# Astrophysical Objects

## Basics of Radio astronomy

Based on Chapter 1-2 of Essential Radio astronomy

**Helga Dénes 2023 S2 Yachay Tech**

hdenes@yachaytech.edu.ec

UNIVERSIDAD YACHAY TECH

SCHOOL OF PHYSICAL SCIENCES AND NANOTECHNOLOGY

# Radio astronomy

- Radio astronomy is the study of natural radio emission from celestial sources.
- The radio band is very broad logarithmically: it spans the five decades between 10 MHz **($\lambda$~30 m)** and 1 THz ($\lambda$=c/$\nu$~**0.3 mm**) at the low-frequency end of the electromagnetic spectrum.
  - The Earth's ionosphere sets a low-frequency limit to ground-based radio astronomy by reflecting extraterrestrial radio waves with frequencies below $\nu$~10 MHz **($\lambda$~30 m).**
- **Nearly everything emits radio waves at some level**, via a wide variety of emission mechanisms. Few astronomical radio sources are obscured because radio waves can penetrate interstellar dust clouds and Compton-thick layers of neutral gas.
- Only optical and radio observations can be made from the ground.
- Coherent amplifiers, which preserve phase information, allow the construction of sensitive multielement aperture-synthesis interferometers that can image complex sources with angular resolution and absolute astrometric accuracies approaching $10^{-4}$ arcsec.
- Quantum noise forever restricts sensitive coherent amplification to the low photon energies E=h$\nu$(where h=Planck's constant ≈6.626×10−27 erg s) of the radio band.
- Also, coherent signals can be shifted to lower frequencies and digitized, permitting the construction of radio **spectrometers with extremely high spectral resolution and frequency accuracy.**

# Radio astronomy

Because the radio window is so broad, (1) **almost all types of astronomical sources**, thermal and nonthermal radiation mechanisms, and propagation phenomena can be observed at radio wavelengths; and (2) a **wide variety of radio telescopes and observing techniques** are needed to cover the radio window effectively.

Major discoveries of radio astronomy include:

1. nonthermal radiation from our Galaxy and many other astronomical sources;
2. the “violent universe” of powerful radio galaxies and quasars (quasi-stellar radio sources) powered by supermassive black holes (SMBHs);
3. spectral-line emission from cold interstellar gas atoms, ions, and molecules;
4. maser (microwave amplification by stimulated emission of radiation) emission from interstellar molecules;
5. cosmic microwave background radiation from the hot big bang;
6. pulsars and neutron stars;
7. indirect evidence for gravitational radiation;
8. the supermassive black hole at the center of our Galaxy;
9. evidence for dark matter in galaxies, deduced from their HI (neutral hydrogen) rotation curves;
10. extrasolar planets;

# Radio astronomy

Many unique scientific and technical features of radio astronomy result from radio waves occupying the long-wavelength end of the electromagnetic spectrum.

At macroscopic wavelengths large groups of charged particles moving together in volumes $<\lambda^3$ may produce strong coherent emission, accounting for the astounding radio brightnesses of pulsars at $\lambda \sim 1$m.

Dust scattering is negligible because **interstellar dust grains are much smaller than radio wavelengths**, so the **dusty interstellar medium (ISM) is nearly transparent**. This allowed radio astronomers to see through the dusty disk of our Galaxy and discover the compact radio source Sgr A$*$ powered by the supermassive black hole at its center.

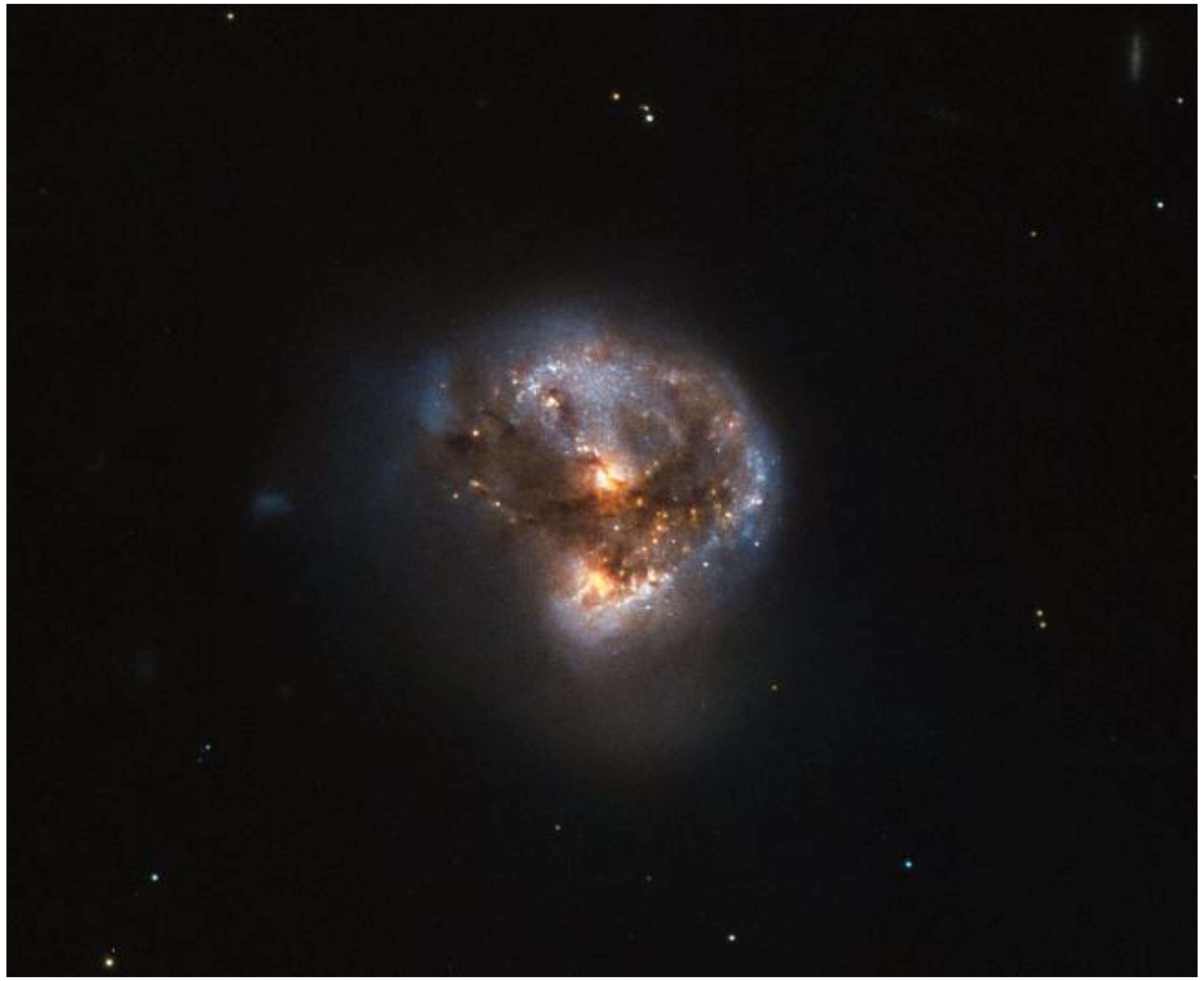
**Low frequencies imply low photon energies $E=h\nu$.** Thus radio spectral lines trace extremely low-energy transitions produced by atomic hyperfine splitting (the ubiquitous 21-cm line of neutral hydrogen at $\nu \approx 1.420$ GHz generates photons of energy $E \approx 6 \times 10^{-6}$eV), the quantized rotation rates of polar molecules such as carbon monoxide in interstellar space, and high-level recombination lines from interstellar atoms.

# Radio astronomy

At radio frequencies, the dimensionless ratio $h\nu/(kT)$ of photon energy to the mean kinetic energy of particles at temperature T is very small ($\ll 1$). In this limit, the brightness of a blackbody emitter is proportional to $\nu^2$, ensuring that nearly every astronomical object is a thermal radio source at some low level.

On the negative side, the fact that nearly everything emits radio radiation means radio astronomers must deal with **large and fluctuating natural foregrounds of emission from the ground, from the atmosphere, and even from their own antennas and receivers**.

Also, **stimulated emission** (negative absorption) becomes comparable with absorption when $h\nu/(kT) \ll 1$. This greatly lowers the opacities of radio spectral lines, makes their emission strengths nearly independent of the temperature of the emitting gas, and allows **maser emission** with only a small population inversion.



