

№
Контрольна робота №3
з математики аналізу
Кравець Олена
ТМО - 11

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$$B. y' = \frac{3-4t^3}{2-5t^4}$$

$$x = 2t - t^5$$

$$y'' = \frac{60t^3 - 20t^6 - 24t^2}{(2-5t^4)^2}$$

$$x = 2t - t^5$$

③ $y^{(98)}$

$$y = (x^2 + 2x - 3) \sin 3x$$

$$y^{(0)} = x^2 + 2x - 3$$

$$y^{(1)} = 2x + 2$$

$$y^{(2)} = 2$$

$$y^{(3)} = 0$$

$$y^{(4)} = 0$$

$$\dots$$

$$y^{(100)} = 0$$

$$y^{(98)} = 2^{98} \sin 3x$$

$$y^{(97)} = (y^{(98)})' = -2^{97} \cos 3x$$

$$y^{(96)} = -2^{96} \sin 3x$$

$$y^{(0)} =$$

$$y^{(98)} = \sum_{m=0}^{98} C_{98}^{100} y^{(m)} U^{(98-m)} = C_{98}^0 y^{(0)} U^{(98)} + C_{98}^1 y^{(1)} U^{(97)} +$$

$$+ C_{98}^2 y^{(2)} U^{(96)} + \dots + C_{98}^{98} y^{(98)} U^{(0)} =$$

$$= 1 \cdot (x^2 + 2x - 3) \cdot 2^{98} \sin 3x + 98 (2x + 2) (-2^{97} \cos 3x) +$$

$$+ \frac{97 \cdot 98}{2} 2 (-2^{96} \sin 3x) = x^2 + 2x - 3 \cdot 2^{98} \sin 3x + 98 (2x + 2) \times$$

$$\times 2^{97} \cos 3x + 97 \cdot 98 (-2^{96} \sin 3x)$$

④ $y = \arcsin \frac{1}{x}$

$$dy = \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \left(\frac{1}{x}\right)' = \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)$$

$$d^2y = d(dy) = d\left(\frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)\right) =$$

$$= \left(d\left(\frac{1}{\sqrt{1 - \frac{1}{x^2}}}\right) \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot d\left(-\frac{1}{x^2}\right)\right) dx =$$

$$= -\frac{2x}{(x^2 - 1)^2} \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \frac{2}{x^3}$$

$$a) y = \ln^3(\operatorname{tg}^3(\cos(2^x + x^2)))$$

$$(u^3)' = 3u^2 u'$$

$$y' = 3\ln^2(\operatorname{tg}^3(\cos(2^x + x^2))) \cdot (\ln(\operatorname{tg}^3(\cos(2^x + x^2))))' =$$

$$= 3\ln^2(\operatorname{tg}^3(\cos(2^x + x^2))) \cdot \frac{1}{\operatorname{tg}^3(\cos(2^x + x^2))} \cdot$$

$$\times 3\operatorname{tg}^2(\cos(2^x + x^2)) \cdot \frac{1}{\cos^2(\cos(2^x + x^2))} \cdot (-\sin(2^x + x^2)) \cdot \ln 2 \cdot 2^x$$

$$+ 2x$$

$$b) y = (\arcsin \frac{3}{5}x)^{x^2} = e^{\ln x^2 (\arcsin \frac{3}{5}x)}$$

$$y' = e^{\ln \arcsin \frac{3}{5}x - x^2} \cdot \frac{1}{\arcsin \frac{3}{5}x} \cdot \frac{1}{(1 - x^2)^{\frac{1}{2}}} \cdot \frac{1}{5}x^{-\frac{2}{3}} \cdot x^2 +$$

$$+ \ln(\arcsin(x^{\frac{2}{3}})) \cdot 2x$$

$$② \begin{cases} x = 2t - t^5 \\ y = 3t - t^4 \end{cases}$$

$$y_x = \frac{(3t - t^4)'_t}{(2t - t^5)'_t} = \frac{3 - 4t^3}{2 - 5t^4}$$

$$y_{xx} = \frac{(\frac{3-4t^3}{2-5t^4})'_t}{(2t - t^5)'_t} = \frac{60t^2 - 20t^6 - 24t^2}{(2-5t^4)^2} = \frac{60t^2 - 20t^6 - 24t^2}{(2-5t^4)^2}$$

$$\left(\frac{3-4t^3}{2-5t^4}\right)' = \frac{(3-4t^3)'(2-5t^4) - (2-5t^4)'(3-4t^3)}{(2-5t^4)^2} = \frac{-4 \cdot 3t^2(2-5t^4) - (3-4t^3)(-20t^3)}{(2-5t^4)^2} =$$

$$= \frac{-12t^2(2-5t^4) - (3-4t^3)(-20t^3)}{(2-5t^4)^2} = \frac{-24t^2 + 60t^6 + (3-4t^3) \cdot 20t^3}{(2-5t^4)^2} =$$

$$= \frac{-24t^2 - 20t^6 + 60t^3}{(2-5t^4)^2} = \frac{60t^3 - 20t^6 - 24t^2}{(2-5t^4)^2}$$

