Домашня робота (2.1)

№1.9, с.5

$$\sum_{i=1}^{n} i^3 = 1^3+2^3+\dots+n^3 = \frac{n^2(n+1)^2}{4}$$

$T(n): 1^3+2^3+\dots+n^3 = \frac{n^2(n+1)^2}{4}$, $n \in N$

$\nabla T(1): 1^3 = \frac{(1+1)^2 1}{4} = \frac{4}{4} = 1; 1 = 1$ – прав.

$T(k): 1^3+2^3+\dots+k^3 = \frac{k^2(k+1)^2}{4}$ – припуск., що пр.

$T(k+1): 1^3+2^3+\dots+k^3+(k+1)^3 = \frac{(k+1)^2((k+1)+1)^2}{4}$ – доведемо

$(1^3+2^3+\dots+k^3)+(k+1)^3 = \frac{(k+1)^2((k+1)+1)^2}{4} = \frac{(k+1)^2(k+2)^2}{4}$ Δ

№1.10, с.5

$$\sum_{i=1}^{n}(2i-1) = 1+3+\dots+(2n-1) = n^2$$

$T(n): 1+3+\dots+(2n-1) = n^2$; $n \in N$

$\nabla T(1): 2\cdot 1-1 = 1^2$; $1=1$ – прав.

$T(k): 1+3+\dots+(2k-1) = k^2$ – припуск., що пр.

$T(k+1): 1+3+\dots+(2(k+1)-1) = (2k+1)^2$ – доведемо

$1+3+\dots+(2(k+1)-1) = (k+1)^2$;

$2k+2-1 = (k+1)^2$; $(2k+1) = (k+1)^2$ Δ

№1.11, с.5

$$\sum_{i=1}^{n} 2^{i-1} = 1+2+\dots+2^{n-1} = 2^n-1$$

$T(n): 1+2+\dots+2^{n-1} = 2^n-1$, $n \in N$

$\nabla T(1): 2^{1-1} = 2^1-1$; $2^0 = 1$; $1 = 1$ – прав.

$T(k): 1+2+\dots+2^{k-1} = 2^k-1$ – припуск., що пр.

$T(k+1)$: $1+2+\dots+2^{(k+1)-1} = 2^{k+1}-1$ – доведемо

$1+2+\dots+2^{(k+1)-1} = 2^{k+1}-1$; $2^{k+1-1} = 2^{k+1}-1$;

$2^k = 2^{k+1}-1$ Δ

№ 1.13, с.5

$\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{n+1}}$

$T(n): \sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{n+1}}, n \in \mathbb{N}$

∇ $T(1): \sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{1+1}} = 2\cos\frac{\pi}{2^2} = 2\cos\frac{\pi}{4}$

$\sqrt{2} = 2\frac{\sqrt{2}}{2}$; $\sqrt{2} = \sqrt{2}$ – прав.

$T(k): \sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{k+1}}$ – припуск., що пр.

$T(k+1): \sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{(k+1)+1}} = 2\cos\frac{\pi}{2^{k+2}}$ – доведемо

$\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}} = 2\cos\frac{\pi}{2^{k+2}}$; $\sqrt{2\left(1+\cos\frac{\pi}{2^{k+1}}\right)} = 2\cos\frac{\pi}{2^{k+2}}$

$\sqrt{2\cdot 2\cos^2\frac{\pi}{2^{k+2}}} = 2\cos\frac{\pi}{2^{k+2}}$

$2\cos\frac{\pi}{2^{k+2}} = 2\cos\frac{\pi}{2^{k+2}}$ Δ

№ 1.1, с.5

n^3+5n на 6

$T(n): (n^3+5n) \vdots 6, n \in \mathbb{N}$

∇ $T(1): (1^3+5\cdot 1) \vdots 6$, бо $6:6=1$ – прав.

$T(k): (k^3+5k) \vdots 6, k \in \mathbb{N}$ – припуск., що пр.

