Контрольна робота №3.
з матем аналізу
Кравець Ольги
ПМО - 11

№	а 1	б	2	3	4	5	6	а 7	б	+1 бал за оформ	Σ
бали											

B. $\begin{cases} y'_x = \dfrac{3-4t^3}{2-5t^4} \\ x = 2t - t^5 \end{cases}$ $\begin{cases} y''_{xx} = \dfrac{\dfrac{60t^3 - 20t^6 - 24t^2}{(2-5t^4)^2}}{2-5t^4} \\ x = 2t - t^5 \end{cases}$

③ $y^{(98)}$

$y = (x^2+2x-3)\sin 3x$

$v^{(0)} = x^2+2x-3$

$v^{(1)} = 2x+2$ $\quad$ $u^{(98)} = 2^{98} \cdot \sin 3x$

$v^{(2)} = 2$ $\quad$ $u^{(97)} = (u^{(96)})' = -2^{97}\cos 3x$

$v^{(3)} = 0$ $\quad$ $u^{(96)} = -2^{96}\sin 3x$

$v^{(4)} = 0$

...

$v^{(100)} = 0$ $\quad$ $u^{(0)} =$

). $\ln 2 \cdot 27$

$$y^{(98)} = \sum_{m=0}^{98} C_{98}^{m} v^{(m)} u^{(98-m)} = C_{98}^{0} v^{(0)} u^{(98)} + C_{98}^{1} v^{(1)} u^{(97)} +$$

$$+ C_{98}^{2} v^{(2)} u^{(96)} + \underbrace{C_{98}^{3} v^{(3)} u^{(95)} + \ldots + C_{98}^{98} v^{(98)} u^{(0)}}_{0} =$$

$$= 1 \cdot (x^2+2x-3) \cdot 2^{98} \sin 3x + 98(2x+2)(-2^{97}\cos 3x) +$$

$$+ \frac{97 \cdot 98}{2} \cdot 2(-2^{96}\sin 3x) = x^2+2x-3 \cdot 2^{98}\sin 3x + 98(2x+2) \times$$

$$\times 2^{97}\cos 3x + 97 \cdot 98(-2^{96}\sin 3x)$$

④ $y = \arcsin\frac{1}{x}$

$$dy = \frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(\frac{1}{x}\right)' = \frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)$$

$$d^2y = d(dy) = d\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)\right) =$$

$$= \left(d\left(\frac{1}{1-\frac{1}{x^2}}\right) \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot d\left(-\frac{1}{x^2}\right)\right) dx =$$

$$= -\frac{2x}{(x^2-1)^2} \cdot \left(-\frac{1}{x^2}\right) + \frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \frac{2}{x^3}$$

$-20t^3 =$

3 $=$

а) ① $y = \ln^3(tg^3(\cos(2^x+x^2)))$

$(u^3)' = 3u^2u'$

$y' = 3\ln^2(tg^3(\cos(2^x+x^2))) \cdot (\ln(tg^3(\cos(2^x+x^2))))' =$

$= 3\ln^2(tg^3(\cos(2^x+x^2))) \cdot \dfrac{1}{tg^3(\cos(2^x+x^2))} \times$

$\times\ 3tg^2(\cos(2^x+x^2)) \cdot \dfrac{1}{\cos^2(\cos 2^x+x^2)} \cdot (-\sin(2^x+x^2)) \cdot \ln 2 \cdot 2^x$

$+2x$

б) $y = (\arcsin\sqrt[3]{x})^{x^2} = e^{\ln x^2(\arcsin\sqrt[3]{x})}$

$y' = e^{\ln\arcsin\sqrt[3]{x}\cdot x^2} \cdot \dfrac{1}{\arcsin\sqrt[3]{x}} \cdot \dfrac{1}{(1-x^{\frac{2}{3}})^2} \cdot \frac{1}{3}x^{-\frac{2}{3}} \cdot x^2 +$

$+\ln(\arcsin(x^{\frac{1}{3}})) \cdot 2x$

② $\begin{cases} x = 2t - t^5 \\ y = 3t - t^4 \end{cases}$

$y'_x = \dfrac{(3t-t^4)'_t}{(2t-t^5)'_t} = \dfrac{3-4t^3}{2-5t^4}$

$y''_{xx} = \dfrac{\left(\frac{3-4t^3}{2-5t^4}\right)'_t}{(2t-t^5)'_t} = \dfrac{\frac{60t^3-20t^6-24t^2}{(2-5t^4)^2}}{2-5t^4}$

$\left(\dfrac{3-4t^3}{2-5t^4}\right)' = \dfrac{(3-4t^3)'(2-5t^4)-(2-5t^4)'(3-4t^3)}{(2-5t^4)^2} = \dfrac{-4\cdot 3t^2(2-5t^4)-(3-4t^3)(-20t^3)}{(2-5t^4)^2} =$

