

# Домашня робота (2.1)

№ 20, c.5

$$\sum_{i=1}^n i^3 = 1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$T(n): 1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}, \text{ new}$$

$$\nabla T(1): 1^3 = \frac{(1+1)^2}{4} = \frac{4}{4} = 1; 1 = 1 - \text{прав.}$$

$$T(k): 1^3 + 2^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4} - \text{припускаємо, що пр.}$$

$$T(k+1): 1^3 + 2^3 + \dots + k^3 + (k+1)^3 = \frac{(k+1)^2((k+1)+1)^2}{4} - \text{доведемо}$$

$$(1^3 + 2^3 + \dots + k^3) + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4} \quad \Delta$$

№ 10, c.5

$$\sum_{i=1}^n (2i-1) = 1 + 3 + \dots + (2n-1) = n^2$$

$$T(n): 1 + 3 + \dots + (2n-1) = n^2, \text{ new}$$

$$\nabla T(1): 2 \cdot 1 - 1 = 1^2; 1 = 1 - \text{прав.}$$

$$T(k): 1 + 3 + \dots + (2k-1) = k^2 - \text{припускаємо, що пр.}$$

$$T(k+1): 1 + 3 + \dots + (2(k+1)-1) = (k+1)^2 - \text{доведемо}$$

$$1 + 3 + \dots + (2(k+1)-1) = k^2 + (2k+1) = (k+1)^2;$$

$$2k+2-1 = (k+1)^2; (2k+1) = (k+1)^2 - k^2 \quad \Delta$$

№ 11, c.5

$$\sum_{i=1}^n 2^{i-1} = 1 + 2 + \dots + 2^{n-1} = 2^n - 1$$

$$T(n): 1 + 2 + \dots + 2^{n-1} = 2^n - 1, \text{ new}$$

$$\nabla T(1): 2^{1-1} = 2^0 = 1; 2^1 - 1 = 1; 1 = 1 - \text{прав.}$$

$$T(k): 1 + 2 + \dots + 2^{k-1} = 2^k - 1 - \text{припускаємо, що пр.}$$

$$T(k+1): 1 + 2 + \dots + 2^{k+1-1} = 2^{k+1} - 1 - \text{доведемо}$$

$$1 + 2 + \dots + 2^{k+1-1} = 2^k - 1 + 2^k = 2^{k+1} - 1;$$

$$2^k < 2^{k+1} - 1 \quad \Delta$$



### N 1.3, c.5

$$\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{n+1}}$$

$$T(n): \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{n+1}}, \text{ new}$$

$$\nabla T(1): \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{1+1}} = 2 \cos \frac{\pi}{2^2} = 2 \cos \frac{\pi}{4}$$

$$\sqrt{2} = 2 \cos \frac{\pi}{4}; \sqrt{2} = \sqrt{2} - \text{npab.}$$

$$T(k): \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{k+1}} - \text{npunyk, uo np.}$$

$$T(k+1): \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{(k+1)+1}} = 2 \cos \frac{\pi}{2^{k+2}} - \text{pobegemo}$$

$$\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}} = 2 \cos \frac{\pi}{2^{k+2}}; \sqrt{2 - 2 \cos \frac{\pi}{2^{k+1}}} = 2 \cos \frac{\pi}{2^{k+2}}$$

### N 2.2, c.5

$$n^3 + 5n : 6 \text{ na } 6$$

$$T(n): (n^3 + 5n) : 6, \text{ new}$$

$$\nabla T(1): (1^3 + 5 \cdot 1) : 6, \text{ do } 6 : 6 = 1 - \text{npab.}$$

$$T(k): (k^3 + 5k) : 6, k \in \mathbb{N} - \text{npunyk, uo np.}$$

$$T(k+1): ((k+1)^3 + 5(k+1)) : 6 - \text{pobegemo}$$

$$k^3 + 3k^2 + 3k + 1 + 5k + 5 = k^3 + 5k + 3k(k+1) + 6 = (k^3 + 5k) : 6$$

zugno z mnozhe: 6  $\Delta$

### N 2.2, c.5

$$T(n): n^2 (n^4 - 1) : 60, \text{ new}$$

$$\nabla T(1): 1^2 (1^4 - 1) : 60 - \text{npab.}$$

$$T(k): k^2 (k^4 - 1) : 60, k \in \mathbb{N} - \text{npunyk, uo np.}$$

$$T(k+1): (k+1)^2 ((k+1)^4 - 1) : 60 - \text{pobegemo}; (k+1)^2 (k^4 + 4k^3 + 6k^2 + 4k + 1 - 1) : 60$$