$T(k+1): ((k+1)^3+5(k+1)) \vdots 6$ – доведемо

$k^3+3k^2+3k+1+5k+5 = k^3+5k+3k(k+1)+6 = (k^3+5k) \vdots 6$

згідно з гіпотези $\vdots 6$ Δ

№ 1.2, с.5

$T(n): n^2(n^4-1) \vdots 60, n \in \mathbb{N}$

∇ $T(1): 1^2(1^4-1) \vdots 60$ – прав.

$T(k): k^2(k^4-1) \vdots 60, k \in \mathbb{N}$ – припуск., що пр.

$T(k+1): (k+1)^2((k+1)^4-1) \vdots 60$ – доведемо; $(k+1)^2(k^2+2k+2)(k^2+2k+1-1) =$

$= (k+1)^2(k^2+2k+2)k(k+2) = k(k+1)(k+2)(k^2+2k+2)(k+1)$ –

– згідно з гіпотези $\vdots 60$ Δ

№ 1.3, с.5

$T(n): (5^{n+3}+11^{3n+1}) \vdots 17, n \in \mathbb{N}$

∇ $T(1): (5^{1+3}+11^{3+1}) \vdots 17$, бо $17:17=1$ – прав.

$T(k): (5^{k+3}+11^{3k+1}) \vdots 17, k \in \mathbb{N}$ – припуск., що пр.

$T(k+1): (5^{(k+1)+3}+11^{3k+3+1}) \vdots 17$ – доведемо

5^{k+4} $5^k \cdot 5^4 + 11^{3k+4}$ $11^{3k} \cdot 11^4$ – згідно з гіпотези $\vdots$ 17 Δ

№ 1.4, с.5

$7^{2n} - 4^{2n}$ на 33

$T(n): (7^{2n} - 4^{2n}) \vdots 33, n \in \mathbb{N}$

∇ $T(1): (7^2 - 4^2) \vdots 33$, об $33 : 33 = 1$ – прав

$T(k): (7^{2k} - 4^{2k}) \vdots 33, k \in \mathbb{N}$ – припуск., що пр.

$T(k+1): (7^{2(k+1)} - 4^{2(k+1)}) \vdots 33$ – доведемо

$7^{2k+2} - 4^{2k+2}$ згідно з гіпотези $\vdots$ 33 Δ

№ 1.19, с.5

$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n} (n \geq 1)$

∇ $T(1): 2 - \frac{1}{1} (1 \geq 1) = 1$; $2 - 1 (1 \geq 1) = 1$; $1 \geq 1 = 1$ – прав

$T(k): 2 - \frac{1}{k} (k \geq 1)$ – припуск., що пр.

$T(k+1): 2 - \frac{1}{k+1} (k+1 \geq 1)$ доведемо

$\frac{2k+2-1}{k+1} (k+1 \geq 1) = \frac{2k+1}{k+1} (k+1) \geq 1$ – згідно з гіпотез. Δ

№ 1.22, с.5

$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > 2(\sqrt{n+1} - 1) (n > 1)$

∇ $T(1): 2(\sqrt{1+1} - 1)(1 > 1) = 1$; $2(\sqrt{2} - 1)(1 > 1) = 1$ – прав

$T(k): 2(\sqrt{k+1} - 1)(k > 1)$ – припуск., що пр.

$T(k+1): 2(\sqrt{(k+1)+1} - 1)(k+1 > 1)$ – доведемо

$2(\sqrt{k+2} - 1)(k+1 > 1) =$

$= (2\sqrt{k+2} - 2)(k+1) > 1$ – згідно з гіпотези Δ

№ 1.23, с.5

$n^{n+1} > (n+1)^n (n \geq 3)$

$T(n): n^{n+1} > (n+1)^n (n \geq 3)$, $n \in \mathbb{N}$

∇ $T(3)$: $3^{3+1} > (3+1)^3 \ (3 \geq 3)$; $3^4 > 4^3 \ (3 \geq 3)$ - прав

$T(k)$: $k^{k+1} > (k+1)^k \ (k \geq 3), k \in \mathbb{N}$ – припуск, що пр

$T(k+1)$: $(k+1)^{k+1+1} > ((k+1)+1)^{k+1} \ (k+1 \geq 3)$ - доведемо

$(k+1)^{k+2} > (k+2)^{k+1} \ (k+1) \geq 3$ – вірно з гіпотези

Домашня робота (з.2)

