Домашня робота (за 8.11)

№2.60

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x+2}-\sqrt{6-x}}=\left[\frac{0}{0}\right]=\cancel{\lim_{x\to 2}\frac{(x-2)(x+2)}{(\sqrt{x+2}-\sqrt{6-x})(x+2)}}=$$

$$=\lim_{x\to 2}\frac{x-2}{\sqrt{x+2}-\sqrt{6-x}}\cdot\frac{\sqrt{x+2}+\sqrt{6-x}}{\sqrt{x+2}+\sqrt{6-x}}=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+2}+\sqrt{6-x})}{x+2-6+x}=$$

$$=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+2}+\sqrt{6-x})}{2x-4}=\lim_{x\to 2}\frac{\cancel{(x-2)}(\sqrt{x+2}+\sqrt{6-x})}{2\cancel{(x-2)}}=$$

$$=\lim_{x\to 2}\frac{\sqrt{x+2}+\sqrt{6-x}}{2}=\frac{2+2}{2}=\boxed{2}$$

№2.62

$$\lim_{x\to 0}\frac{1-\sqrt{x^2+1}}{x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\left(\frac{1-\sqrt{x^2+1}}{x^2}\cdot\frac{1+\sqrt{x^2+1}}{1+\sqrt{x^2+1}}\right)=$$

$$=\lim_{x\to 0}\frac{1-(x^2+1)}{x^2(1+\sqrt{x^2+1})}=\lim_{x\to 0}\frac{-\cancel{x^2}}{\cancel{x^2}(1+\sqrt{x^2+1})}=\lim_{x\to 0}\frac{-1}{1+\sqrt{x^2+1}}=$$

$$=\frac{-1}{1+1}=\boxed{-\frac{1}{2}}$$

№2.64

$$\lim_{x\to 8}\frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}=\left[\frac{0}{0}\right]=\lim_{x\to 8}\frac{\frac{1}{\sqrt{9+2x}}}{\frac{1}{3\sqrt[3]{x^2}}}=\lim_{x\to 8}\frac{3\sqrt[3]{x^2}}{\sqrt{9+2x}}=$$

$$=\lim_{x\to 8}\frac{3\sqrt[3]{8^2}}{\sqrt{9+2\cdot 8}}=\frac{3\cdot 2^2}{\sqrt{25}}=\frac{12}{5}=\boxed{2{,}4}$$

№2.66

$$\lim_{x\to 2}\frac{\sqrt[3]{4x}-2}{\sqrt{2+x}-\sqrt{2x}}=\left[\frac{0}{0}\right]=\lim_{x\to 2}\frac{\frac{\sqrt[3]{4}}{3\sqrt[3]{x^2}}}{\frac{1}{2\sqrt{2+x}}-\frac{1}{\sqrt{2x}}}=$$

$$=\lim_{x\to 2}\frac{\frac{\sqrt[3]{4}}{3\sqrt[3]{x^2}}}{\frac{\sqrt{2x}-2\sqrt{2+x}}{2\sqrt{2x}\cdot(2+x)}}=\lim_{x\to 2}\frac{2\sqrt[3]{4}\sqrt{2x(2+x)}}{3\sqrt[3]{x^2}\cdot(\sqrt{2x}-2\sqrt{2+x})}=$$

$$=\lim_{x\to 2}\left(\frac{2\sqrt[6]{4^2}\sqrt[6]{(2\cdot(2+x))^3}}{3\sqrt[3]{x^2}\cdot(\sqrt{2x}-2\sqrt{2+x})}\right)=\lim_{x\to 2}\frac{2\sqrt[6]{2^4\cdot 8x^3\cdot(2+x)^3}}{3\sqrt[3]{x^2}(\sqrt{2x}-2\sqrt{2+x})}=$$

$$= \lim_{x\to 2} \frac{2\sqrt[6]{2^4x^3(2+x)^3}}{3\sqrt[3]{x^2}\cdot(\sqrt{2x}-2\sqrt{2+x})} = \lim_{x\to 2}\frac{4\sqrt[6]{2x^3(2+x)^3}}{3\sqrt[3]{x^2}(\sqrt{2x}-2\sqrt{2+x})} =$$

$$= \lim_{x\to 2}\frac{4\sqrt[6]{16x^3+24x^4+12x^5+2x^6}}{3\sqrt[3]{x^2}\cdot(\sqrt{2x}-2\sqrt{2+x})} =$$

$$= \frac{4\sqrt[6]{16\cdot 2^3+24\cdot 2^4+12\cdot 2^5+2\cdot 2^6}}{3\sqrt[3]{2^2}(\sqrt{4}-2\sqrt{4})} =$$