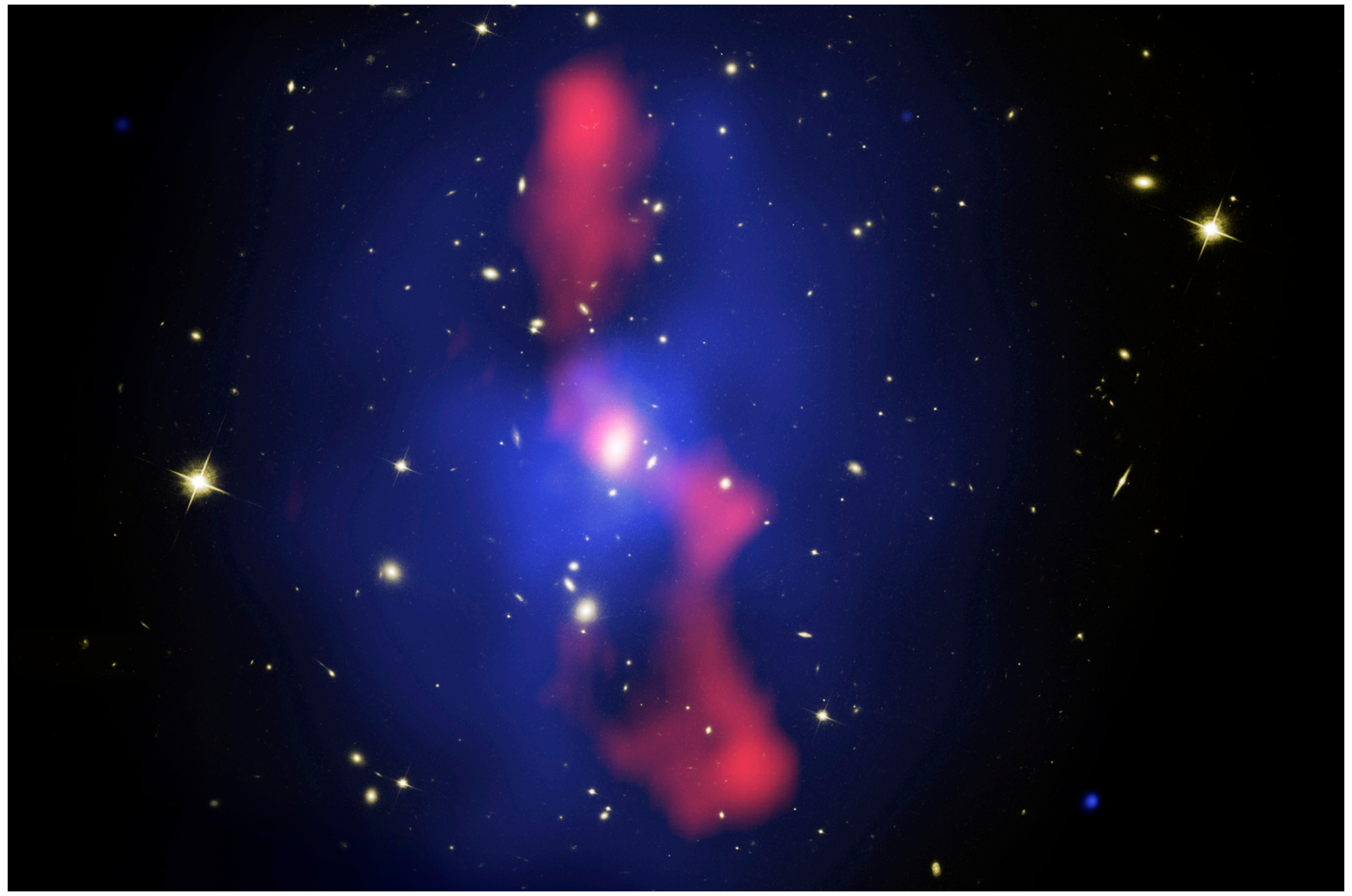
"megamaser" called IRAS 16399-0937
The merger of two galaxies causes rapid star-formation, which produces intense microwave radiation.

# Radio astronomy

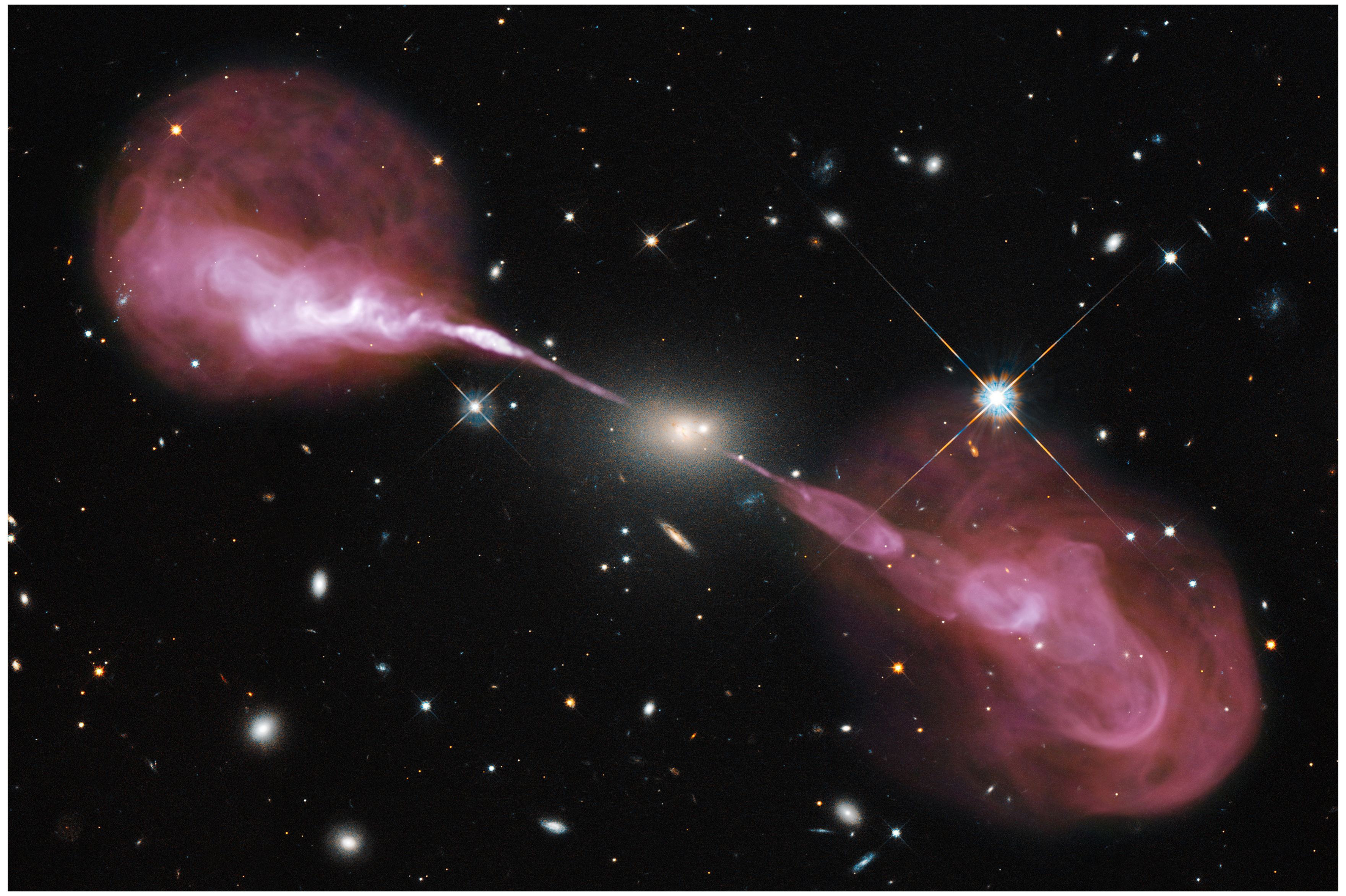
In contrast, $h\nu/(kT) \gg 1$ for cold sources at optical frequencies, where the exponential high-frequency cutoff of the blackbody radiation spectrum ensures that essentially no optical photons are emitted. **Cold thermal emitters** (e.g., the 2.73 K cosmic microwave background, or interstellar gas at temperatures below 100 K) are completely invisible. For example, a person can be approximated by a 300 K blackbody with a surface area $\sim 1\,\mathrm{m}^2$ . Such a blackbody emits $\sim 10^{16}$ photons per second at radio frequencies below 10 GHz but only 0.01 photons per second at visible wavelengths $\lambda < 0.75\,\mu\mathrm{m}$.

Radio **synchrotron sources live long** after their emitting electrons were accelerated to relativistic energies, so they can provide long-lasting archaeological records of past energetic phenomena (e.g., see Figures). Likewise, neutral hydrogen stripped from colliding galaxies continues emitting at $\lambda$=21cm for tens of millions of years.