№1.32, с.7

$(\forall x)(\exists y)$: $x+y<3$ x

$(\exists x)(\forall y)$: $x+y \neq 3$ і

№1.34, с.7

$(\exists x)(\exists y)$: $x+y=3$ x

$(\forall x)(\forall y)$: $x+y \neq 3$ x

№1.36, с.7

$(\exists x>0)(\exists y>0)$: $x+y=0$ x

$(\forall x>0)(\forall y>0)$: $x+y \neq 0$ і

№1.38, с.7

$(\forall x>0)(\exists y<0)$: $x+y=0$ x

$(\exists x>0)(\forall x<0)$: $x+y \neq 0$ і

№1.40, с.7

$(\forall x)(\forall y)$: $\overset{p}{x<y} \Rightarrow \overset{q}{x^3<y^3}$ x

$(\exists x)(\exists y)$: $x<y \wedge x^3 \geq y^3$ і

№1.42, с.8

$(\forall x)$: $x^2>x \Leftrightarrow x>1 \vee x<0$ і

$(\exists x)$: $x^2>x \wedge x \geq 1 \wedge x \geq 0$ x

№1.44, с.8

$(\forall x)$: $x^2>x \Leftrightarrow x>1 \wedge x<0$ і

$(\exists x)$: $x^2>x \wedge x \leq 1 \vee x \geq 0$ x

№ 159, с.8

$A=\{x\in R : x^2+6x+8<0\}$, $B=\{x\in R : x^2+3x<0\}$

$x^2+6x+8=0$

$D=36-4\cdot 8=4$

$x=\frac{-6+2}{2}=-2$

$x=\frac{-6-2}{2}=-4$

$(x+2)(x+4)<0$

$A\in(-4;-2)$

$\bar{A}=(-\infty;-4]\cup[2;+\infty)$

$x^2+3x=0$

$x(x+3)=0$

$x=0 \quad x=-3$

$x(x+3)<0$

$B\in(-3;0)$

$\bar{B}=(-\infty;-3]\cup[0;+\infty)$

$A\cup B=(-4;0)$

$A\cap B=(-3;-2)$

$A\setminus B=(-4;-3)$

$B\setminus A=(-2;0)$

$A\,\Delta\,B=(-4;-3)\cup(-2;0)$

№ 160, с.8

$A \Delta B = (-4; -3) \cup (-2; 0)$

№ 1.60, с 8

$A = \{x \in R : 1 < |x - 3| \le 2\}$, $B = \{x \in R ; 2|x| < 3\}$

$1 < x - 3 \le 2$

$\begin{cases} x - 3 > 1 \\ x - 3 \le 2 \end{cases}$ $\left[\begin{array}{l} \begin{cases} x \ge 3 \\ x > 4 \end{cases} \\ \begin{cases} x < 2 \\ x < 3 \end{cases} \end{array}\right.$ $\left[\begin{array}{l} \begin{cases} x \le 5 \\ x \ge 3 \end{cases} \\ \begin{cases} x \ge 1 \\ x < 3 \end{cases} \end{array}\right.$ $[1; 2) \cup (4; 5]$

$\left[\begin{array}{l} \begin{cases} 2x < 3 \\ x \ge 0 \end{cases} \\ \begin{cases} 2(-x) < 3 \\ x < 0 \end{cases} \end{array}\right.$ $\left[\begin{array}{l} \begin{cases} x < \frac{3}{2} \\ x \ge 0 \end{cases} \\ \begin{cases} x > -\frac{3}{2} \\ x < 0 \end{cases} \end{array}\right.$

$[0; \frac{3}{2})$

$(-\frac{3}{2}; 0)$

$B \in (-\frac{3}{2}; \frac{3}{2})$

$A \in [1; 2) \cup (4; 5]$

$\bar{A} = (-\infty; 1) \cup (1; 2] \cup [2; 4] \cup (5; +\infty)$

$\bar{B} = (-\infty; -\frac{3}{2}] \cup [\frac{3}{2}; +\infty)$

$A \cup B = (-\infty; -\frac{3}{2}] \cup (1; +\infty)$

$A \cap B = [\frac{3}{2}; 2) \cup (4; 5]$

$A \setminus B = (1; \frac{3}{2}]$

$B \setminus A = (-\infty; \frac{3}{2}] \cup (2; 4)$

$A \Delta B = [\frac{3}{2}; 1] \cup [5; +\infty)$

Біном Ньютона (доведення)

$$(a+b)^n = \sum_{k=0} C_n^k a^{n-k} b^k = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + C_n^2 a^{n-2} b^2 + \dots + C_n^k a^{n-k} b^k + \dots + C_n^n a^0 b^n$$

