Special Topics in Particle Physics

Acceleration mechanisms

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Acceleration mechanisms

The origin of cosmic rays is one of the major unsolved astrophysical problems. **The highest-energy cosmic rays possess macroscopic energies and their origin is likely to be associated with the most energetic processes in the universe.**

When discussing cosmic-ray origin, one must in principle **distinguish between the power source and the acceleration mechanism**. One **generally assumes that in most cases cosmic-ray particles are not only produced in the sources but also accelerated to higher energies in or near the source**.

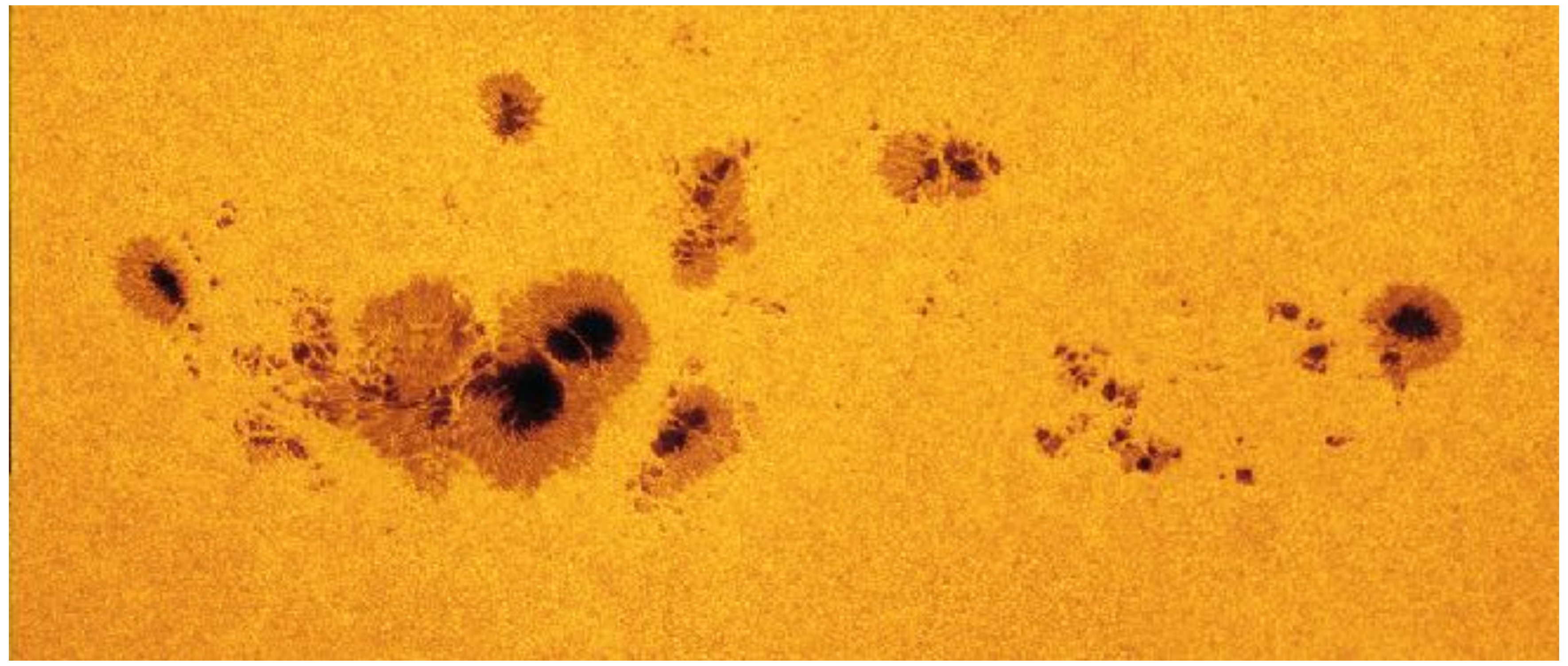
Candidate sites for cosmic-ray production and acceleration are **supernova explosions, highly magnetized spinning neutron stars, i.e., pulsars, accreting black holes, and active galactic nuclei.** However, it is also possible that cosmic-ray particles powered by some source **experience acceleration during the propagation in the interstellar or intergalactic medium by interactions with extensive gas clouds.** These gas clouds can be associated with magnetic-field irregularities and charged particles can gain energy while they scatter off the constituents of these ‘magnetised clouds’.

In *top–down scenarios* energetic cosmic rays can also be produced by the decay of topological defects, domain walls, or cosmic strings, which could be relics of the Big Bang.

Acceleration mechanisms

- Cyclotron mechanism
- Sunspots and solar flares (magnetic reconnection)
- Shock acceleration
- Fermi mechanisms (reflection from magnetised gas clouds)
- Pulsars
- Binaries

Cyclotron mechanism



Even **normal stars can accelerate charged particles up to the GeV range**. This acceleration can occur in **time-dependent magnetic fields**. These magnetic sites appear as **starspots or sunspots**, respectively.

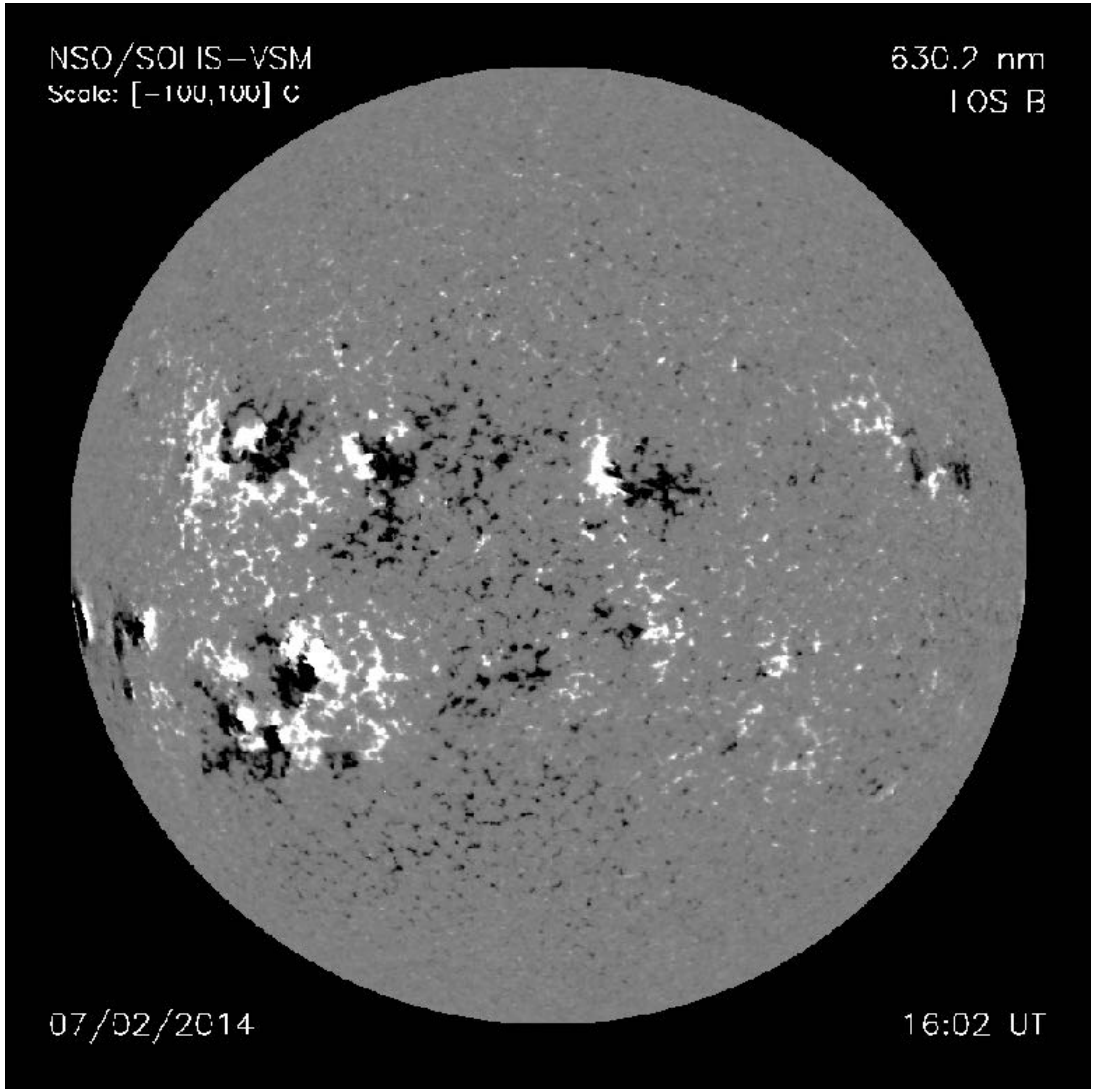
The **temperature of sunspots is slightly lower** compared to the surrounding regions. They appear darker, because part of the **thermal energy has been transformed into magnetic field energy.** Sunspots in typical stars can be associated with magnetic field strengths of up to1000 gauss (1tesla=10^4 gauss). The lifetime of such sunspots can exceed several rotation periods (Sun's rotation period is 25-30 days). The spatial extension of sunspots on the Sun can be as large as 10^4 km.

The observed **Zeeman splitting of spectral lines** has shown beyond any doubt that magnetic fields are responsible for the sunspots. Since the Zeeman splitting of spectral lines depends on the magnetic field strength, this fact can also be used to **measure the strength of the magnetic fields** on stars.

The magnetic fields in the Sun are **generated by turbulent plasma motions**, where the plasma consists essentially of protons and electrons. The motions of this plasma constitute currents, which produce magnetic fields. **When these magnetic fields are generated and when they decay, electric fields are created, in which protons and electrons can be accelerated**.

Solar magnetic field

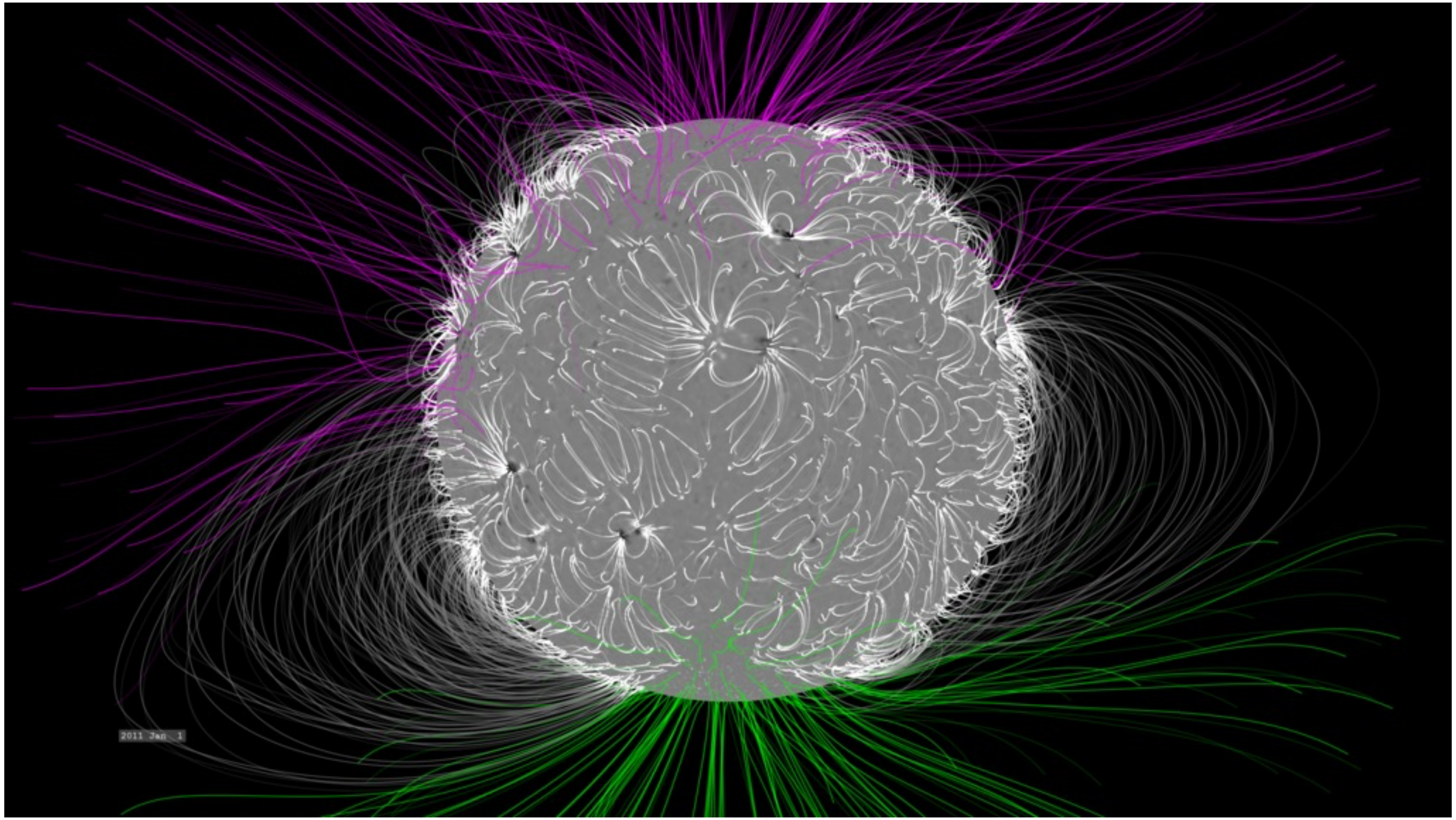
- **A magnetogram is an image taken by an instrument (magnetograph) that shows the strength, polarity, and location of the magnetic fields on the Sun.**
- Figure is a magnetogram image of the whole solar disk, where regions of positive polarity are indicated by white and regions of negative polarity by black, the regions without appreciable magnetic field being represented in grey.
- One notes that most bipolar magnetic regions are roughly aligned parallel to the solar equator.
- In the magnetic bipolar regions in the northern hemisphere, one finds the positive polarity (white) to appear on the left side of the negative polarity (black).
- This is reversed in the southern hemisphere, where white appears to the right of black.



https://nso.edu/data/nisp-data/magnetograms/

Solar magnetic field

- Magnetic field model of the Sun



https://svs.gsfc.nasa.gov/4623

Cyclotron mechanism

Fig. 5.1 Principle of particle acceleration by variable sunspots

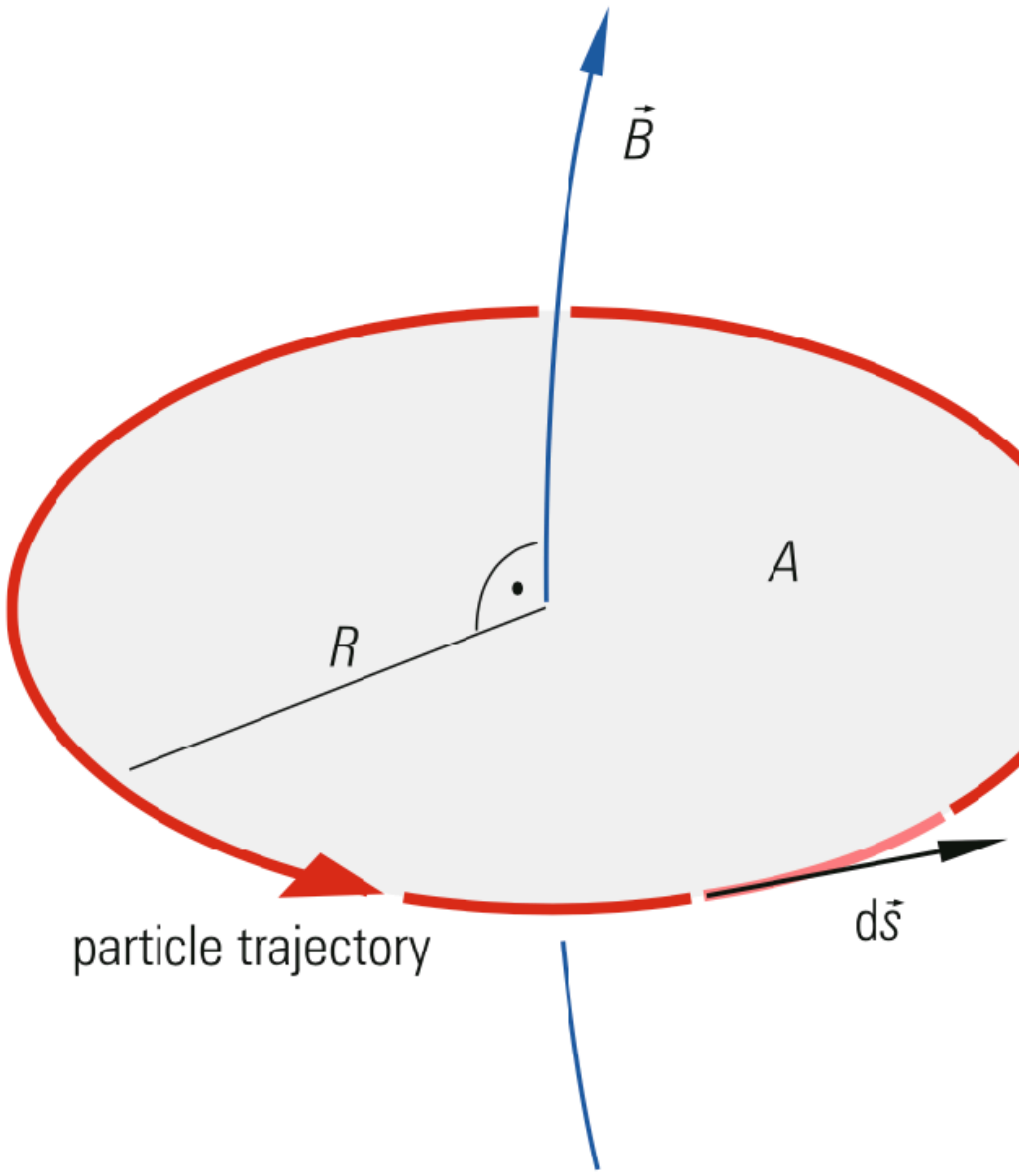


Figure 5.1 shows schematically a sunspot of extension $A = \pi R^2$ with a variable magnetic field $\boldsymbol{B}$.

The time-dependent change of the magnetic flux ϕ produces a potential U ,

$$-\frac{d\phi}{dt} = \oint \boldsymbol{E} \cdot d\boldsymbol{s} = U$$

($\boldsymbol{E}$—electrical field strength, d$\boldsymbol{s}$—infinitesimal distance along the particle trajectory).

Cyclotron mechanism

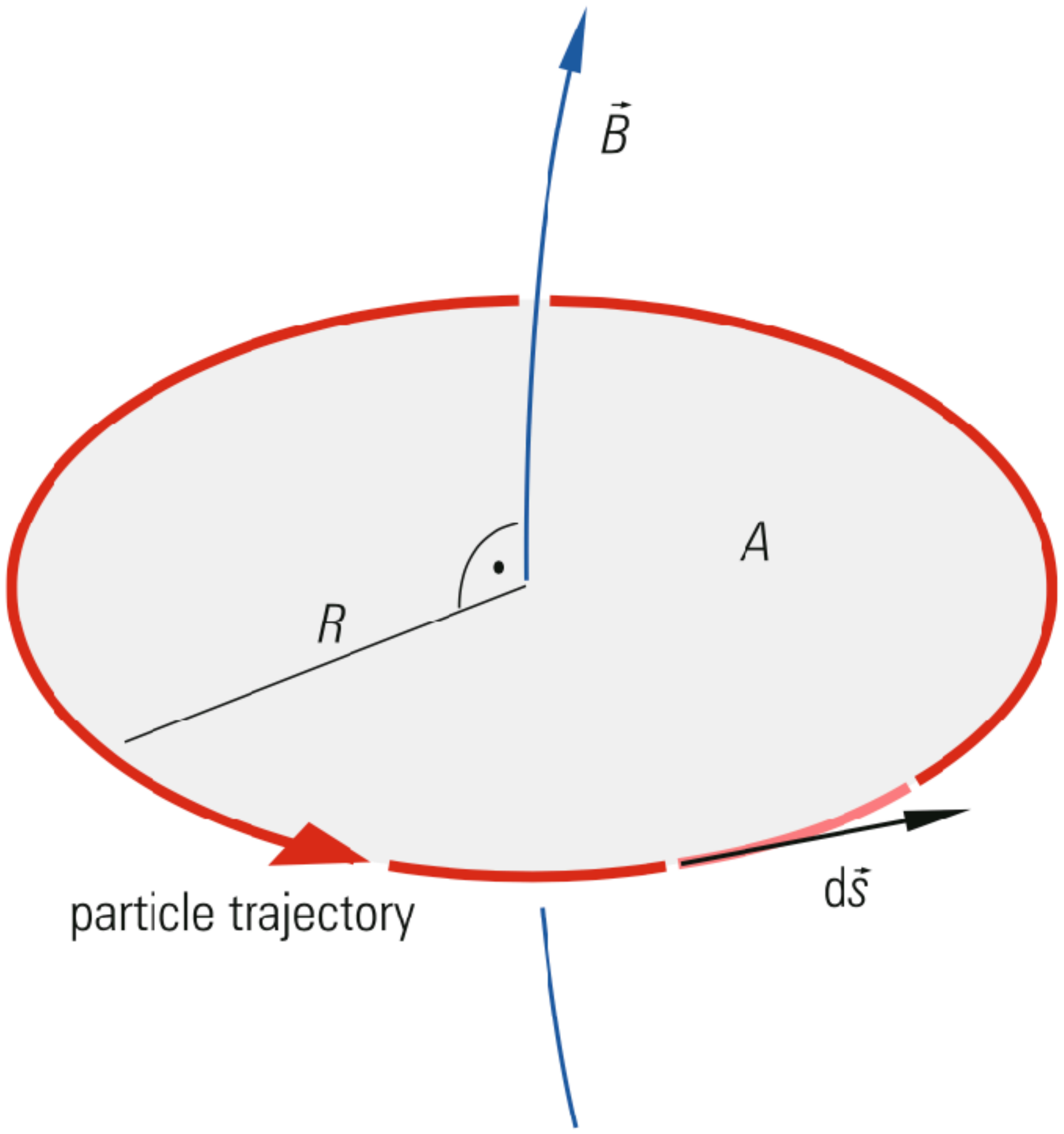
The magnetic flux is given by

$$\phi = \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} = B\pi R^2 \,,$$

where d $\boldsymbol{A}$ is the infinitesimal area element. In this equation it is assumed that $\boldsymbol{B}$ is perpendicular to the area, i.e., $\boldsymbol{B} \parallel \boldsymbol{A}$, (the vector $\boldsymbol{A}$ is always perpendicular to the area). One turn of a charged particle around the time-dependent magnetic field leads to an energy gain of

$$E = eU = e\pi R^2 \frac{\mathrm{d}B}{\mathrm{d}t} \,.$$

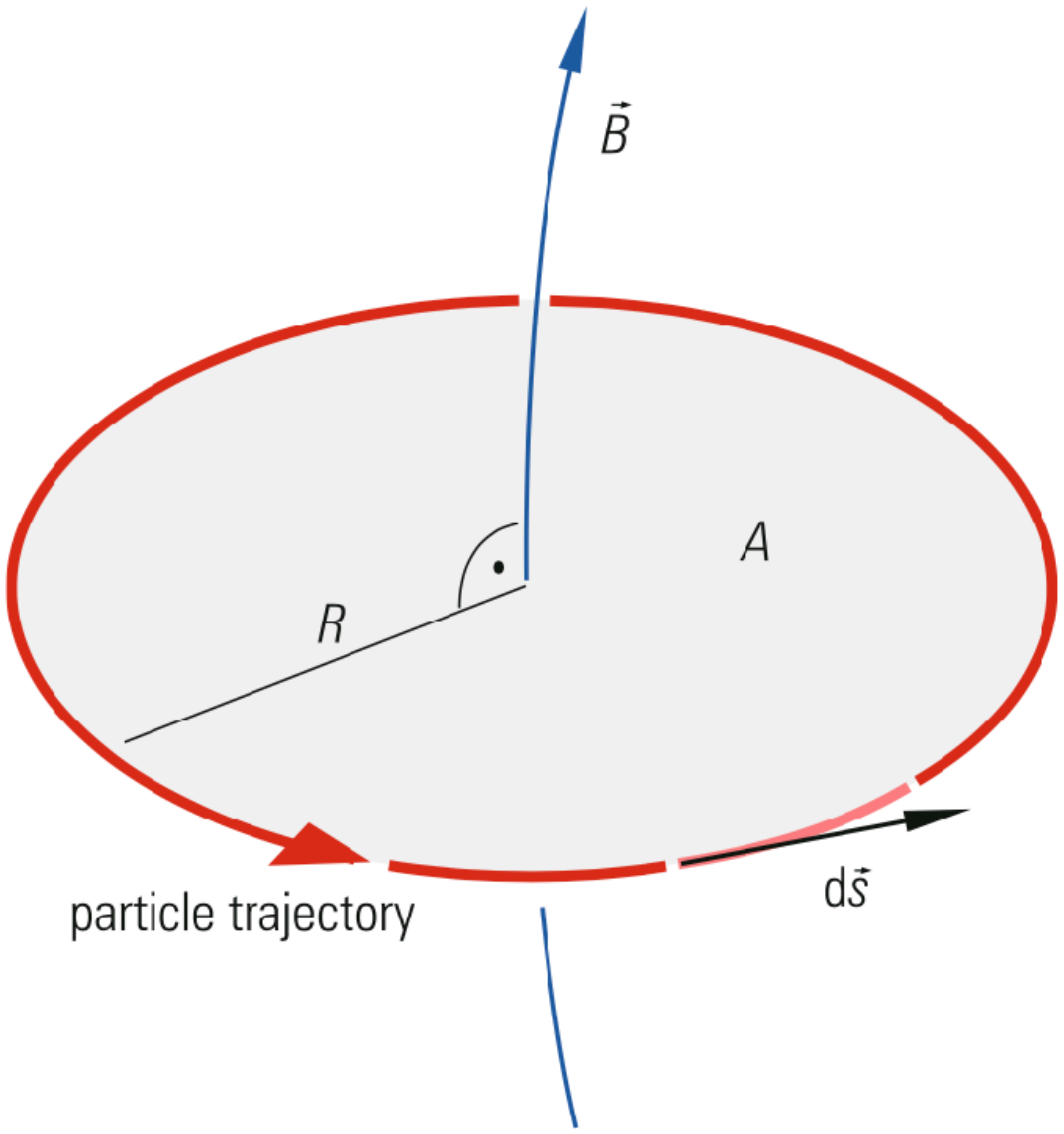
Fig. 5.1 Principle of particle acceleration by variable sunspots



Cyclotron mechanism

$$E = eU = e\pi R^2 \frac{\mathrm{d}B}{\mathrm{d}t} .$$

Fig. 5.1 Principle of particle acceleration by variable sunspots



A sunspot of an extension $R = 10^9$ cm and magnetic field $B = 2000$ gauss at a lifetime of one day ($\frac{dB}{dt} = 2000$ gauss/day) leads to

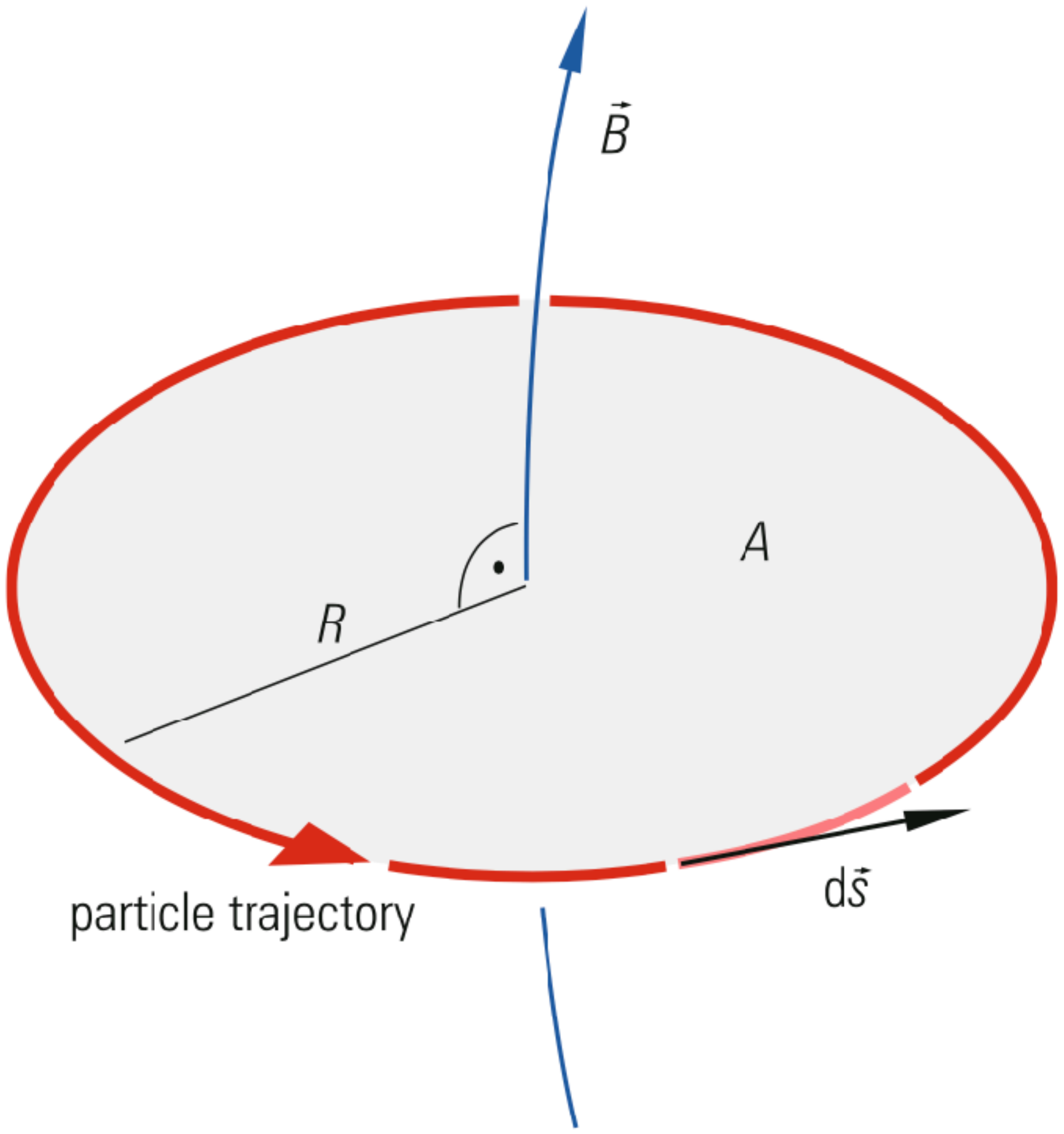
$$E = 1.6 \times 10^{-19}\,\mathrm{A\,s}\,\pi\ 10^{14}\,\mathrm{m}^2\ \frac{0.2\,\mathrm{V\,s}}{86\,400\,\mathrm{s\,m}^2}$$

$$= 1.16 \times 10^{-10}\,\mathrm{J} = 0.73\,\mathrm{GeV} .$$

Particles from the Sun with energies beyond 10 GeV have been observed. This, however, might also represent the limit for the acceleration power of stars based on the cyclotron mechanism.