$$= \frac{4\sqrt[6]{(\sqrt[3]{16}\cdot 2+\sqrt[3]{2}\cdot 2^2)^3}}{3\sqrt[3]{2^2}\cdot(2-4)} = \frac{-2\sqrt{4\sqrt[3]{2}+4\sqrt[3]{2}}}{3\sqrt[3]{2^2}} =$$

$$= -\frac{\sqrt[6]{2^{12}2^2}}{3} = -\frac{2^2}{3} = \boxed{-\frac{4}{3}}$$

№2.68

$$\lim_{x\to\infty} x(\sqrt{x^2+1}-x) = \lim_{x\to\infty}(x\sqrt{x^2+1}-x^2) =$$

$$= \lim_{x\to\infty}\left((x\sqrt{x^2+1}-x^2)\cdot\frac{x\sqrt{x^2+1}+x^2}{x\sqrt{x^2+1}+x^2}\right) = \lim_{x\to\infty}\frac{x^2(x^2+1)-x^4}{x\sqrt{x^2+1}+x^2} =$$

$$= \lim_{x\to\infty}\left(\frac{x^4+x^2-x^4}{\sqrt{x^4+x^2}+x^2}\right) = \lim_{x\to\infty}\frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{1+\frac{1}{x^2}}+1)} = \lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1} =$$

$$= \frac{1}{\sqrt{1+0}+1} = \boxed{\frac{1}{2}}$$

№2.70

$$\lim_{x\to\infty}(\sqrt{x^2+1}-\sqrt{x^2-1}) = [\infty-\infty] = \lim_{x\to\infty}(\sqrt{x^2+1}-\sqrt{x^2-1})\left(\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}\right) =$$

$$= \lim_{x\to\infty}\frac{x^2+1-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{\cancel{x}\cdot\frac{2}{x}}{\cancel{x}(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}})} =$$

$$= \lim_{x\to\infty}\frac{\frac{2}{x}}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}} = \frac{2\cdot 0}{\sqrt{1+0}+\sqrt{1-0}} = \boxed{0}$$

№2.72

$$\lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty}\frac{\cancel{\sqrt{x}}(1+\sqrt[3]{\frac{1}{\sqrt{x}}}+\sqrt[4]{\frac{1}{x}})}{\cancel{\sqrt{x}}\cdot\sqrt{2+\frac{1}{x}}} =$$

$$= \lim_{x\to+\infty} \frac{1+\sqrt[3]{\frac{1}{\sqrt{x}}}+\sqrt[4]{\frac{1}{x}}}{\sqrt{2+\frac{1}{x}}} = \frac{1+0+0}{\sqrt{2+0}} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

№2.74

$$\lim_{x\to-8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}} = \lim_{x\to-8} \frac{-\frac{1}{2\sqrt{1-x}}}{\frac{1}{3\sqrt[3]{x^2}}} = \lim_{x\to-8} \frac{-3\sqrt[3]{x^2}}{2\sqrt{1-x}} = \boxed{-2}$$

№2.76

$$\lim_{x\to3} \frac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9} = \left[\frac{0}{0}\right] = \lim_{x\to3} \frac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}\cdot\frac{\sqrt{x+13}+2\sqrt{x+1}}{\sqrt{x+13}+2\sqrt{x+1}} =$$

$$= \lim_{x\to3} \frac{x+13-4(x+1)}{(x^2-9)(\sqrt{x+13}+2\sqrt{x+1})} \left[\frac{0}{0}\right] = \lim_{x\to3} \frac{x+13-4x-4}{(x-3)(x+3)(\sqrt{x+13}+2\sqrt{x+1})} =$$

$$= \lim_{x\to3} \frac{-3x+9}{(x-3)(x+3)(\sqrt{x+13}+2\sqrt{x+1})} = \lim_{x\to3} \frac{-3(x-3)}{\cancel{(x-3)}(x+3)(\sqrt{x+13}+2\sqrt{x+1})} =$$

$$= \lim_{x\to3} -\frac{3}{x\sqrt{x+13}+2x\sqrt{x+1}+3\sqrt{x+13}+6\sqrt{x+1}} = \boxed{-\frac{1}{16}}$$