$$= (k+1)^2 (k^4 + 4k^3 + 6k^2 + 4k) : 60 = k(k+1)(k+2)(k^2 + 2k + 2) : 60$$

- zugno z mnozhe: 60  $\Delta$

### N 2.3, c.5

$$T(n): (5^{n+3} + 11^{3n+1}) : 17, \text{ new}$$

$$\nabla T(1): (5^{1+3} + 11^{3 \cdot 1 + 1}) : 17, \text{ do } 17 : 17 = 1 - \text{npab.}$$

$$T(k): (5^{k+3} + 11^{3k+1}) : 17, k \in \mathbb{N} - \text{npunyk, uo np.}$$

$$T(k+1): (5^{(k+1)+3} + 11^{3(k+1)+1}) : 17 - \text{pobegemo}$$



$$5^{k+4} \cdot 5^k \cdot 5^4 + \frac{1}{11^{3k}} \cdot 11^4 - \text{згідно з нерівністю: } 17 \Delta$$

N 2.4, c.5

$$7^{2n} - 4^{2n} \text{ на } 33$$

$$T(n): (7^{2n} - 4^{2n}) : 33, \text{ new}$$

$$\nabla T(1): (7^2 - 4^2) : 33, \text{ об } 33 : 33 = 1 - \text{прав}$$

$$T(k): (7^{2k} - 4^{2k}) : 33, k \in \mathbb{N} - \text{припустимо, що пр.}$$

$$T(k+1): (7^{2(k+1)} - 4^{2(k+1)}) : 33 - \text{доведемо}$$

$$7^{2k+2} - 4^{2k+2} \text{ згідно з нерівністю: } 33 \Delta$$

N 2.19, c.5

$$\sum_{i=1}^n \frac{1}{i^2} \leq 2 - \frac{1}{n} \quad (n \geq 1)$$

$$\nabla T(1): 2 - \frac{1}{1} (1 \geq 1) \leq 1; 2 - 1 (1 \geq 1) \leq 1; 1 \geq 1 - \text{прав}$$

$$T(k): 2 - \frac{1}{k} (k \geq 1) - \text{припустимо, що пр.}$$

$$T(k+1): 2 - \frac{1}{k+1} (k+1 \geq 1) \text{ доведемо}$$

$$\frac{2k+2-1}{k+1} (k+1 \geq 1) \leq \frac{2k+1}{k+1} (k+1) \geq 1 - \text{згідно з нерівністю } \Delta$$

N 2.22, c.5

$$\sum_{i=1}^n \frac{1}{\sqrt{i}} > 2(\sqrt{n+1} - 1) \quad (n \geq 1)$$

$$\nabla T(1): 2(\sqrt{1+1} - 1) (1 \geq 1) \geq 1; 2(\sqrt{2} - 1) (1 \geq 1) \geq 1 - \text{прав}$$

$$T(k): 2(\sqrt{k+1} - 1) (k \geq 1) - \text{припустимо, що пр.}$$

$$T(k+1): 2(\sqrt{k+1+1} - 1) (k+1 \geq 1) - \text{доведемо}$$

$$2(\sqrt{k+2} - 1) (k+1 \geq 1) =$$

$$= (2\sqrt{k+2} - 2) (k+1) \geq 1 - \text{згідно з нерівністю } \Delta$$

N 2.23, c.5

$$n^{n+1} > (n+1)^n \quad (n \geq 3)$$

$$T(n): n^{n+1} > (n+1)^n \quad (n \geq 3), \text{ new}$$



$$\nabla T(3): 3^{3+1} > (3+1)^3 \quad (3 > 3); \quad 3^4 > 4^3 \quad (3 > 3) - \text{правда}$$

$$T(k): k^{k+1} > (k+1)^k \quad (k > 3, k \in \mathbb{N}) - \text{предположим, что это верно}$$