Cyclotron mechanism

Fig. 5.1 Principle of particle acceleration by variable sunspots

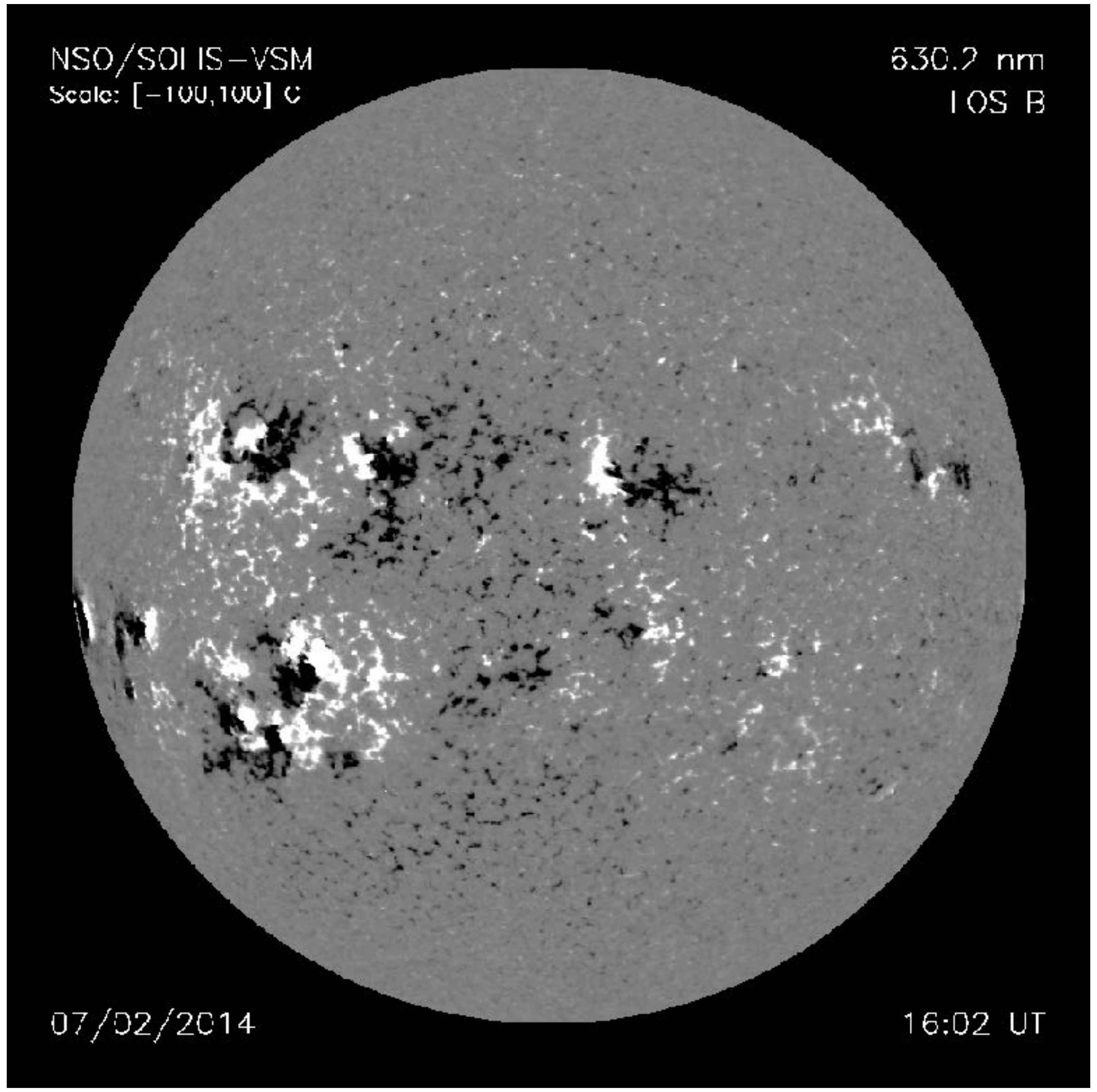


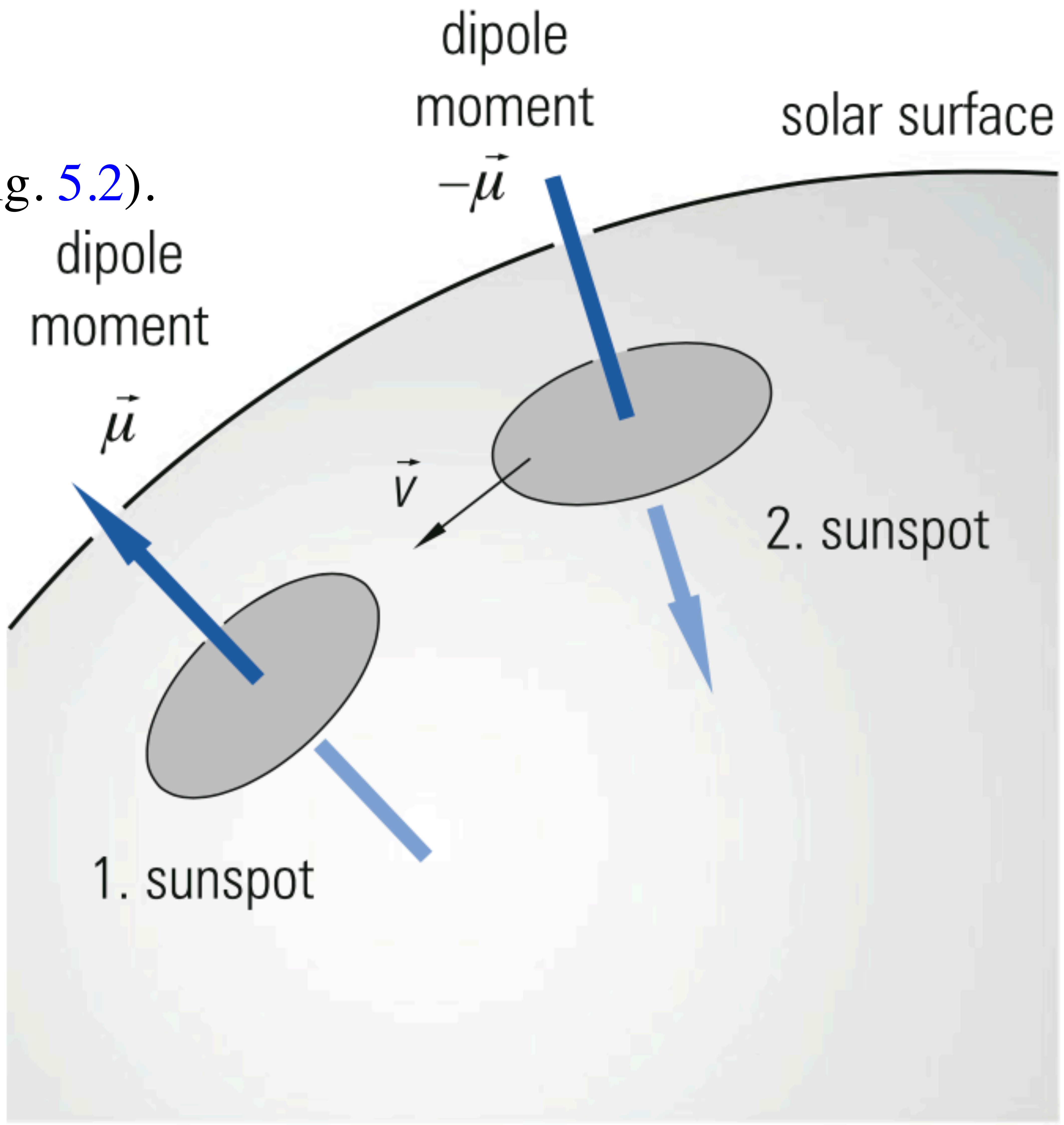
The cyclotron model can explain the correct energies, however, it **does not explain why charged particles propagate in circular orbits around time-dependent magnetic fields.** Circular orbits are only stable in the presence of guiding forces such as they are used in earthbound accelerators.

Acceleration by Sunspots

Fig. 5.2 Sketch of a sunspot pair

Sunspots often come in pairs of opposite magnetic polarity (see Fig. 5.2).





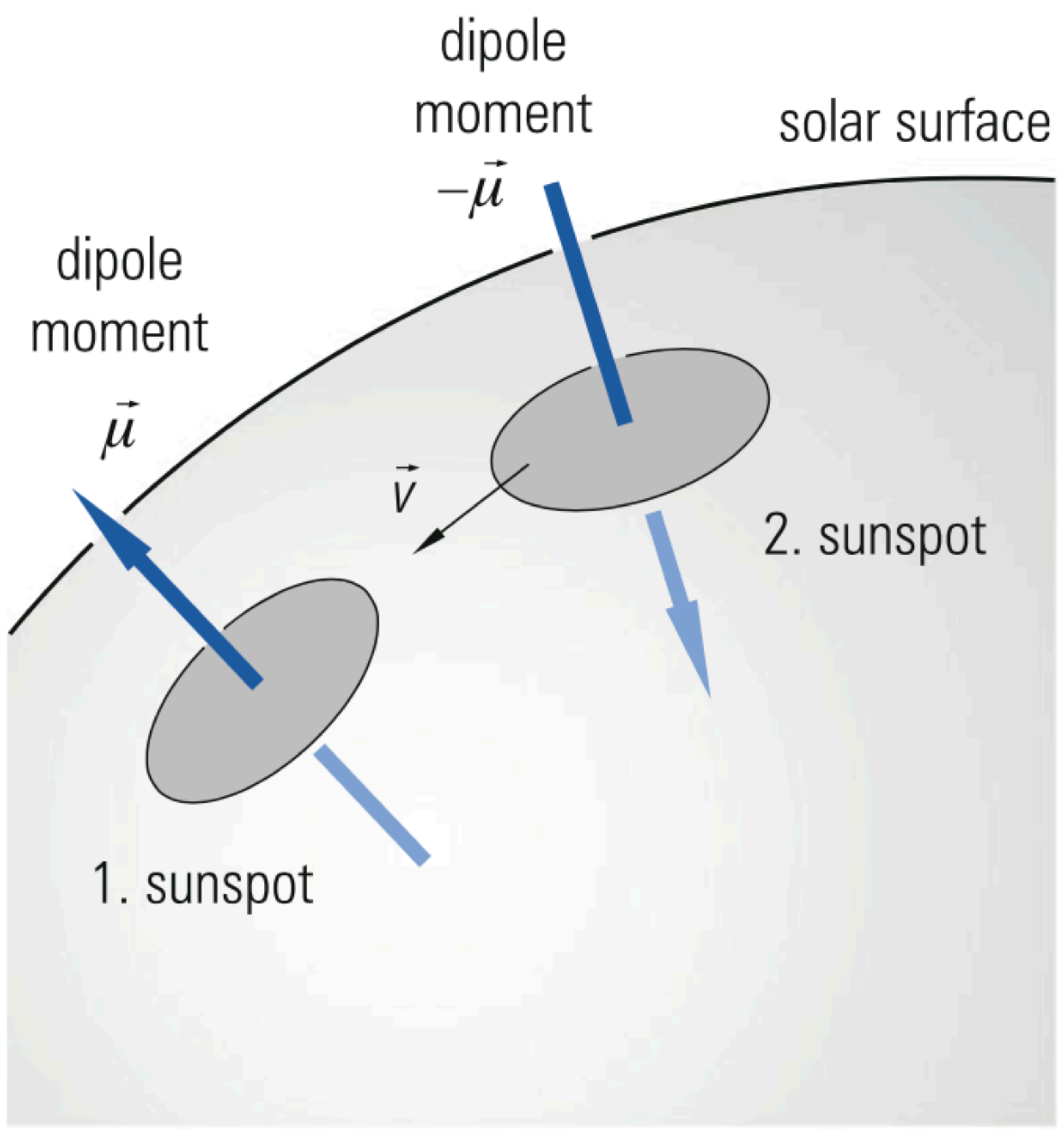
Acceleration by Sunspots

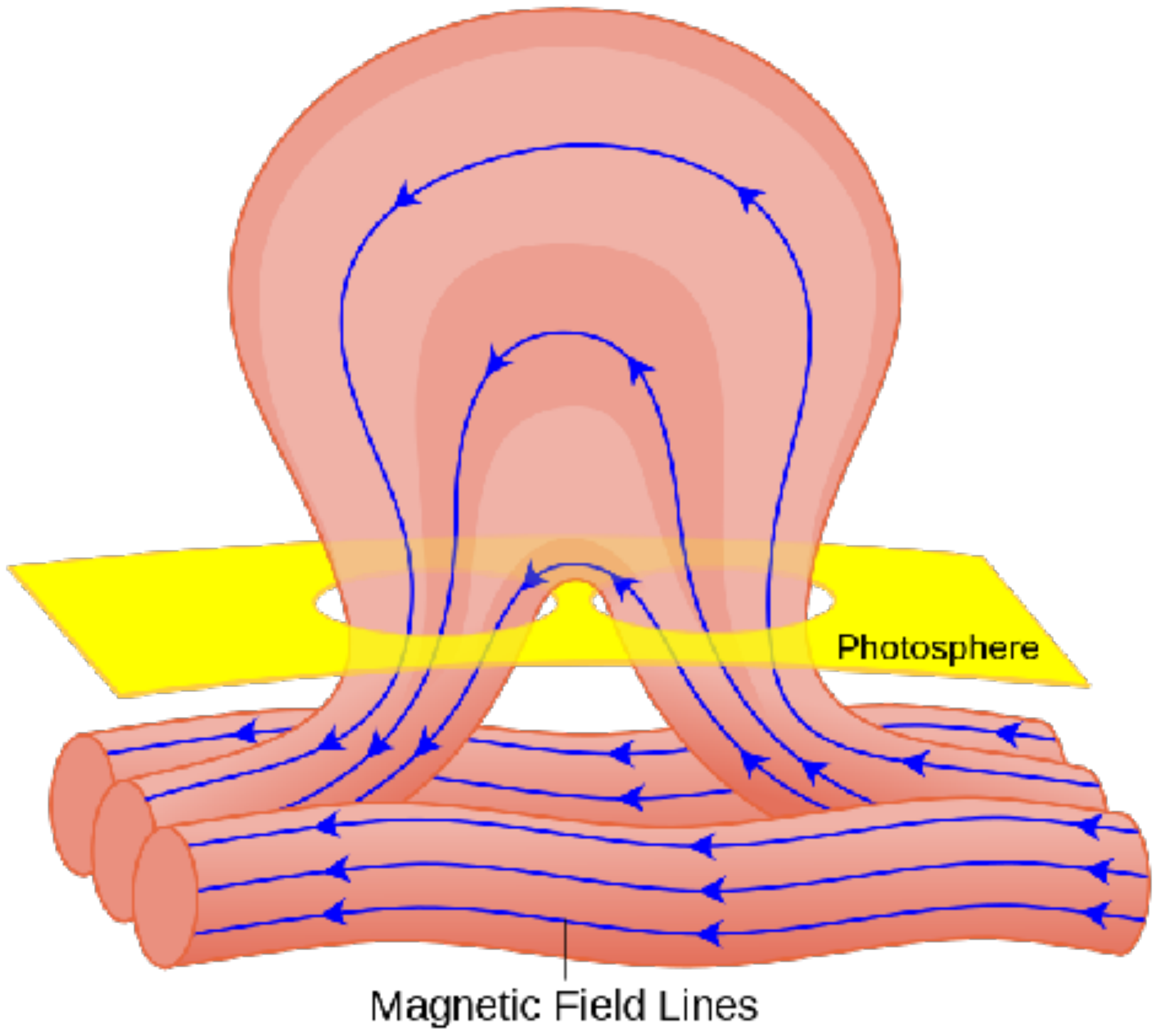
The sunspots normally approach each other and merge at a later time.

Let us assume that the left sunspot is at rest and the right one approaches the first sunspot with a velocity v. **The moving magnetic dipole produces an electric field perpendicular to the direction of the dipole and perpendicular to its direction of motion** v, i.e., parallel to v × ***B***.

Typical solar magnetic sunspots can create **electrical fields of 10 V/m**. In spite of such a low field strength, **protons can be accelerated** since the collision energy loss is smaller than the energy gain in the low-density chromosphere. Under realistic assumptions (distance of sunspots 10^7 m, magnetic field strengths 2000 gauss, relative velocity v = 10^7 m/day) **particle energies in the GeV range are obtained.** This shows that the model of particle acceleration in approaching magnetic dipoles can only explain energies, which can also be provided by the cyclotron mechanism. The mechanism of approaching sunspots, however, sounds more plausible because in this case **no guiding forces** (like in the cyclotron model) **are required.**

Fig. 5.2 Sketch of a sunspot pair





Magnetic loops associated with sunspots

Acceleration by Sunspots

In addition to merging sunspots, particle acceleration is **also possible during strong solar flares** (see Fig. 5.3), in which **electric fields of several 100 V/m** can be created.

In these fields, which can extend over large distances, like 10^7 m, **electrons and ions can be accelerated up to several GeV.** Massive stars with strong stellar eruptions may reach even higher energies.

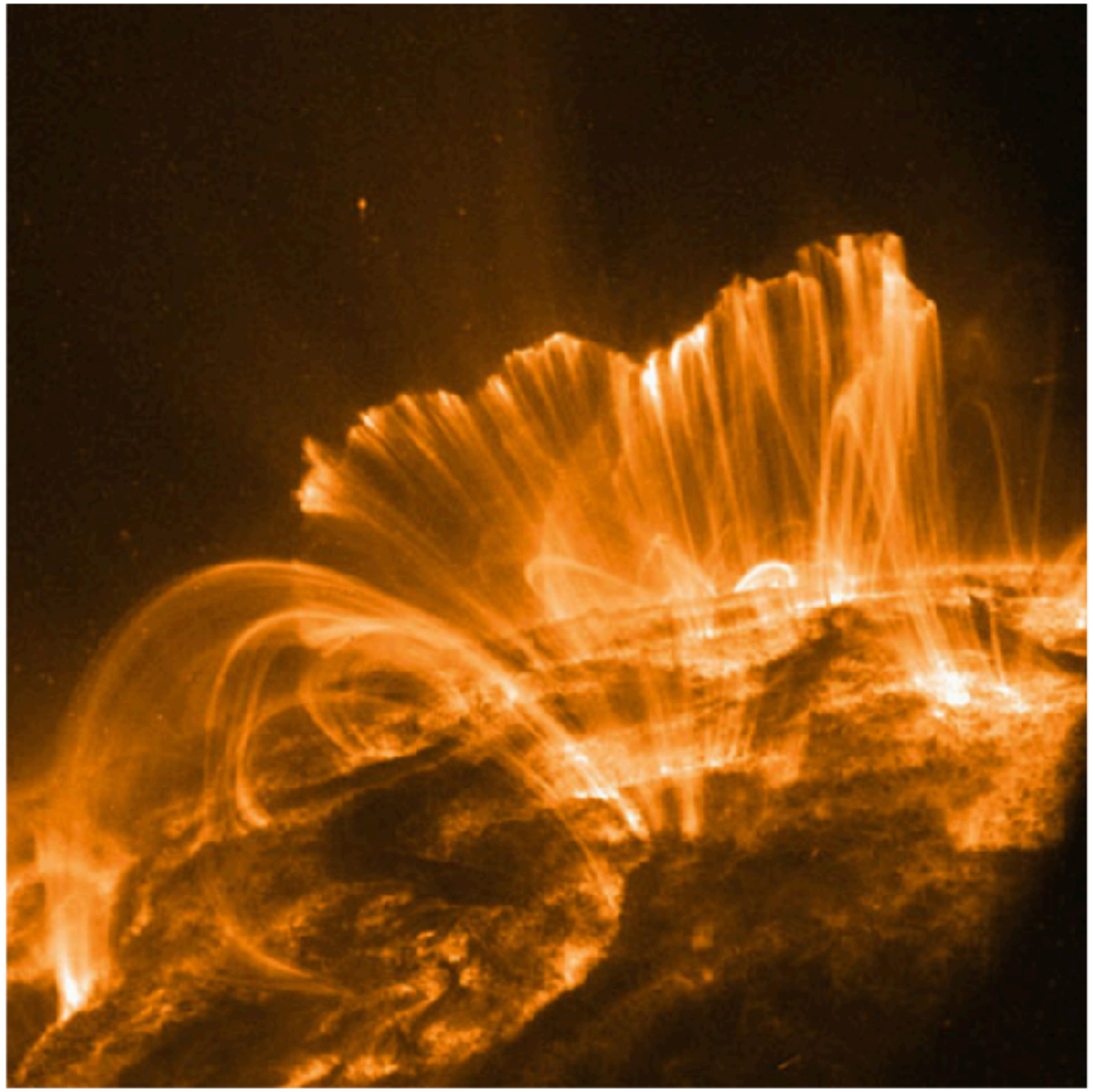
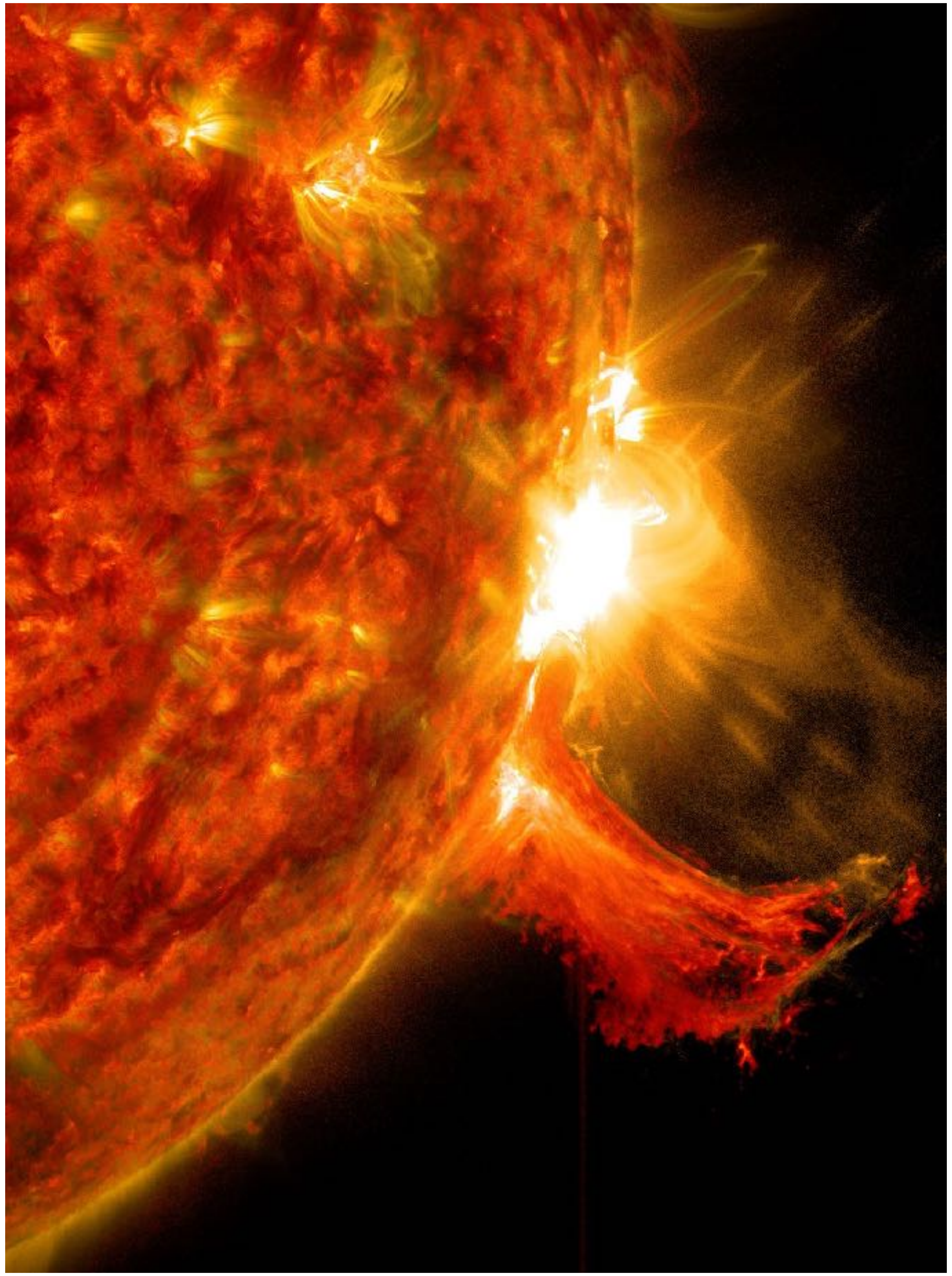


Fig. 5.3 Post-eruptive loops in the wake of a solar flare recorded by the TRACE satellite [49]

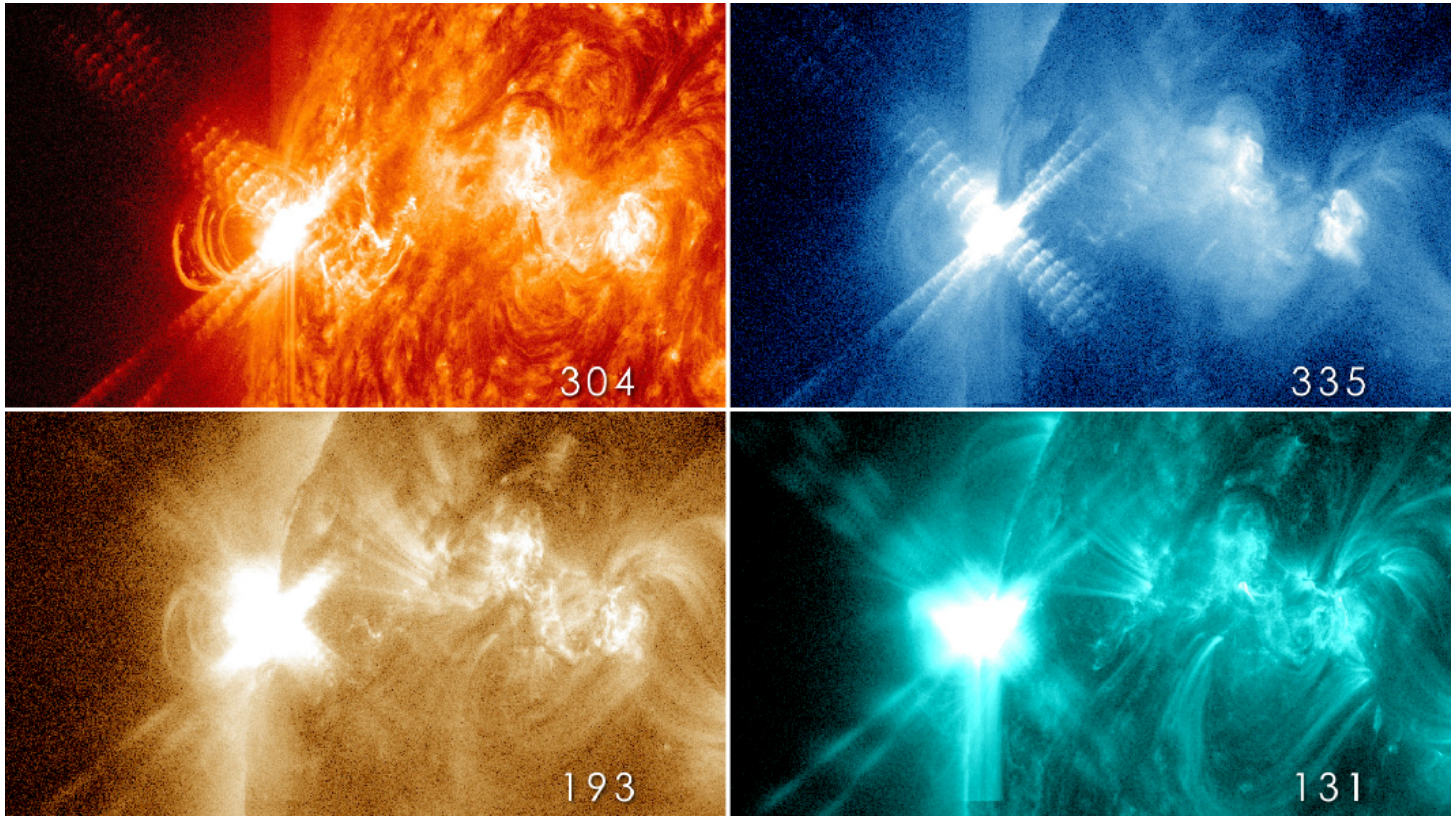
Solar flares

- **A solar flare is an intense localized eruption of electromagnetic radiation in the Sun's atmosphere.**
- Flares occur **in active regions** and are often, but not always, accompanied by coronal mass ejections, solar particle events, and other solar phenomena.
- The occurrence of solar flares varies with the 11-year solar cycle.
- Solar flares are thought to occur **when stored magnetic energy in the Sun's atmosphere accelerates charged particles in the surrounding plasma.** This results in the emission of electromagnetic radiation across the electromagnetic spectrum.