$= \dfrac{-12t^2(2-5t^4)-(3-4t^3)(-20t^3)}{(2-5t^4)^2} = \dfrac{-24t^2+60t^6+(3-4t^3)\cdot 20t^3}{(2-5t^4)^2} =$

$= \dfrac{-24t^2-20t^6+60t^3}{(2-5t^4)^2} = \dfrac{60t^3-20t^6-24t^2}{(2-5t^4)^2}$

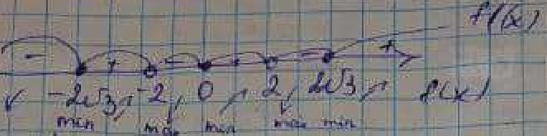
Кравець Ольги ПМО-11

$f'(x)=0$ при $\frac{12x^2-x^4}{(4-x^2)^2}=0 \Leftrightarrow x=0$ або

$12x^2-x^4=0$

$x^2(12-x^2)=0$ $\qquad 12-x^2=0 \quad x^2=12 \quad x=\pm 2\sqrt{3}$

$x_1=0 \quad x_2=-2\sqrt{3} \quad x_3=2\sqrt{3}$



$\nexists f'(x)$ при $x=2$, $x=-2$ не є стац. т.

⑥ Опуклість, т. перегину

$$f''(x)=\frac{96x+8x^3}{(4-x^2)^3}$$

$96x+8x^3=0$

$8x(12+x^2)=0$

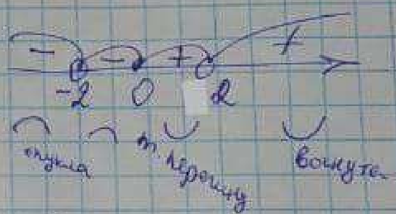
$x(12+x^2)=0$

$x=0 \qquad 12+x^2=0 \ \notin R$

$x=0$ – т. перегину

$[0,+\infty)$ – вгнута

$(-\infty;0]$ – опукла



$f(0)=0$

$x_1=-2\sqrt{3}$ – т. min

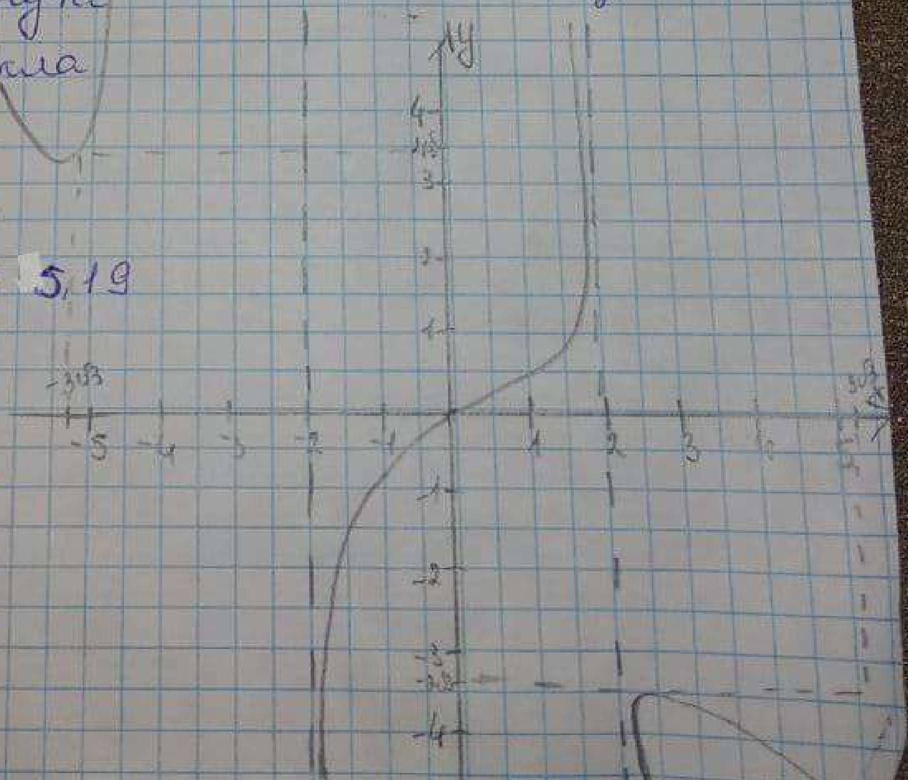
$f_{min}=f(3\sqrt{3})\approx 5{,}19$

$x_2=0$ – т. min

$f(0)=0$

$x_3=2\sqrt{3}$ – т. min

$f_{min}=f(2\sqrt{3})\approx -5{,}19$



⑤ $y = \frac{x^3}{4-x^2}$

① $D(y) = \mathbb{R} \setminus \{-2, 2\}$

$E(f)$

② Неперіодична, непарна $\Rightarrow$ симетр. відносно поч. коорд.