The radio source (red) in the galaxy cluster MS0735.6+7421 has displaced the X-ray emitting gas (blue)



The radio galaxy Hercules A (3C 348).



# Radio astronomy

Most **plasma effects** (scattering, dispersion, Faraday rotation, etc.) have strengths proportional to $\nu^{-2}$ and are strong enough at low radio frequencies to be useful tools for **tracing interstellar electron densities and magnetic field strengths.**

# Radio astronomy

Radio telescopes must have very large aperture diameters D to achieve good diffraction-limited angular resolution $\theta \approx \lambda/D$ radians at radio wavelengths. Even the biggest precision radio telescopes (e.g., telescopes with small rms reflector surface errors $\sigma < \lambda/16$) such as the Green Bank Telescope (GBT) with $\sigma \approx 0.2$mm and D=100m are limited to $\theta \gg 1$arcsec.

On the other hand, huge multielement interferometers spanning up to $D \sim 10^4$ km are practical.

Paradoxically, the **finest angular resolution** for imaging faint and complex sources is **obtainable at the long-wavelength (radio)** end of the electromagnetic spectrum. Interferometers also yield extremely accurate astrometry because interferometric positions depend on measuring time delays which can be done extremely accurately.

The Very Long Baseline Array (VLBA) of 10 25-m telescopes extending 8000 km, yields angular resolution as fine as $\theta$=0.00017 arcsec, surpassing the resolution of the Hubble Space Telescope by two orders of magnitude.

# Radio astronomy

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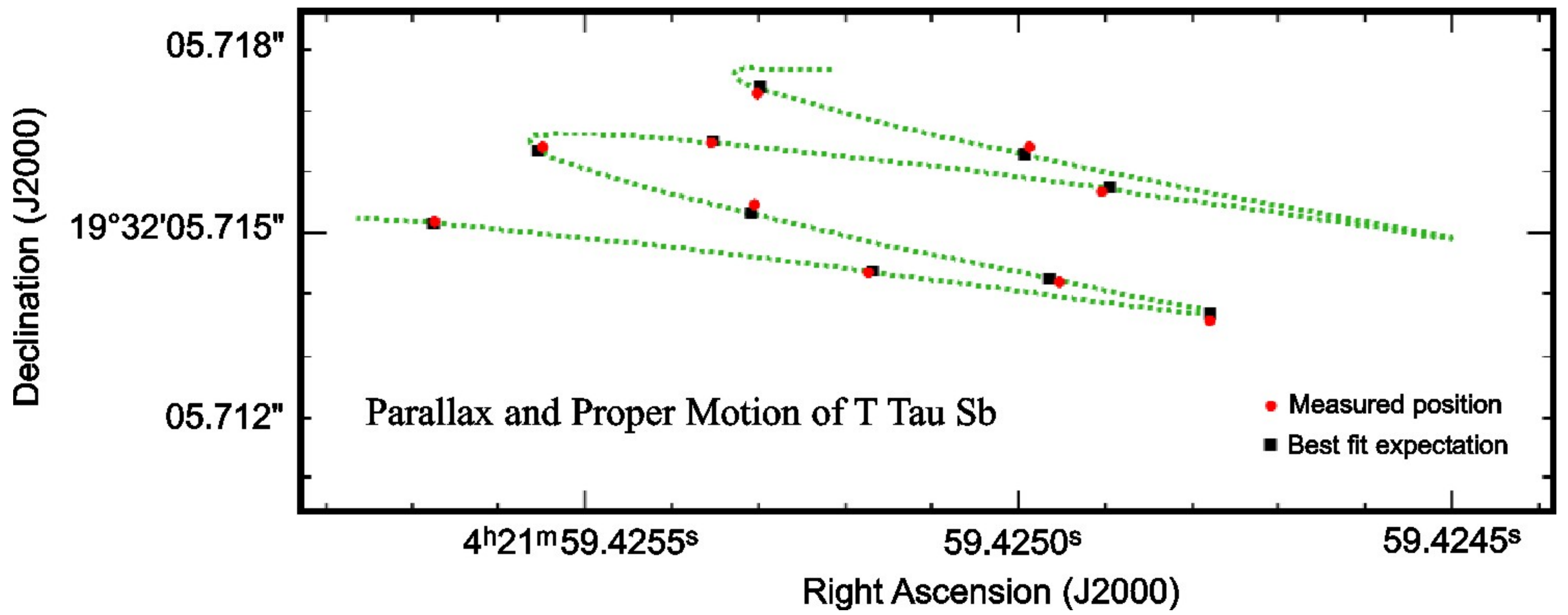



Multiepoch VLBA position measurements of T Tau Sb, young stellar object, allowed to determine its parallax distance with unprecedented accuracy: d=146.7±0.6 pc, and even to **detect accelerated proper motion.**

# Radio astronomy

Coherent (phase-preserving) amplifiers are required for accurate interferometric imaging of faint extended sources because they allow the signal from each telescope in a multielement interferometer to be amplified before it is split and combined with the signals from the other telescopes.

The minimum possible noise temperature of a coherent receiver is $T \approx h\nu/k$ owing to quantum noise. Quantum noise is proportional to frequency, so even the best possible coherent amplifiers at visible-light frequencies must have noise temperatures $T > 10^4$ K. Aperture-synthesis interferometers at radio wavelengths provide unparalleled sensitivity, image fidelity, angular resolution, and absolute position accuracy.

# Radiation Fundamentals

Astronomers study an astronomical source by measuring the strength of its radiation as a function of direction on the sky (by mapping or imaging) and frequency (spectroscopy), plus other quantities (time, polarization). Clear and quantitative definitions are needed to describe the strength of radiation and how it varies with direction, frequency, and distance between the source and the observer.

Consider the simplest possible case of radiation traveling from a source through empty space, so there is no **absorption**, **scattering**, or **emission** along the path to an observer.

In the **ray-optics approximation**, radiated energy flows in straight lines. This approximation is valid only for systems much larger than the wavelength $\lambda$ of the radiation, a criterion easily met by astronomical sources.

The "brightness" of the Sun appears to be about the same over most of the Sun's surface, which looks like a nearly uniform disk even though it is actually a sphere. This means that a photograph of the Sun would be uniformly exposed across the Sun's disk. It also turns out that the exposure would not change if photographs were made at different distances from the Sun, from points near Mars, the Earth, and Venus, for example.

# Radiation Fundamentals

The angular size of the Sun depends on the distance between the Sun and the camera, but the number of photons falling on the detector per unit area per unit time per unit solid angle does not.

The photo taken from near Venus would not be overexposed, and the one from near Mars would not be underexposed. The total number of solar photons from all directions reaching the camera per unit area per unit time (or the total energy absorbed per unit area per unit time) does decrease with increasing distance, but only because the solid angle subtended by the Sun decreases.

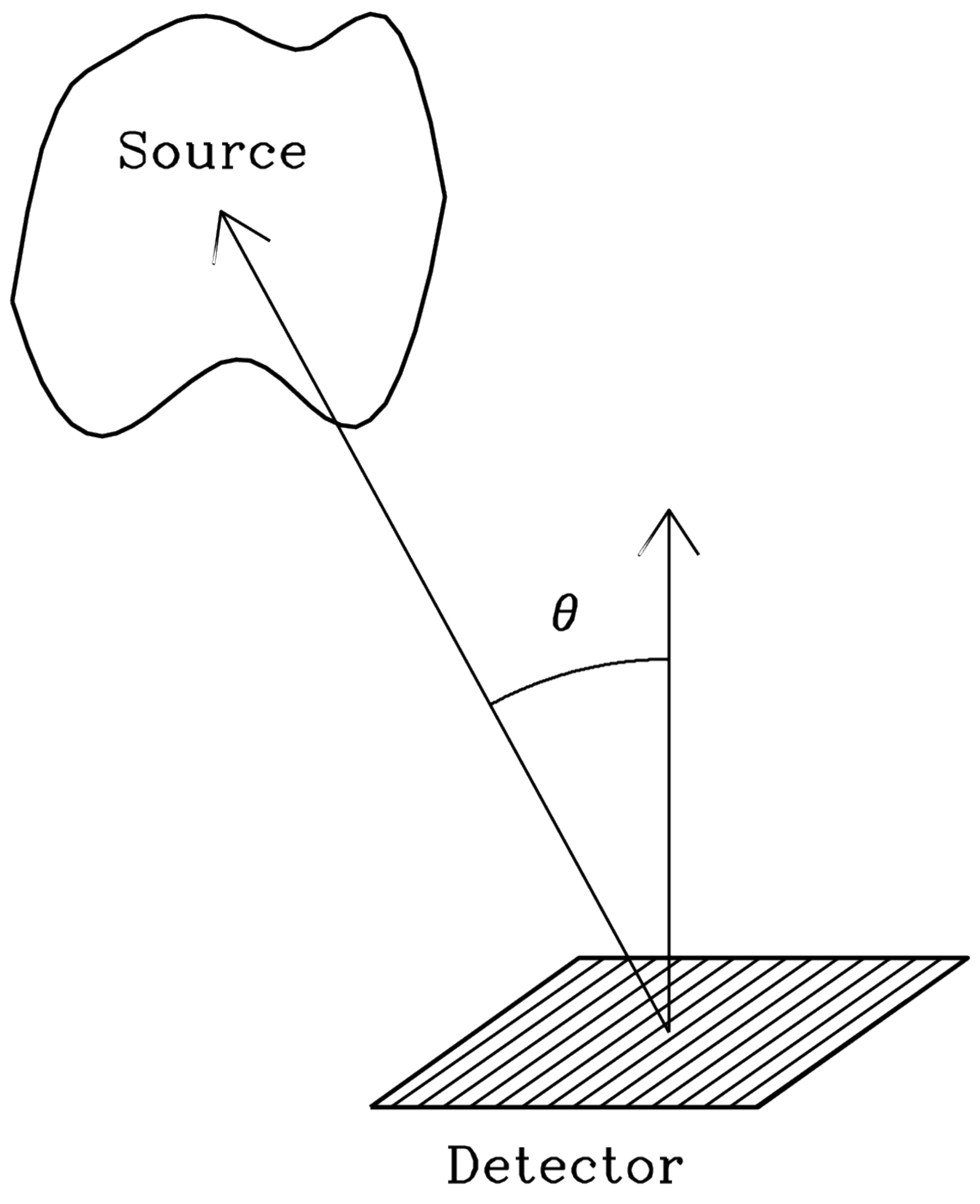
Thus we distinguish between the **brightness or intensity** of the Sun's radiation, **which does not depend on distance, and the apparent flux, which does**.