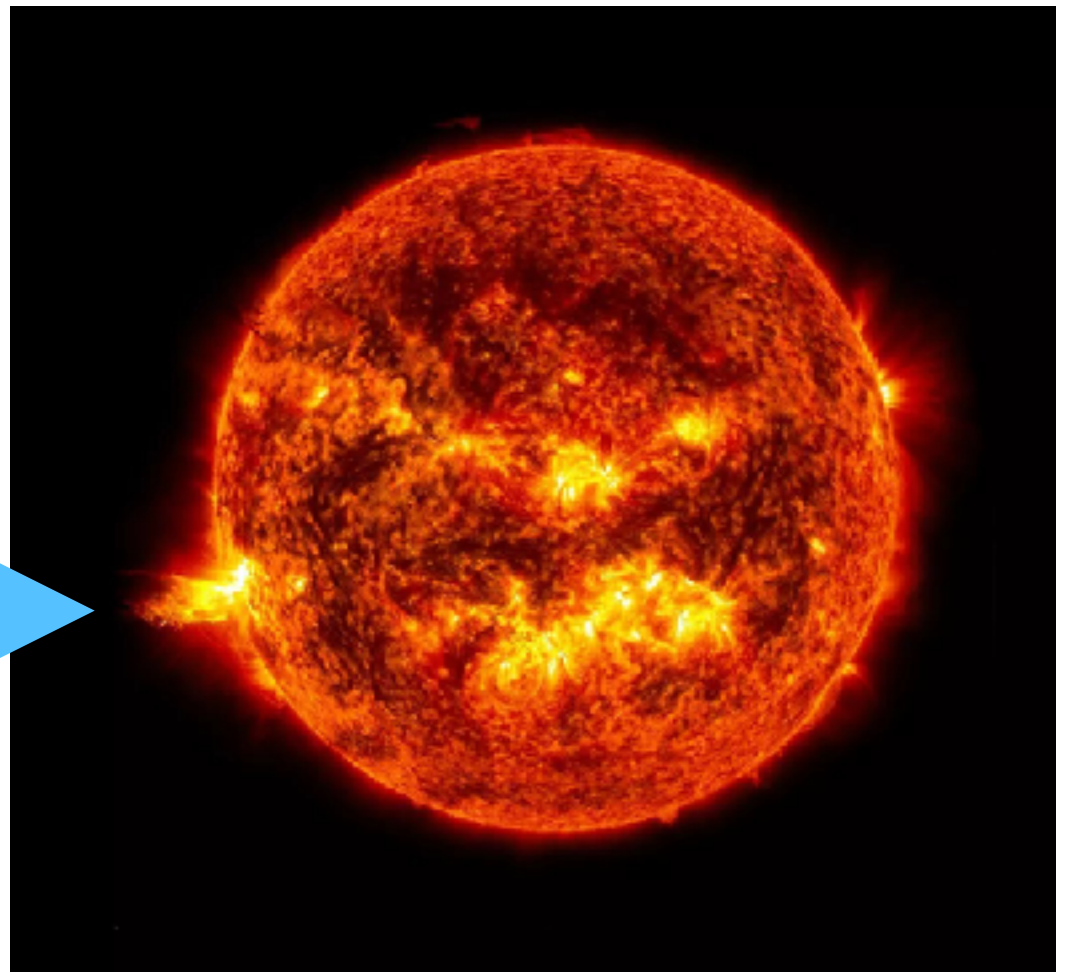


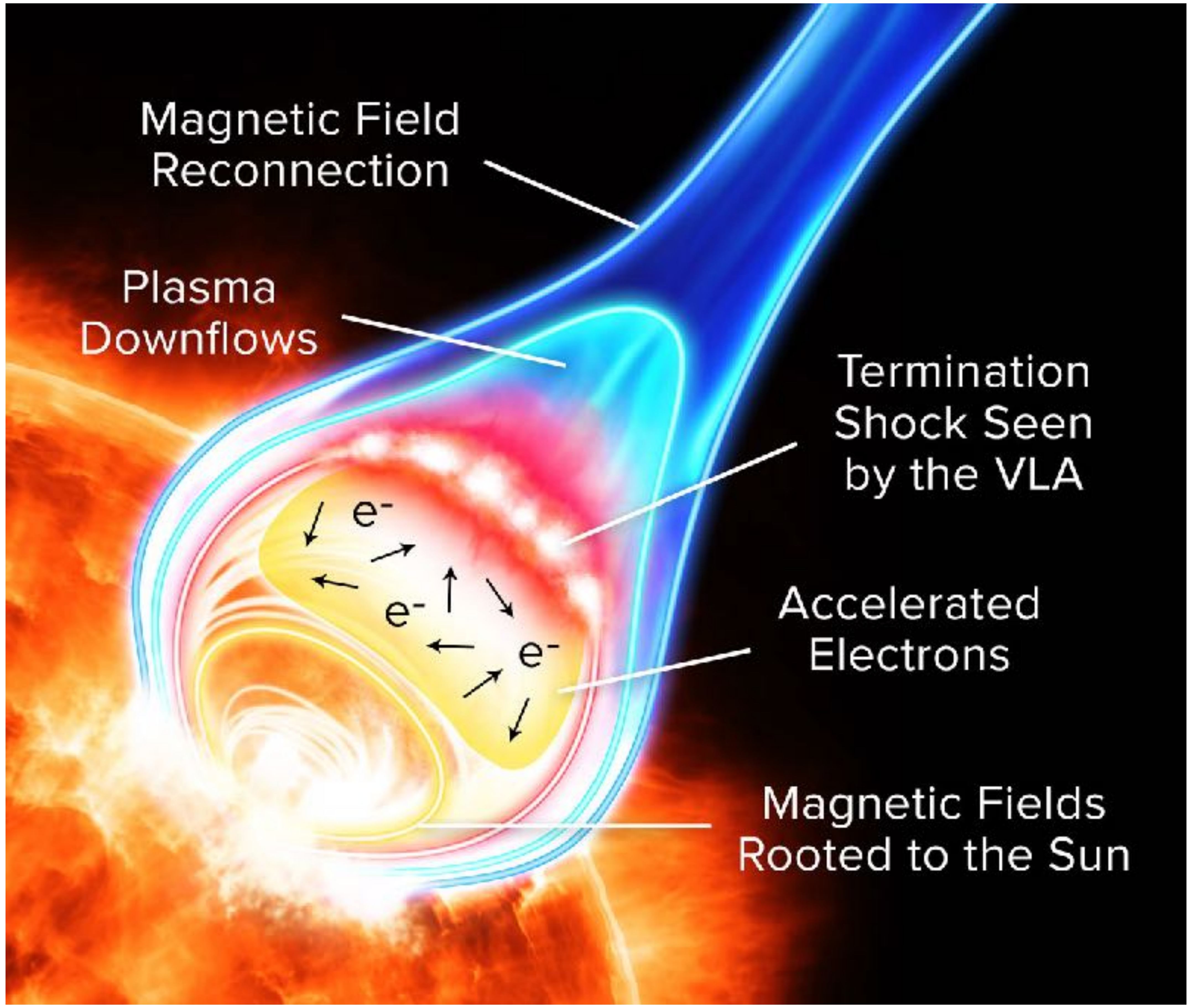
https://en.wikipedia.org/wiki/Solar_flare

Solar flares



An X3.2-class solar flare observed in different wavelengths. Clockwise from top left: 304, 335, 131, and 193 Å

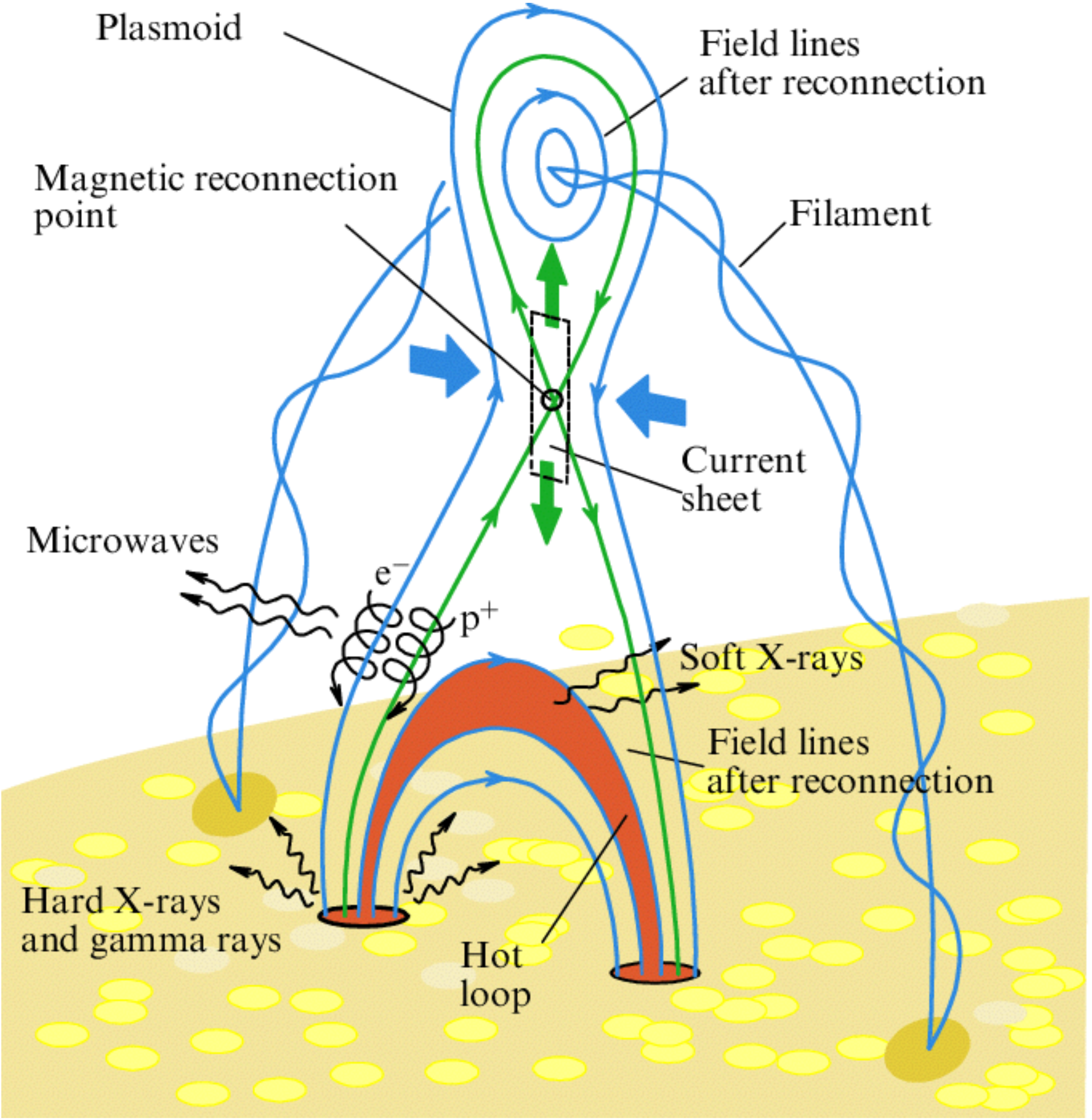




https://en.wikipedia.org/wiki/Solar_flare

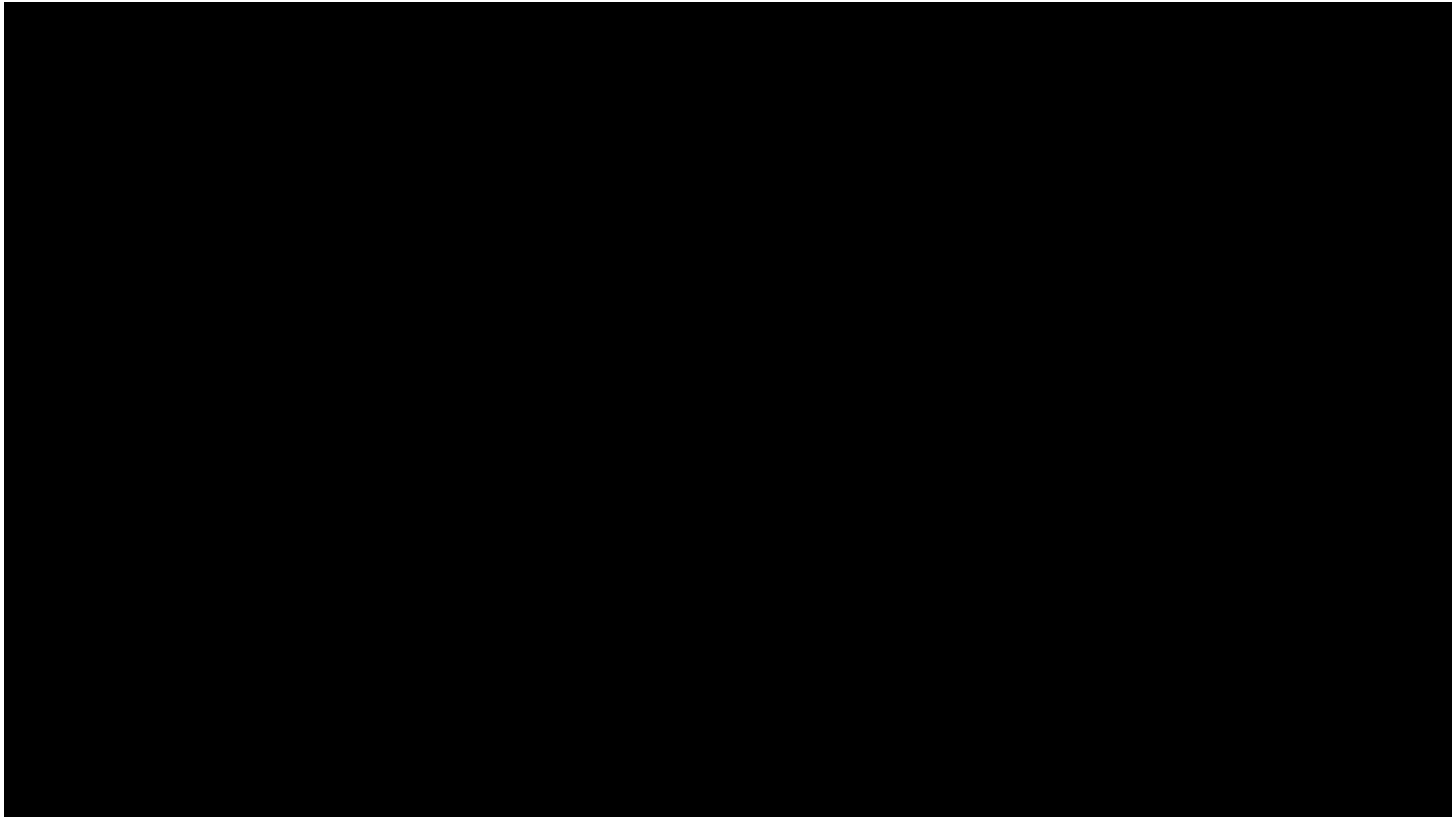
Solar flares

Standard model of a solar flare. The flare is triggered by the ascension of a filament, which results in **magnetic reconnection**, the inflow of cool plasma from the loop sides (blue arrows), and the outflow of hot plasma upwards and downwards (green arrows). The helices show the motion of electrons and ions, accelerated due to the reconnection, along the field lines towards the loop footpoints, where they give rise to hard X-rays (XRs) and gamma rays. As a result of deceleration of the charged particles, the plasma of the solar atmosphere heats up, evaporates, fills the post-flare loops, and emits in the soft X-ray range.



https://www.researchgate.net/figure/Color-online-Standard-model-of-a-solar-flare-The-flare-is-triggered-by-the-ascension_fig2_341075910

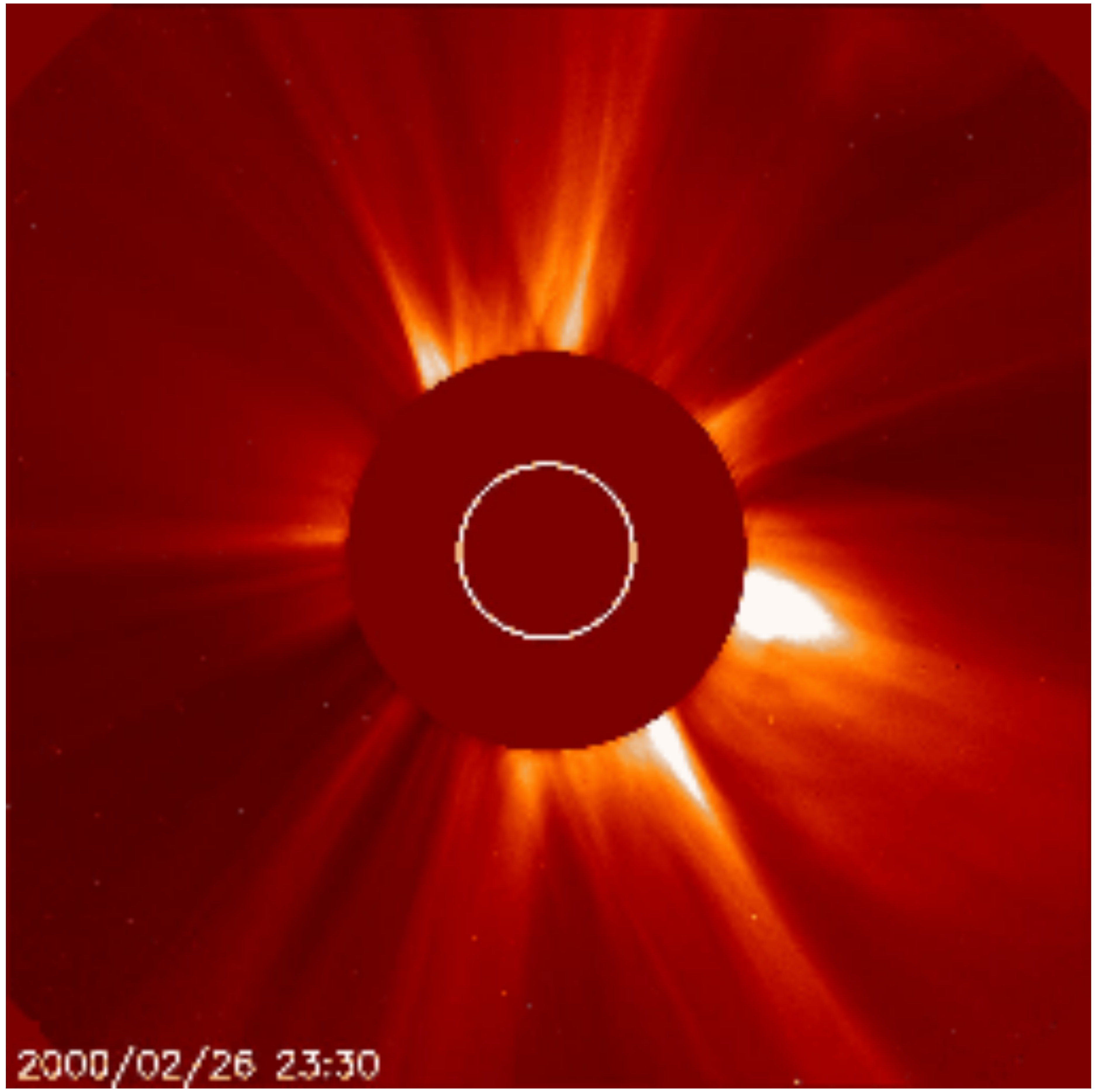
Solar flares



https://www.youtube.com/watch?v=P2AHHx7U4t4

Coronal Mass Ejection - CME

- **A coronal mass ejection (CME) is a significant release of plasma and accompanying magnetic field from the Sun's corona into the heliosphere.**
- CMEs are **often associated with solar flares** and other forms of solar activity, but a broadly accepted theoretical understanding of these relationships has not been established.
- If a CME enters interplanetary space, it is referred to as an interplanetary coronal mass ejection (ICME).
- ICMEs are capable of reaching and colliding with Earth's magnetosphere, where they **can cause geomagnetic storms, aurorae, and in rare cases damage to electrical power grids**.



https://en.wikipedia.org/wiki/Coronal_mass_ejection

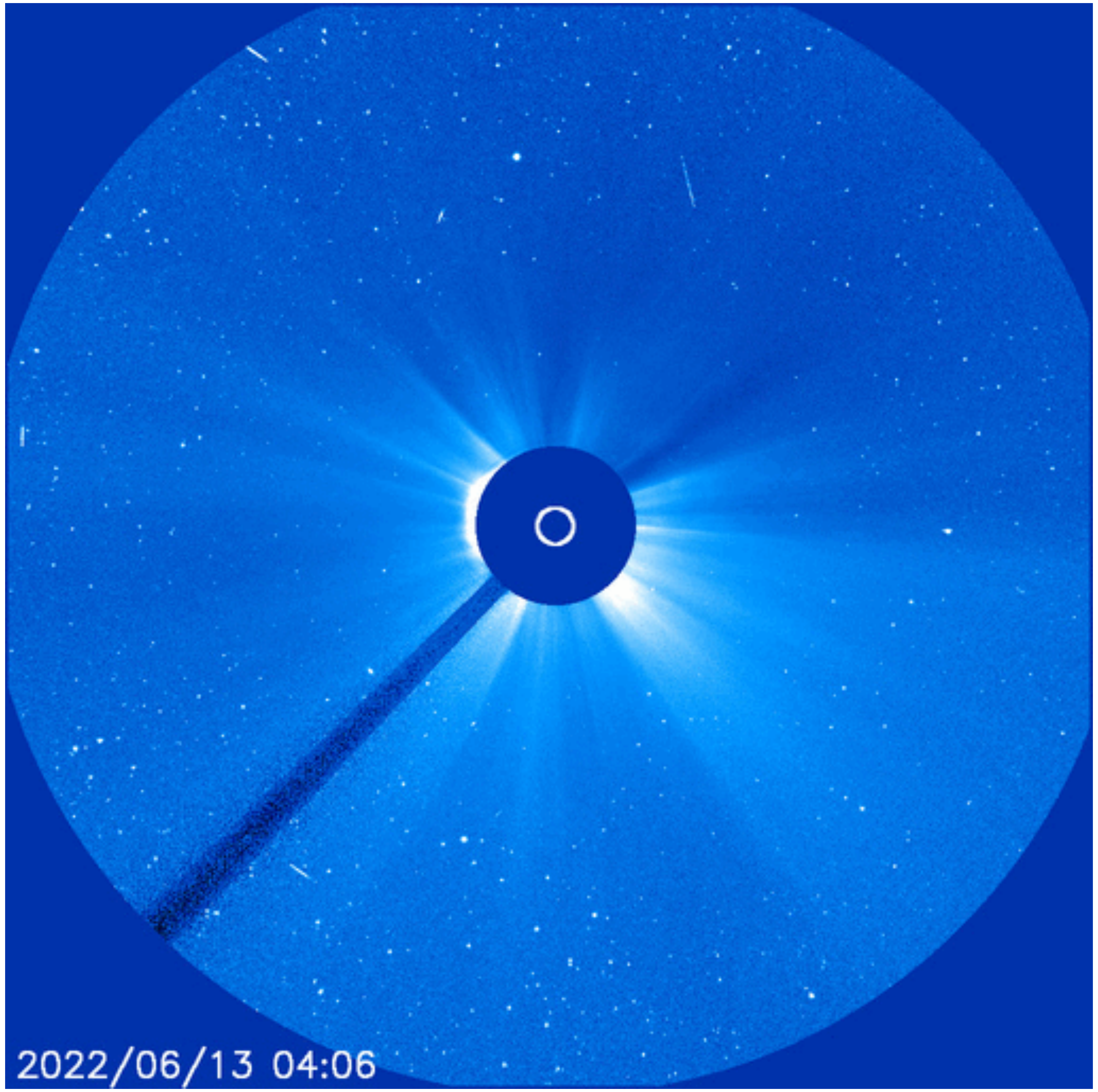
Solar flares

Note, how the green light is created in the upper atmosphere.

Photo from the ISS of **aurora australis** during a **geomagnetic storm** on 29 May 2010. The storm was most likely **caused by a CME** that had erupted from the Sun on 24 May 2010, five days prior to the storm.