$x=2$: $\lim\limits_{x\to 2-0} \frac{x^3}{4-x^2} = \frac{(2-0)^3}{4-(2-0)^2} = \frac{8}{+0} = +\infty$

$\lim\limits_{x\to 2+0} \frac{x^3}{4-x^2} = \frac{(2+0)^3}{4-(2+0)^2} = \frac{8}{+0} = +\infty$

$x=-2$: $\lim\limits_{x\to -2-0} \frac{x^3}{4-x^2} = \frac{(-2-0)^3}{4-(-2-0)^2} = \frac{-8}{+0} = -\infty$

$x=-2$: $\lim\limits_{x\to -2+0} \frac{x^3}{4-x^2} = \frac{(-2+0)^3}{4-(-2+0)^2} = \frac{-8}{+0} = -\infty$

③ $\left.\begin{array}{l} т.\ x=-2 \\ т.\ x=2 \end{array}\right\}$ — т. розриву II роду

а) вертикал. асимптоти

б) $\exists \lim\limits_{x\to\pm\infty} \frac{f(x)}{x} = \lim\limits_{x\to\pm\infty} \frac{x^3}{x(4-x^2)} = \lim\limits_{x\to\pm\infty} \frac{x^3}{4x-x^3} = -1 = k$

$\exists \lim\limits_{x\to\pm\infty} (f(x)-kx) = \lim\limits_{x\to\pm\infty} \left(\frac{x^3}{4-x^2} + 1\cdot x\right) =$

$= \lim\limits_{x\to\pm\infty} \left(\frac{\cancel{x^3}+4x-\cancel{x^3}}{4-x^2}\right) = \lim\limits_{x\to\pm\infty} \frac{4x}{4-x^2} = \lim\limits_{x\to\pm\infty} \left(\frac{\cancel{x^2}\cdot\frac{4}{x}}{\cancel{x^2}\left(\frac{4}{x^2}-1\right)}\right) =$

$= \lim\limits_{x\to\pm\infty} \left(\frac{\frac{4}{x}}{\frac{4}{x^2}-1}\right) = 0$; $\Rightarrow$ $y = kx+b = -1\cdot x + 0 = -x$

$y = -x$ — похила асимп. при $x\to\pm\infty$

④ $x=0 \Rightarrow f(x)=0$ $(0;0)$

$y=0 \Rightarrow \frac{x^3}{4-x^2} = 0$ $x^3=0$ $x=0$ $x\neq\pm2$ $(0;0)$

⑤ Extr.:

$f'(x) = \frac{(x^3)'(4-x^2) - (4-x^2)'x^3}{(4-x^2)^2} = \frac{3x^2(4-x^2)+2x\cdot x^3}{(4-x^2)^2} =$

$= \frac{12x^2 - x^4}{(4-x^2)^2}$

⑥ $\lim\limits_{x\to 0} \frac{\ln(1+x^2)}{\cos x - e^{-x^2}}$

$\ln(1+x^2)$ – ф-ла Маклорена с.66 IV

$\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} + \ldots + \frac{(-1)^{n+1}}{n}(x^2)^n + (o(x^2))^n, x\to 0$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n}{(2n)!} \cdot x^{2n} + o(x^{2n}), x\to 0$

$e^{-x^2} = 1 - x^2 + \frac{x^4}{16} + \ldots + (-1)^n \frac{(x^2)^n}{n!} + o((x^2)^n), x\to 0$

$\lim\limits_{x\to 0} \frac{x^2 - \frac{x^4}{2} + \frac{x^6}{3} + \ldots + \frac{(-1)^{n+1}}{n}(x^2)^n + (o(x^2))^n}{\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n}{(2n)!} \cdot x^{2n} + o(x^{2n}) - (\cancel{1} - x^2 + \frac{x^4}{16} + \ldots + \frac{(-1)^n (x^2)^n}{n!} + \ldots}$

$\frac{\ldots}{+ \; o((x^2))^n}$

⑦ а) $\lim\limits_{x\to +0} (\pi x^2)^{\sin x} = [1^\infty] = \lim\limits_{x\to +0} e^{\ln(\pi x^2)^{\sin x}} =$

$= \lim\limits_{x\to +0} e^{\sin x \ln(\pi x^2)} = e^{\lim\limits_{x\to +0} \ln(\pi x^2)\cdot \sin x} = [e^{\infty\cdot 0}] =$

$= e^{\lim\limits_{x\to +0} \frac{\ln(\pi x^2)}{\sin x}} \overset{L}{=} e^{\lim\limits_{x\to +0} \frac{\frac{1}{\pi x^2}\cdot 2\pi x}{\cos x}} = e^{[\frac{-\infty}{\infty}]} = \varnothing$