Кравець Анна ДМО-11

$$f'(x) = 0 \quad \text{н/е} \quad \frac{12x^2 - x^4}{(1-x^2)^2} = 0 \Rightarrow x=0 \quad \text{адб}$$

$$12x^2 - x^4 = 0$$

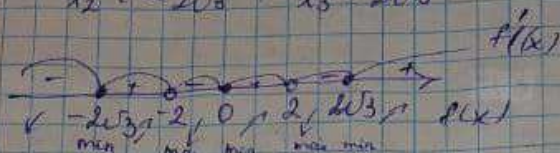
$$x^2(12 - x^2) = 0$$

$$12 - x^2 = 0$$

$$x^2 = 12$$

$$x = \pm 2\sqrt{3}$$

$$x_1 = 0 \quad x_2 = -2\sqrt{3} \quad x_3 = 2\sqrt{3}$$



$\exists f'(x)$ н/е $x=2, x=-2$ не є стая. т.

⑥ Опукисть, т. перешку

$$f''(x) = \frac{96x + 8x^3}{(4-x^2)^3}$$

$$96x + 8x^3 = 0$$

$$8x(12 + x^2) = 0$$

$$x(12 + x^2) = 0$$

$$x=0 \quad 12+x^2=0 \in \mathbb{R}$$

$x=0$ — т. перешку

$[0, +\infty)$ — вогнута

$(-\infty, 0]$ — опукла

$$f(0) = 0$$

$$x_1 = -2\sqrt{3} \text{ — т. min}$$

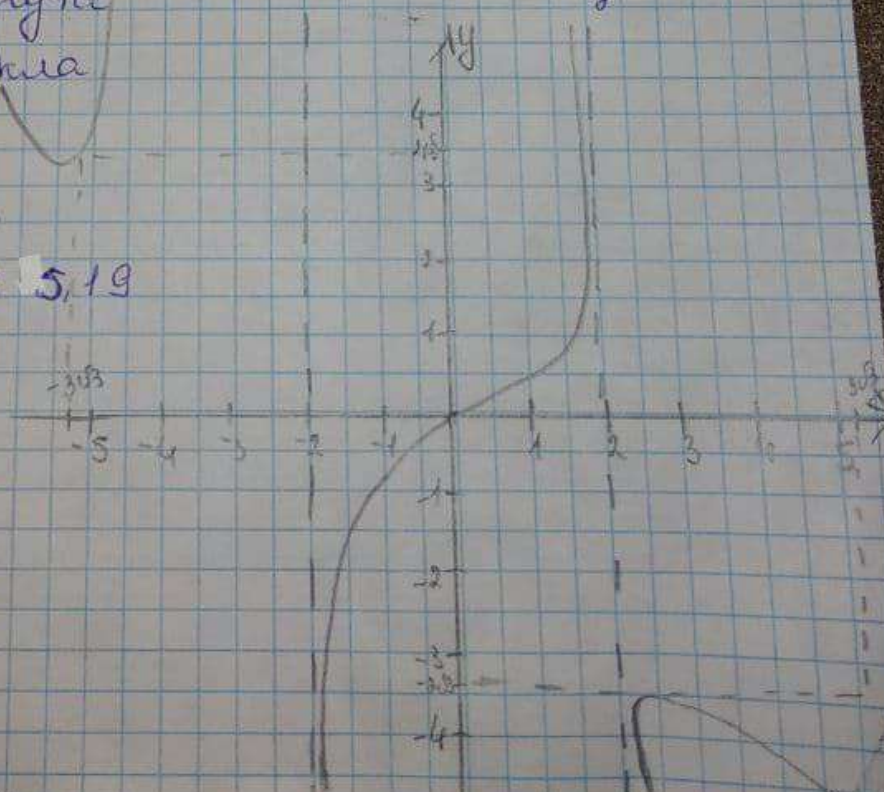
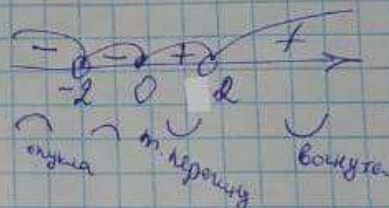
$$f_{\min} = f(-2\sqrt{3}) \approx 5,19$$

$$x_2 = 0 \text{ — т. min}$$

$$f(0) = 0$$

$$x_3 = 2\sqrt{3} \text{ — т. min}$$

$$f_{\min} = f(2\sqrt{3}) \approx -5,19$$



$$⑤ \quad y = \frac{x^3}{4-x^2}$$

$$⑥ \quad D(y) = \mathbb{R} \setminus \{-2, 2\}$$

⑦ Неперіодична, кепарка \rightarrow симетр. відносно поч. коорд.