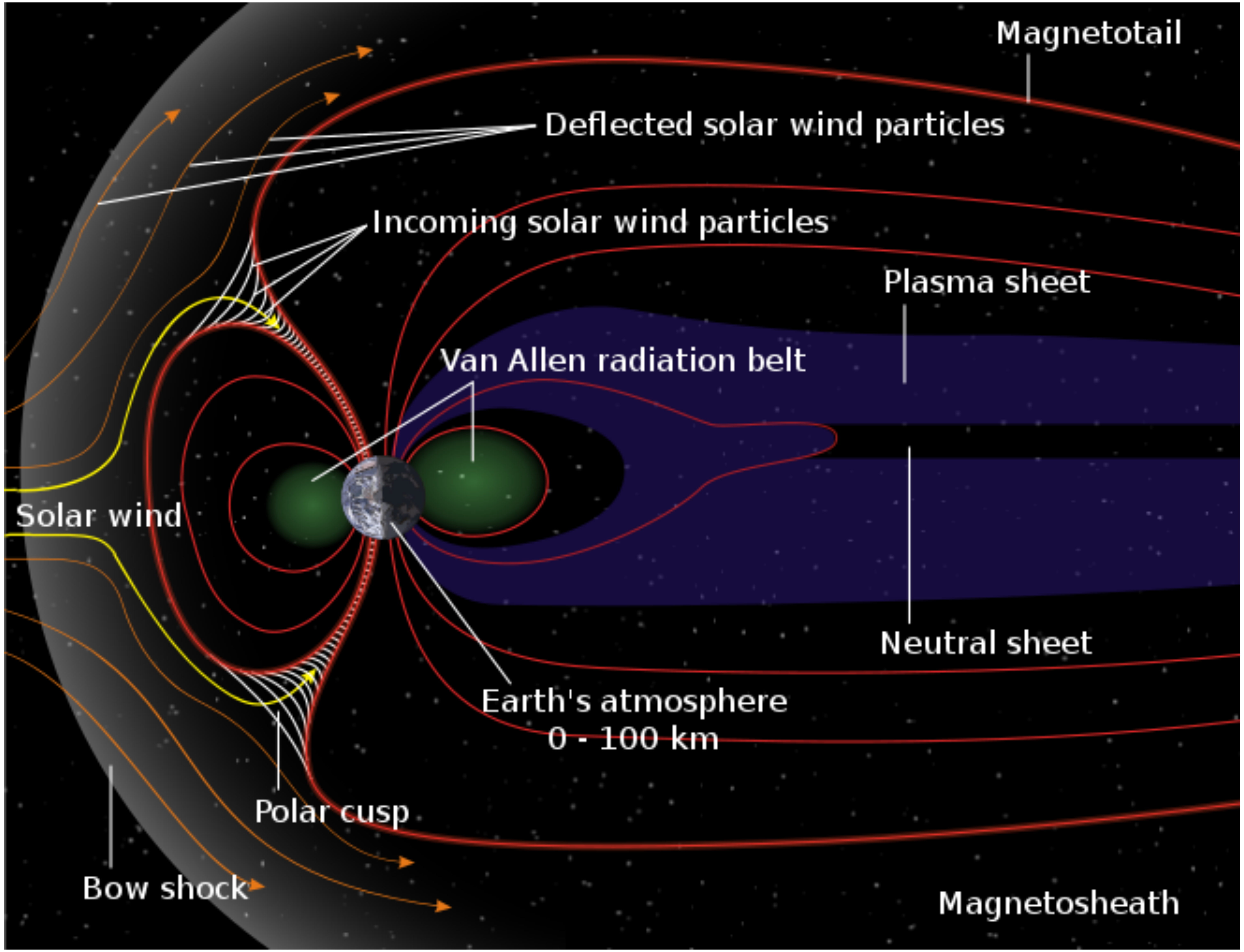
Coronal Mass Ejection - CME

- The **largest recorded geomagnetic perturbation,** resulting presumably from a CME, was the **solar storm of 1859**. Also known as the **Carrington Event,** it disabled parts of the at the time newly created United States telegraph network, starting fires and shocking some telegraph operators.
- Near solar maxima, the Sun produces about three CMEs every day, whereas near solar minima, there is about one CME every five days.
- These types of **Solar activity are causing space wether**
- It is important to monitor space weather since it can disturb our power grids, radio network, satellites or harm astronauts



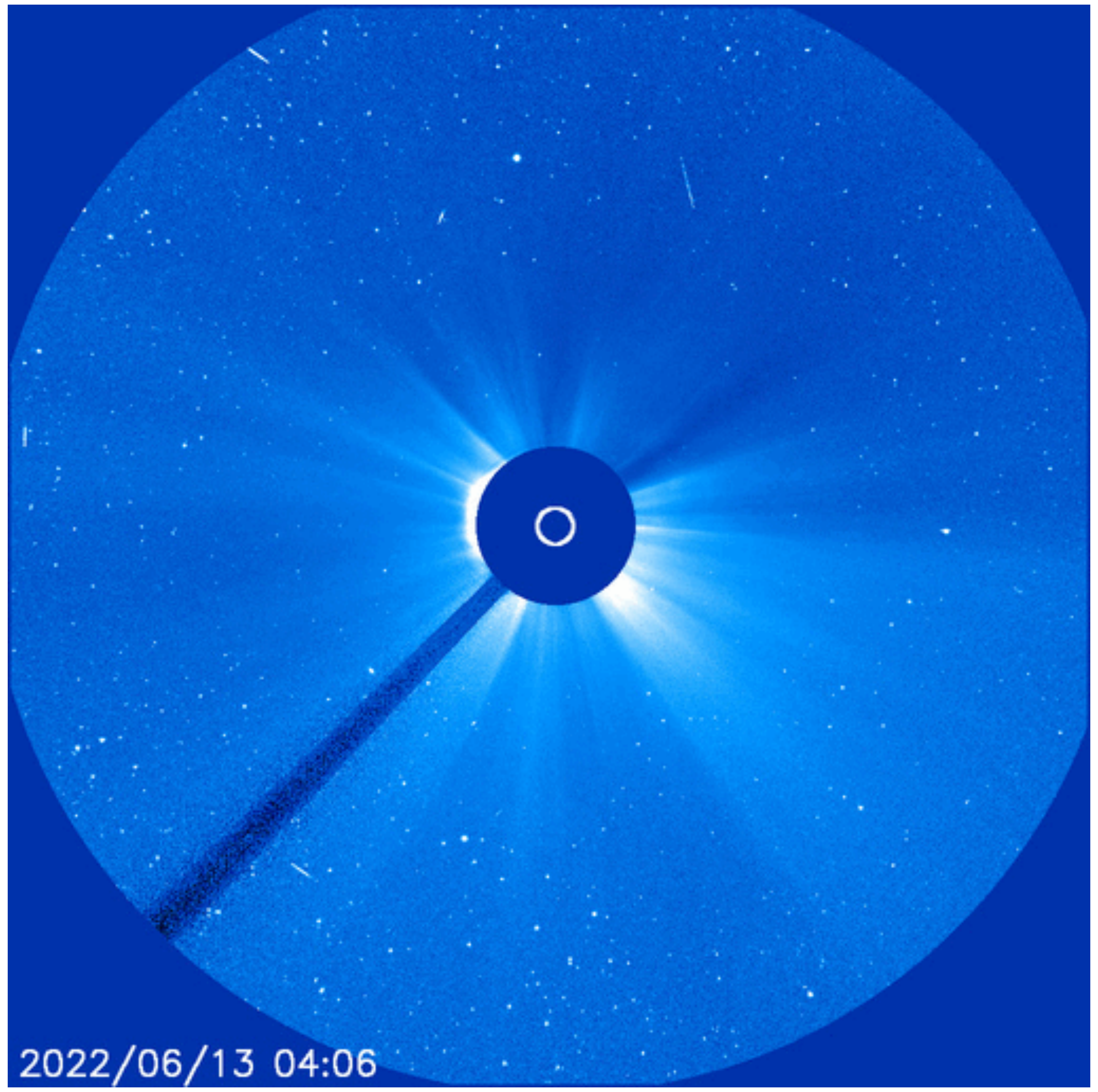
https://en.wikipedia.org/wiki/Coronal_mass_ejection

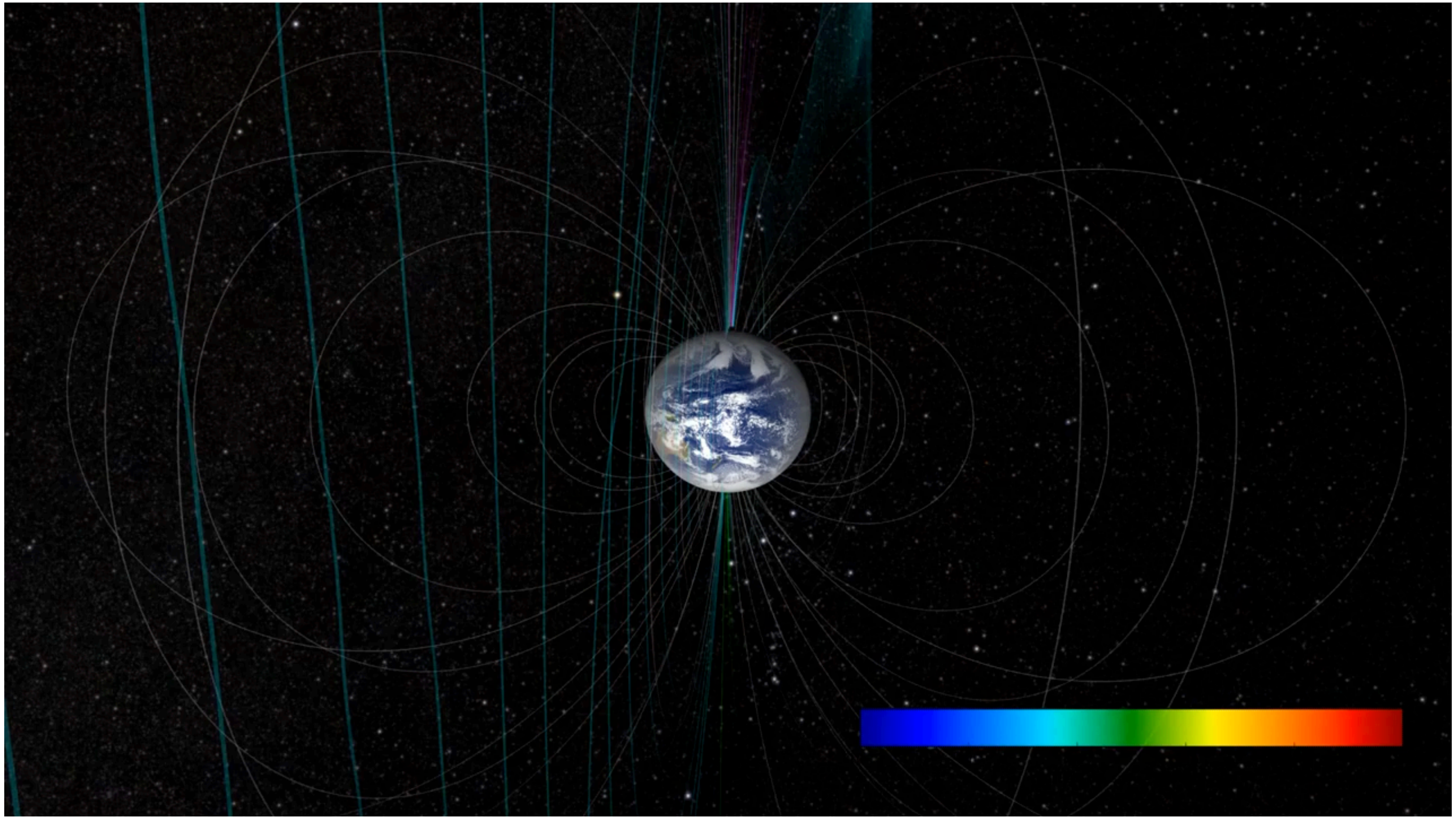
Discoveries in the 20th Century



Earth’s magnetosphere

Discoveries in the 20th Century



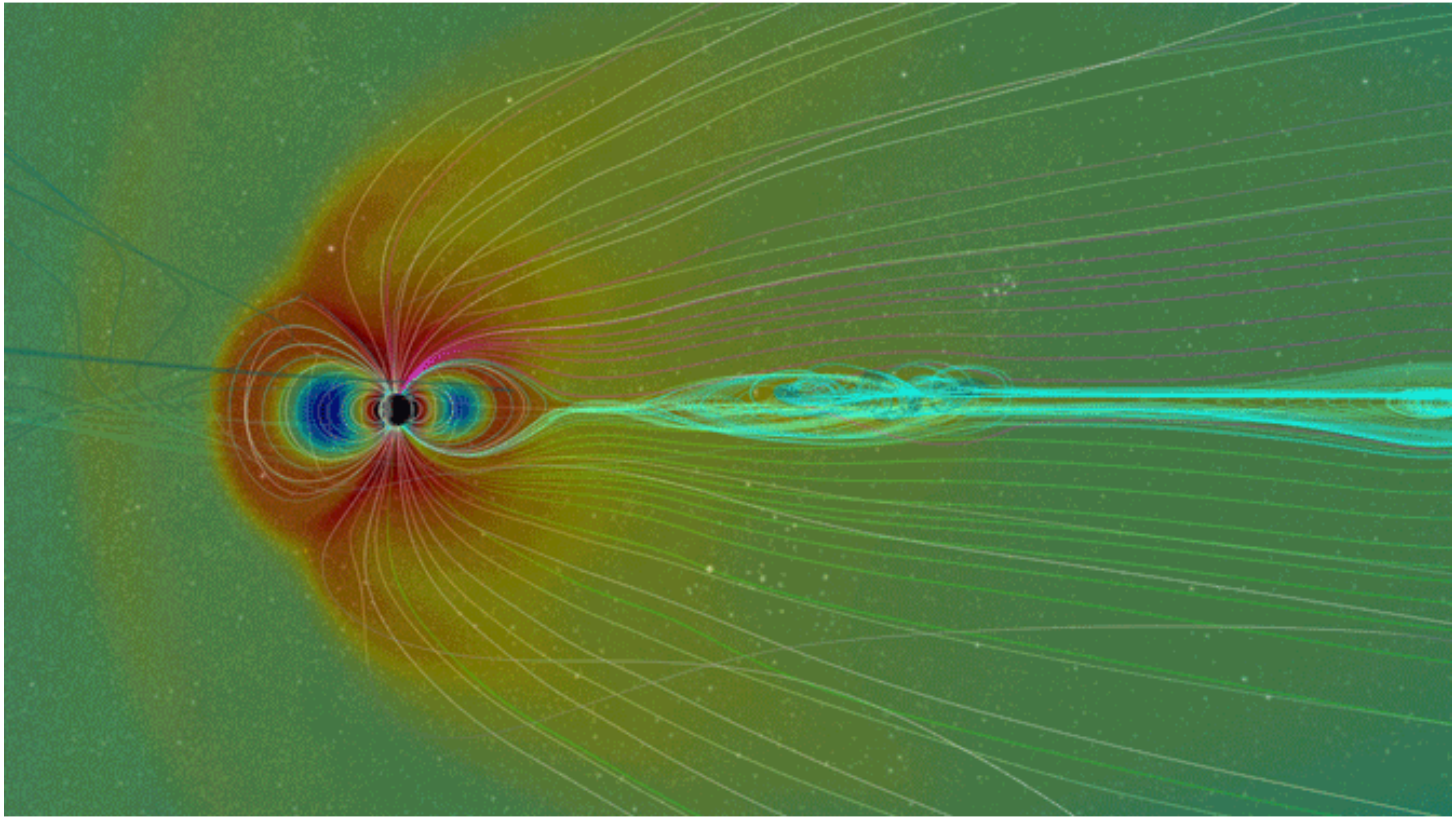


Earth's magnetosphere during a CME

https://www.youtube.com/watch?v=DyuTyEw3etk

Coronal Mass Ejection - CME

- Simulation of a very intense CME interacting with Earth's magnetosphere
- This CME could have caused the **Carrington Event**



https://svs.gsfc.nasa.gov/11660

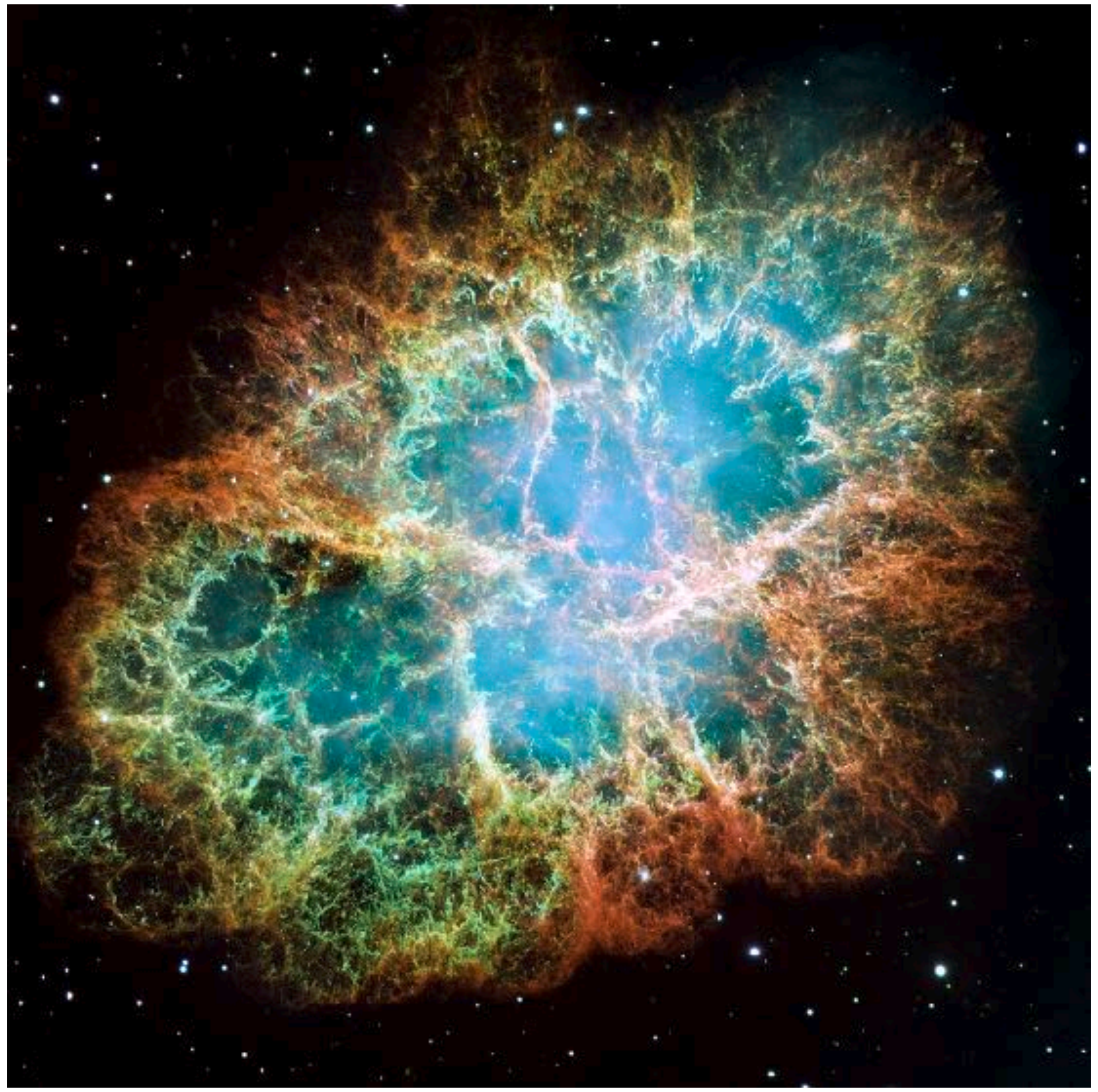
Shock Acceleration

If a **massive star has exhausted its hydrogen, the radiation pressure can no longer withstand the gravitational pressure and the star will collapse** under its own gravity.

The liberated gravitational energy increases the central temperature of a massive star to such an extent that **helium burning** can start.

If the **helium reservoir is used up**, the process of gravitational infall of matter repeats itself until the temperature is further increased so that the products of helium themselves can initiate fusion processes. These **successive fusion processes can lead at most to elements of the iron group (Fe, Co, Ni)**. For higher nuclear charges the fusion reaction is endotherm, which means that without providing additional energy heavier elements cannot be synthesized.

When the fusion process stops at iron, the massive star will implode. In this process **part of its mass will be ejected into interstellar space**. This material can be recycled for the production of a new star generation, which will contain—like the Sun—some heavy elements.



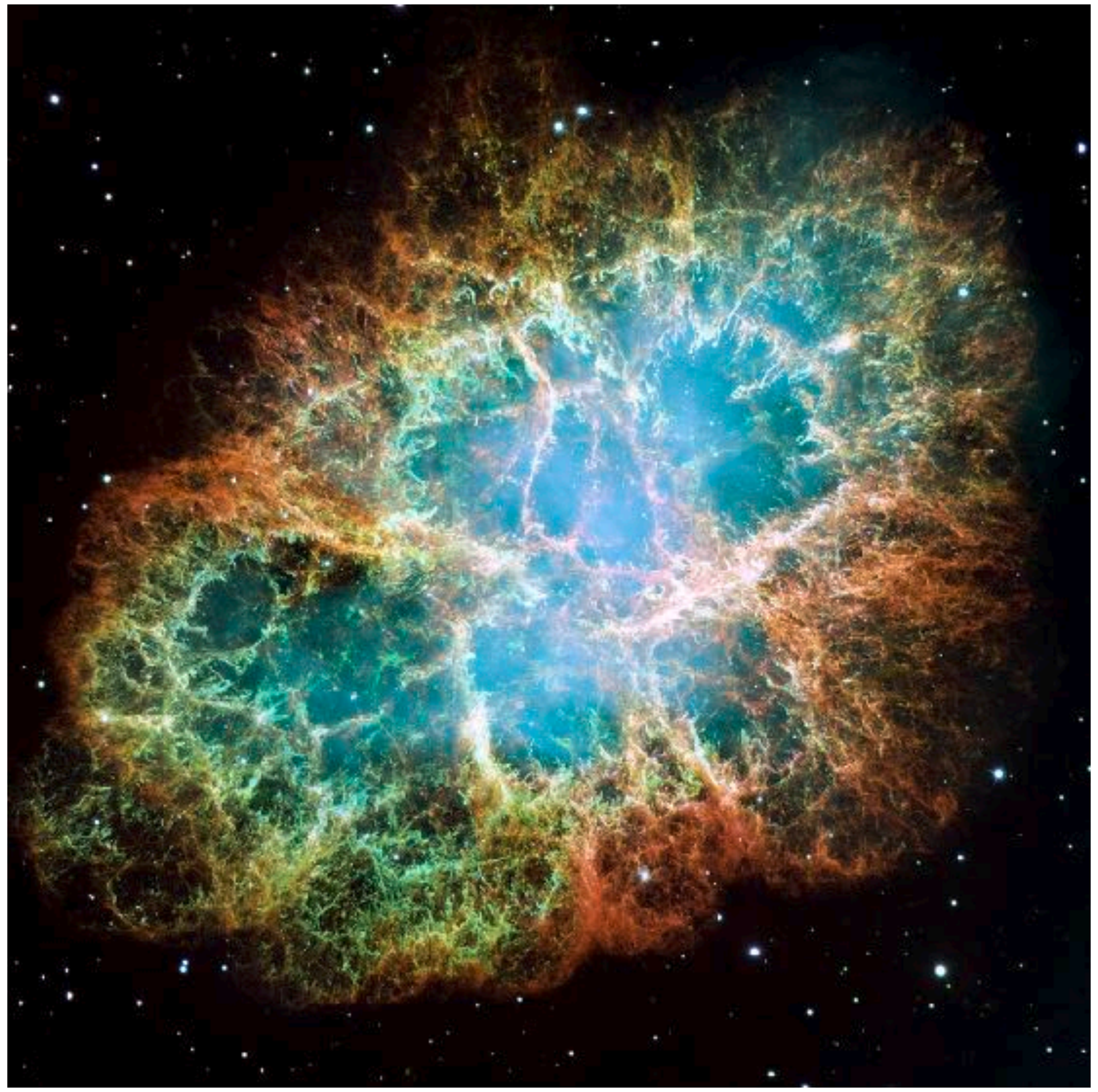
SN 1054 remnant (Crab Nebula).

Shock Acceleration

As a result of the implosion a **compact neutron star will be formed** that has a density, which is comparable to the density of atomic nuclei (~2 times as heavy as the Sun in an object of the size of a city).

In the course of a **supernova explosion** some **elements heavier than iron are produced** if the copiously available neutrons are attached to the elements of the iron group, which—with successive β- decays—allows elements with higher nuclear charge to be formed:

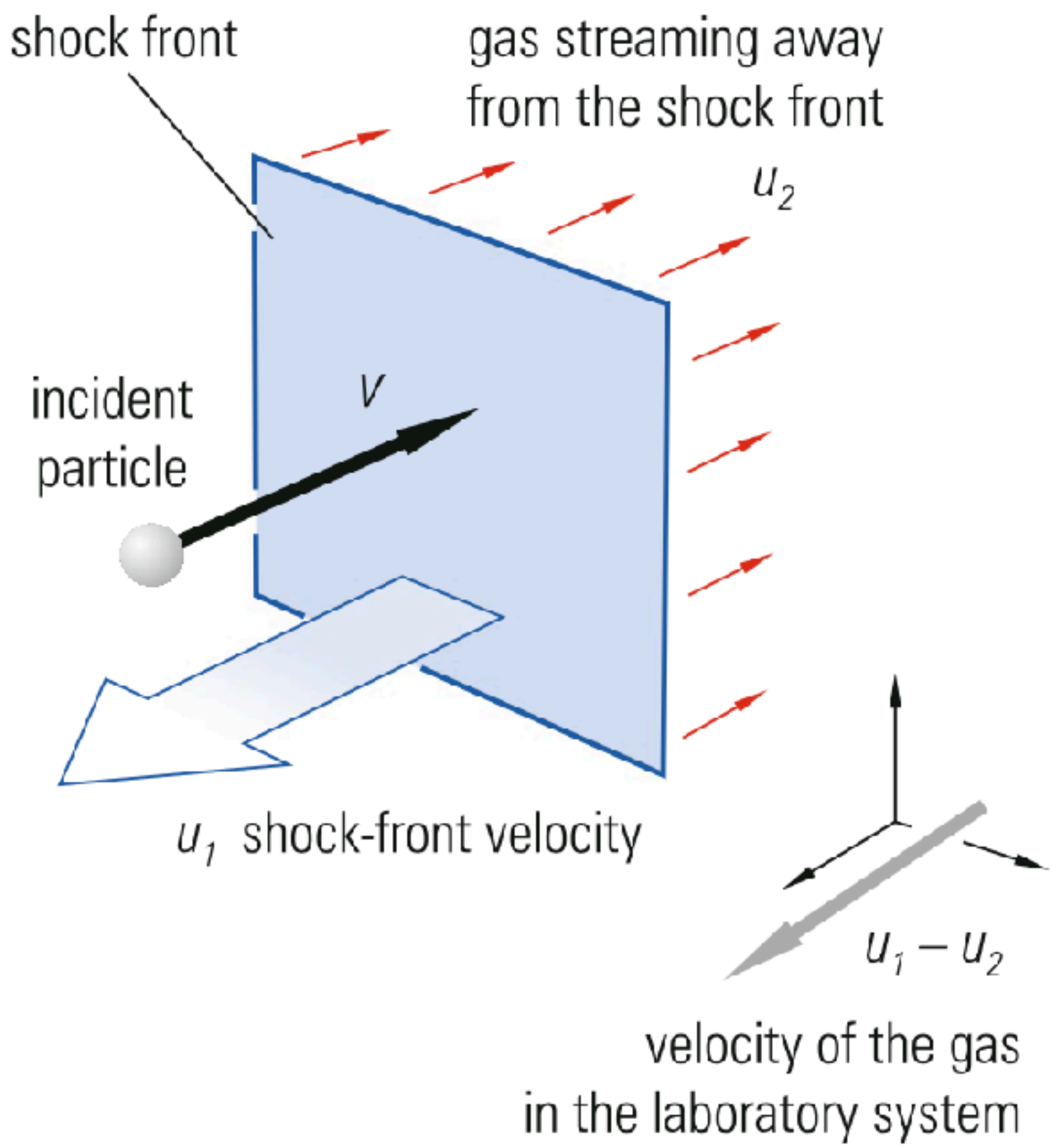
$$
\begin{aligned}
{}^{56}_{26}\mathrm{Fe} + n &\rightarrow {}^{57}_{26}\mathrm{Fe}\,, \\
{}^{57}_{26}\mathrm{Fe} + n &\rightarrow {}^{58}_{26}\mathrm{Fe}\,, \\
{}^{58}_{26}\mathrm{Fe} + n &\rightarrow {}^{59}_{26}\mathrm{Fe}^{*} \\
&\quad \hookrightarrow {}^{59}_{27}\mathrm{Co} + e^{-} + \bar{\nu}_e\,, \\
{}^{59}_{27}\mathrm{Co} + n &\rightarrow {}^{60}_{27}\mathrm{Co}^{*} \\
&\quad \hookrightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e\,.
\end{aligned}
$$



SN 1054 remnant (Crab Nebula).

Shock Acceleration

Fig. 5.4 Schematics of shock-wave acceleration



The **ejected envelope of a supernova represents a *shock front*** with respect to the interstellar medium. Let us assume that the shock front moves at a velocity u_1. Behind the shock front the gas recedes with a velocity u_2. This means that the gas has a velocity $u_1 - u_2$ in the laboratory system (see Fig. 5.4).