№2.78

$$\lim_{x\to16} \frac{\sqrt[4]{x}-2}{\sqrt{x}-4} = \left[\frac{0}{0}\right] = \lim_{x\to16} \frac{\sqrt[4]{x}-2}{\sqrt{x}-4}\cdot\frac{\sqrt[4]{x}+2}{\sqrt[4]{x}+2} =$$

$$= \lim_{x\to16} \frac{\cancel{\sqrt{x}-4}}{(\sqrt[4]{x}+2)\cancel{(\sqrt{x}-4)}} = \lim_{x\to16} \left(\frac{1}{\sqrt[4]{x}+2}\right) = \frac{1}{\sqrt[4]{16}+2} = \boxed{\frac{1}{4}}$$

№2.80

$$\lim_{x\to0} \frac{\sqrt[n]{1+x}-1}{x} \quad (n\in\mathbb{Z}) = \lim_{x\to0} \frac{(\sqrt[n]{1+x}-1)\left((\sqrt[n]{1+x})^{n-1}+(\sqrt[n]{1+x})^{n-2}+\ldots+\sqrt[n]{1+x}+1\right)}{x\left((\sqrt[n]{1+x})^{n-1}+(\sqrt[n]{1+x})^{n-2}+\ldots+\sqrt[n]{1+x}+1\right)}$$

$$= \lim_{x\to0} \frac{x}{\cancel{x}\left((\sqrt[n]{1+x})^{n-1}+(\sqrt[n]{1+x})^{n-2}+\ldots+\sqrt[n]{1+x}+1\right)} =$$

$$= \lim_{x\to0} \frac{1}{(\sqrt[n]{1+x})^{n-1}+(\sqrt[n]{1+x})^{n-2}+\ldots+\sqrt[n]{1+x}+1} = \boxed{\frac{1}{n}}$$

№2.82

$$\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x^2+x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{\sqrt[3]{-x^2+3x+8}-2}{x^2+x}=$$

$$=\lim_{x\to 0}\frac{\sqrt[3]{-x^2+3x+8}-2}{x^2+x}\cdot\frac{\sqrt[3]{(-x^2+3x+8)^2}+2\sqrt[3]{-x^2+3x+8}+4}{\sqrt[3]{(-x^2+3x+8)^2}+2\sqrt[3]{-x^2+3x+8}+4}=$$

$$=\lim_{x\to 0}\frac{-x^2+3x+8-8}{x(x+1)\left(\sqrt[3]{(-x^2+3x+8)^2}+2\sqrt[3]{-x^2+3x+8}+4\right)}=$$

$$=\lim_{x\to 0}\frac{x(-x+3)}{x(x+1)\left(\sqrt[3]{(-x^2+3x+8)^2}+2\sqrt[3]{-x^2+3x+8}+4\right)}$$

$$=\lim_{x\to 0}\frac{-x+3}{(x+1)\left(\sqrt[3]{(-x^2+3x+8)^2}+2\sqrt[3]{-x^2+3x+8}+4\right)}=\frac{3}{1\cdot 4+4+4}=\frac{3}{12}=\boxed{\frac{1}{4}}$$

№2.81

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}=\lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}}+\frac{1}{2\sqrt{1+x}}}{\frac{1}{3\sqrt[3]{(1+x)^2}}+\frac{1}{3\sqrt[3]{(1-x)^2}}}=$$

$$=\lim_{x\to 0}\frac{\frac{\sqrt{1-x}+\sqrt{1+x}}{2\sqrt{(1+x)(1-x)}}}{\frac{\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}}{3\sqrt[3]{(1+x)^2(1-x)^2}}}=\lim_{x\to 0}\frac{(\sqrt{1-x}+\sqrt{1+x})\cdot 3\cdot((1+x)\cdot(1-x))^{2/3}}{2((1+x)(1-x))^{1/2}\left(\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}\right)}$$

$$=\lim_{x\to 0}\frac{(\sqrt{1-x}+\sqrt{1+x})\,3((1+x)(1-x))^{1/6}}{2\left(\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}\right)}=\lim_{x\to 0}\frac{(\sqrt{1-x}+\sqrt{1+x})\,3(1-x^2)^{1/6}}{2\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}}=$$

$$=\lim_{x\to 0}\frac{(3\sqrt{1-x}+3\sqrt{1+x})\,(1-x^2)^{1/6}}{2\left(\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}\right)}=\boxed{\frac{3}{2}}$$