$$T(k+1): (k+1)^{(k+1)+1} > ((k+1)+1)^{k+1} \quad (k+1 > 3) - \text{покажем}$$

$$k+1^{k+2} > (k+2)^{k+1} \quad (k+1 > 3) - \text{знаем из индукции}$$

Задача 3.2

N1.32, c. 2

$$(\forall x)(\exists y): x+y < 3 \quad (\times)$$

$$(\exists x)(\forall y): x+y \neq 3 \quad (i)$$

N1.34, c. 2

$$(\exists x)(\exists y): x+y = 3 \quad (\times)$$

$$(\forall x)(\forall y): x+y \neq 3 \quad (\times)$$

N1.36, c. 2

$$(\exists x \geq 0)(\exists y \geq 0): x+y = 0 \quad (\times)$$

$$(\forall x \geq 0)(\forall y \geq 0): x+y \neq 0 \quad (i)$$

N1.38, c. 2

$$(\forall x \geq 0)(\exists y \leq 0): x+y = 0 \quad (\times)$$

$$(\exists x \geq 0)(\forall y \leq 0): x+y \neq 0 \quad (i)$$

N1.40, c. 2

$$(\forall x)(\forall y): x < y \Rightarrow x^3 < y^3 \quad (\times)$$

$$(\exists x)(\exists y): x < y \wedge x^3 \geq y^3 \quad (i)$$

N1.42, c. 8

$$(\forall x): x^2 > x \Leftrightarrow x > 1 \vee x < 0 \quad (i)$$

$$(\exists x): x^2 > x \wedge x \geq 1 \wedge x \geq 0 \quad (\times)$$

N1.44, c. 8

$$(\forall x): x^2 > x \Leftrightarrow x > 1 \wedge x < 0 \quad (i)$$

$$(\exists x): x^2 > x \wedge x \leq 1 \vee x \geq 0 \quad (\times)$$



N 159, c.8

$$A = \{x \in \mathbb{R} : x^2 + 6x + 8 < 0\}, \quad B = \{x \in \mathbb{R} : x^2 + 3x < 0\}$$

$$x^2 + 6x + 8 = 0$$

$$\Delta = 36 - 4 \cdot 8 = 4$$

$$x = \frac{-6 \pm 2}{2} = -2$$

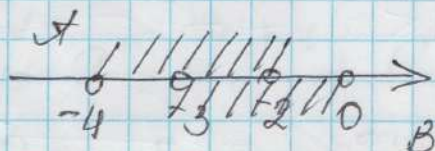
$$x = \frac{-6 - 2}{2} = -4$$

$$(x+2)(x+4) < 0$$



$$A \in (-4; -2)$$

$$\bar{A} = (-\infty; -4] \cup [2; +\infty)$$



$$A \cup B = (-4; 0)$$

$$A \cap B = (-3; -2)$$

$$A \setminus B = (-4; -3)$$

$$B \setminus A = (-2; 0)$$

$$A \Delta B = (-4; -3) \cup (-2; 0)$$

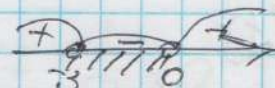
17.60 c.8

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

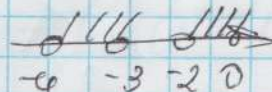
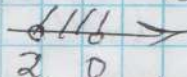
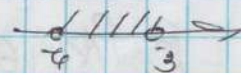
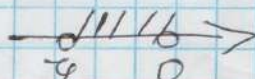
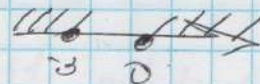
$$x = 0 \quad x = -3$$

$$x(x+3) < 0$$



$$B \in (-3; 0)$$

$$\bar{B} = (-\infty; -3] \cup [0; +\infty)$$





$$A \Delta B = (-4; -3) \cup (-2; 0)$$

U1.60, c8

$$A = \{x \in \mathbb{R} : 1 < |x-3| \leq 2\}, B = \{x \in \mathbb{R} : 2|x| < 3\}$$

$$1 < |x-3| \leq 2$$

$$\begin{cases} x-3 > 1 \\ x-3 \leq 2 \end{cases}$$

$$\begin{cases} x > 4 \\ x \leq 5 \end{cases} \quad \begin{cases} x < 3 \\ x \geq 2 \end{cases} \quad [1; 2) \cup (4; 5]$$

$$\begin{cases} x < 3 \\ x > 0 \\ x < 0 \\ x \geq 0 \end{cases}$$

$$\begin{cases} x < \frac{3}{2} \\ x > 0 \\ x < -\frac{3}{2} \\ x \geq 0 \end{cases}$$

$$\begin{bmatrix} 0 & 3 \\ 3 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & 1.5 \\ 1.5 & 0 \end{bmatrix}$$

$$B \in \left(-\frac{3}{2}; \frac{3}{2}\right)$$

$$A \in [4; 2) \cup (4; 5]$$

$$A = (-\infty; 1) \cup [2; 2] \cup [2; 5] \cup (5; +\infty)$$

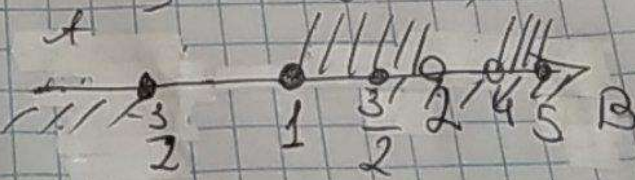
$$A \cup B = (-\infty; \frac{3}{2}) \cup (4; +\infty)$$