A particle of velocity v colliding with the shock front and being reflected gains the energy

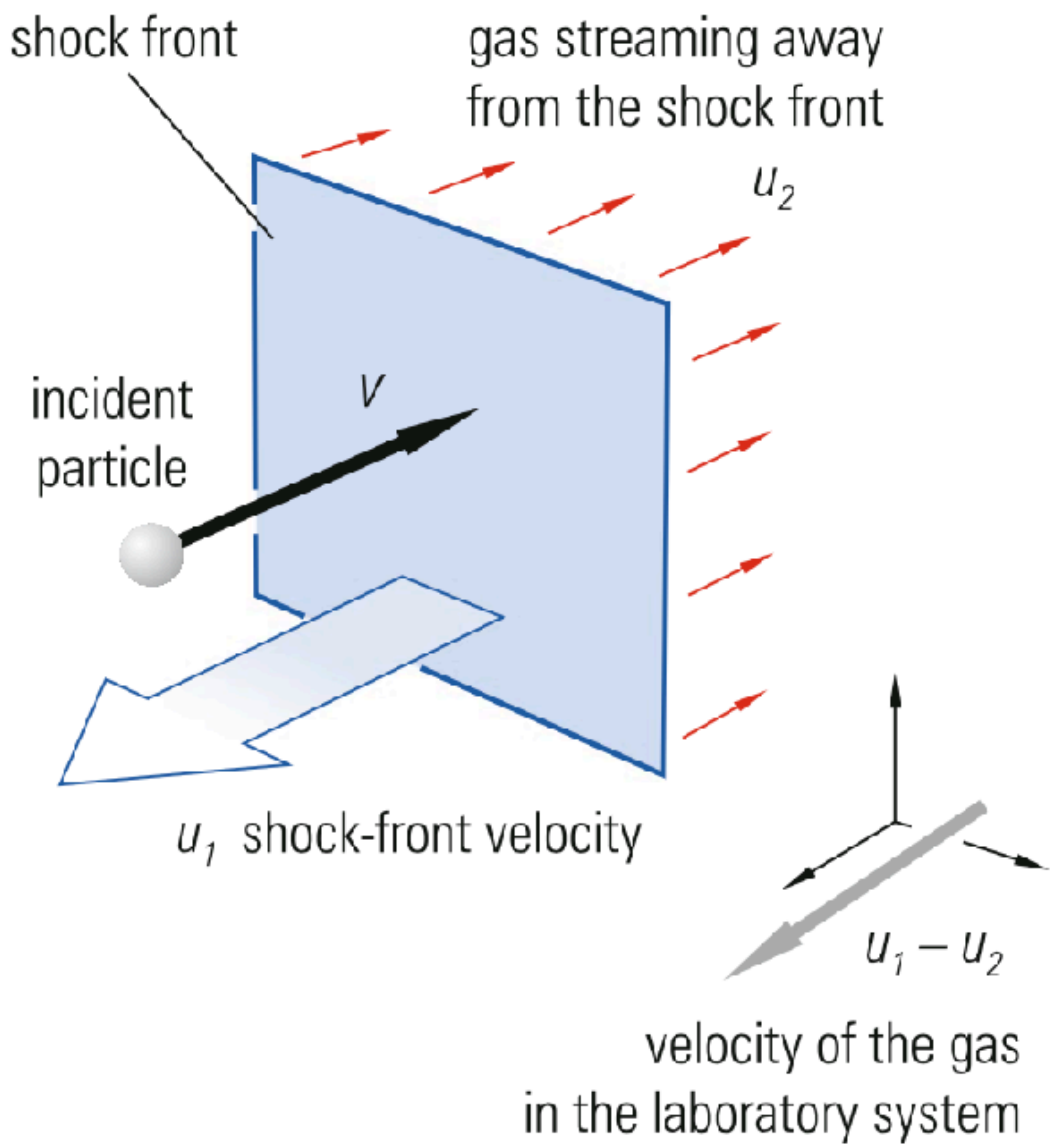
$$\Delta E = \frac{1}{2}m(v + (u_1 - u_2))^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}m(2v(u_1 - u_2) + (u_1 - u_2)^2) .$$

Since the linear term dominates ($v \gg u_1, u_2$; $u_1 > u_2$), this simple model provides a relative energy gain of

$$\frac{\Delta E}{E} \approx \frac{2(u_1 - u_2)}{v}$$

Shock Acceleration

Fig. 5.4 Schematics of shock-wave acceleration



A more general, relativistic treatment of shock acceleration including also variable scattering angles leads to

$$\frac{\Delta E}{E} = \frac{4}{3} \frac{u_1 - u_2}{c}$$

where it has been assumed that the particle velocity v can be approximated by the speed of light c.

In physics, a **shock wave**, or shock, is **a type of propagating disturbance that moves faster than the local speed of sound in the medium**. Like an ordinary wave, a shock wave carries energy and can propagate through a medium but is characterized by an abrupt, nearly discontinuous, change in pressure, temperature, and density of the medium.

Shock Acceleration

Similar results are obtained if one assumes that particles are trapped between two shock fronts and are reflected back and forth from the fronts (see also Fig. 5.5).

Usually the inner front will have a much higher velocity (v_2) compared to the outer front (v_1), which is decelerated already in interactions with the interstellar material (Fig. 5.5).

The inner shock front can provide **velocities up to 20 000 km/s**, as obtained from measurements of the Doppler shift of the ejected gas. The outer front spreads into the interstellar medium with velocities between some 100 km/s up to 1000 km/s. For shock accelerations in active galactic nuclei even superfast shocks with $v_2 = 0.9\,c$ are discussed.

A particle of velocity v being reflected at the inner shock front gains the energy

$$\Delta E_1 = \frac{1}{2}m(v + v_2)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(v_2^2 + 2vv_2)\,.$$

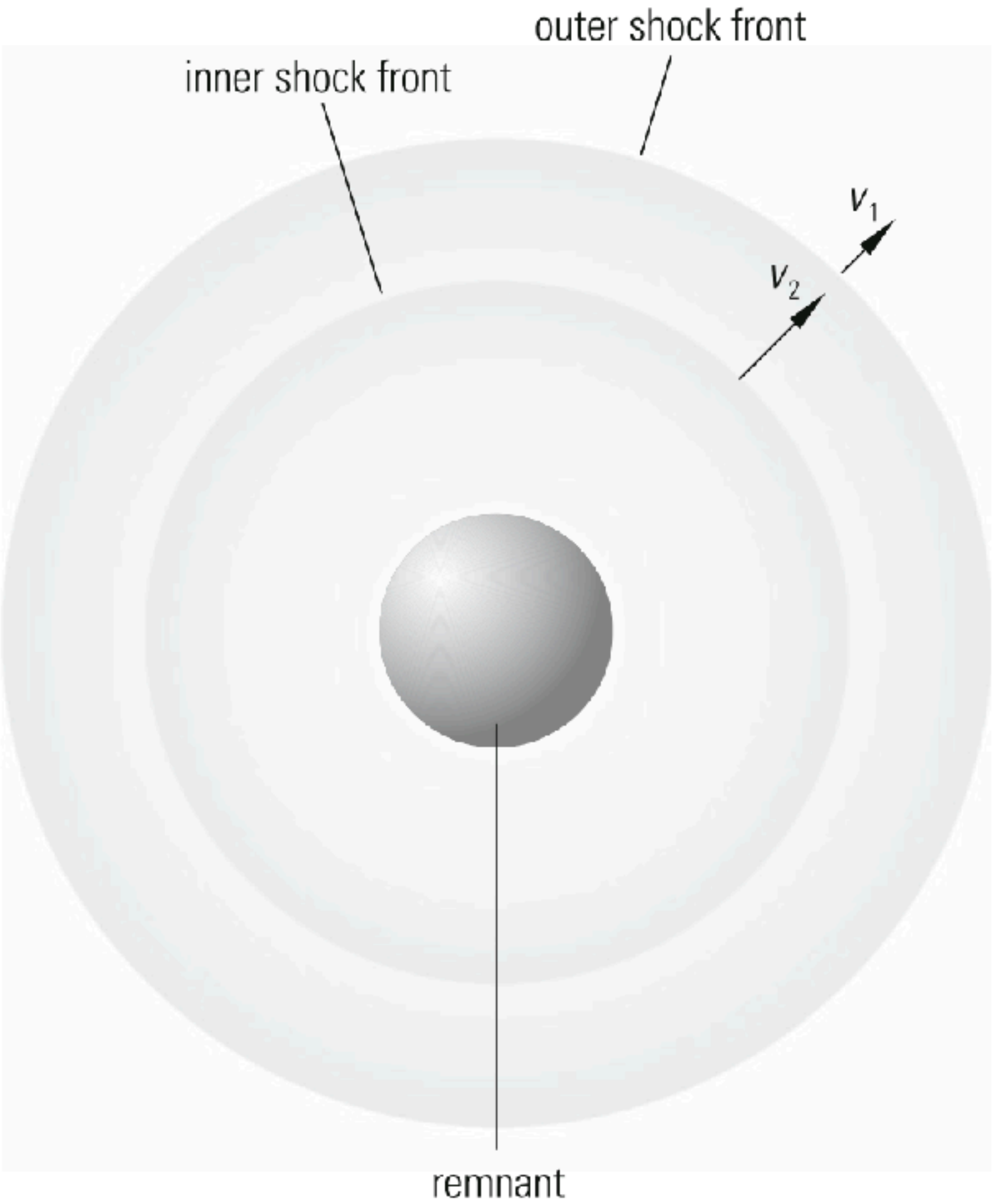


Fig. 5.5 Particle acceleration by multiple reflections between two shock fronts

Shock Acceleration

Reflection at the outer shock front leads to an energy loss

$$\Delta E_2 = \frac{1}{2}m(v - v_1)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(v_1^2 - 2vv_1)\,.$$

On average, the particle gains an energy

$$\Delta E = \frac{1}{2}m(v_1^2 + v_2^2 + 2v(v_2 - v_1))\,.$$

Since the quadratic terms can be neglected and because of $v_2 > v_1$, one gets

$$\Delta E \approx mv\Delta v\,, \quad \frac{\Delta E}{E} \approx 2\frac{\Delta v}{v}$$

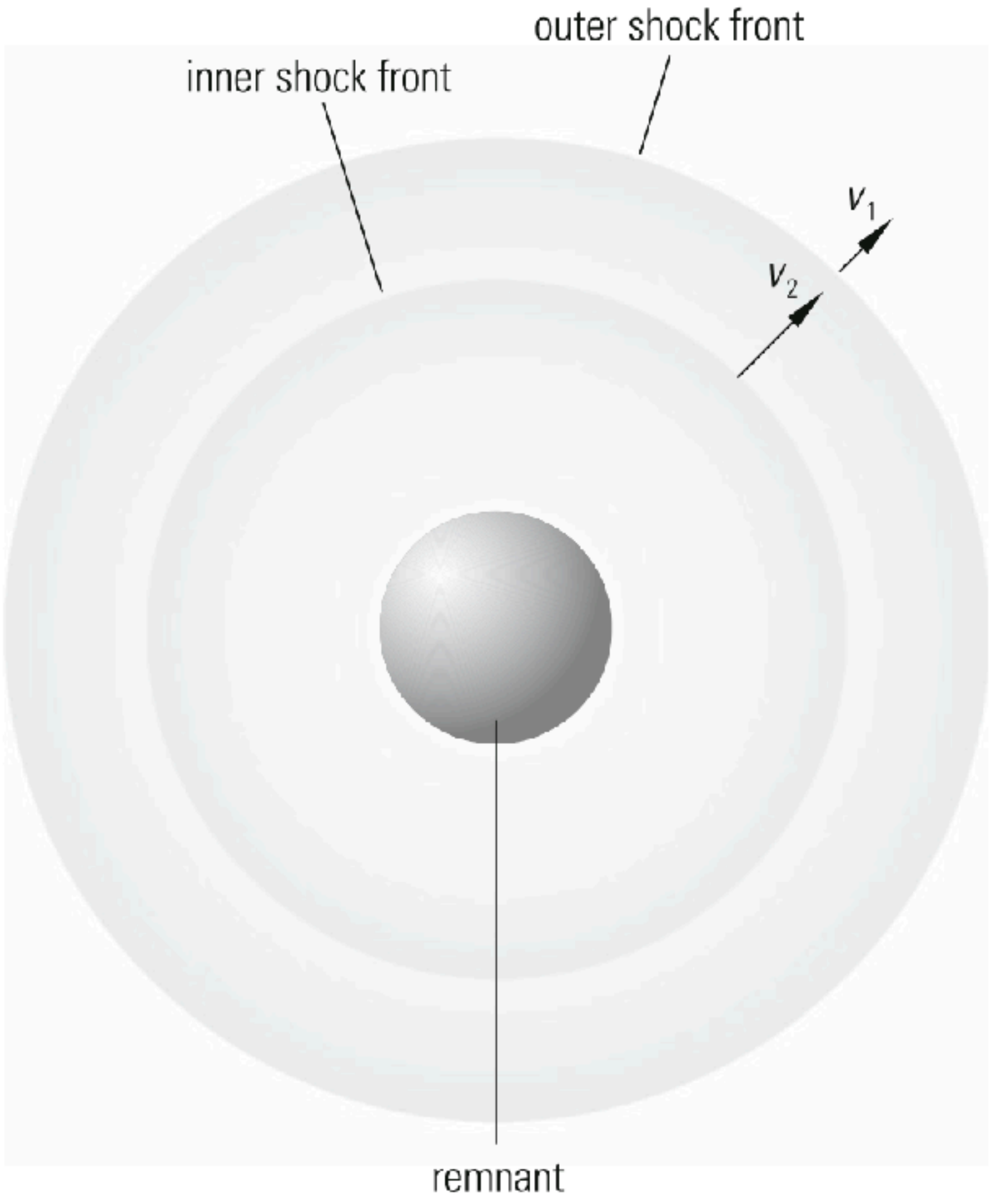


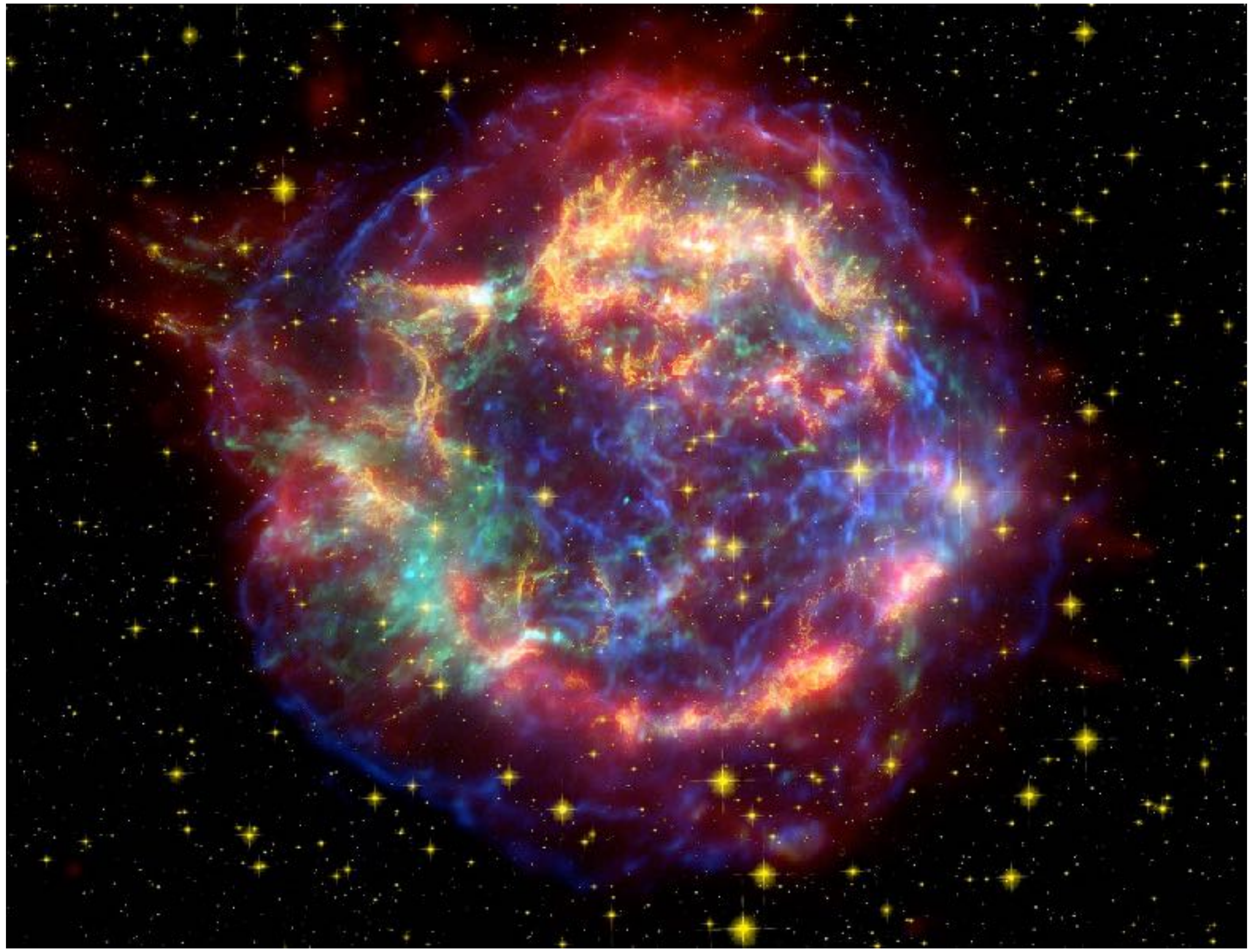
Fig. 5.5 Particle acceleration by multiple reflections between two shock fronts

Shock Acceleration

Both presented shock acceleration mechanisms are linear in the relative velocity. Sometimes this type of shock acceleration is **called Fermi mechanism of first order**.

Under plausible conditions using the **relativistic treatment, maximum energies of about 100 TeV can be explained in this way**. It seems even possible that energies up to the galactic cosmic-ray knee at $\approx 10^{15}$eV are attainable.

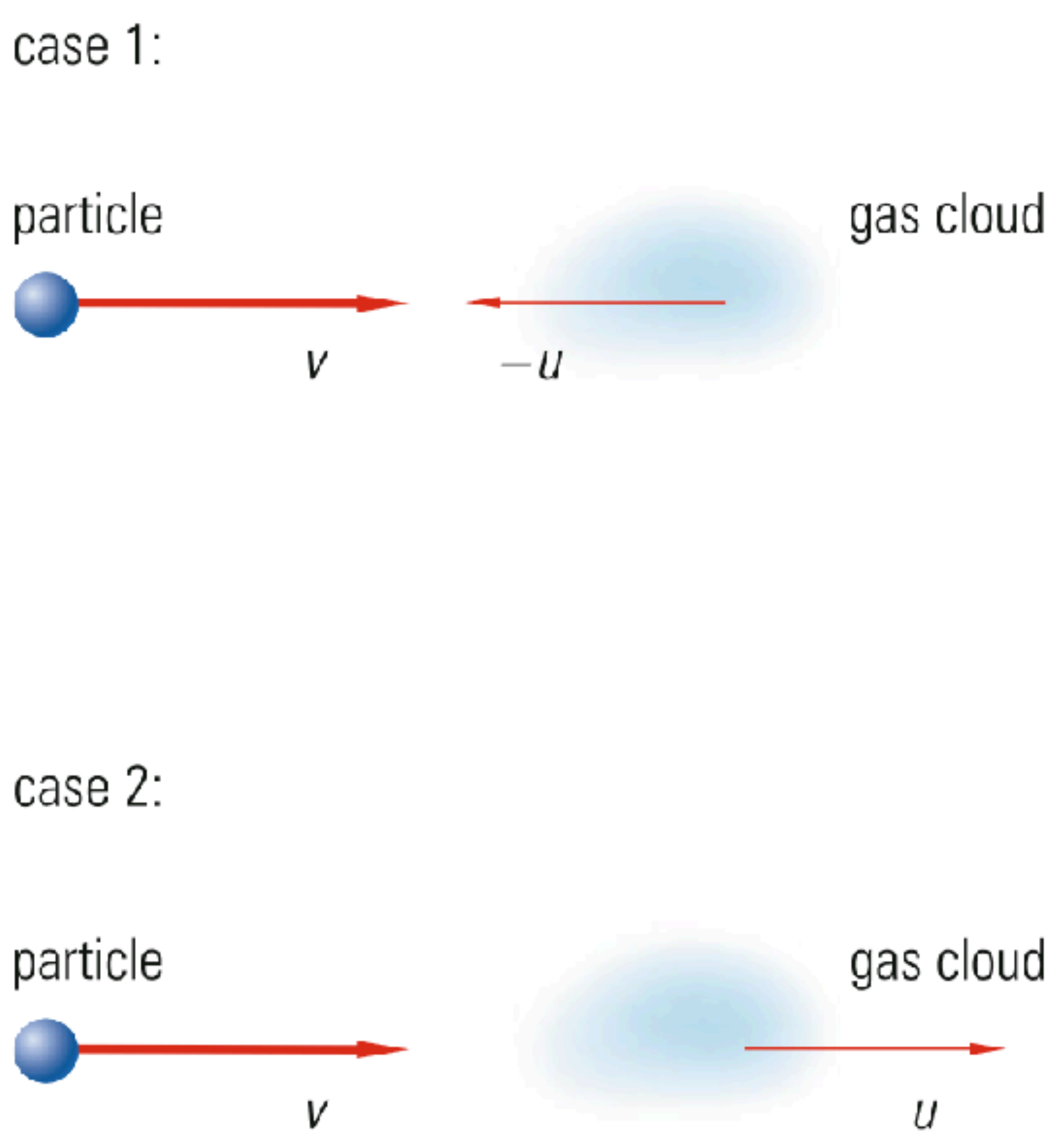
The Supernova rate of the Milky Way is approximately one every 50 years. However, the last Supernova explosion in the Galaxy was about 300 years ago, around the year 1667 or 1680. The remnant of this explosion, Cassiopeia A.



A false color image of Cassiopeia A (Cas A) using observations from both the Hubble and Spitzer telescopes as well as the Chandra X-ray Observatory.

Fermi Mechanisms

Fig. 5.6 Energy gain of a particle by a reflection from a magnetic gas cloud



Fermi mechanism* of second order** (or more general Fermi mechanism) describes the **interaction of cosmic-ray particles with magnetised clouds**. At first sight it appears improbable that particles can gain energy in this way. Let us assume that a particle (with velocity **v**) is reflected from a gas cloud, which moves with a velocity ***u (Fig. 5.6).

If **v** and ***u*** are **antiparallel, the particle gains the energy**

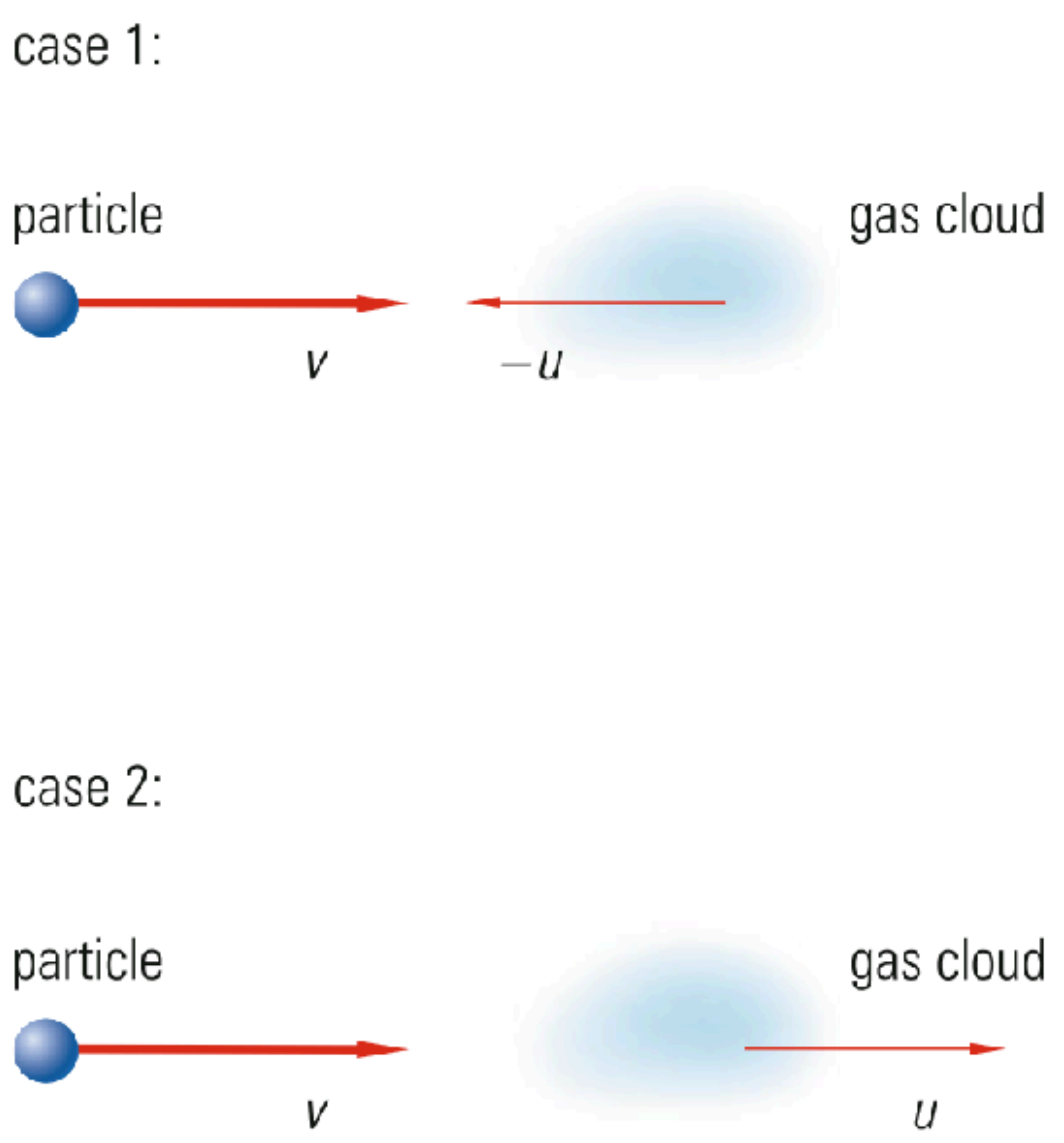
$$\Delta E_1 = \frac{1}{2}m(v+u)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(2uv + u^2)$$

In case that **v** and ***u*** are **parallel, the particle loses an energy**

$$\Delta E_2 = \frac{1}{2}m(v-u)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(-2uv + u^2)\,.$$

Fermi Mechanisms

Fig. 5.6 Energy gain of a particle by a reflection from a magnetic gas cloud



On average a net energy gain of

$$\Delta E = \Delta E_1 + \Delta E_2 = mu^2$$

results, leading to the relative energy gain of

$$\frac{\Delta E}{E} = 2\frac{u^2}{v^2}$$

Since this acceleration mechanism is quadratic in the cloud velocity, this variant is **often called Fermi mechanism of 2nd order**. The **result remains correct even under relativistic treatment.** Since the cloud velocity is rather low compared to the particle velocities ($u \ll$ v $\approx c$), the **energy gain per collision (~u^2) is very small**. Therefore, the acceleration of particles by the Fermi mechanism **requires a very long time.**

Fermi Mechanisms

In this acceleration type one assumes that magnetised clouds act as collision partners—and not normal gas clouds—because the gas density and thereby the interaction probability is larger in magnetised clouds.

Another important aspect is that **cosmic-ray particles will lose some of their gained energy by interactions** with the interstellar or intergalactic gas between two collisions. This is why **this mechanism requires a minimum injection energy, above which particles can only be effectively accelerated**. These injection energies could be provided by the Fermi mechanism of 1st order, that is, by shock acceleration.

It appears possible that second-order Fermi acceleration is **capable of accelerating cosmic rays up to ultra-high energies.**

Pulsars

Spinning magnetised neutron stars (pulsars) are **remnants of supernova explosions**. While **stars typically have radii of 10^6 km**, they shrink under a gravitational collapse to a size of just about **20 km.** This process leads to **densities** of 6×10^{13} g/cm^3 **comparable to nuclear densities**.

- **degeneracy pressure of neutrons supports a neutron star.**

In this process electrons and protons are so closely packed that in processes of weak interactions neutrons are formed:

$$p + e^- \rightarrow n + \nu_e \, .$$

What is the name of this process?

- Since the neutron mass is more than the combined mass of a proton and an electron, the reaction can take place **only if some energy is supplied** to make up for this mass deficit. Therefore, under ordinary laboratory circumstances, is an unlikely reaction and free neutrons decay away.

Neutron stars

- When matter is compressed to very high densities, things change drastically. For simplicity, let us assume that the highly compressed matter consists of electrons, protons and neutrons.
- The **electrons become degenerate** with the rise of density while the other heavier particles still remain non-degenerate. Suppose we want to put an additional electron in a region of high density. We know that all the **levels are filled up to the Fermi momentum** p_F, which is **related to the number density** n_e **of electrons**

$$E_{\mathrm{F}} = \sqrt{p_{\mathrm{F}}^2 c^2 + m_{\mathrm{e}}^2 c^4}$$

- E_F is the **Fermi energy associated with this Fermi momentum** p_F.
- Unless an energy $E_F - m_e c^2$ is added to an electron, it is not possible to put the electron in the region of high density, since all the lower energy states are filled.
- When this excess energy required becomes equal to or larger than $(m_n - m_p - m_e)c^2$, the amount by which the neutron mass exceeds the sum of the proton mass and the electron mass. In this situation, it will be **energetically favourable for the electron to combine with a proton to produce a neutron.**

Neutron stars

The condition is:

$$\sqrt{p_{\mathrm{F,c}}^2 c^2 + m_{\mathrm{e}}^2 c^4} - m_{\mathrm{e}} c^2 = (m_{\mathrm{n}} - m_{\mathrm{p}} - m_{\mathrm{e}}) c^2,$$

where $p_{F,c}$ **is the critical Fermi momentum.** From this

$$m_{\mathrm{e}} c^2 \left(1 + \frac{p_{\mathrm{F,c}}^2}{m_{\mathrm{e}}^2 c^2}\right)^{1/2} = Qc^2,$$

where Q = $m_n - m_p$. We can also **express the critical Fermi momentum** from this:

$$p_{\mathrm{F,c}} = m_{\mathrm{e}} c \left[\left(\frac{Q}{m_{\mathrm{e}}}\right)^2 - 1\right]^{1/2}$$

Neutron stars

Example problem 8

- Since the **Fermi momentum increases with density**, we expect the Fermi momentum to be less than $p_{F,c}$ **when the density is below a critical density**. In this situation, **free electrons are energetically favoured** and we do not expect any neutrons to be present.
- The **critical density, at which the Fermi momentum becomes equal to** $p_{F,c}$, can be obtained by putting the values of fundamental constants into the equation to get $p_{F,c}$, then obtaining n_e and multiplying n_e by $m_p + m_e$. This gives:

$$\rho_c = 1.2 \times 10^{10}\ \mathrm{kg\ m^{-3}}$$

- **When the density is made higher than this, the electrons start combining with protons** to give neutrons. This phenomenon is called the ***neutron drip*.**
- At densities well above the critical density, matter would mainly consist of neutrons. **These neutrons do not decay, since there are no free states for the product electron** to occupy.