Some authors prefer to use brightness for the power per unit area per unit solid angle emitted at the source and specific intensity for the power per unit area per unit solid angle along the path to the detector. The two are identical if there is no absorption or emission between the source and the detector, and we will use these terms interchangeably.

# Radiation Fundamentals

Note also that the number of photons per unit area hitting the detector is proportional to $\cos\theta$ if the normal to the detector is tilted by an angle $\theta$ from the direction of the incoming rays.

The total brightness is contributed by photons of all frequencies. The **brightness per unit frequency is called the specific intensity** (also spectral intensity or spectral brightness). The notation for specific intensity is $I_\nu$, where the subscript $\nu$ is used to indicate "per unit frequency." In the ray-optics approximation, specific intensity can be defined quantitatively in terms of




- $d\sigma$ = an infinitesimal surface area (e.g., of a detector);
- $\theta$ = the angle between a "ray" of radiation and the normal to the surface;
- $d\Omega$ = an infinitesimal solid angle measured from the observer's location.

# Radiation Fundamentals

The surface containing dσ can be any surface, real or imaginary; that is, it could be the physical surface of the detector, the source, or an imaginary surface anywhere along the ray. If **energy** dE from within the solid angle dΩ flows through the projected area cosθdσ in time dt and in a narrow frequency band of width dν, then

$$dE = I_\nu \cos\theta \, d\sigma \, d\Omega \, dt \, d\nu.$$

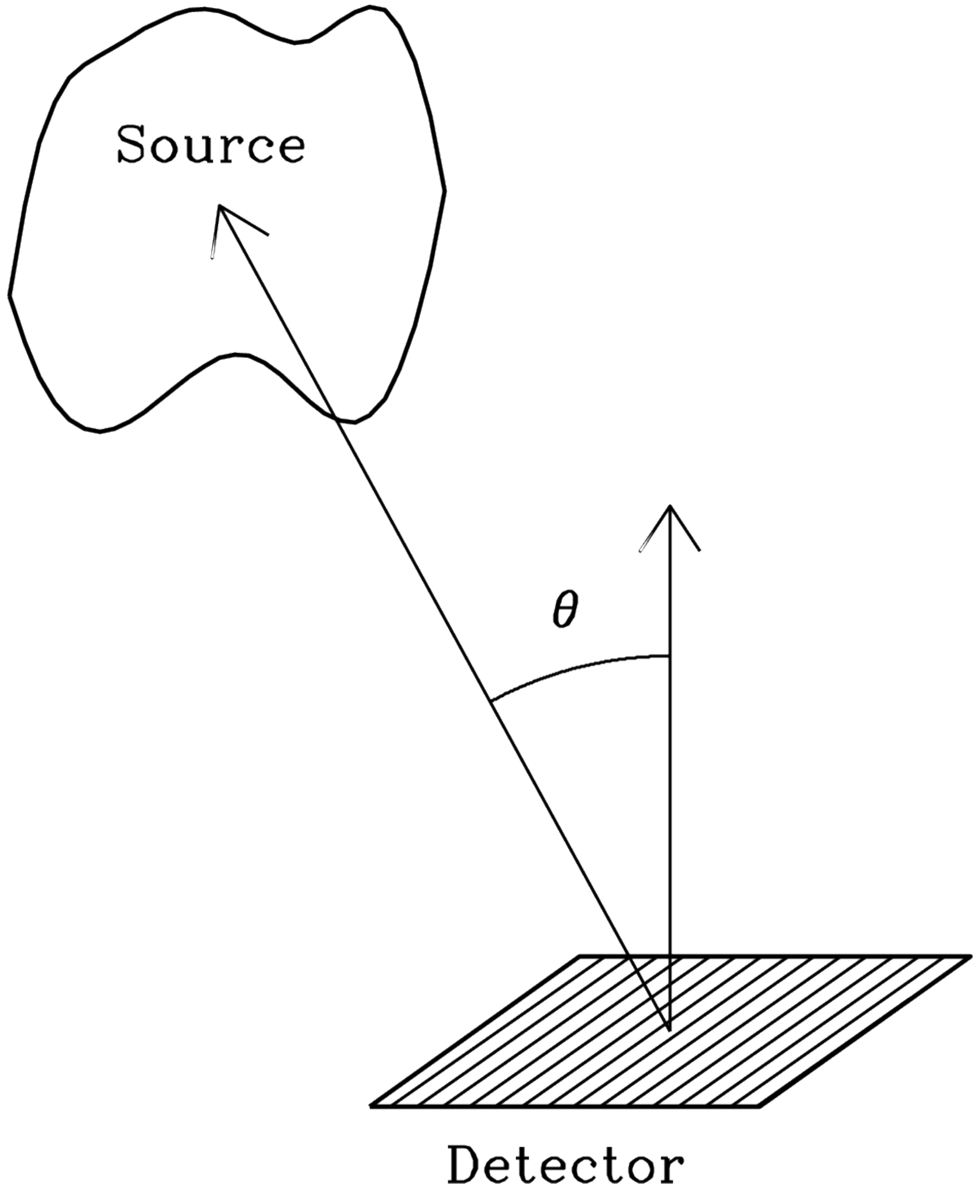
Power is defined as the flow of energy per unit time, so the corresponding power dP is

$$\underbrace{dP}_{\text{(watts)}} = \frac{dE}{dt} = I_\nu \, (\underbrace{\cos\theta \, d\sigma}_{(\mathrm{m}^2)} \, \underbrace{d\Omega}_{\text{sr}} \, \underbrace{d\nu}_{\text{Hz}}) \, .$$