№2.92

$$\lim_{x\to\infty}\left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3-x^2+1}\right)=\left[\frac{\infty}{\infty}\right]=$$

$$=\lim_{x\to\infty}\left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3-x^2+1}\cdot\frac{\sqrt[3]{(x^3+x^2+1)^2}+\sqrt[3]{x^6+x^5+x^3-x^5-x^4-x^2+x^3-x^2+1}+\sqrt[3]{(x^3-x^2+1)^2}}{\sqrt[3]{(x^3+x^2+1)^2}+\sqrt[3]{x^6+x^5+x^3-x^5-x^4-x^2+x^3-x^2+1}+\sqrt[3]{(x^3-x^2+1)^2}}\right.$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3-x^2+1}\right)\left(\sqrt[3]{(x^3+x^2+1)^2}+\sqrt[3]{x^6+2x^3-x^4+1}+\sqrt[3]{(x^3-x^2+1)^2}\right)}{\sqrt[3]{x^6+x^4+1+2x^5+2x^3+2x^2}+\sqrt[3]{x^6+2x^3-x^4+1}+\sqrt[3]{x^6+x^4+1-2x^5+2x^3-2x^2}}=$$

$$\lim_{x\to\infty}\frac{x^3+x^2+1-(x^3-x^2+1)}{\sqrt[3]{x^6+x^4+1+2x^5+2x^3+2x^2}+\sqrt[3]{x^6+2x^3-x^4+1}+\sqrt[3]{x^6+x^4+1-2x^5+2x^3-2x^2}}=$$

$$=\lim_{x\to\infty}\frac{2x^2}{x^2\left(\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^6}+\frac{2}{x}+\frac{2}{x^3}+\frac{2}{x^4}}+\sqrt[3]{1+\frac{2}{x^3}-\frac{1}{x^2}+\frac{1}{x^6}}+\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^6}-\frac{2}{x}+\frac{2}{x^3}-\frac{2}{x^4}}\right)}=$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt[3]{1+\underbrace{\frac{1}{x^2}}_{0}+\underbrace{\frac{1}{x^6}}_{0}+\underbrace{\frac{2}{x}}_{0}+\underbrace{\frac{2}{x^3}}_{0}+\underbrace{\frac{2}{x^4}}_{0}}+\sqrt[3]{1+\underbrace{\frac{2}{x^3}}_{0}-\underbrace{\frac{1}{x^2}}_{0}+\underbrace{\frac{1}{x^6}}_{0}}+\sqrt[3]{1+\underbrace{\frac{1}{x^2}}_{0}+\underbrace{\frac{1}{x^6}}_{0}-\underbrace{\frac{2}{x}}_{0}+\underbrace{\frac{2}{x^3}}_{0}-\underbrace{\frac{2}{x^4}}_{0}}}=$$

$$=\frac{2}{\sqrt[3]{1}+\sqrt[3]{1}+\sqrt[3]{1}}=\boxed{\frac{2}{3}}$$

№293

$$\lim_{x\to+0}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)=[\infty-\infty]=$$

$$=\lim_{x\to+0}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\frac{1}{\sqrt{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\frac{1}{\sqrt{x}}}}\right)=$$

$$=\lim_{x\to+0}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{\sqrt{x}+x}{x\sqrt{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{\sqrt{x}+x}{x\sqrt{x}}}}\right)=$$

$$=\lim_{x\to+0}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{x+x\sqrt{x}}{x^2}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{x+x\sqrt{x}}{x^2}}}\right)=$$

$$=\lim_{x\to+0}\left(\sqrt{\frac{1}{x}+\frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}}-\sqrt{\frac{1}{x}-\frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}}\right)=\lim_{x\to+0}\left(\sqrt{\frac{(\sqrt{x}+x\sqrt{1+\sqrt{x}})\sqrt{x}}{x^2}}-\sqrt{\frac{(\sqrt{x}-x\sqrt{1+\sqrt{x}})\sqrt{x}}{x^2}}\right)=$$

$$=\lim_{x\to+0}\left(\sqrt{\frac{x+x\sqrt{x+\sqrt{x}\cdot x}}{x^2}}-\sqrt{\frac{x-x\sqrt{x+\sqrt{x}\cdot x}}{x^2}}\right)=\lim_{x\to+0}\left(\sqrt{\frac{x(1+\sqrt{x+\sqrt{x}\cdot x})}{x^2}}-\sqrt{\frac{x(1-\sqrt{x+\sqrt{x}\cdot x})}{x^2}}\right)=$$

$$=\lim_{x\to+0}\frac{\sqrt{1+\sqrt{x+\sqrt{x}\cdot x}}-\sqrt{1-\sqrt{x+\sqrt{x}\cdot x}}}{\sqrt{x}}=\boxed{0}$$