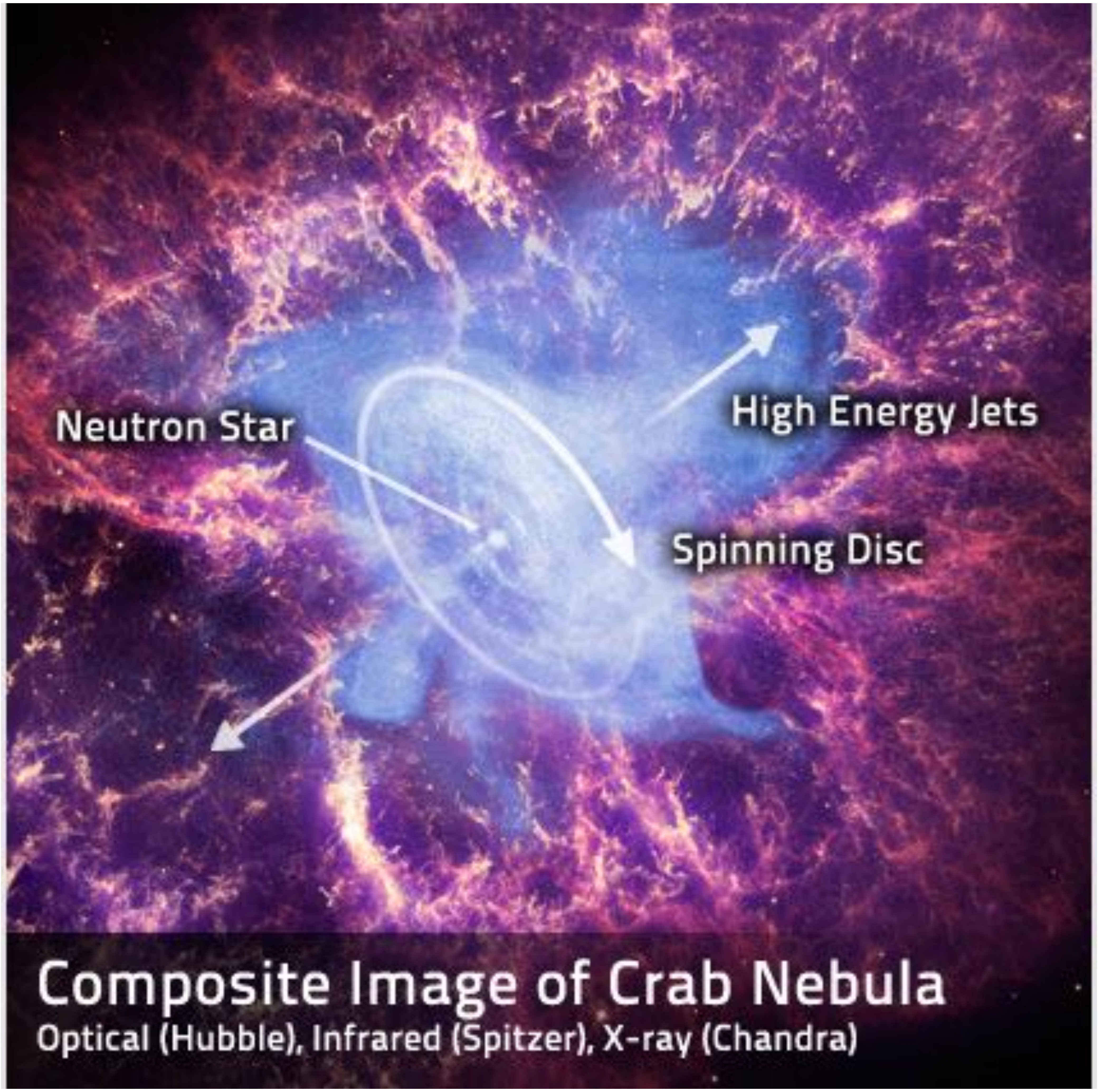
Neutron stars

- This simplified calculation of neutron drip without considering the possible formation of nuclei. When the existence of nuclei is taken into account, the calculation becomes much harder.
- On making various reasonable assumptions, the more realistic value of the critical density for neutron drip is found to be **3.2×10^{14} kg m^{-3}**.
- Strictly speaking, the term **'neutron drip' refers to neutrons getting out of nuclei when the density is raised above the critical density.** If a stellar core is compressed to densities higher than what is needed for the neutron drip, the core will essentially consist of neutrons.
- Since neutrons are **Fermi particles** like electrons and obey the Pauli exclusion principle, **neutrons also can give rise to a degeneracy pressure**.
- **When neutrons are packed to densities close to the density inside an atomic nucleus, the neighbouring neutrons interact with each other through nuclear forces**. Because of this finding an **accurate equation of state for matter at such high densities is very difficult** and the subject of ongoing research.
- **Neutron stars have a mass limit**. One can get an **absolute theoretical limit of 3.2 $M_{\odot}$**, it is generally believed that the actual mass limit is somewhat less than this and **most likely around 2.9 $M_{\odot}$.**

Neutron stars - radius

- Calculations suggest that a neutron star typically has a radius of order 10 km and internal density close to 10^{18} kg m^{-3}. -> **star of mass $M_\odot$ and radius 10 km** (keep in mind that this mass refers to the **leftover of the core after the supernova explosion** and not the original mass of the star)
- When a star of mass $M_\odot$ collapses to a radius of 10 km, the gravitational potential energy lost is of order 10^{46}J, which is approximately the energy output of a supernova. If the gravitational energy lost in the collapse of the inner core to form a neutron star is dumped into the outer layers of the star, then the outer layers can explode with this energy.

Example problem 4



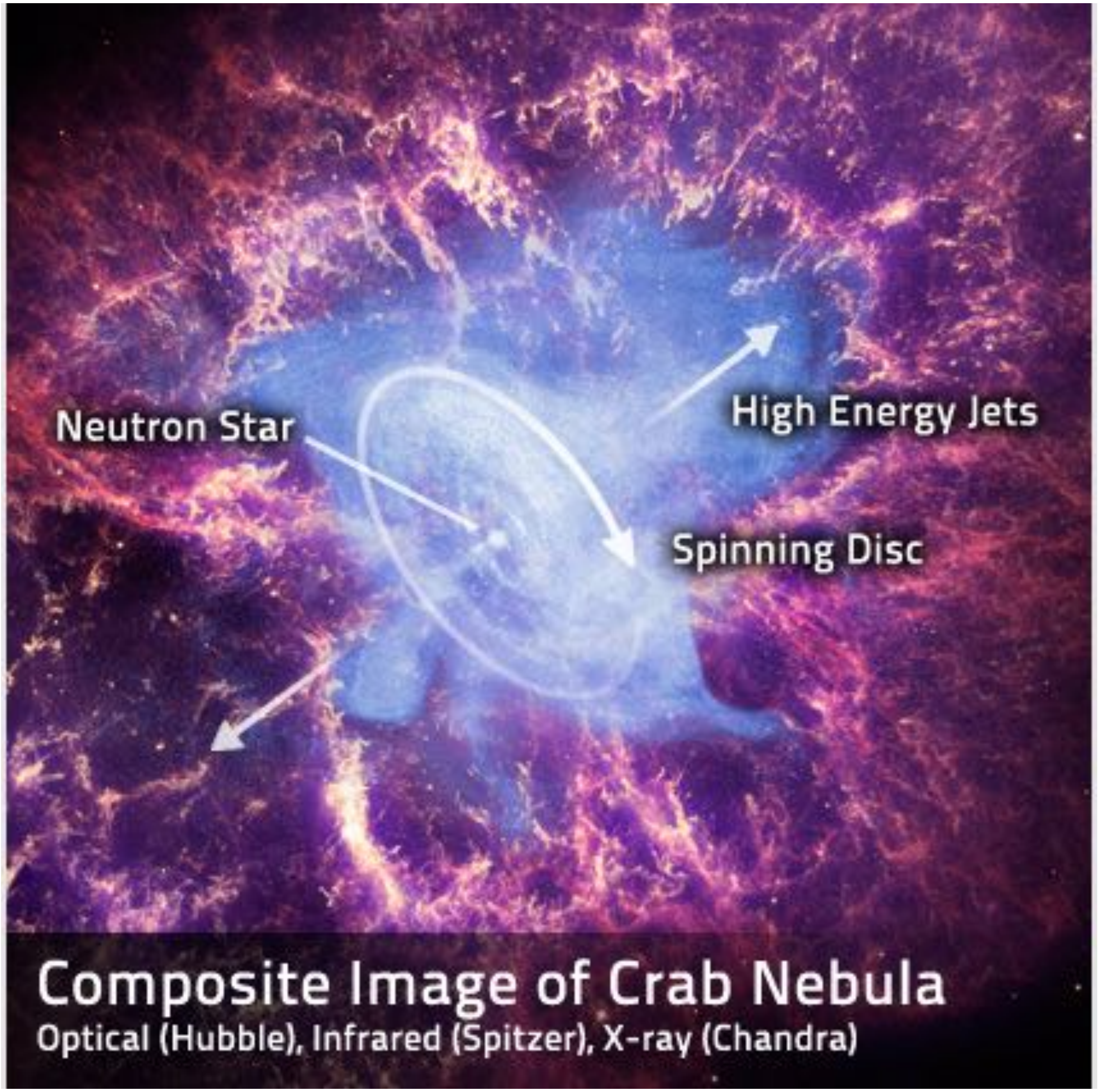
The image is a combination of optical, infrared, and X-ray images. The X-ray image reveals evidence of a spinning disc of super hot gas with high speed jets shooting out in opposite directions of it. It is believed that the explosion of a massive star leaves behind a fast spinning neutron star that could produce this phenomenon.

https://ecuip.lib.uchicago.edu/multiwavelength-astronomy/astrophysics/02.html

Pulsars

Since the Fermi energy (E_F) of electrons in such a neutron star amounts to several hundred MeV, the **formed neutrons cannot decay because of the Pauli principle**, since the maximum energy of electrons in neutron beta decay is only 0.78 MeV and all energy levels in the Fermi gas of electrons up to this energy and even beyond are occupied.

The gravitational collapse of stars **conserves the angular momentum.** Therefore, because of their small size, rotating neutron stars possess extraordinarily **short rotational periods.**



Example problem 2

$$E_F = \sqrt{p_F^2 c^2 + m_e^2 c^4}$$

Pulsars

Assuming orbital periods of a normal star of about one month like for the Sun, one obtains—if the mass loss during contraction can be neglected—**pulsar frequencies** ω_{Pulsar} of (Θ—moment of inertia)

$$\Theta_{\text{star}}\,\omega_{\text{star}} = \Theta_{\text{pulsar}}\,\omega_{\text{pulsar}}\,,$$

$$\omega_{\text{pulsar}} = \frac{R^2_{\text{star}}}{R^2_{\text{pulsar}}}\,\omega_{\text{star}}$$

corresponding to pulsar periods of

$$T_{\text{pulsar}} = T_{\text{star}}\,\frac{R^2_{\text{pulsar}}}{R^2_{\text{star}}}\,.$$

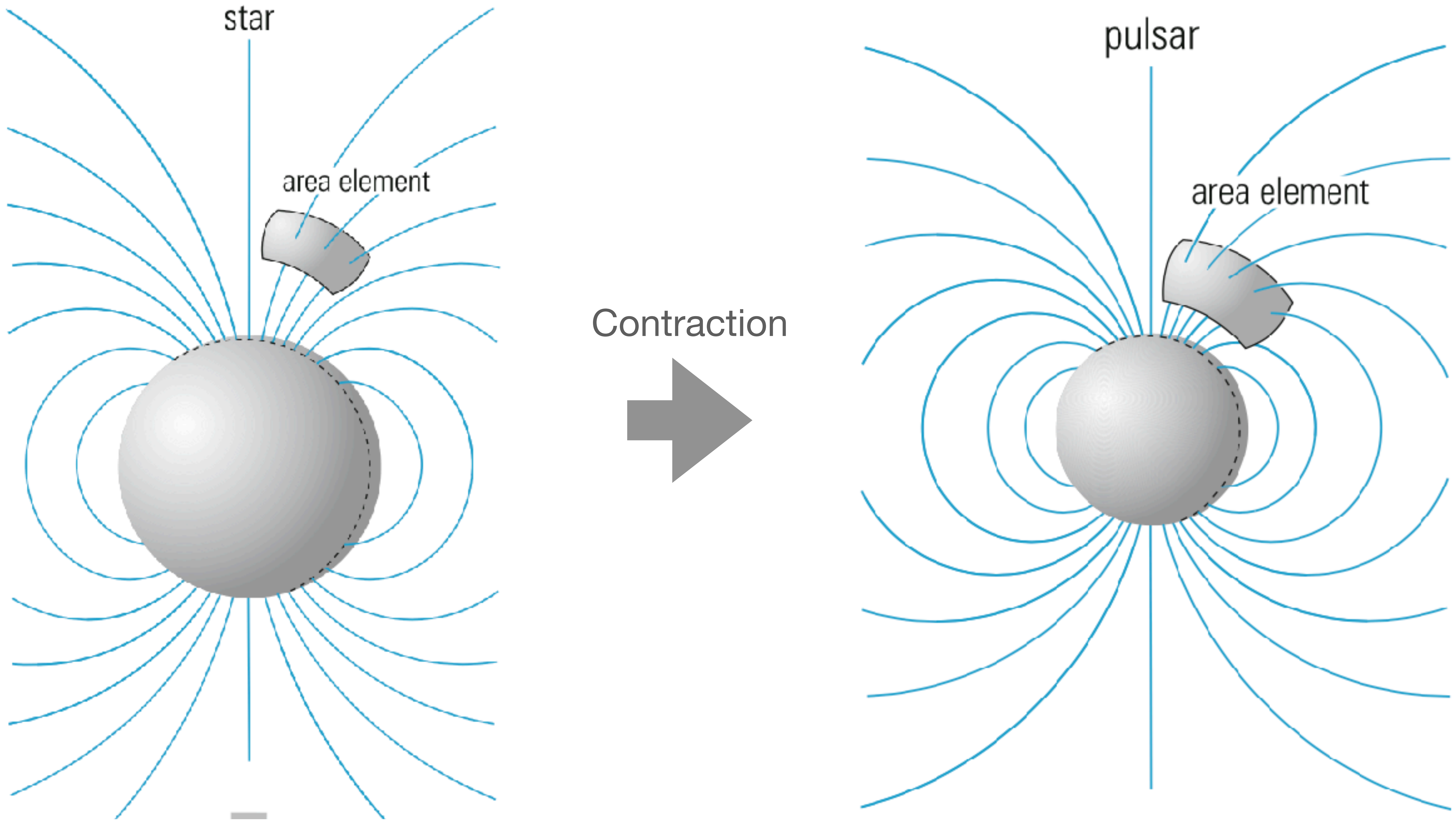
For a stellar size R_{star} =10^6km, a pulsar radius R_{pulsar} =20km, and a rotation period of T_{star} = 1 month one obtains

$$\boxed{T_{\text{pulsar}} \approx 1\,\text{ms}\,.}$$

Pulsars

The gravitational collapse **amplifies the original magnetic field extraordinarily.** If one assumes that the magnetic flux, e.g., through the upper hemisphere of a star, is conserved during the contraction, the **magnetic field lines will be tightly squeezed**. One obtains (see Fig. 5.7)

Fig. 5.7 Increase of a magnetic field during the gravitational collapse of a star

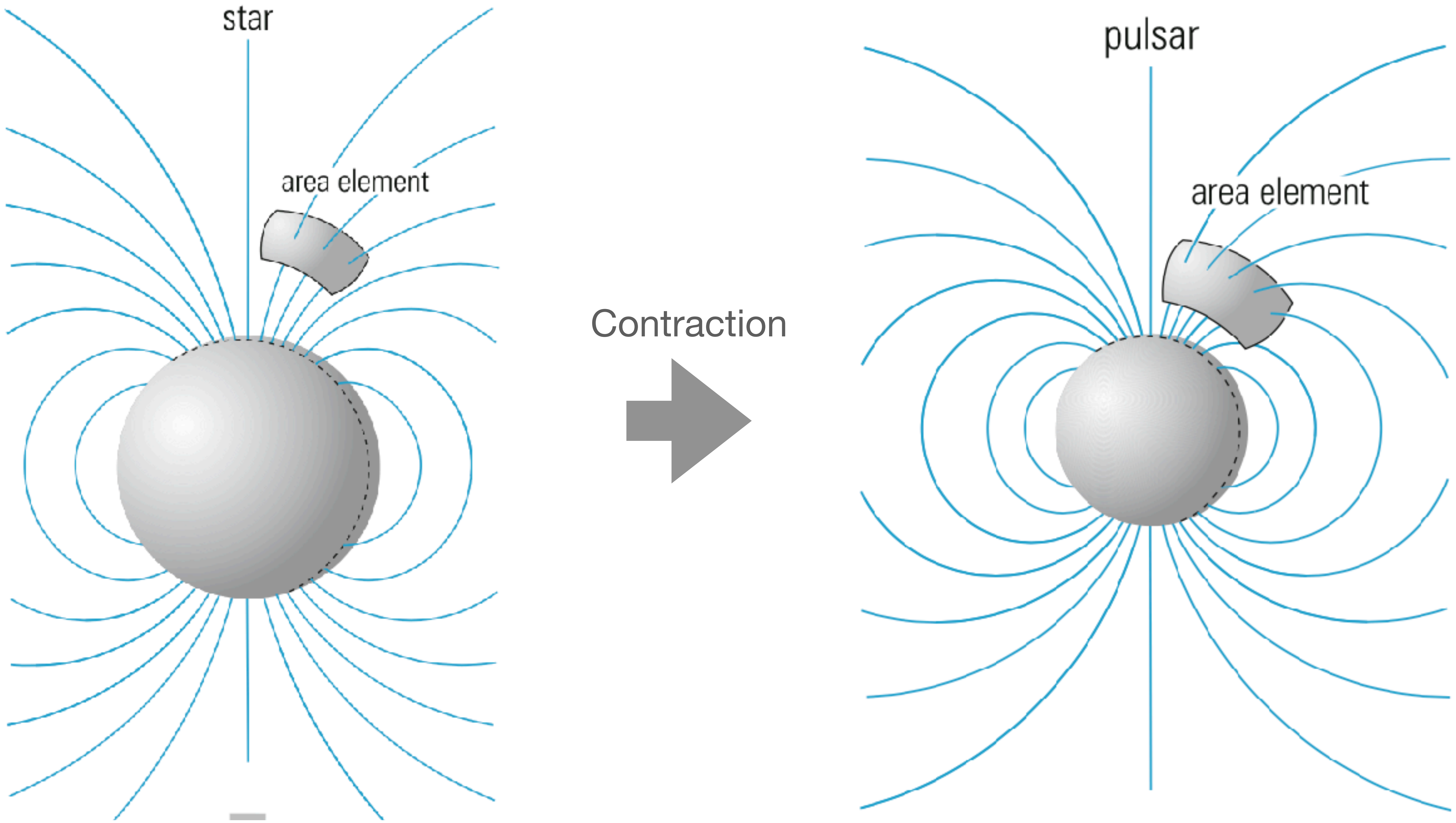


Pulsars

$$\int_{\text{star}} \boldsymbol{B}_{\text{star}} \cdot \mathrm{d}\boldsymbol{A}_{\text{star}} = \int_{\text{pulsar}} \boldsymbol{B}_{\text{pulsar}} \cdot \mathrm{d}\boldsymbol{A}_{\text{pulsar}} ,$$

$$B_{\text{pulsar}} = B_{\text{star}} \frac{R_{\text{star}}^2}{R_{\text{pulsar}}^2} .$$

Fig. 5.7 Increase of a magnetic field during the gravitational collapse of a star



Pulsars

For B_{star} = 1000 gauss magnetic pulsar fields of 2.5×10^{12} gauss = $2.5 \times$ **10^8 T are obtained!**

These theoretically expected extraordinarily high magnetic field strengths have been **experimentally confirmed** by measuring quantized energy levels of free electrons in strong magnetic fields ('Landau levels').

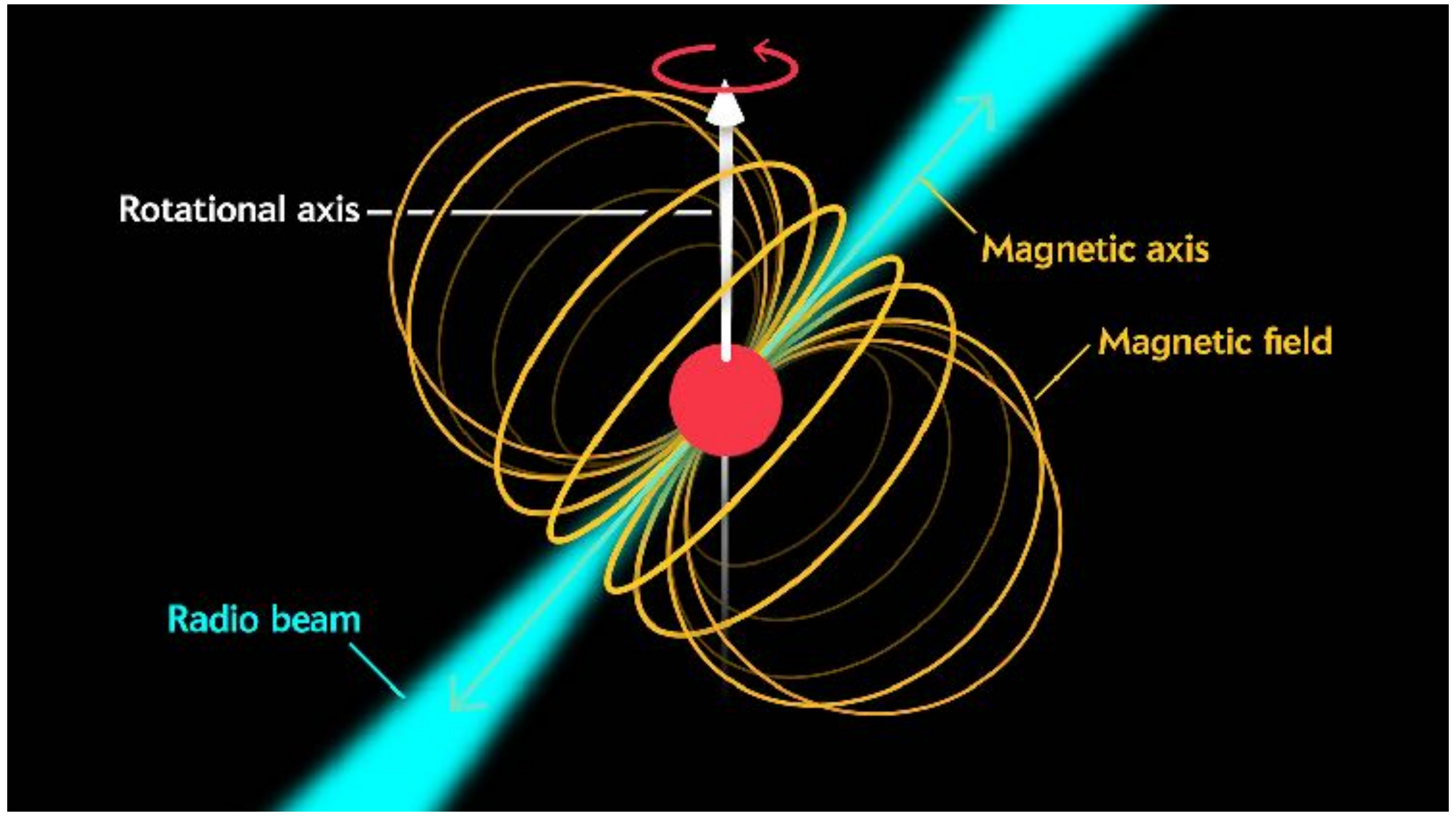
The **rotational axis of pulsars usually does not coincide with the direction of the magnetic field.** It is obvious that the vector of these high magnetic fields spinning around the non-aligned axis of rotation **will produce strong electric fields, in which particles can be accelerated.**

For a 30-ms pulsar with rotational velocities of

$$v = \frac{2\pi R_{\text{pulsar}}}{T_{\text{pulsar}}} = \frac{2\pi \times 20 \times 10^3\,\text{m}}{3 \times 10^{-2}\,\text{s}} \approx 4 \times 10^6\,\text{m/s}$$

one obtains, using $\boldsymbol{E} = \text{v} \times \boldsymbol{B}$ with $\text{v} \perp \boldsymbol{B}$, electrical field strengths of

$$|\boldsymbol{E}| = v\,B \approx 10^{15}\,\text{V/m}\,.$$



Pulsars

This implies that singly charged particles **can gain 1 PeV** = 1000 TeV **per meter.** However, it is not at all obvious how pulsars manage in detail to transform the rotational energy into the acceleration of particles. Pulsars possess a rotational energy of

$$E_{\mathrm{rot}} = \frac{1}{2}\,\Theta_{\mathrm{pulsar}}\,\omega_{\mathrm{pulsar}}^2 = \frac{1}{2}\,\frac{2}{5}\,m\,R_{\mathrm{pulsar}}^2\,\omega_{\mathrm{pulsar}}^2$$
$$\approx 7 \times 10^{42}\,\mathrm{J} \approx 4.4 \times 10^{61}\,\mathrm{eV}$$

($T_{\mathrm{pulsar}} = 30$ ms, $M_{\mathrm{pulsar}} = 2 \times 10^{30}$ kg, $R_{\mathrm{pulsar}} = 20$ km, $\omega = 2\pi/T$)

If the pulsars succeed to convert a fraction of only **1% of this enormous energy into the acceleration** of cosmic-ray particles, one obtains an injection rate of

$$\frac{\mathrm{d}E}{\mathrm{d}t} \approx 1.4 \times 10^{42}\,\mathrm{eV/s}\,,$$

if a pulsar lifetime of 10^{10} years is assumed.

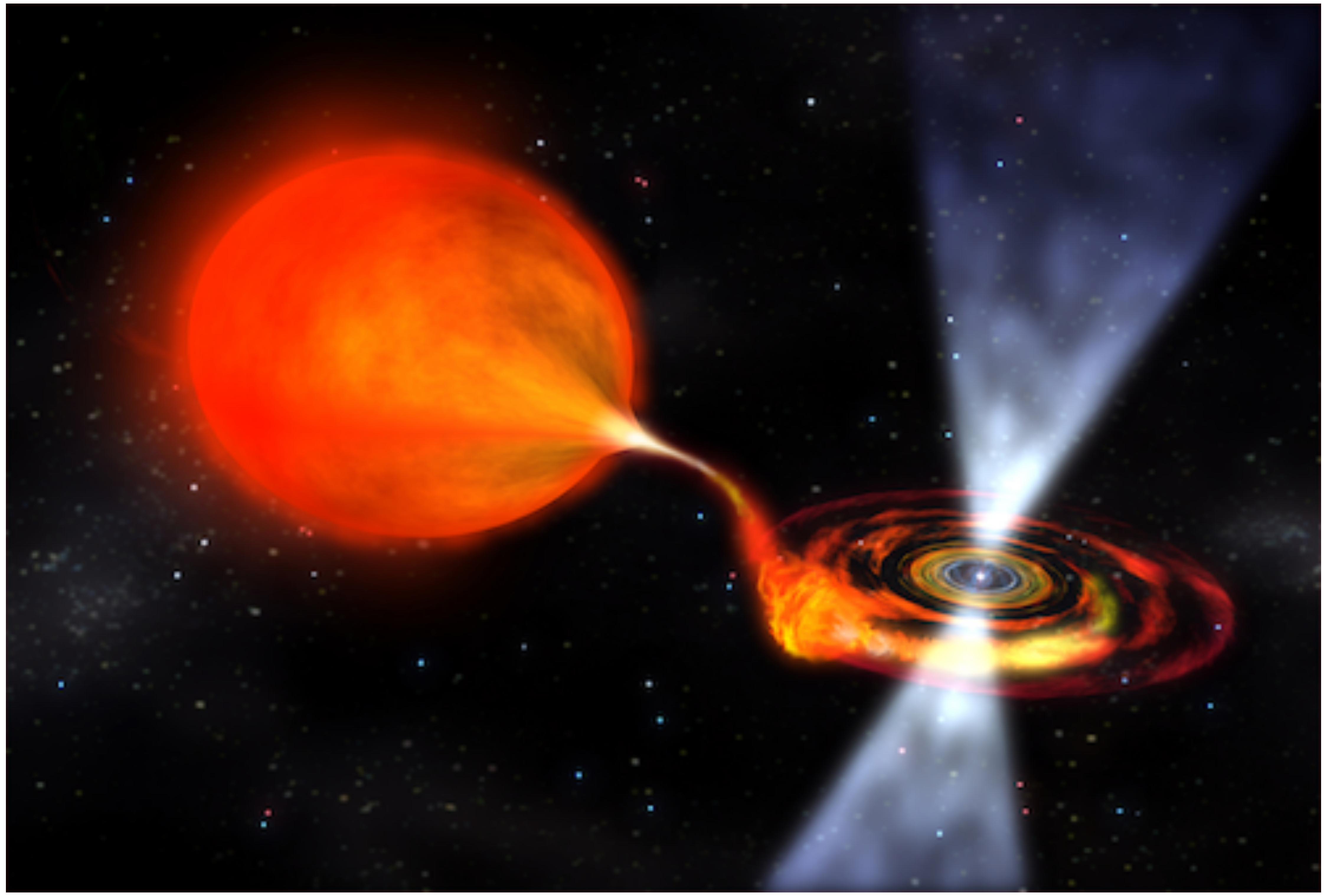
Pulsars

If one considers that our galaxy contains 10^{11} stars and if the supernova explosion rate (pulsar creation rate) is assumed to be 1 per century, a total number of 10^8 pulsars have provided energy for the acceleration of cosmic-ray particles since the creation of our galaxy (age of the galaxy $\approx 10^{10}$ years). This leads to a total energy of 2.2×10^{67} eV for an average pulsar injection time of 5×10^9 years.