$$A \cap B = \left[\frac{3}{2}; 2\right) \cup (4; 5]$$

$$A \setminus B = \left(1; \frac{3}{2}\right) \cup (5; +\infty)$$

$$B \setminus A = \left(-\frac{3}{2}; -1\right) \cup \left(-\frac{1}{2}; 2\right)$$

$$A \Delta B = \left[-\frac{3}{2}; -1\right) \cup \left(-\frac{1}{2}; 2\right) \cup (5; +\infty)$$





# Бинетова теорема (доказательство)

$$(a+b)^n = \sum C_n^k a^{n-k} b^k = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + C_n^2 a^{n-2} b^2 + \dots + C_n^k a^{n-k} b^k + \dots + C_n^n a^0 b^n$$

$$T(n): (a+b)^n = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + \dots + C_n^k a^{n-k} b^k + \dots + C_n^n a^0 b^n$$

$$\nabla T(1): (a+b)^1 = C_1^0 a^1 b^0 + C_1^1 a^0 b^1 = C_1^0 a + C_1^1 b = 1 \cdot a + 1 \cdot b = a+b$$

$$T(m): (a+b)^m = C_m^0 a^m b^0 + C_m^1 a^{m-1} b^1 + C_m^2 a^{m-2} b^2 + \dots + C_m^{k-1} a^{m-k+1} b^{k-1} + C_m^k a^{m-k} b^k + C_m^{k+1} a^{m-k-1} b^{k+1} + \dots + C_m^{m-1} a^1 b^{m-1} + C_m^m a^0 b^m - \text{нужны, что нр.}$$

$$T(m+1): (a+b)^{m+1} = C_{m+1}^0 a^{m+1} b^0 + C_{m+1}^1 a^m b^1 + \dots + C_{m+1}^{k-1} a^{m-k+2} b^{k-1} + C_{m+1}^k a^{m-k+1} b^k + C_{m+1}^{k+1} a^{m-k} b^{k+1} + \dots + C_{m+1}^m a^1 b^m + C_{m+1}^{m+1} a^0 b^{m+1} - \text{добавим}$$

$$\begin{aligned} (a+b)^{m+1} &= (a+b)^m (a+b) = (C_m^0 a^m b^0 + C_m^1 a^{m-1} b^1 + \dots + C_m^{k-1} a^{m-k+1} b^{k-1} + C_m^k a^{m-k} b^k + C_m^{k+1} a^{m-k-1} b^{k+1} + \dots + C_m^{m-1} a^1 b^{m-1} + C_m^m a^0 b^m) (a+b) \\ &= C_m^0 a^{m+1} b^0 + C_m^1 a^m b^1 + \dots + C_m^{k-1} a^{m-k+2} b^{k-1} + C_m^k a^{m-k+1} b^k + C_m^{k+1} a^{m-k} b^{k+1} + \dots + C_m^{m-1} a^2 b^{m-1} + C_m^m a^1 b^m + C_m^0 a^m b^1 + C_m^1 a^{m-1} b^2 + \dots + C_m^{k-1} a^{m-k+1} b^k + C_m^k a^{m-k} b^{k+1} + C_m^{k+1} a^{m-k-1} b^{k+1} + \dots + C_m^{m-1} a^1 b^m + C_m^m a^0 b^{m+1} \\ &= C_m^0 a^{m+1} b^0 + (C_m^0 + C_m^1) a^m b^1 + \dots + (C_m^{k-1} + C_m^k) a^{m-k+1} b^k + (C_m^k + C_m^{k+1}) a^{m-k} b^{k+1} + \dots + (C_m^{m-1} + C_m^m) a^1 b^m + C_m^m a^0 b^{m+1} \end{aligned}$$

Так как  $C_m^k + C_m^{k+1} = C_{m+1}^{k+1}$   
 $C_m^{m-1} + C_m^m = C_{m+1}^m$

$$C_m^0 + C_m^1 = C_{m+1}^1$$

$$C_m^{k-1} + C_m^k = C_{m+1}^k$$



$$(a+b)^{m+1} = C_{m+1}^0 a^{m+1} b^0 + C_{m+1}^1 a^m b^1 + \dots + C_{m+1}^k a^{m+1-k} b^k + \dots + C_{m+1}^m a^1 b^m + C_{m+1}^{m+1} b^{m+1}$$