} =$
$= \lim\limits_{n\to\infty} e^{-2} = \boxed{\frac{1}{e^2}}$

(17) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n+2}\right)^{\frac{2n-1}{3}} = \lim\limits_{n\to\infty}\left(\frac{(n+2)-6}{n+2}\right)^{\frac{2n-1}{3}} = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{-6}{n+2}\right)\right)^{\frac{-n+2}{6}}}_{e}\right)^{\frac{-6}{n+2}\cdot\frac{2n-1}{3}} =$
$= \lim\limits_{n\to\infty} e^{\frac{-4n+2}{n+2}} = e^{-4} = \boxed{\frac{1}{e^4}}$

(18) $\lim\limits_{n\to\infty}\left(\frac{n-2}{n+1}\right)^n = \lim\limits_{n\to\infty}\left(\frac{(n+1)-3}{n+1}\right)^n = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{-3}{n+1}\right)\right)^{\frac{-n+1}{3}}}_{e}\right)^{\frac{-3}{n+1}\cdot n} = \lim\limits_{n\to\infty} e^{\frac{-3n}{n+1}} =$
$= e^{-3} = \boxed{\frac{1}{e^3}}$

(19) $\lim\limits_{n\to\infty}\left(\frac{n^2+1}{n^2+n}\right)^{3n-2} = \lim\limits_{n\to\infty}\left(\frac{n^2+1}{n(n+1)}\right)^{3n-2} = \lim\limits_{n\to\infty}\left(\left(1+\left(\frac{-n+1}{n^2+n}\right)\right)^{\frac{-n^2+n}{n+1}}\right)^{\frac{-n+1}{n^2+n}\cdot 3n-2} =$
$= \lim\limits_{n\to\infty} e^{\frac{-n+1}{n^2+n}\cdot 3n-2} = \lim\limits_{n\to\infty} e^{\frac{-5n+1}{n+1}} = e^{-5} = \boxed{\frac{1}{e^5}}$

(16) $\lim\limits_{n\to\infty}\left(1-\frac{5}{3n}\right)^{10} = \boxed{1}$

(20) $\lim\limits_{n\to\infty}\left(1+\frac{2}{n}\right)^{\frac{n^2-1}{n+2}} = \lim\limits_{n\to\infty}\left(\left(\left(1+\frac{2}{n}\right)^{\frac{n+2}{n^2-1}}\right)^{\frac{n^2-1}{n+2}}\right)^{\frac{n^2-1}{n+2}} = e^{\frac{n^2-1}{n+2}} = e^1 = \boxed{e}$

(21) $\lim\limits_{n\to\infty}\left(\underbrace{2-\frac{1}{n^2}}_{4}\right)^n = \lim\limits_{n\to\infty} 2^n = \boxed{+\infty}$

(22) $\lim\limits_{n\to\infty}\left(2+\frac{3}{n}\right)^{\frac{2}{n}} = \boxed{\infty}$

(23) $\lim\limits_{n\to\infty}\left(\frac{n^2+2}{n^2-1}\right)^{\frac{2n-1}{2}} = \lim\limits_{n\to\infty}\left(\frac{(n^2-1)+3}{n^2-1}\right)^{\frac{2n-1}{2}} = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{3}{n^2-1}\right)\right)^{\frac{n^2-1}{3}}}_{e}\right)^{\frac{3}{n^2-1}\cdot\frac{2n-1}{2}} =$
$= \lim\limits_{n\to\infty} e^{\frac{6n-3}{2n^2-2}} = \boxed{e^3}$

(24) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{3n} = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{e}\right)^{3} = \boxed{e^3}$

(25) $\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n} = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{1}{3n}\right)^{3n}}_{e}\right)^{\frac{1}{3}} = e^{\frac{1}{3}} = \boxed{\sqrt[3]{e}}$

(26) $\lim\limits_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n = \lim\limits_{n\to\infty}\left(1+\left(-\frac{1}{n^2}\right)\right)^n =$

$= \lim\limits_{n\to\infty}\left(\left(1+\left(-\frac{1}{n^2}\right)\right)^{-n^2}\right)^{-\frac{1}{n^2}\cdot n} = \lim\limits_{n\to\infty} e^{-\frac{n}{n^2}} = \lim\limits_{n\to\infty} e^{\left(-\frac{1}{n}\right)} = [e^{\infty}] = \boxed{\infty}$

(27) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n+4}\right)^{n+2} = \lim\limits_{n\to\infty}\left(\frac{(n+4)-4}{n+4}\right)^{n+2} =$

$= \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{-4}{n+4}\right)\right)^{-\frac{n+4}{4}}}_{e}\right)^{\frac{-4}{n+4}\cdot n+2} = \lim\limits_{n\to\infty} e^{\frac{-4n-8}{n+4}} = e^{\frac{-2(n+4)}{n+4}} = e^{-2} = \boxed{\frac{1}{e^2}}$