№295

$$\lim_{x\to+\infty} x^{3/2}\left(\sqrt{x+1}-2\sqrt{x+2}+\sqrt{x+3}\right) = \lim_{x\to+\infty}\left(\sqrt{x^3}\sqrt{x+1}-2\sqrt{x^3}\sqrt{x+2}+\sqrt{x^3}\sqrt{x+3}\right) =$$

$$= \lim_{x\to+\infty} \sqrt{x^3(x+1)} - 2x\sqrt{x}\sqrt{x+2} + \sqrt{x^3\cdot(x+3)} =$$

$$= \lim_{x\to+\infty}\left(x\sqrt{x(x+1)} - 2x\sqrt{x(x+2)} + x\sqrt{x(x+3)}\right) =$$

$$= \lim_{x\to+\infty}\left(x\sqrt{x^2+x} - 2x\sqrt{x^2+2x} + x\sqrt{x^2+3x}\right) =$$

$$= \lim_{x\to+\infty} x^{3/2}\frac{\left((\sqrt{x+1}+2\sqrt{x+2})+\sqrt{x+3}\right)\left((\sqrt{x+1}-2\sqrt{x+2})-\sqrt{x+3}\right)}{(\sqrt{x+1}-2\sqrt{x+2})-\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} \frac{x^{3/2}(\sqrt{x+1}+2\sqrt{x+2})^2-(x-3)}{\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} x^{3/2}\frac{\cancel{x}+1+4\sqrt{x^2+3x+2}+4x+8-\cancel{x}+3}{\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} x^{3/2}\frac{12+4x+4\sqrt{x^2+3x+2}}{\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3}} = \lim_{x\to+\infty} x^{3/2}\frac{4(3+x+\sqrt{x^2+3x+2})}{\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3}}$$

$$= 2\lim_{x\to+\infty} x^{3/2}\frac{(6+2x+2\sqrt{x^2+3x+2})(6-2x+2\sqrt{x^2+3x+2})}{(\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3})(6-2x+2\sqrt{x^3+3x+2})} =$$

$$= 2\lim_{x\to+\infty} \frac{-x^{3/2}}{(\sqrt{x+1}-2\sqrt{x+2}-\sqrt{x+3})(6-2x+2\sqrt{x^3+3x+2})}\frac{:x^{3/2}}{:x^{3/2}} =$$

$$= 2\lim_{x\to+\infty} \frac{-1}{\left(\sqrt{1+\frac{1}{x}}+2\sqrt{1+\frac{2}{x}}-\sqrt{1-\frac{3}{x}}\right)\left(\sqrt{\frac{6}{x^2}}-2+2\right)} =$$

$$= 2\cdot\left(-\frac{1}{3}\right) = \boxed{-\frac{2}{3}}$$

№2.96

$$\lim_{x\to+\infty} x^{3/2}\left(\sqrt{x+2}+\sqrt{x+1}-\sqrt{x}-\sqrt{x+3}\right) =$$

$$= \lim_{x\to+\infty} x^{3/2} \frac{(\sqrt{x+2}+\sqrt{x+1}-\sqrt{x}-\sqrt{x+3})(\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}+\sqrt{x+3})}{(\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} x^{3/2} \frac{(\sqrt{x+2}+\sqrt{x+1})^2-2x-2\sqrt{x^2+3x}-3}{(\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} x^{3/2} \frac{2x+3+2\sqrt{x^2+3x+2}-2x-2\sqrt{x^2+3x}-3}{(\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3}} =$$

$$= \lim_{x\to+\infty} x^{3/2} \frac{2\sqrt{x^2+3x+2}-2\sqrt{x^2+2x}}{(\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3}} =$$

$$= 2\lim_{x\to+\infty} x^{3/2} \frac{(\sqrt{x^2+3x+2}-\sqrt{x^2+2x})(\sqrt{x^2+3x+2}+\sqrt{x^2+2x})}{((\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3})(\sqrt{x^2+3x+2}+\sqrt{x^2+2x})} =$$

$$= 2\lim_{x\to+\infty} \frac{-x^{3/2}}{((\sqrt{x+2}+\sqrt{x+1})+\sqrt{x}+\sqrt{x+3})(\sqrt{x^3+3x+2}+\sqrt{x^2+2x})} \begin{matrix} : x^{3/2} \\ : x^{3/2} \end{matrix} =$$

$$= 2\lim_{x\to+\infty} \frac{-1}{\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}+1+\sqrt{1+\frac{3}{x}}\right)\left(\sqrt{x\sqrt{x}+\frac{3}{\sqrt{x}}+\frac{2}{x}}+\sqrt{\sqrt{x}+\frac{2}{\sqrt{x}}}\right)} =$$

$$= 2\cdot\frac{-1}{4} = -\frac{2}{4} = \boxed{-\frac{1}{2}}$$