$$x \rightarrow 2: \lim_{x \rightarrow 2-0} \frac{x^3}{4-x^2} = \frac{(2-0)^3}{4-(2-0)^2} = \frac{8}{+0} = +\infty$$

$$\lim_{x \rightarrow 2+0} \frac{x^3}{4-x^2} = \frac{(2+0)^3}{4-(2+0)^2} = \frac{8}{+0} = +\infty$$

$$x \leftarrow -2: \lim_{x \rightarrow -2-0} \frac{x^3}{4-x^2} = \frac{(-2-0)^3}{4-(-2-0)^2} = \frac{-8}{+0} = -\infty$$

$$x \leftarrow -2: \lim_{x \rightarrow -2+0} \frac{x^3}{4-x^2} = \frac{(-2+0)^3}{4-(-2+0)^2} = \frac{-8}{+0} = -\infty$$

⑧ $m \cdot x = -2$
 $m \cdot x = 2$ } — т. розриву II роду

а) Вертикал. асимптоти

$$б) \exists \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{x^3}{x(4-x^2)} = \lim_{x \rightarrow \pm\infty} \frac{x^3}{4x-x^3} = -1 = k$$

$$\exists \lim_{x \rightarrow \pm\infty} (f(x) - kx) = \lim_{x \rightarrow \pm\infty} \left(\frac{x^3}{4-x^2} + 1 \cdot x \right) =$$

$$= \lim_{x \rightarrow \pm\infty} \left(\frac{x^3 + 4x - x^3}{4-x^2} \right) = \lim_{x \rightarrow \pm\infty} \frac{4x}{4-x^2} = \lim_{x \rightarrow \pm\infty} \left(\frac{x \cdot \frac{4}{x}}{x \cdot (\frac{4}{x^2} - 1)} \right) =$$

$$= \lim_{x \rightarrow \pm\infty} \left(\frac{\frac{4}{\frac{1}{x}}}{\frac{4}{x^2} - 1} \right) = 0; \Rightarrow y = kx + b = -1 \cdot x + 0 = -x$$

$y = -x$ — похил. асимпт. при $x \rightarrow \pm\infty$

$$④ \quad x=0 \Rightarrow f(x)=0 \quad (0;0)$$

$$y=0 \Rightarrow \frac{x^3}{4-x^2}=0 \quad x^3=0 \quad x=0 \quad x \neq \pm 2 \quad (0;0)$$

⑤ Extr:

$$f'(x) = \frac{(x^3)'(4-x^2) - (4-x^2)'x^3}{(4-x^2)^2} = \frac{3x^2(4-x^2) + 2x \cdot x^3}{(4-x^2)^2} =$$

$$= \frac{12x^2 - x^4}{(4-x^2)^2}$$

$$\textcircled{6} \quad \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{\cos x - e^{-x^2}}$$

$\ln(1+x^2)$ - по формуле Маклорена c. 66 IV

$$\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \dots + \frac{(-1)^{n+1}}{n} (x^2)^n + o((x^2)^n), x \rightarrow 0$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n}{(2n)!} x^{2n} + o(x^{2n}), x \rightarrow 0$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \dots + \frac{(-1)^n}{n!} (x^2)^n + o((x^2)^n), x \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \dots + \frac{(-1)^{n+1}}{n} (x^2)^n + o((x^2)^n)}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n}{(2n)!} x^{2n} + o(x^{2n}) - (1 - x^2 + \frac{x^4}{2!} - \dots + \frac{(-1)^n}{n!} (x^2)^n + o((x^2)^n))}$$

$$\textcircled{7} \quad a) \quad \lim_{x \rightarrow +0} (\pi x^2)^{\sin x} = [1^\infty] = \lim_{x \rightarrow +0} e^{\ln(\pi x^2) \sin x} =$$

$$= \lim_{x \rightarrow +0} e^{\sin x \ln(\pi x^2)} = e^{\lim_{x \rightarrow +0} \ln(\pi x^2) \cdot \sin x} = [e^{-\infty \cdot 0}] =$$

$$= e^{\lim_{x \rightarrow +0} \frac{\ln(\pi x^2)}{\sin x}} = e^{\lim_{x \rightarrow +0} \frac{\frac{1}{\pi x^2} \cdot 2\pi x}{\cos x}} = e^{\lim_{x \rightarrow +0} \frac{2}{x \cos x}} = \varnothing$$