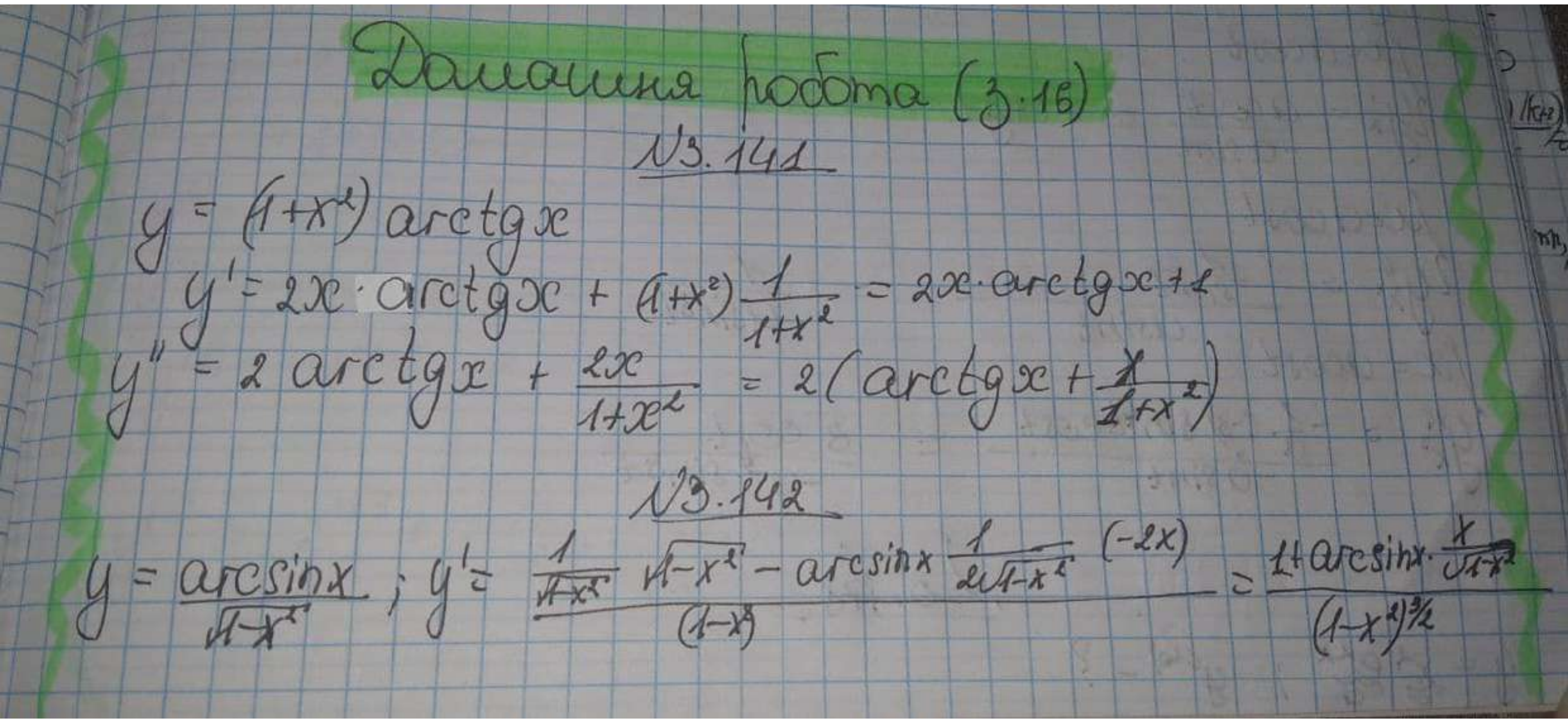

Домашня робота (3.16)

№3.141

$$y = (1+x^2)\,\mathrm{arctg}\,x$$

$$y' = 2x\cdot\mathrm{arctg}\,x + (1+x^2)\frac{1}{1+x^2} = 2x\cdot\mathrm{arctg}\,x + 1$$

$$y'' = 2\,\mathrm{arctg}\,x + \frac{2x}{1+x^2} = 2\left(\mathrm{arctg}\,x + \frac{x}{1+x^2}\right)$$