$T(n): (a+b)^n = C_n^0 a^n b^0 + C_n^1 a^{n-1} b^1 + \dots + C_n^k a^{n-k} b^k + \dots + C_n^n a^0 b^n$

$\nabla T(1): (a+b)^1 = C_1^0 a^1 b^0 + C_1^1 a^{1-1} b^1 = C_1^0 a + C_1^1 b$ – прав.

$T(m): (a+b)^m = C_m^0 a^m b^0 + C_m^1 a^{m-1} b^1 + C_m^2 a^{m-2} b^2 + \dots + C_m^{k-1} a^{m-k+1} b^{k-1} + C_m^k a^{m-k} b^k + C_m^{k+1} a^{m-k+1} + \dots + C_m^{m-1} a^1 b^{m-1} + C_m^m a^0 b^m$ – припуск, що пр.

$T(m+1): (a+b)^{m+1} = C_{m+1}^0 a^{m+1} b^0 + C_{m+1}^1 a^m b^1 + \dots + C_{m+1}^{k-1} a^{m-k+2} b^{k-1} + C_{m+1}^k a^{m-k+1} b^k + C_{m+1}^{k+1} a^{m-k} b^{k+1} + \dots + C_{m+1}^m a^1 b^m + C_{m+1}^{m+1} a^0 b^{m+1}$ – довед. що

$(a+b)^{m+1} = (a+b)^m (a+b) = (C_m^0 a^m b^0 + C_m^1 a^{m-1} b^1 + \dots + C_m^{k-1} a^{m-k+1} b^{k-1} + C_m^k a^{m-k} b^k + C_m^{k+1} a^{m-k-1} b^{k+1} + \dots + C_m^{m-1} a^1 b^{m-1} + C_m^m a^0 b^m)(a+b) = C_m^0 a^{m+1} b^0 + C_m^1 a^m b^1 + \dots + C_m^{k-1} a^{m-k+2} b^{k-1} + C_m^k a^{m-k+1} b^k + C_m^{k+1} a^{m-k} b^{k+1} + \dots + C_m^{m-1} a^2 b^{m-1} + C_m^m a^1 b^m + C_m^0 a^m b^1 + C_m^1 a^{m-1} b^2 + \dots + C_m^{k-1} a^{m-k+1} b^k + C_m^k a^{m-k} b^{k+1} + C_m^{k+1} a^{m-k-1} b^{k+1} + \dots + C_m^{m-1} a^1 b^m + C_m^m a^0 b^{m+1} = C_m^0 a^{m+1} + (C_m^0 + C_m^1) a^m b^1 + \dots + (C_m^{k-1} + C_m^k) a^{m-k+1} b^k + (C_m^k + C_m^{k+1}) a^{m-k} b^{k+1} + \dots + (C_m^{m-1} + C_m^m) a^1 b^{m-1} + C_m^m a^0 b^{m+1}$

$C_m^0 + C_m^1 = C_{m+1}^1$

$C_m^{k-1} + C_m^k = C_{m+1}^k$

Так як $C_m^k + C_m^{k+1} = C_{m+1}^{k+1}$

$C_m^{m-1} + C_m^m = C_{m+1}^m$

$$(a+b)^{m+1} = C_{m+1}^{0} a^{m+1} b^{0} + C_{m+1}^{1} a^{m} b^{1} + \dots + C_{m+1}^{k} a^{m+1-k} b^{k} +$$

$$+ C_{m+1}^{k+1} a^{m-k} b^{k+1} + \dots + C_{m+1}^{m} a^{1} b^{m} + C_{m+1}^{m+1} b^{m+1} \quad \Delta$$