Thus the quantitative definition of **specific intensity or spectral brightness** is

$$\boxed{I_\nu \equiv \frac{dP}{(\cos\theta \, d\sigma) \;\; d\nu \, d\Omega}}$$

Units: $\mathrm{W m^{-2} Hz^{-1} sr^{-1}}$



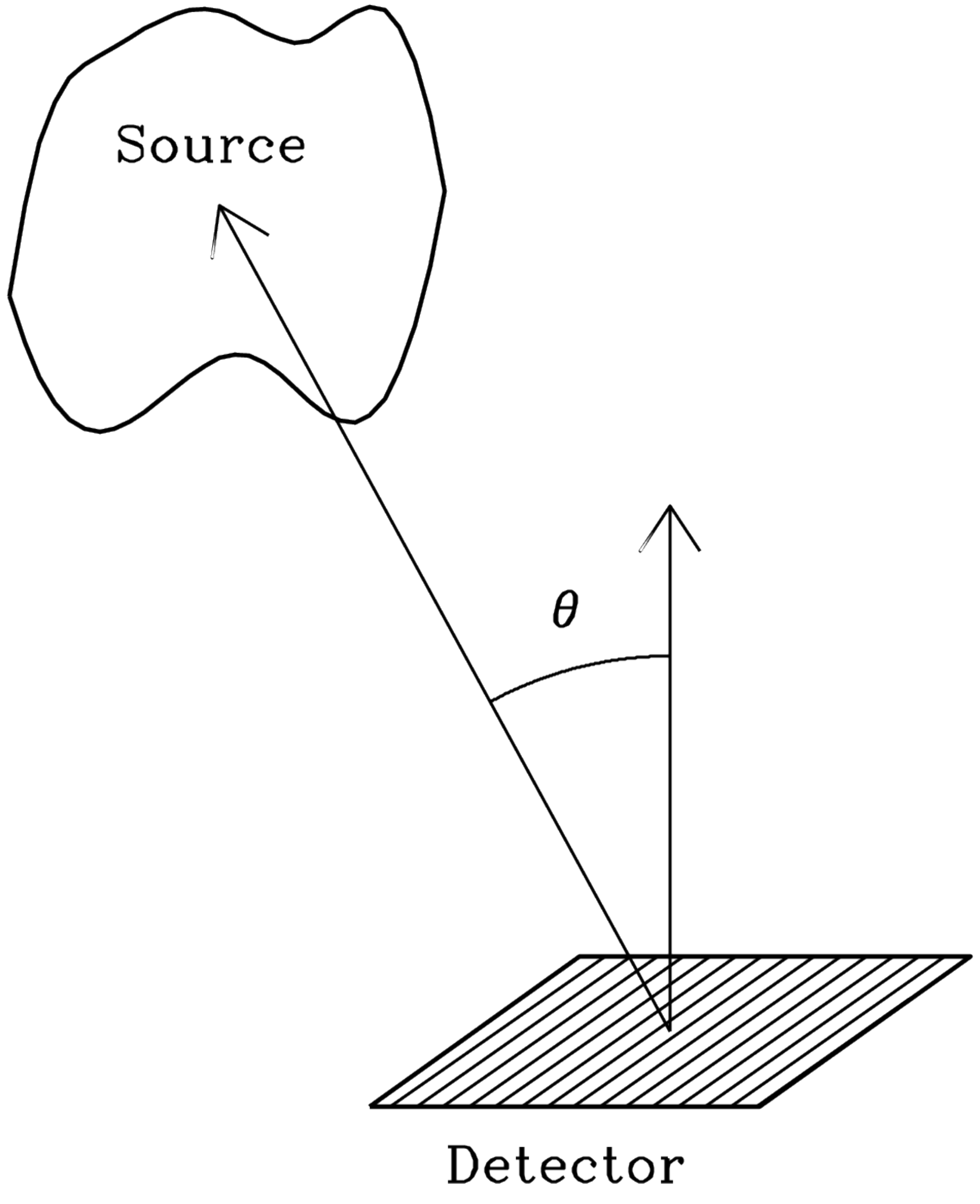
# Radiation Fundamentals

The brightness per unit wavelength

$$\boxed{I_\lambda \equiv \frac{dP}{(\cos\theta\, d\sigma)\quad d\lambda\, d\Omega}}$$

Relationship between frequency and wavelength

$$|I_\nu d\nu| = |I_\lambda d\lambda| \qquad \boxed{\frac{I_\lambda}{I_\nu} = \left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2} = \frac{\nu^2}{c}.}$$




The reasons for specifying the brightness in an infinitesimal frequency range $d\nu$ or wavelength range $d\lambda$ are

(1) the detailed spectra of sources carry astrophysically important information

(2) source properties (e.g., opacity) may vary with frequency, and

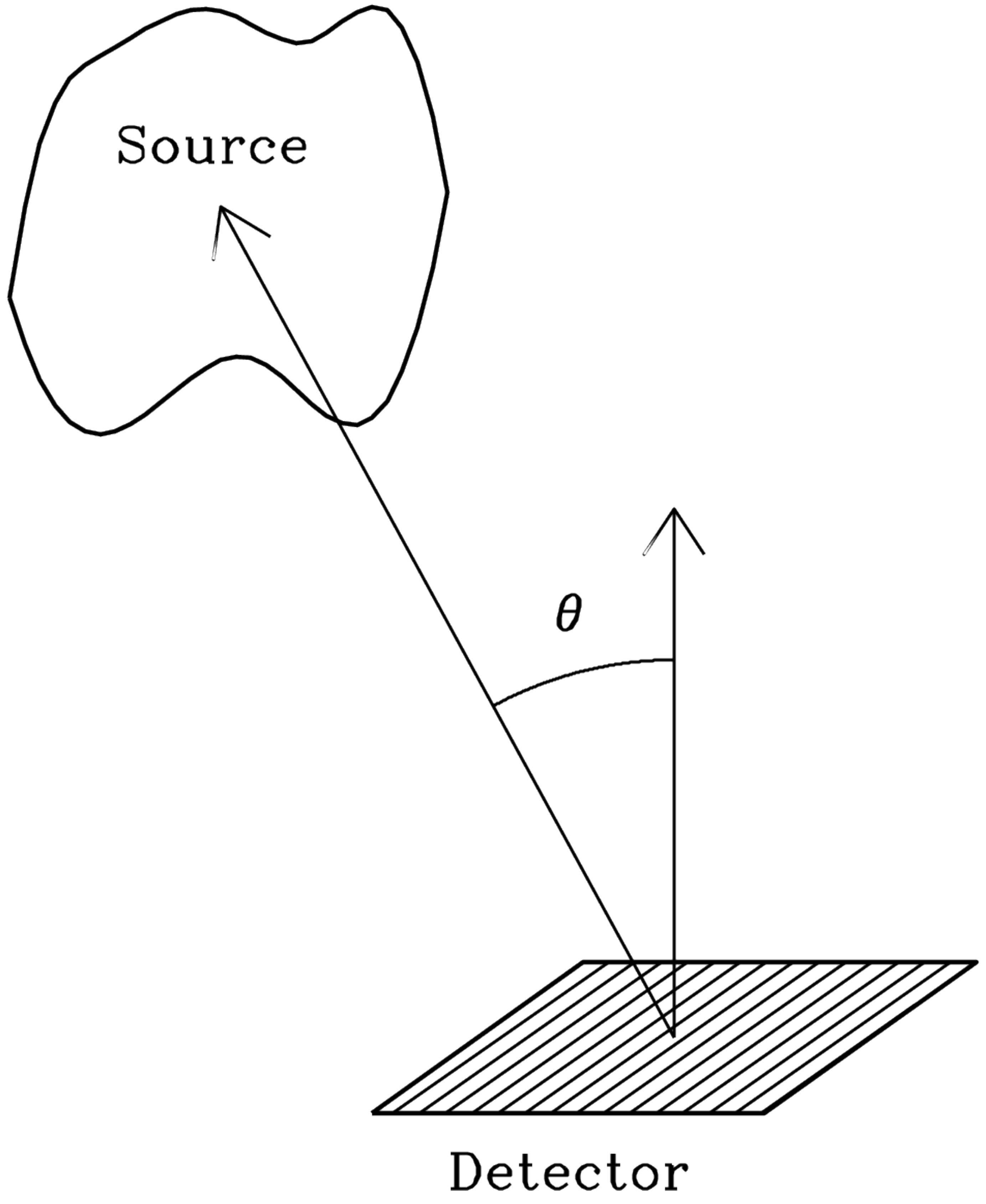
(3) most general theorems about radiation are true for all narrow frequency ranges (e.g., specific intensity is conserved along a ray path in empty space), so they are also true for any wider frequency range.