(28) $\lim\limits_{n\to\infty}\left(\frac{3n+5}{3n+7}\right)^{\frac{2n^2+1}{n}} = \lim\limits_{n\to\infty}\left(\frac{3n+7-2}{3n+7}\right)^{\frac{2n^2+1}{n}} =$

$= \lim\limits_{n\to\infty}\left(\left(1+\left(\frac{-2}{3n+7}\right)\right)^{-\frac{3n+7}{2}}\right)^{\frac{-2}{3n+7}\cdot\frac{2n^2+1}{n}} = \lim\limits_{n\to\infty} e^{\frac{-2}{3n+7}\cdot\frac{2n^2+1}{n}} =$

$= \lim\limits_{n\to\infty} e^{\frac{-4n^2+2}{3n^2+7n}} = e^{-4/3} = \boxed{\frac{1}{\sqrt[3]{e^4}}}$

(29) $\lim\limits_{n\to\infty}\left(\frac{n^2+3n+4}{n^2}\right)^{4n+1} = \lim\limits_{n\to\infty}\left(\frac{n^2-n^2+n^2+3n+4}{n^2}\right)^{4n+1} =$

$= \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{3n+4}{n^2}\right)^{\frac{n^2}{3n+4}}}_{e}\right)^{\frac{3n+4}{n^2}\cdot 4n+1} = \lim\limits_{n\to\infty} e^{\frac{12n+16}{n}} = \boxed{e^3}$

(30) $\lim\limits_{n\to\infty}\left(\frac{2n^2-n}{3n+2n^2}\right)^{n+5} = \lim\limits_{n\to\infty}\left(\frac{\cancel{n}(2n-1)}{\cancel{n}(3+2n)}\right)^{n+5} = \lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+5} =$

$= \lim\limits_{n\to\infty}\left(\frac{(2n+3)-4}{2n+3}\right)^{n+5} = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\left(\frac{-4}{2n+3}\right)\right)^{-\frac{2n+3}{4}}}_{e}\right)^{\frac{-4}{2n+3}\cdot n+5} =$

$= \lim\limits_{n\to\infty} e^{\frac{6n+15}{2n+3}} = e^{6/2} = \boxed{e^3}$

II. Знайти границі послідовностей.

11) $\lim\limits_{n\to\infty}\left(4+\frac{1}{n}\right)^{n}$

12) $\lim\limits_{n\to\infty}\left(\frac{2n-3}{n+1}\right)^{n}$

13) $\lim\limits_{n\to\infty}\left(\frac{2n-1}{3n+4}\right)^{\frac{n}{3}}$

14) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+2}$

15) $\lim\limits_{n\to\infty}\left(\frac{n+1}{n+3}\right)^{n}$

16) $\lim\limits_{n\to\infty}\left(1-\frac{5}{3n}\right)^{10}$

17) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n+2}\right)^{\frac{2n-1}{3}}$

18) $\lim\limits_{n\to\infty}\left(\frac{n-2}{n+1}\right)^{n}$

19) $\lim\limits_{n\to\infty}\left(\frac{n^2+1}{n^2+n}\right)^{3n-2}$

20) $\lim\limits_{n\to\infty}\left(1+\frac{2}{n}\right)^{\frac{n^2-1}{n+2}}$

21) $\lim\limits_{n\to\infty}\left(2-\frac{1}{n^2}\right)^{n}$

22) $\lim\limits_{n\to\infty}\left(2+\frac{3}{n}\right)^{\frac{1}{n}}$

23) $\lim\limits_{n\to\infty}\left(\frac{n^2+2}{n^2-1}\right)^{\frac{2n-1}{2}}$

24) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{3n}$

25) $\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n}$

26) $\lim\limits_{n\to\infty}\left(1-\frac{1}{n^2}\right)^{n}$

27) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n+4}\right)^{n+2}$

28) $\lim\limits_{n\to\infty}\left(\frac{3n+5}{3n+7}\right)^{\frac{2n^2+1}{n}}$

29) $\lim\limits_{n\to\infty}\left(\frac{n^2+3n+4}{n^2}\right)^{4n+1}$

30) $\lim\limits_{n\to\infty}\left(\frac{2n^2-n}{3n+2n^2}\right)^{n+5}$

I. Знайти усі часткові границі послідовності $\{x_n\}$ а також $\overline{\lim\limits_{n\to\infty}} x_n$ та $\underline{\lim\limits_{n\to\infty}} x_n$.

1) $x_n = (-1)^n \cdot \frac{2}{3} + 1$

2) $x_n = (-1)^{n+2} (3+2n)$

3) $x_n = (-1)^{n+1} \cdot n + n$

4) $x_n = \sin \pi n + 2$

5) $x_n = \dfrac{\cos \frac{n^2}{2} + 1}{n+1}$

6) $x_n = (-1)^{n+1} \cdot \frac{1}{2} + 3$

7) $x_n = ((-1)^n + 1) \cdot 2n$

8) $x_n = 1 + \sin \frac{\pi n}{2}$

9) $x_n = \cos \pi n + 2$

10) $x_n = \frac{1}{2n} ((-1)^n + \sin \pi n)$