№3.142

$$y = \frac{\arcsin x}{\sqrt{1-x^2}}\,;\quad y' = \frac{\frac{1}{\sqrt{1-x^2}}\sqrt{1-x^2} - \arcsin x\,\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)} = \frac{1+\arcsin x\cdot\frac{x}{\sqrt{1-x^2}}}{(1-x^2)^{3/2}}$$

$$y'' = \frac{\left(\frac{1}{\sqrt{1-x^2}}\cdot\frac{x}{\sqrt{1-x^2}} + \arcsin x\,\frac{\sqrt{1-x^2}-x\frac{1}{2\sqrt{1-x^2}}(-2x)}{1-x^2}\right)(1-x^2)^{3/2}}{(1-x^2)^3} -$$

$$- \frac{\left(1+\arcsin x\cdot\frac{1}{\sqrt{1-x^2}}\right)\frac{3}{2}(1-x^2)^{\frac{1}{2}}(-2x)}{(1-x^2)^3} =$$

$$= \frac{\left(\frac{x}{1-x^2} + \arcsin x\cdot\frac{1-x^2+x^2}{(1-x^2)^{3/2}}\right)(1-x^2)^{3/2} - \left(-3x\sqrt{1-x^2} - 3x^2\arcsin x\right)}{(1-x)^3}$$

$$= \frac{x\sqrt{1-x^2} + \arcsin x + 3x\sqrt{1-x^2} + 3x^2\arcsin x}{(1-x^2)^3} =$$

$$= \frac{4x\sqrt{1-x^2} + (1+3x^2)\arcsin x}{(1-x^2)^3}$$

№3.157

$y = x^x$

$d^2y - ?$ $\qquad dy = y'dx$

$dy = (x'x^{x-1} + x^x\ln x)dx = (x^x + x^x\ln x)\,dx = x^x(1+\ln x)\,dx = y'$

$d^2y = y''(dx)^2$

$d^2y = \left((x\cdot x^{x-1} + x^x\ln x)(1+\ln x) + x^x\frac{1}{x}\right)(dx)^2 =$

$= \left(x^x(1+\ln x)^2 + x^{x-1}\right)(dx)^2 = \left(x^2 + 2x^x\ln x + x^x(\ln x)^2 + x^{x-1}\right)(dx)^2$

№3.165

$$\begin{cases} x = a\cos t \\ y = a\sin t \end{cases}$$

$$\begin{cases} x = a\cos t \\ y'_x = \frac{a\cos t}{-a\sin t} = -\mathrm{ctg}\, t \end{cases}$$

$$\begin{cases} x = a\cos t \\ y''_{x^2} = \dfrac{\frac{1}{\sin^2 t}}{-a\sin t} = -\dfrac{1}{a\sin^3 t} \end{cases}$$

$$\begin{cases} x = a\cos t \\ y'''_{x^3} = \dfrac{-\frac{1}{a}(-3)\sin^{-4}t\cos t}{-a\sin t} = \dfrac{3\cos t}{-a^2\sin^5 t} \end{cases}$$

№3.178

$y = x^2 e^{2x}$; $y^{(10)} - ?$

$\nabla\ y^{(20)} = \left[(u \cdot v)^{(20)} = \sum_{k=0}^{20} C_{20}^{k} u^{(n-k)} v^{k} \right] =$

$= 1 \cdot x^2 (e^{2x})^{(20)} + 20 \cdot 2x (e^{2x})^{(19)} + \frac{20!}{2!18!} \cdot 2 (e^{2x})^{(18)} + \frac{20!}{3!17!} \cdot 0 =$

$= x^2 (e^{2x})^{(20)} \cdot 2^{20} + 20 \cdot 2x (e^{2x})^{(19)} \cdot 2^{19} + 20 \cdot 19 \cdot (e^{2x})^{(18)} \cdot 2^{18} =$