# Radiation Fundamentals

Thus the conservation of specific intensity implies that **total intensity** defined as

$$I \equiv \int_0^\infty I_\nu(\nu)\, d\nu = \int_0^\infty I_\lambda(\lambda)\, d\lambda$$

**Specific intensity is conserved (is constant) along any ray in empty space** (where there is no absorption or emission)**.**

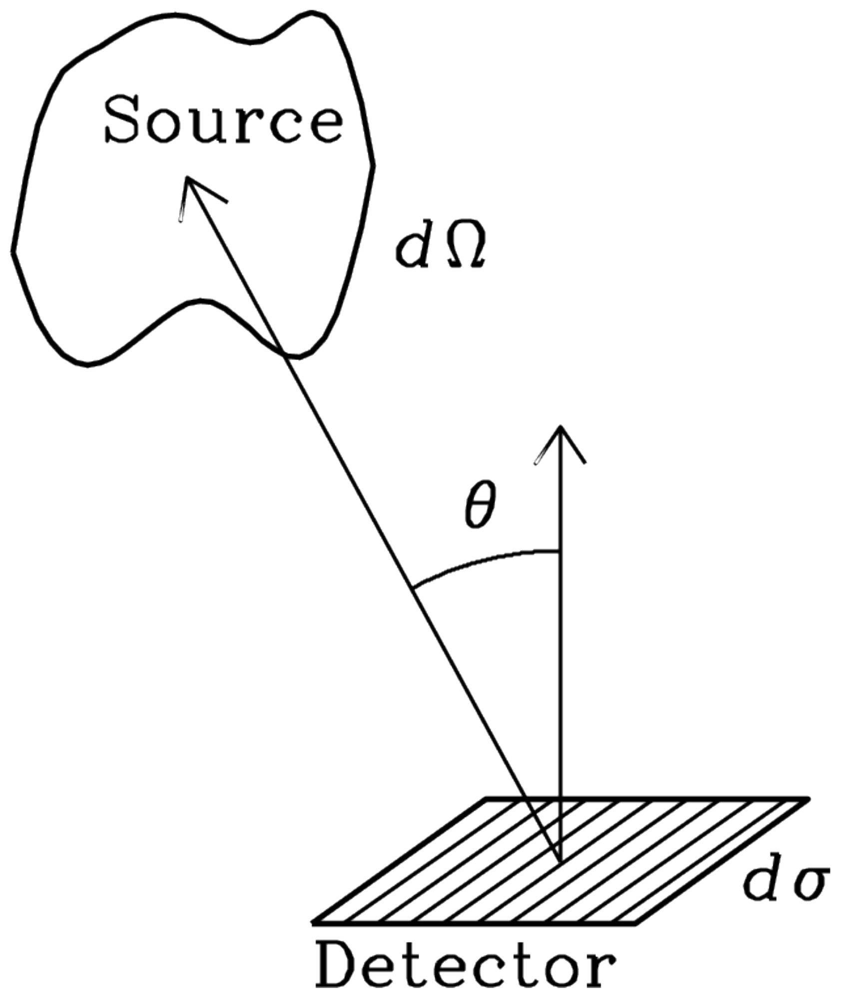


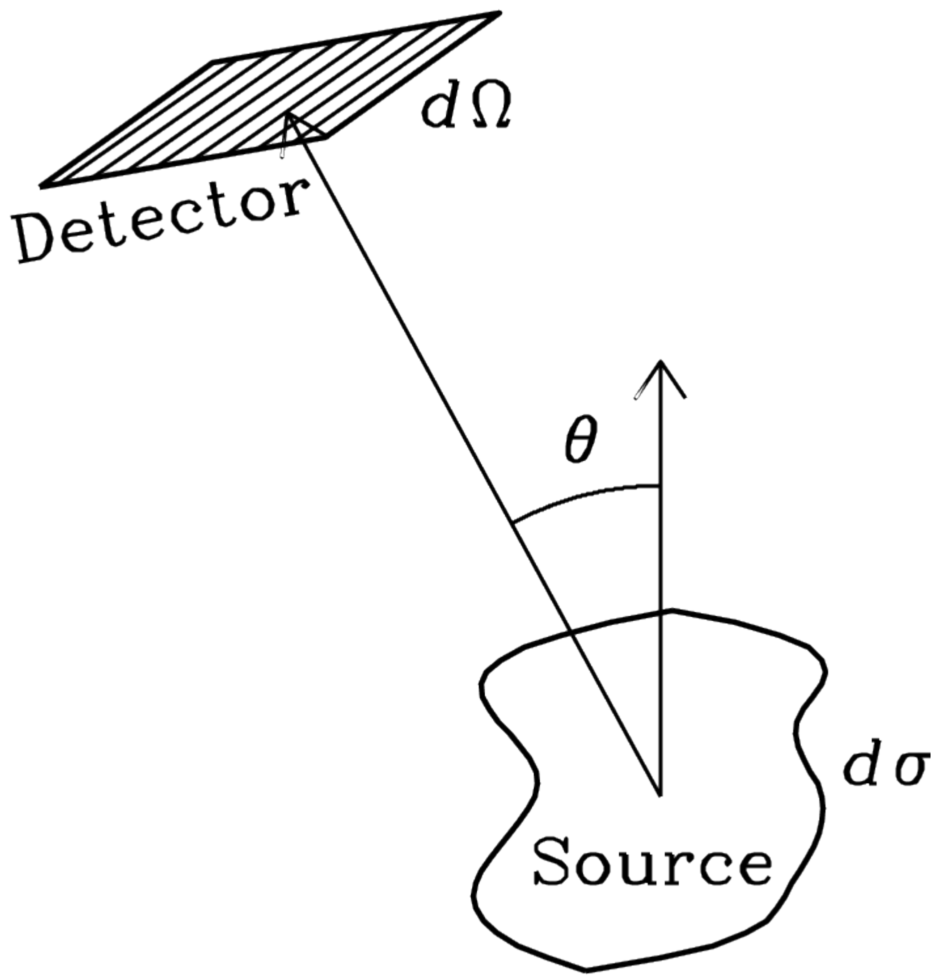
# Radiation Fundamentals

The conservation of specific intensity has two important consequences:

1. **Brightness is independent of distance.** Thus the camera setting for a good exposure of the Sun would be the same, regardless of whether the photograph was taken close to the Sun (from near Venus, for example) or far away from the Sun (from near Mars, for example), so long as the Sun is resolved in the photograph.

2. **Brightness is the same at the source and at the detector.** Thus you can think of brightness in terms of energy flowing out of the source or as energy flowing into the detector.



# Radiation Fundamentals

The conservation of brightness applies to any lossless optical system (a system of lenses and mirrors, for example) that can change the direction of a ray. N**o passive optical system can increase the specific intensity or total intensity of radiation.**

If you look at the Moon through a large telescope, the Moon will appear bigger (in angular size) but not brighter.

Many people are disappointed when they see a large galaxy through a telescope because it looks so dim; they expected to see a brilliantly glowing disk of stars, as in the photograph of Andromeda. The difference is not in the telescope; it is in the detector—the photograph appears brighter only because the photograph has accumulated more light over a long exposure time.

# Radiation Fundamentals

If a source is discrete, meaning that it subtends a well-defined solid angle, the spectral power received by a detector of unit projected area (Figure 2.6) is called the **flux density $S_\nu$** of the source. Equation (2.2) implies

$$\frac{dP}{d\sigma d\nu} = I_\nu \cos\theta \, d\Omega,$$

so integrating over the solid angle subtended by the source yields

$$\boxed{S_\nu \equiv \int_{\text{source}} I_\nu(\theta, \phi) \cos\theta \, d\Omega.}$$

If the source angular size is $\ll$1rad, $\cos\theta \approx 1$ and the expression for flux density is much simpler:

$$\boxed{S_\nu \approx \int_{\text{source}} I_\nu(\theta, \phi) \, d\Omega.}$$

This is usually the case for astronomical sources, and astronomers rarely use flux densities to describe sources so extended that the $\cos\theta$ factor must be retained (e.g., the diffuse emission from our Galaxy).

# Radiation Fundamentals

In practice, when should **spectral brightness and when should flux density** be used to describe a source?

If a source is **unresolved**, meaning that it is much smaller in angular size than the point-source response of the eye or telescope observing it, its **flux density can be measured** but its spectral brightness cannot.

To the naked eye, the unresolved red giant star Betelgeuse appears to be one of the brightest stars in the sky. Yet calling it a “bright star” is misleading because the total intensity of this relatively cool star is lower than the total intensity of every hotter but more distant star that is scarcely visible to the eye. Betelgeuse appears “brighter” than most other stars only because it subtends a much larger solid angle and therefore its flux is higher.

If a **source is much larger than the point-source response**, its **spectral brightness** at any position on the source **can be measured directly,** but its flux density must be calculated by integrating the observed spectral brightnesses over the source solid angle. Consequently, flux densities are normally used to describe only relatively compact sources.

# Radiation Fundamentals

An illustration of the definition of flux density.