For a total volume of our galaxy (radius 15 kpc, average effective thickness of the galactic disk 1 kpc) of 2×10^{67} cm^3 this **corresponds to an energy density of cosmic rays of 1.1 eV/cm^3.**

One has, of course, to consider that cosmic-ray particles stay only for a limited time in our galaxy and are furthermore subject to **energy-loss processes**. Still, the above presented crude estimate **describes the actual energy density of cosmic rays of ≈1 eV/cm^3 rather well.**

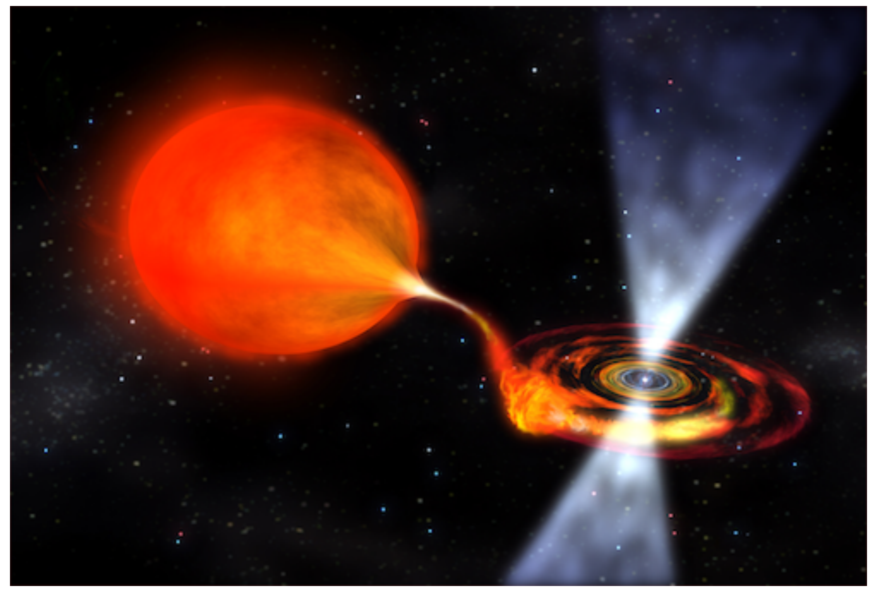
Binaries



Binaries consisting of **a pulsar or neutron star and a normal star** can also be considered as a site of **cosmic-ray-particle acceleration**.

In such a binary system matter is permanently dragged from the normal star and whirled into an **accretion disk around the compact companion**. Due to these enormous **plasma motions very strong electromagnetic fields are produced in the vicinity of the neutron star.** In these fields charged particles can be accelerated to high energies.

Millisecond pulsars



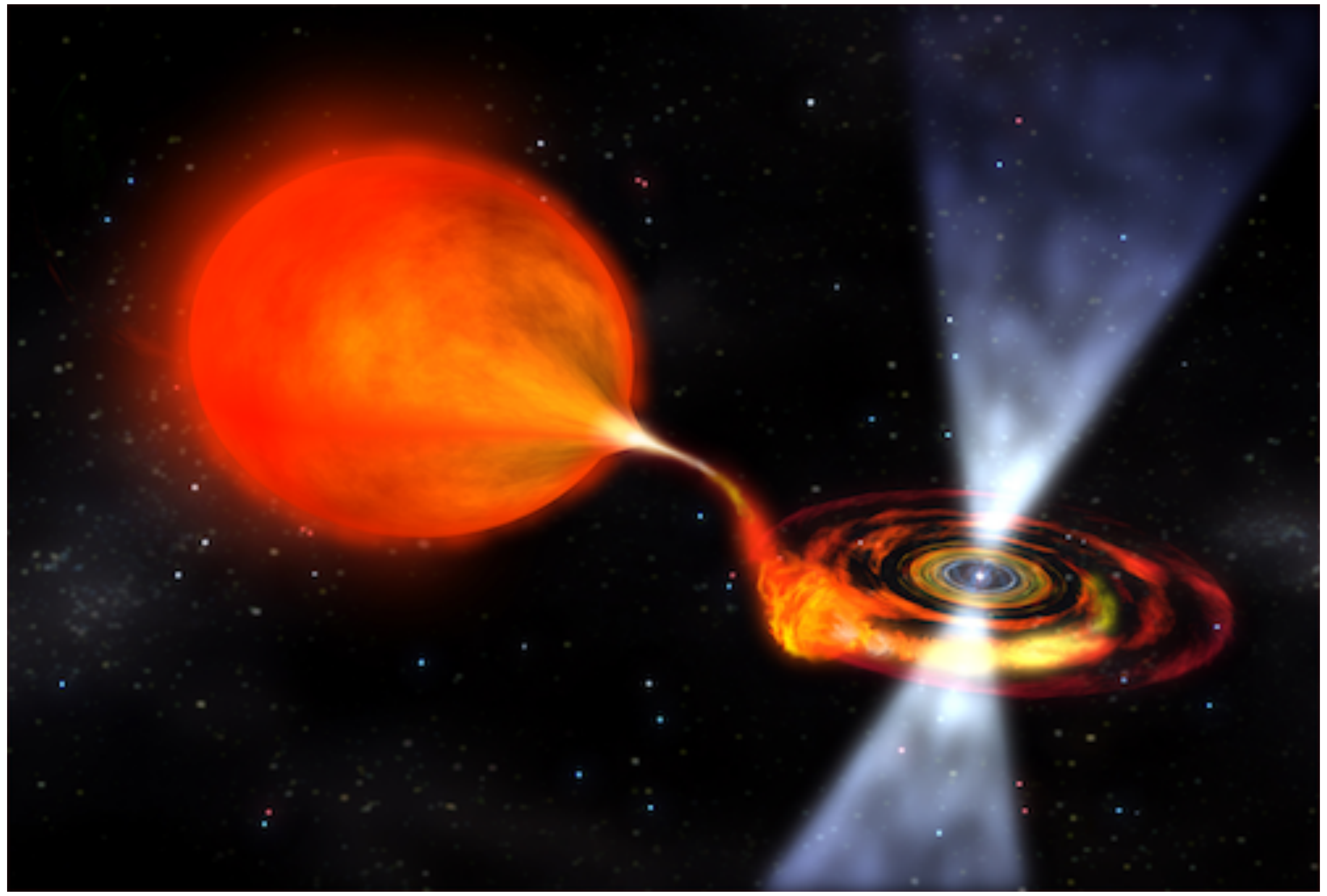
- If the compact object is a pulsar in the binary it is **also called a millisecond pulsars**
- When a neutron star is born, it is expected to have values of rotation period P and magnetic field B typical of an ordinary pulsar.
- Suppose the neutron star is in a **binary system**. At some stage, the binary companion may become a red giant and fill up the Roche lobe. This would lead to a **transfer of mass** from the inflated companion star to the neutron star.
- There are **binary X-ray sources** are believed to be **neutron stars accreting matter** from inflated binary companions.
- Because of the orbital motion of the companion, the matter accreting onto the neutron star from its companion will carry a considerable amount of angular momentum. This is **expected to increase the angular velocity of the accreting neutron star.**
- Eventually, when the red giant phase of the companion star is over (it may become a white dwarf or another neutron star), **the neutron star which has been spun up** by accreting matter with angular momentum **becomes visible as a millisecond pulsar** with a short period P.

Binary X-ray sources

- There can be mass transfer between the two stars in a binary system. If, the mass m is dropped from infinity to a star of mass M and radius R, then the gravitational energy lost is:

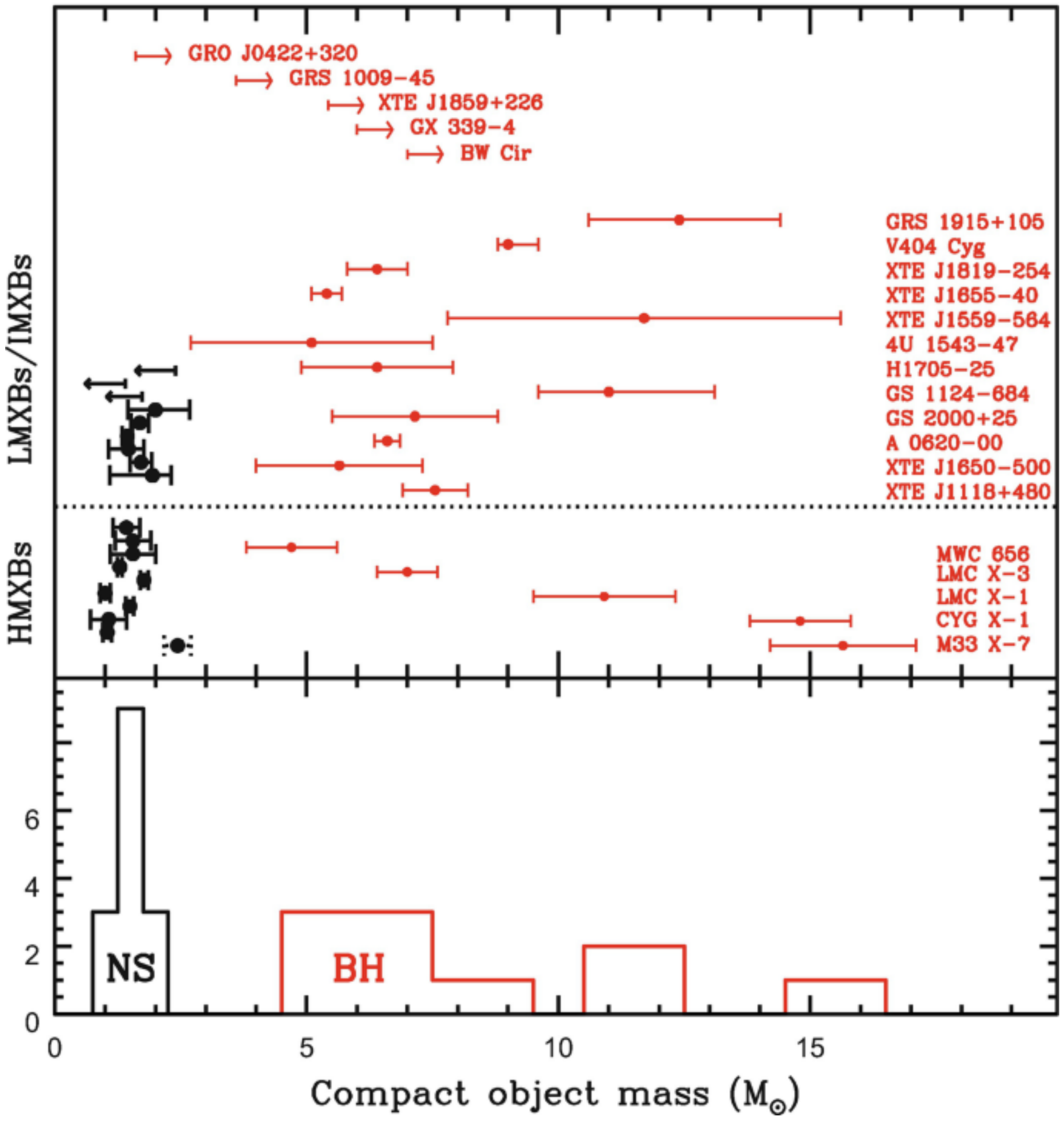
$$\frac{GM}{R}m = \frac{GM}{c^2R}mc^2.$$

- For a typical neutron star of mass $1M_\odot$ and radius 10 km, the factor $GM/c^2\,R$ turns out to be about 0.15.
- Hence the loss of gravitational energy may be a very significant fraction of the rest mass energy, making such an **infall of matter into the deep gravitational well** of a compact object like a neutron star a v**ery efficient process for energy release.**
- Since the **accreting material carries angular momentum, it is unlikely to fall radially inward, but is expected to move inward slowly in the form of a disk**. Such a disk is called an ***accretion disk***.



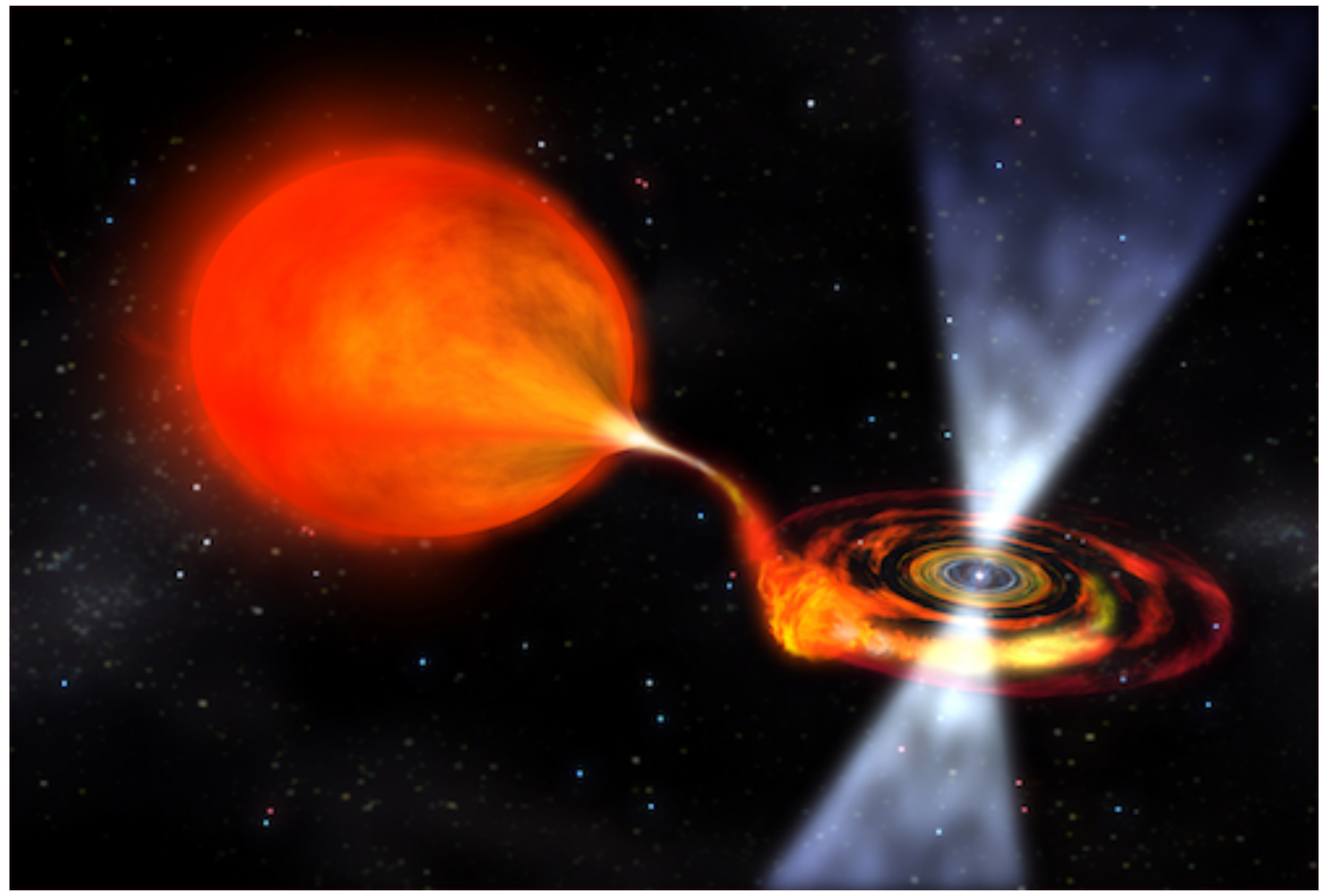
Binary X-ray sources

- **Do all X-ray binaries have neutron stars?**
- However, **there are a few binary X- ray sources with accreting objects which possibly have masses higher than $3M_{\odot}$**.
- The **central accreting object is believed to be a black hole** rather than a neutron star, since its estimated mass is well above what would be the neutron star mass limit based on any reasonable equation of state.



https://link.springer.com/referenceworkentry/10.1007/978-3-319-21846-5_111

Binaries



The energy gain of infalling protons (mass m_p) in the gravitational potential of a pulsar (mass M_{pulsar}) is

$$\Delta E = -\int_{\infty}^{R_{\text{pulsar}}} G \frac{m_p\, M_{\text{pulsar}}}{r^2}\,\mathrm{d}r = G \frac{m_p\, M_{\text{pulsar}}}{R_{\text{pulsar}}}$$

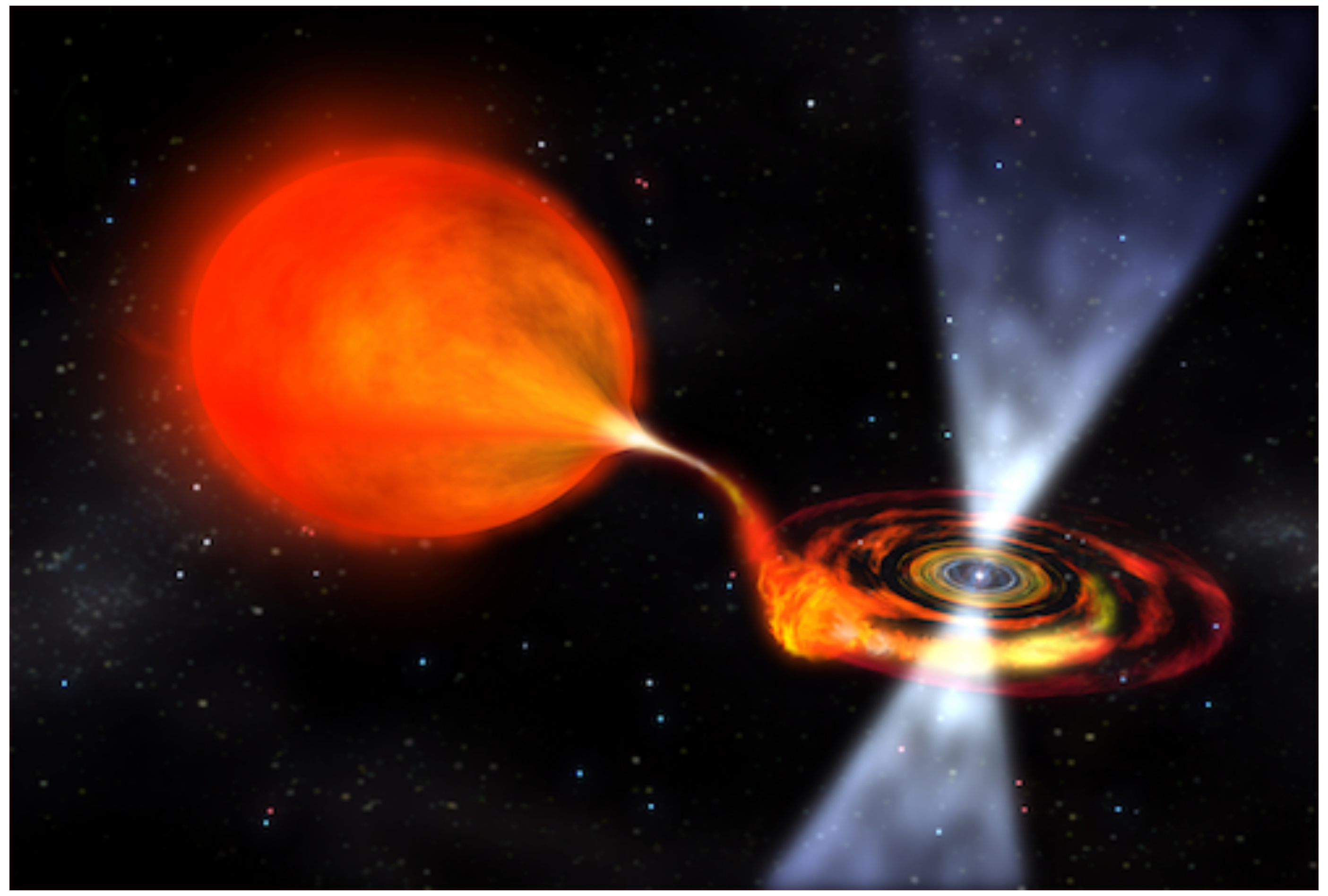
$$\approx 1.1 \times 10^{-11}\,\mathrm{J} \approx 70\,\mathrm{MeV}$$

($m_p \approx 1.67 \times 10^{-27}$ kg, $M_{\text{pulsar}} = 2 \times 10^{30}$ kg, $R_{\text{pulsar}} = 20$ km, $G \approx 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ gravitational constant).

The matter falling into the accretion disk achieves velocities v, which are obtained under classical treatment from

$$\frac{1}{2} m v^2 = \Delta E = G \frac{m\, M_{\text{pulsar}}}{R_{\text{pulsar}}}$$

Binaries



to provide values of

$$v = \sqrt{\frac{2GM_{\text{pulsar}}}{R_{\text{pulsar}}}} \approx 1.2 \times 10^8\ \text{m/s}\,.$$

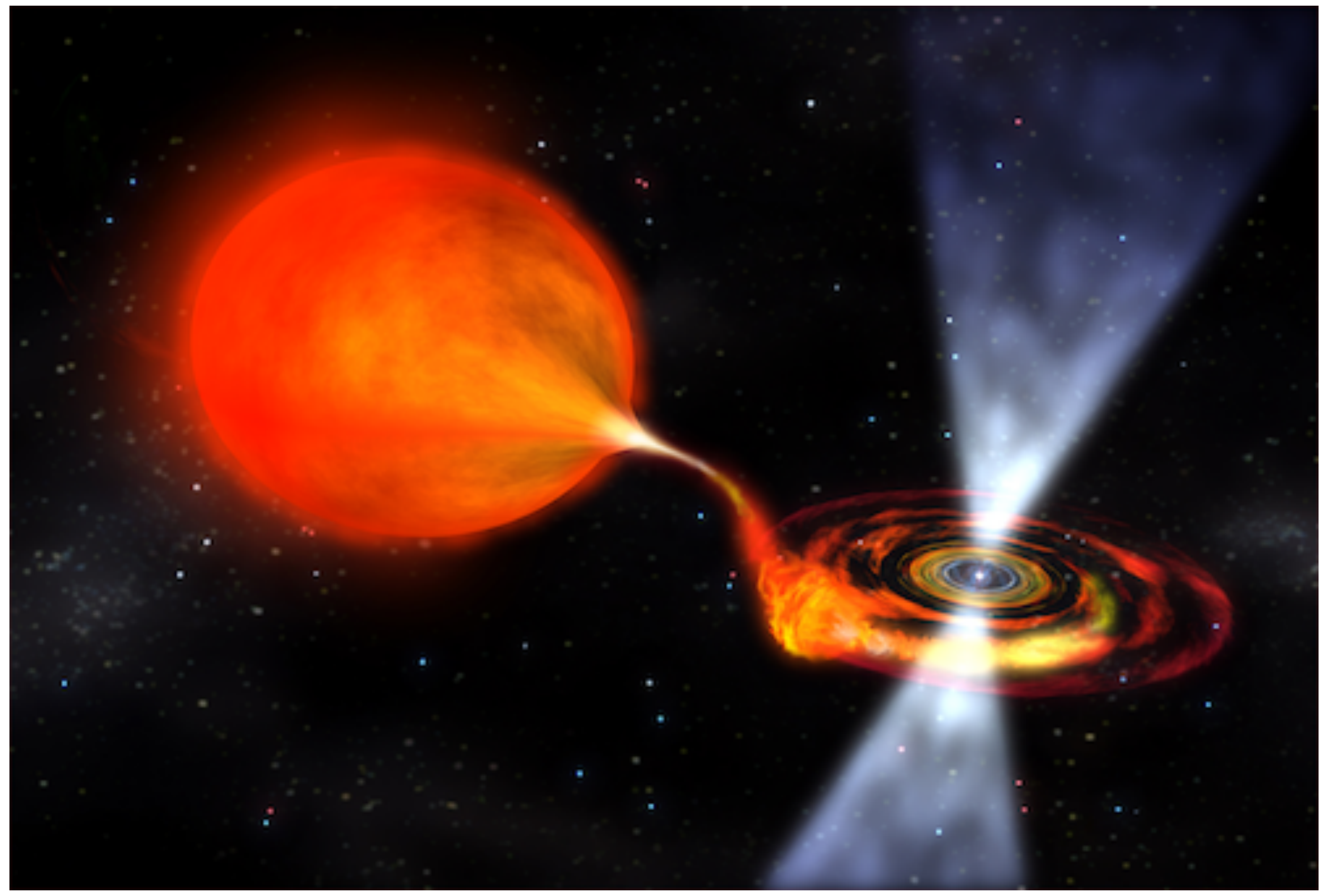
The variable magnetic field of the neutron star, which is perpendicular to the accretion disk, will produce via the Lorentz force a strong electric field. Using

$$\boldsymbol{F} = e(\boldsymbol{v} \times \boldsymbol{B}) = e\boldsymbol{E}$$

the particle energy E is obtained, using v $\perp$ $\boldsymbol{B}$, to

$$E = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = evB\Delta s\,.$$

Binaries



Under plausible assumptions ($v \approx c$, $B = 10^6$ T, $\Delta s = 10^5$ m) particle energies of **3×10^{19} eV are possible.**

Even more powerful are **accretion disks**, which form **around black holes or the compact nuclei of active galaxies.** One assumes that in these active galactic nuclei and in jets ejected from such nuclei, particles can be accelerated to the highest energies observed in primary cosmic rays.

Example problem 3

The **details** of these acceleration processes are **not yet fully understood.**

Sites in the vicinity of black holes—a billion times more massive than the Sun—could possibly provide the environment for the acceleration of the **highest-energy cosmic rays**. Confined **highly relativistic jets** are a common feature of such compact sources. It is assumed that the jets of particles accelerated near a black hole are injected into the radiation field of the source.

Binaries

Electrons and protons **accelerated in the jets via shocks initiate electromagnetic and hadronic cascades. High-energy γ rays** are produced by **inverse Compton scattering** off accelerated electrons.

High-energy neutrinos are created in the **decays of charged pions** in the development of the hadronic cascade. It is assumed that one **detects emission** from these sources **only if the jets are beamed into our line of sight.**

A possible scenario for the acceleration of particles in *beamed jets* from massive compact sources is sketched in Fig. 5.9.