$= 2^{18} e^{36x} (4x^2 e^{4x} + 80x \cdot e^{2x} + 380)$ Δ

№3.190

$y = \underbrace{x}_{v} \underbrace{\cos 2x}_{u};\quad d^{10}y - ?$

$\nabla\ d^{10}y = y^{(10)} (dx)^{10}$

$y^{(10)} = \left[(u \cdot v)^{(10)} = \sum_{k=0}^{10} C_{10}^{k} u^{(10-k)} v^{k} \right] =$

$= 1 \cdot x \cdot (\cos 2x)^{(10)} + \frac{10!}{1!9!} \cdot 1 \cdot (\cos 2x)^{(9)} + \frac{10!}{2!8!} \cdot 0 =$

$= x \cos\left(\frac{\pi}{2} \cdot 10 + 2x\right) \cdot 2^{10} + 10 \cos\left(\frac{5}{2} \cdot 9 + 2x\right) \cdot 2^{9} =$

$= 2^{10} \left(x \cos(\pi + 2x) + 5 \cos\left(\frac{\pi}{2} + 2x\right)\right) = 2^{10} (-x \cdot \cos 2x - 5 \cos 2x)$ Δ.

№3.198

$y = \sin^2 x;\quad y^{(n)} - ?$

$\nabla\ y' = 2 \sin x \cos x = 2^1 \sin x \cos x = 2^0 \sin 2x$

$y'' = 2 \cos x \cdot \cos x - 2 \sin x \sin x = 2\cos^2 x - 2 \sin^2 x = 2^1 \cos 2x$

$y''' = 2(-2 \cos x \sin x - 2 \sin x \cos x) = -8 \sin x \cos x = -2^3 \sin x \cos x = -2^2 \sin 2x$

$y^{IV} = -2^2\, 2 \cos 2x = -2^3 \cos 2x$

$y^{(5)} = +2^3 \sin 2x \cdot 2 = 2^4 \sin 2x$

$y^{(6)} = 2^5 \cos 2x$

$y^{(7)} = -2^6 \sin 2x$

$y^{(8)} = -2^7 \cos 2x$

$\ldots$

$y^{(n)} = -2^{n-1} \cos\left(\frac{\pi}{2} n + 2x\right)$ Δ.

№3.170

$M(3,4)$

$x^2 - xy + y^2 = 1$; $\qquad y'_x, y''_{x^2}, y'''_{x^3}$ − ?

∇

$$y'_x = \frac{-2x+y}{-x+2y}$$

$$y'_x\Big|_{(3,4)} = \frac{-2\cdot3+4}{-3+2\cdot4} = \frac{-6+4}{-3+8} = \frac{-2}{5} = -0{,}4$$

$$y''_{x^2} = \frac{6xy - 6y^2 - 6x^2}{(-x+2y)^3}$$

$$y''_{x^2}\Big|_{(3,4)} = \frac{6\cdot3\cdot4 - 6\cdot4^2\, 6\cdot3^2}{(-3+2\cdot4)^3} = \frac{72-96-54}{5^3} = \frac{-78}{125}$$

$$y'''_{x^3}\Big|_{(3,4)} = \frac{12xy+6y^2+6x^2}{30x^2-12xy+12y^2-(-x+2y)^4}$$

$$y'''_{x^3}\Big|_{(3,4)} = \frac{12\cdot3\cdot4+6\cdot16+6\cdot9}{30\cdot9-12\cdot3\cdot4+12\cdot16-(-3+2\cdot4)^4} =$$

$$= \frac{144+96+54}{270-144+192-3125} = -\frac{294}{2807} \quad \Delta$$

№3.175

$y = \sqrt{x}$ $\qquad y^{(10)}$ − ?