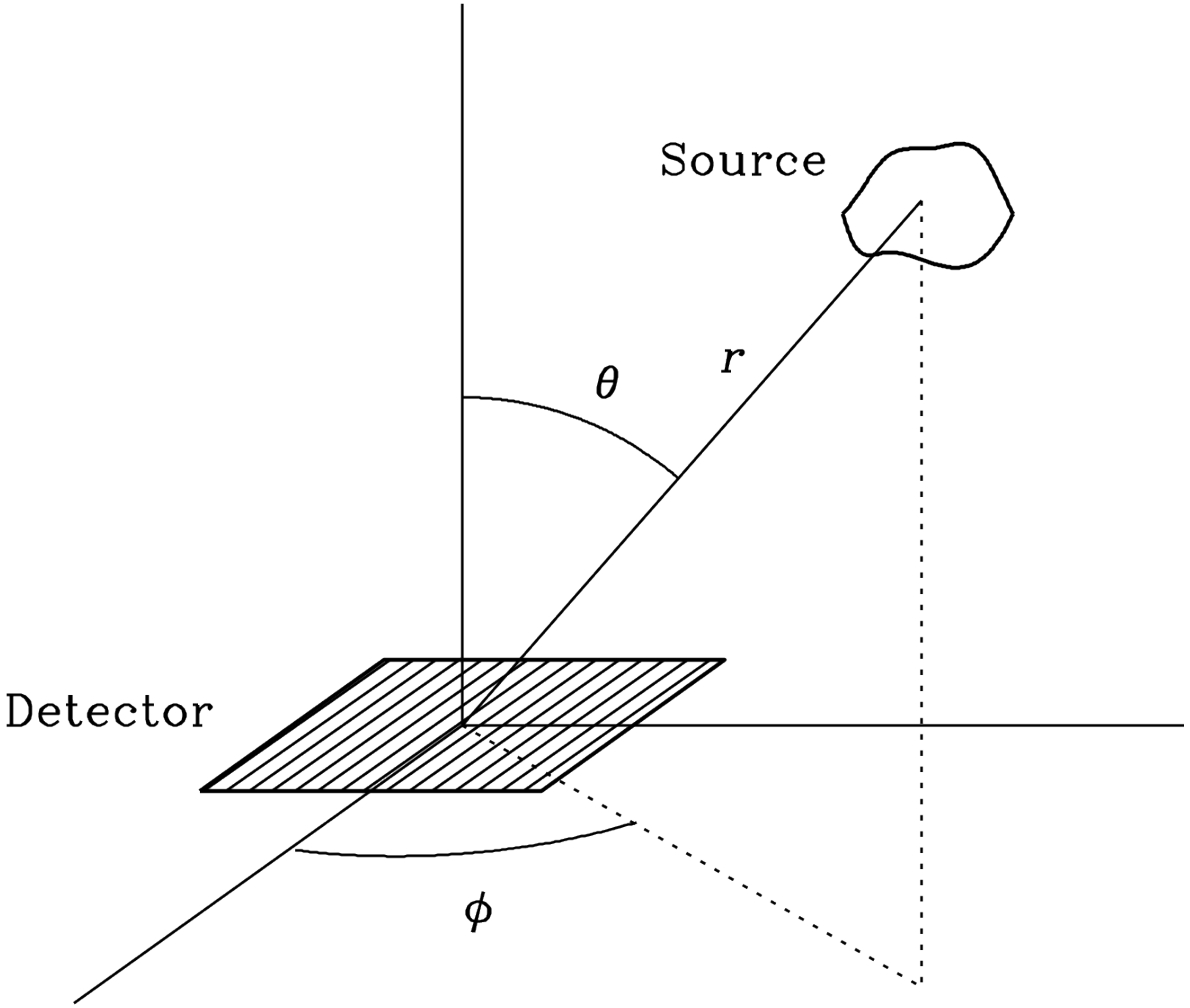
units of flux density, $Wm^{-2}Hz^{-1}$, are much too big for practical astronomical use, so astronomers use smaller ones:

$$1 \text{ jansky} = 1 \text{ Jy} \equiv 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

1 millijansky = 1 mJy $\equiv 10^{-3}$ Jy,
1 microjansky = 1 $\mu$ Jy $\equiv 10^{-6}$ Jy.

Optical astronomers often express flux densities as AB magnitudes defined in terms of Jy by

$$\text{AB magnitude} \equiv -2.5 \log_{10}\left(\frac{S_\nu}{3631 \text{ Jy}}\right)$$



# Radiation Fundamentals

Unlike brightness, flux density depends on source distance d. Because $\int_{\text{source}} d\Omega \propto 1/d^2$ and brightness is conserved

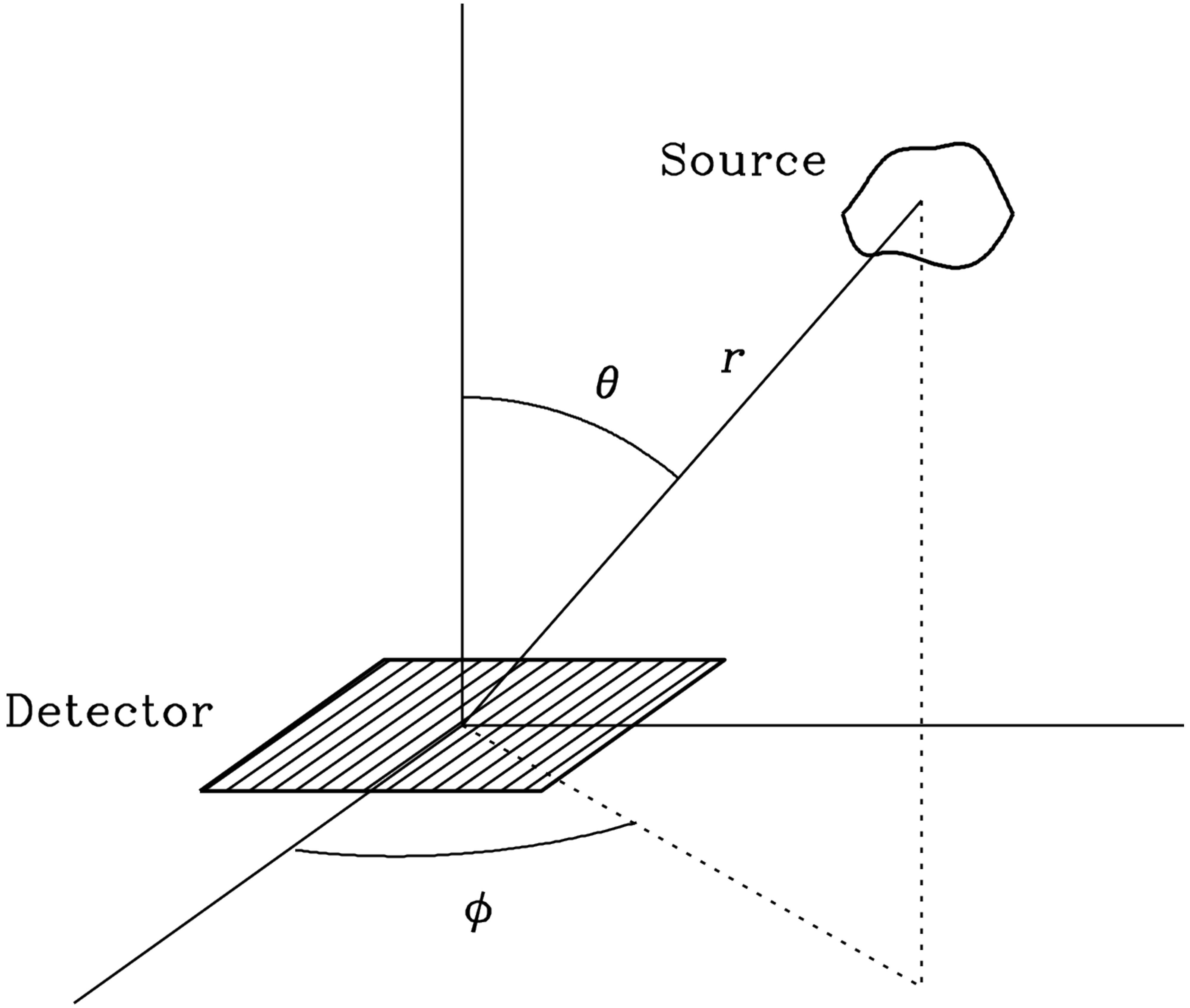
$$S_\nu \propto d^{-2}.$$

The specific intensity or brightness is an intrinsic property of a source, while the **flux density of a source also depends on the distance** between the source and the observer.

The **total flux or flux S** from a source is the integral over frequency of flux density:

$$S \equiv \int_0^\infty S_\nu d\nu.$$

An illustration of the definition of flux density.



# Radiation Fundamentals

$$S \equiv \int_0^\infty S_\nu \, d\nu.$$

Its dimensions are power divided by area, so its units are W $m^{-2}$.
**Total flux is a rarely used quantity** in observational radio astronomy, so radio astronomers often delete the subscript from $S_\nu$ and use the symbol S to indicate flux density. This is convenient but potentially confusing. Likewise, the word "flux" is sometimes used as a shorthand for "flux density" in the literature, even though it is formally incorrect.

The **spectral luminosity** $L_\nu$ of a source is defined as the total power per unit bandwidth radiated by the source at frequency $\nu$; its units are $WHz^{-1}$.
The area of a sphere of radius d is $4\pi d^2$, so the relation between the spectral luminosity and the flux density of an isotropic source radiating in free space is

$$\boxed{L_\nu = 4\pi d^2 S_\nu,}$$

# Radiation Fundamentals

$$\boxed{L_\nu = 4\pi d^2 S_\nu,}$$

where the distance d between the source and the observer is much larger than the dimensions of the source itself.