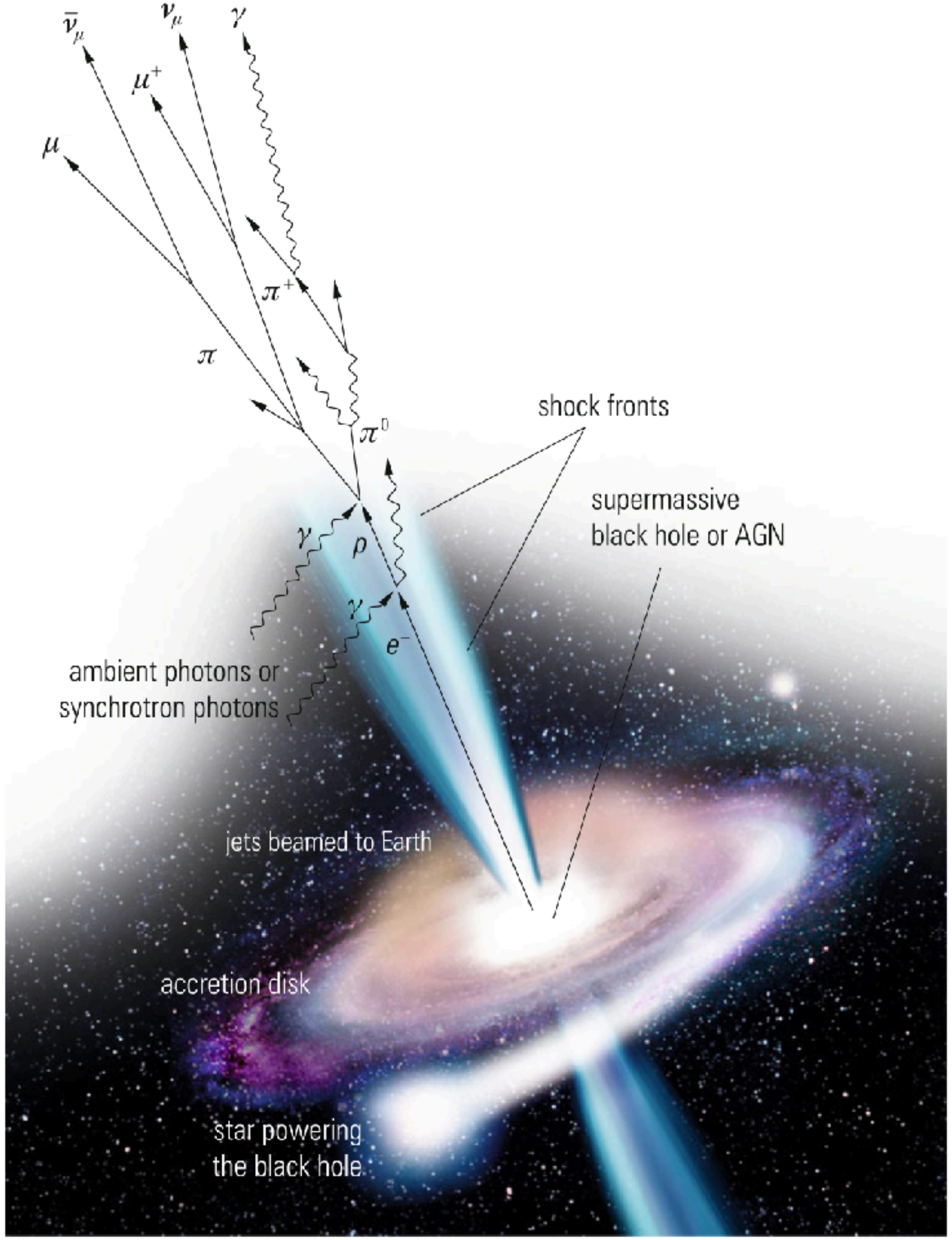


Fig. 5.9 Acceleration model for relativistic jets powered by a black hole or an active galactic nucleus. The various reactions are only sketched

Binaries

Triggered by the fact that no generally accepted acceleration model for very high energies has been put forward, a number of **exotic alternatives** have been proposed.

A **Cannon Ball model** has been discussed, which describes mass ejections from compact sources in the form of cannon balls with relativistic velocities.

In a collapse of a massive object **accretion disks are formed**. The infall of matter onto the accretion disk can lead to the **emission of discrete cannon balls**, which are expected to be **emitted back to back**.

Because **strong, varying time dependent magnetic fields** are involved, **charged particles might be accelerated** to the highest energies.

The Cannon Ball model claims to be able to explain the luminosity, the slopes of the particle spectra and the position of the knee(s) and ankle(s) in terms of simple and standard physics.

Binaries

Another possibility is the **acceleration of energetic particles by** **resonant cyclotron emission.**

Relativistic **shocks with strong, time-dependent magnetic fields,** which in turn produce powerful electric fields, can **accelerate electrons and positrons**.

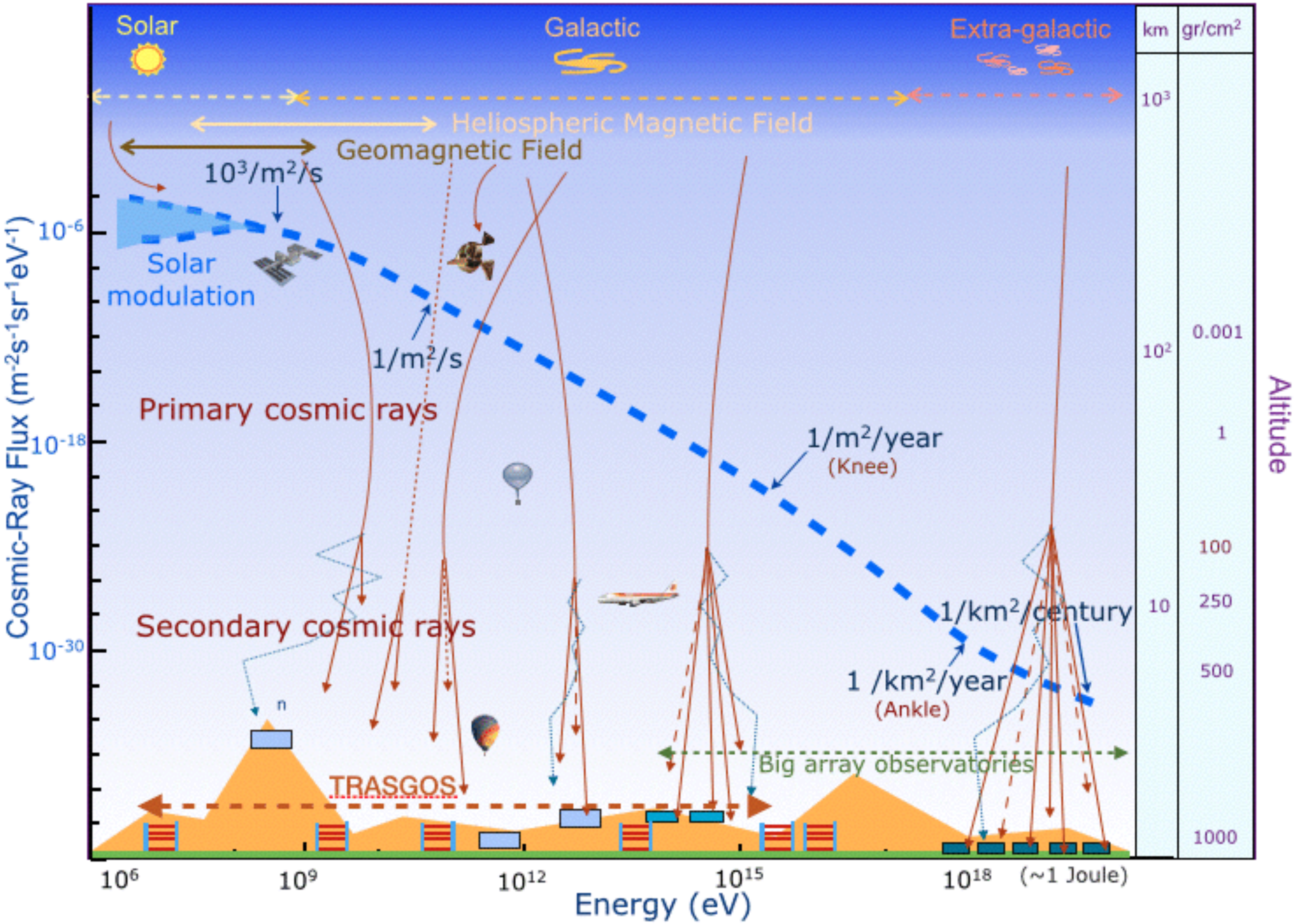
Therefore the **plasma wind** in compact sources can be dominated by ions, where the magnetic field forces the ions to **propagate along helical lines**.

These ions are expected to **emit cyclotron waves** (Alfvén waves) in the magnetosphere of the source, which can be **absorbed in a resonant fashion by electrons and positrons.** In this way the electrons and positrons might be a**ccelerated to high Lorentz factors** with good efficiency. Such a resonant cyclotron emission might be able to produce also high-energy particles.

The large number of proposed mechanisms for high-energy particle acceleration also connected with the jet formation from compact sources shows, that no generally accepted idea about the acceleration of particles with the highest energies of up to 10^{20} eV has emerged.

Energy spectra for primary particles

Energy spectrum of primary cosmic rays with their corresponding estimated sources and the main detection techniques.



https://www.researchgate.net/figure/Energy-spectrum-of-primary-cosmic-rays-with-their-corresponding-estimated-sources-and-the_fig1_323886635

Energy spectra for primary particles

At the present time it is **not at all clear, which of the presented mechanisms contribute predominantly to the acceleration of cosmic-ray particles.**

There are good arguments to assume that the **majority of galactic cosmic rays is produced by shock acceleration,** where the particles emitted from the source are **possibly further accelerated by the Fermi mechanism of 2nd order.**

In contrast, it is likely that the **extremely energetic cosmic rays are predominantly accelerated in pulsars, binaries, or in jets emitted from black holes** or active galactic nuclei.

For shock acceleration in supernova explosions the **shape of the energy spectrum** of cosmic-ray particles can be derived from the acceleration mechanism.

Let E_0 be the initial energy of a particle and εE_0 the energy gain per acceleration cycle. **After the first cycle** one gets

$$E_1 = E_0 + \varepsilon E_0 = E_0(1 + \varepsilon)$$

Energy spectra for primary particles

while after the nth cycle (e.g., due to multiple reflection at shock fronts) one has

$$E_n = E_0(1 + \varepsilon)^n$$

To obtain the final energy $E_n = E$, a number of

$$n = \frac{\ln(E/E_0)}{\ln(1 + \varepsilon)}$$

cycles is required.

Energy spectra for primary particles

Let us assume that the escape probability per cycle is P. The probability that particles still take part in the acceleration mechanism after n cycles is $(1-P)^n$. This leads to the following **number of particles with energies in excess of E** :

$$N(>E) \sim \sum_{m=n}^{\infty} (1-P)^m \,.$$

Because of $\sum_{m=0}^{\infty} x^m = \frac{1}{1-x}$ (for $x < 1$), (5.7.4) can be rewritten as

$$N(>E) \sim (1-P)^n \sum_{m=n}^{\infty} (1-P)^{m-n}$$

$$= (1-P)^n \sum_{m=0}^{\infty} (1-P)^m = \frac{(1-P)^n}{P} \,,$$

where $m-n$ has been renamed m.

Energy spectra for primary particles

The equations can be combined to form the integral energy spectrum

$$N(>E) \sim \frac{1}{P}\left(\frac{E}{E_0}\right)^{-\gamma} \sim E^{-\gamma} \,,$$

where γ is the spectral index. Then we can further combine the equations to obtain:

$$(1-P)^n = \left(\frac{E}{E_0}\right)^{-\gamma} \,,$$

$$n\ln(1-P) = -\gamma\ln(E/E_0) \,,$$

$$\gamma = -\frac{n\ln(1-P)}{\ln(E/E_0)} = \frac{\ln(1/(1-P))}{\ln(1+\varepsilon)} \,.$$

This simple consideration yields a **power law of primary cosmic rays in agreement with observation.**

Energy spectra for primary particles

The energy gain per cycle surely is rather small ($\varepsilon \ll 1$). If also **the escape probability *P* is low** (e.g., at reflections between two shock fronts), (5.7.7) is simplified to

$$\gamma \approx \frac{\ln(1+P)}{\ln(1+\varepsilon)} \approx \frac{P}{\varepsilon} \, .$$

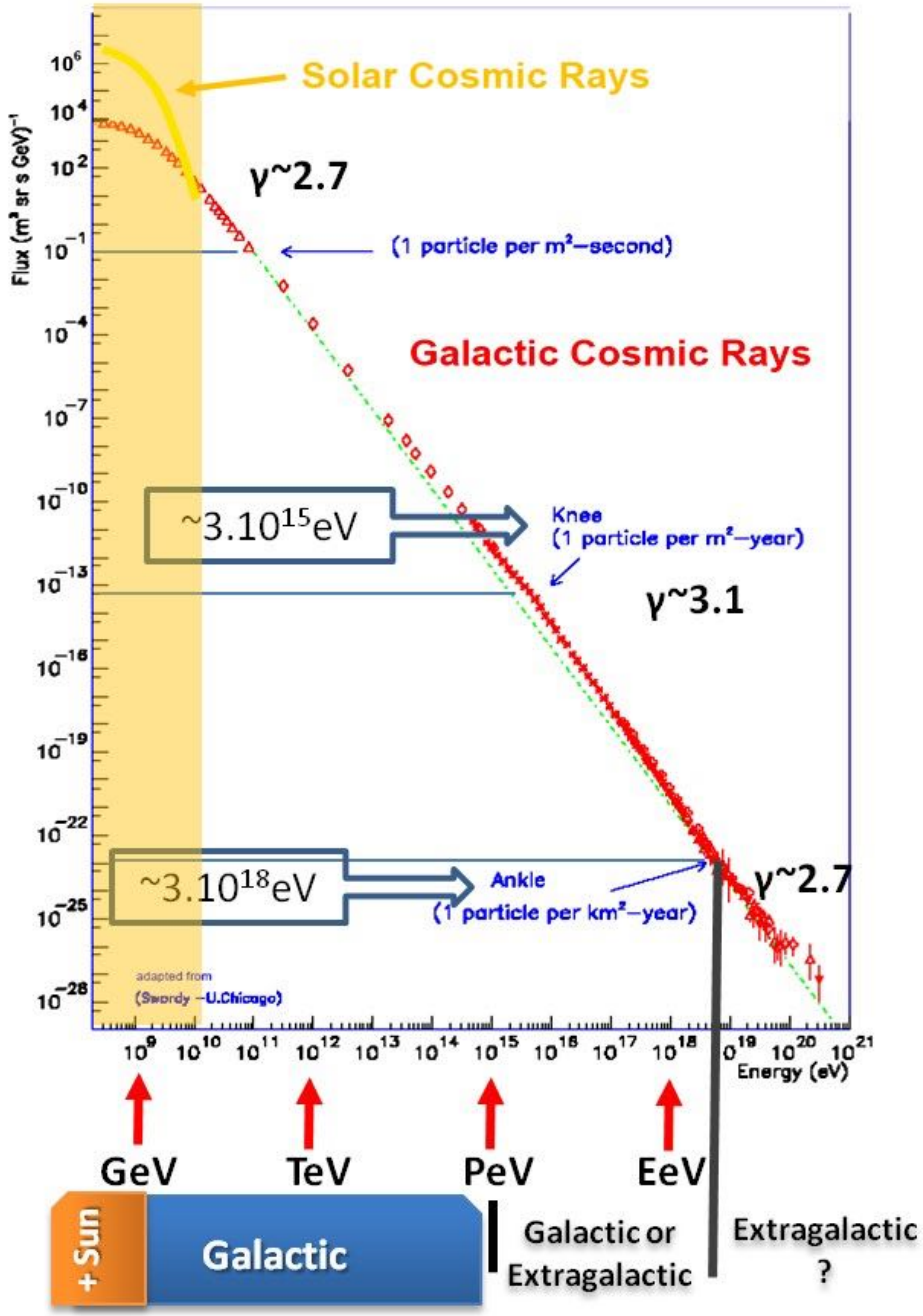
Experimentally one finds that the index of the integral spectrum up to energies of 10^{15} eV is $\gamma = 1.7$. For higher energies the integral primary cosmic-ray-particle spectrum steepens with $\gamma = 2$.

This calculation can only **motivate the shape of the emission spectrum** from the sources.

It cannot describe the details of the primary spectrum as measured at the edge of the atmosphere, in particular, it is asking too much that it should be able to explain the knee(s) and ankle(s). These features **depend on the propagation of cosmic rays** in the galaxy and on the energy-dependent question of the Galactic or extragalactic origin.

Energy spectra

The spectrum of CRs, denoting the different components and sources (Credit: A. Papaioannou, adopted from the CR spectrum by S. Swordy).



https://blogs.egu.eu/divisions/st/2018/03/19/cosmic-rays-messengers-from-space/

Problems

1. Work out the kinetic energy of electrons accelerated in a betatron for the classical ($v \ll c$) and the relativistic case ($B = 1$ tesla, $R = 0.2\,\mathrm{m}$). See also Problem 1.2.

A betatron is a **type of cyclic particle accelerator for electrons.** It consists of a torus-shaped vacuum chamber with an electron source. Circling the torus is an iron transformer core with a wire winding around it. The device functions similarly to a transformer, with the electrons in the torus-shaped vacuum chamber as its secondary coil. An alternating current in the primary coils accelerates electrons in the vacuum around a circular path.

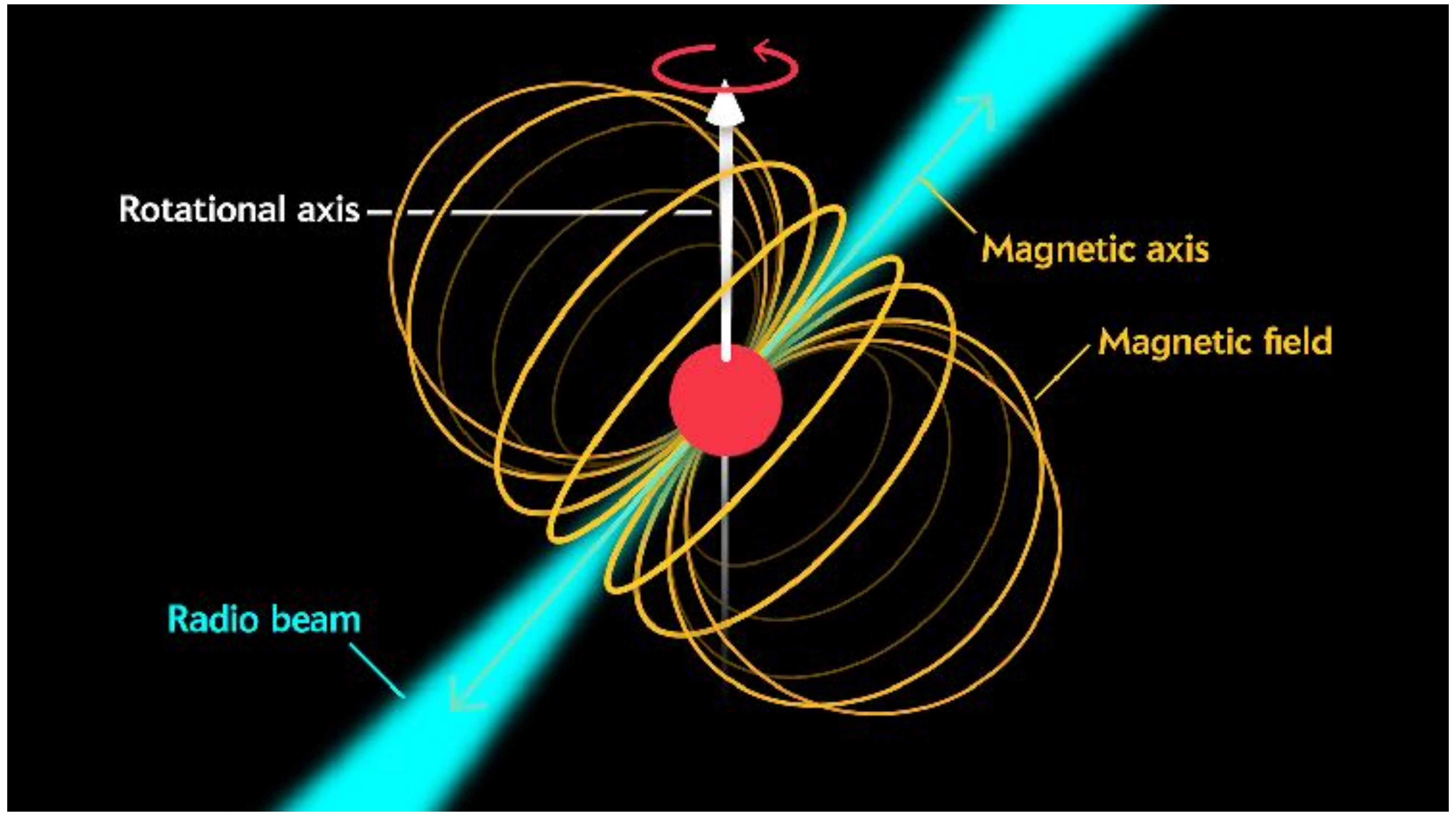
The betatron was the first circular accelerator in which particles orbited at a constant radius.

Problems

3. It is assumed that active galactic nuclei are powered by black holes. What is the energy gain of a proton falling into a one-million-solar-mass black hole down to the event horizon?

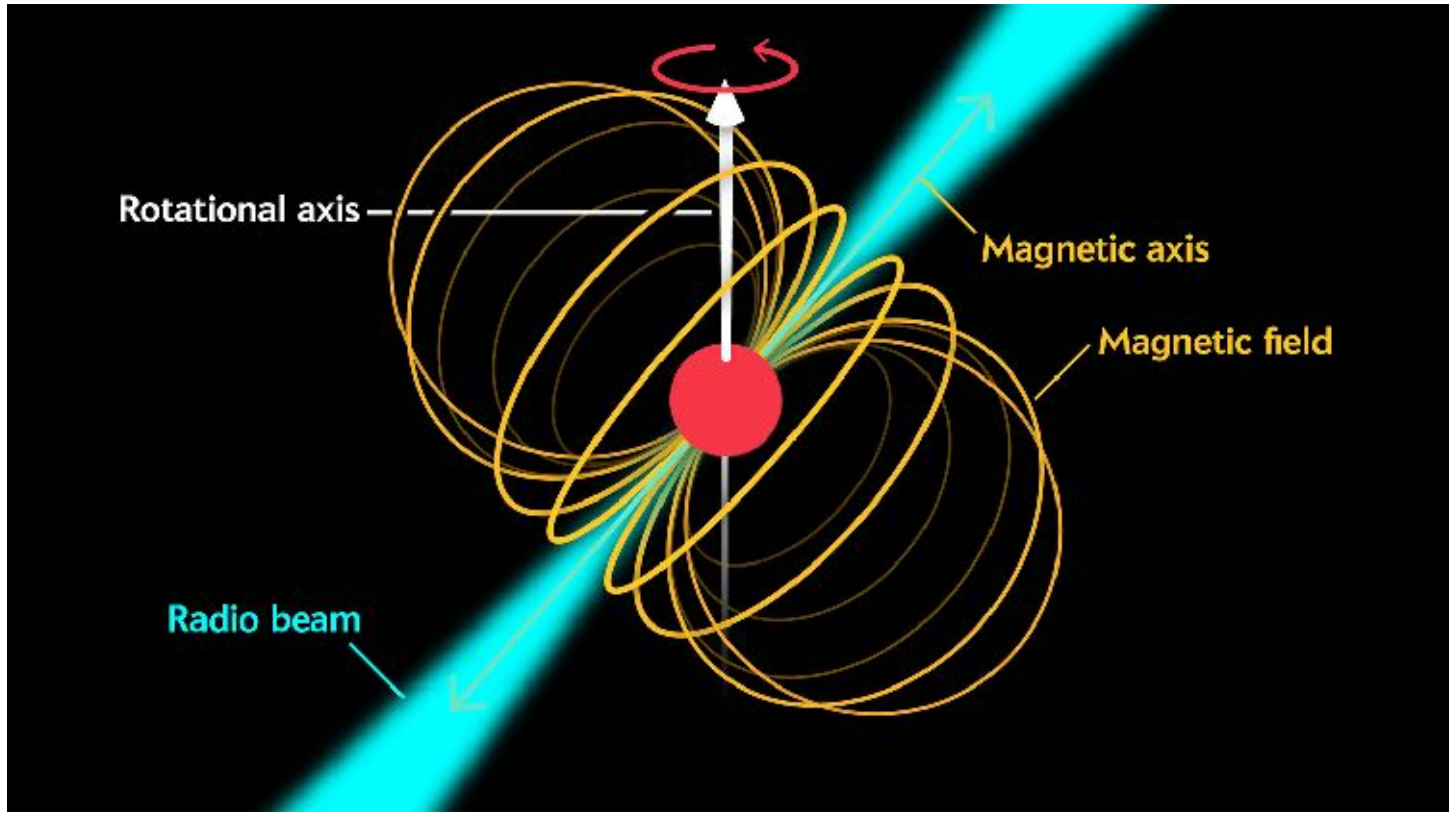
Problems

2. A star of 10 solar masses undergoes a supernova explosion. Assume that 50% of its mass is ejected and the other half ends up in a pulsar of 10 km radius. What is the Fermi energy of the electrons in the pulsar? What is the consequence of it?



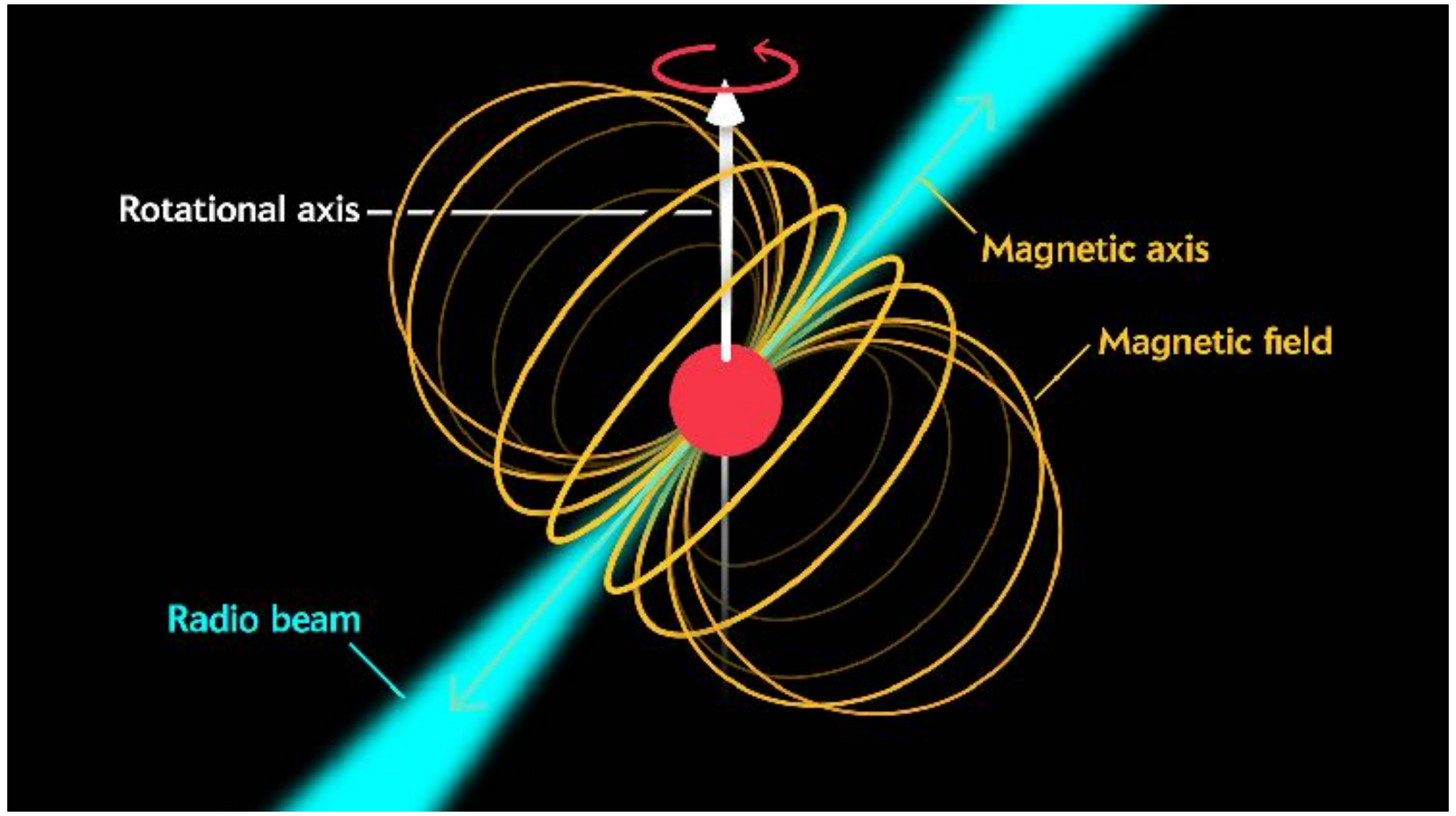
Problems

4. If the Sun were to collapse to a neutron star ($R_{\mathrm{NS}} = 50\,\mathrm{km}$), what would be the rotational energy of such a solar remnant ($M_\odot = 2 \times 10^{30}\,\mathrm{kg}$, $R_\odot = 7 \times 10^8\,\mathrm{m}$, $\omega_\odot = 3 \times 10^{-6}\,\mathrm{s}^{-1}$)?
Compare this rotational energy to the energy that a main-sequence star like the Sun can liberate through nuclear fusion!



Problems

8. Neutron stars are spinning very fast. There are even millisecond pulsars. If they were too fast, they might even disintegrate due to the increasing centrifugal forces. Let us assume that the star has a radius r and an angular velocity ω, and that its density is uniform. Estimate the minimum density of a star with a rotation period of one millisecond so that it does not disintegrate.



Problems

5. In a betatron the change of the magnetic flux $\phi = \int B \, \mathrm{d}A = \pi R^2 B$ induces an electric field,

$$\int E \, \mathrm{d}s = -\dot{\phi} \,,$$

in which particles can be accelerated,

$$E = -\frac{\dot{\phi}}{2\pi R} = -\frac{1}{2} R \dot{B} \,.$$

The momentum increase is given by

$$\dot{p} = -eE = \frac{1}{2} e R \dot{B} \,. \tag{5.8.1}$$

What kind of guiding field would be required to keep the charged particles on a stable orbit?