$\nabla\, y = x^{\frac{1}{2}}$ $\qquad (x^k)^n = k(k-1)(k-2)\dots(k-(n-1))\,x^{k-n}$

$$y^{(10)} = \left(x^{\frac{1}{2}}\right)^{(10)} = \frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\dots\left(\frac{1}{2}-9\right)x^{\frac{1}{2}-10} = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\dots\times$$

$$\times\left(-\frac{17}{2}\right)x^{-\frac{19}{12}} = \frac{1\cdot3\cdot5\cdot7\cdot9\cdot11\cdot13\cdot15\cdot17}{2^{10}\quad x^{\frac{19}{12}}} = \boxed{\frac{17!!}{2^{10}x^{\frac{19}{12}}}}$$

①

$y = \operatorname{arctg}^4\left(\ln^3(3x^2+4)\right)$ $\qquad y'$ − ?

$(x^4)' = 4x^3$; $\quad (u^4)' = 4u^3\cdot u'$

$$y' = 4\operatorname{arctg}^3\left(\ln^3(3x^2+4)\right)\cdot\left(\operatorname{arctg}\left(\ln^3(3x+4)\right)\right)' =$$

$$= 4\operatorname{arctg}^3\left(\ln^3(3x^2+4)\right)\frac{1}{1+\left(\ln^3(3x^2+4)\right)^{2}}\cdot 3\ln^2(3x^2+4)\times$$

$$\times\frac{1}{3x^2+4}\cdot 6x$$

№3.180

$y = \underset{v}{x}\underset{u}{\ln x}$; $y^{(5)} - ?$

$\nabla\, y^{(5)} = \left[(u\cdot v)^{(5)} = \sum_{k=0}^{5} C_5^k\, u^{(n-k)} v^{k}\right] =$

$= x(\ln x)^{(5)} + 5\cdot 1(\ln x)^{(4)} + \frac{5!}{2!\,3!}\cdot 2^{1}(\ln x)^{(3)} + \frac{5!}{3!\,2!}\cdot 0 =$

$= x(\ln x)^5\cdot 5^5 + 5(\ln x)^4\cdot 2^4 + 4.5\,(\ln x)^3\cdot 2^3 =$

$= 5(\ln x)^3\left(5x(\ln x)^2 + (\ln x)16 + 32\right)$

№3.200

$y = \frac{1}{\sqrt{1-2x}}$ $y^{(n)} - ?$

$D(y) = (-\infty; \frac{1}{2})$

$\nabla\, y' = \frac{1}{\sqrt{1-2x}\,(1-2x)}$

$y'' = \frac{3}{\sqrt{1-2x}\,(1-2x)^2}$

$y''' = \frac{15}{\sqrt{1-2x}\,(1-2x)^3}$

$y^{IV} = \frac{105}{\sqrt{1-2x}\,(1-2x)^4}$

$y^{(5)} = \frac{945}{\sqrt{1-2x}\,(1-2x)^5}$

$y^{(6)} = \frac{10395}{\sqrt{1-2x}\,(1-2x)^6}$

$\ldots$

$y^{(n)} = \frac{m}{\sqrt{1-2x}\,(1-2x)^n}$

$\{1 \overset{3}{,} 3 \overset{5}{,} 15 \overset{7}{,} 105 \overset{9}{,} 945 \overset{11}{,} 10395 \ldots\}$

$1(1+2) = 3$

$3(3+2) = 15$

$15(5+2) = 105$

$105(7+2) = 10395$

$\ldots$

m: $n_2 = n_1(n_1+2)$

$n_3 = n_2(n_2+2)$

$n_4 = n_3(n_3+2)$

№3.207

$y = x\cos x$ $y^{(n)} - ?$

$\nabla\, y' = \cos x - x\sin x$

$y'' = -2\sin x - x\cos x$

$y''' = -3\cos x + x\sin x$

$y^{IV} = 4\sin x + x\cos x$

$y^{V} = 5\cos x - x\sin x$

$y^{VI} = -6\sin x - x\cos x$

$y^{(n)} = n\cdot\cos\left(\frac{\pi}{2}\cdot n + x\right) - x\cdot\sin\left(\frac{\pi}{2}n + x\right)$