Beware that some radio sources emit anisotropically, relativistically beamed quasars for example. Unfortunately, the Equation cannot be used to calculate the total (integrated over $4\pi$sr) spectral luminosity of a beamed quasar from a flux-density measurement made from just one direction.

**Spectral luminosity is an intrinsic property of the source** because it **does not depend on the distance** d between the source and the observer—the $d^2$ in the Equation. 2.15 cancels the $d^{-2}$ dependence of $S_\nu$. The luminosity or total luminosity L of a source is defined as the integral over all frequencies of the spectral luminosity:

$$L \equiv \int_0^\infty L_\nu d\nu.$$

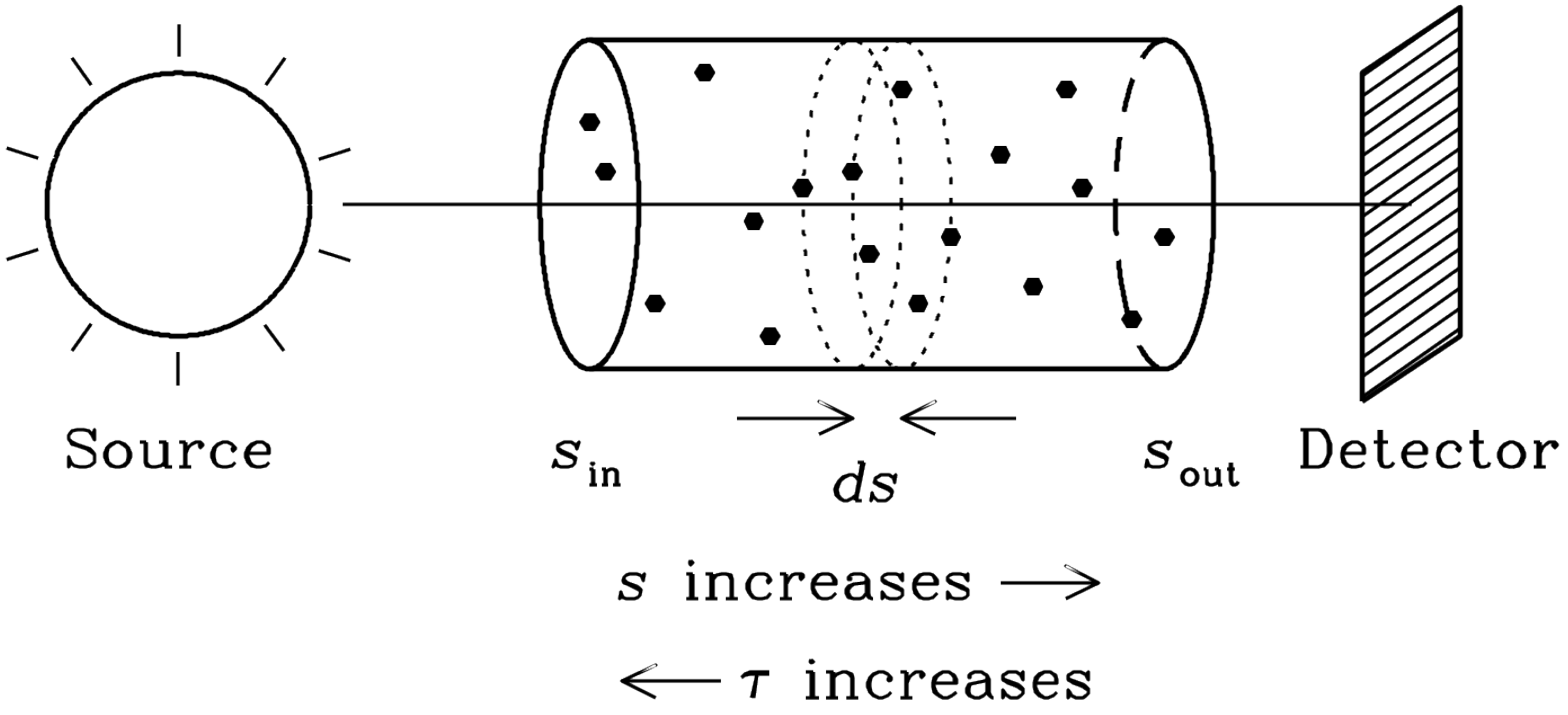
Astronomers sometimes call L the **bolometric luminosity** because a bolometer is a broadband detector that measures the heating power of radiation at all frequencies.

# Radiative transfer

In free space, the specific intensity Iν of radiation is conserved along a ray:

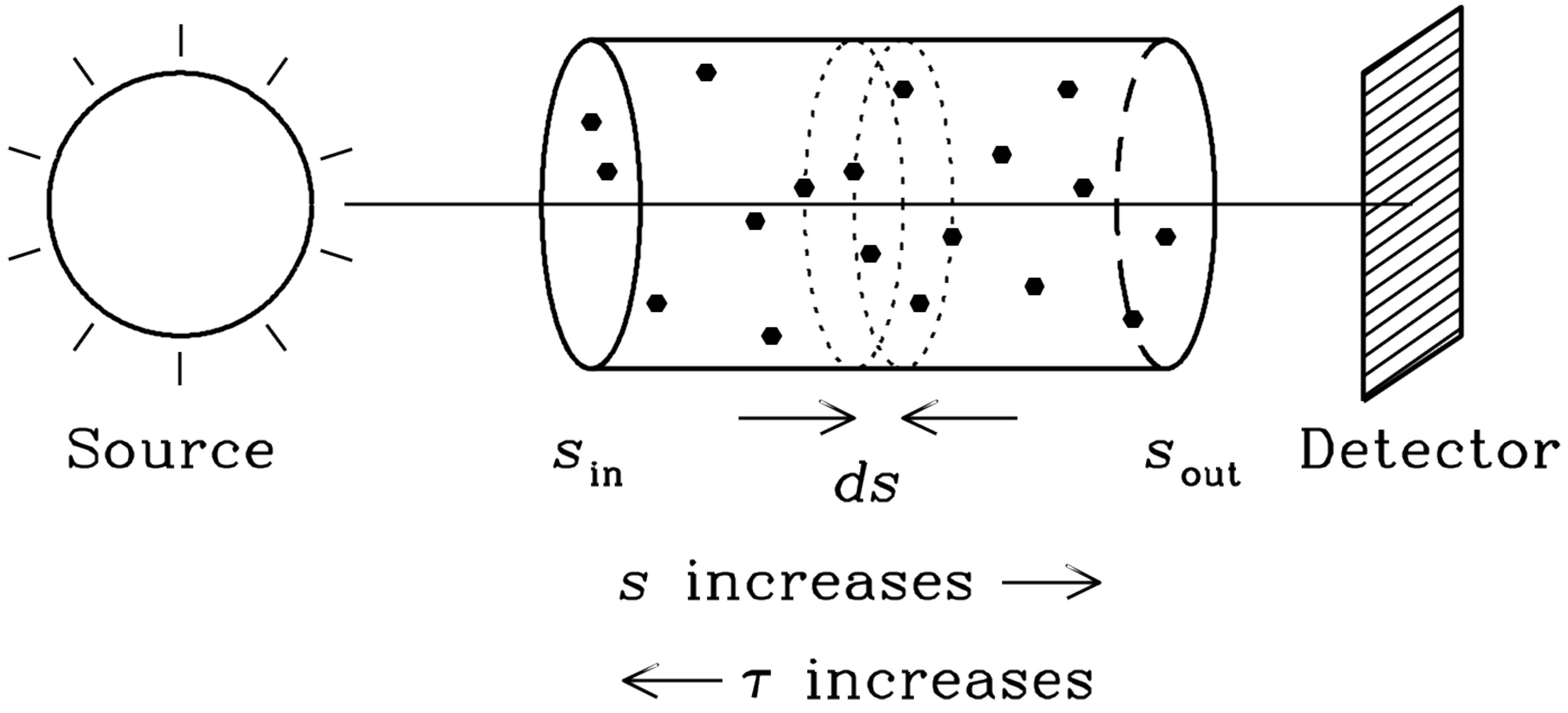
$$\frac{dI_\nu}{ds} = 0,$$

where s is the coordinate along the ray between the source and the detector. What happens if there is an intervening medium between $s_{in}$ and $s_{out}$




Absorption between a source and a detector. The optical depth τ is measured in the opposite direction, starting from τ=0 at $s_{out}$ and increasing as s decreases.

# Absorption



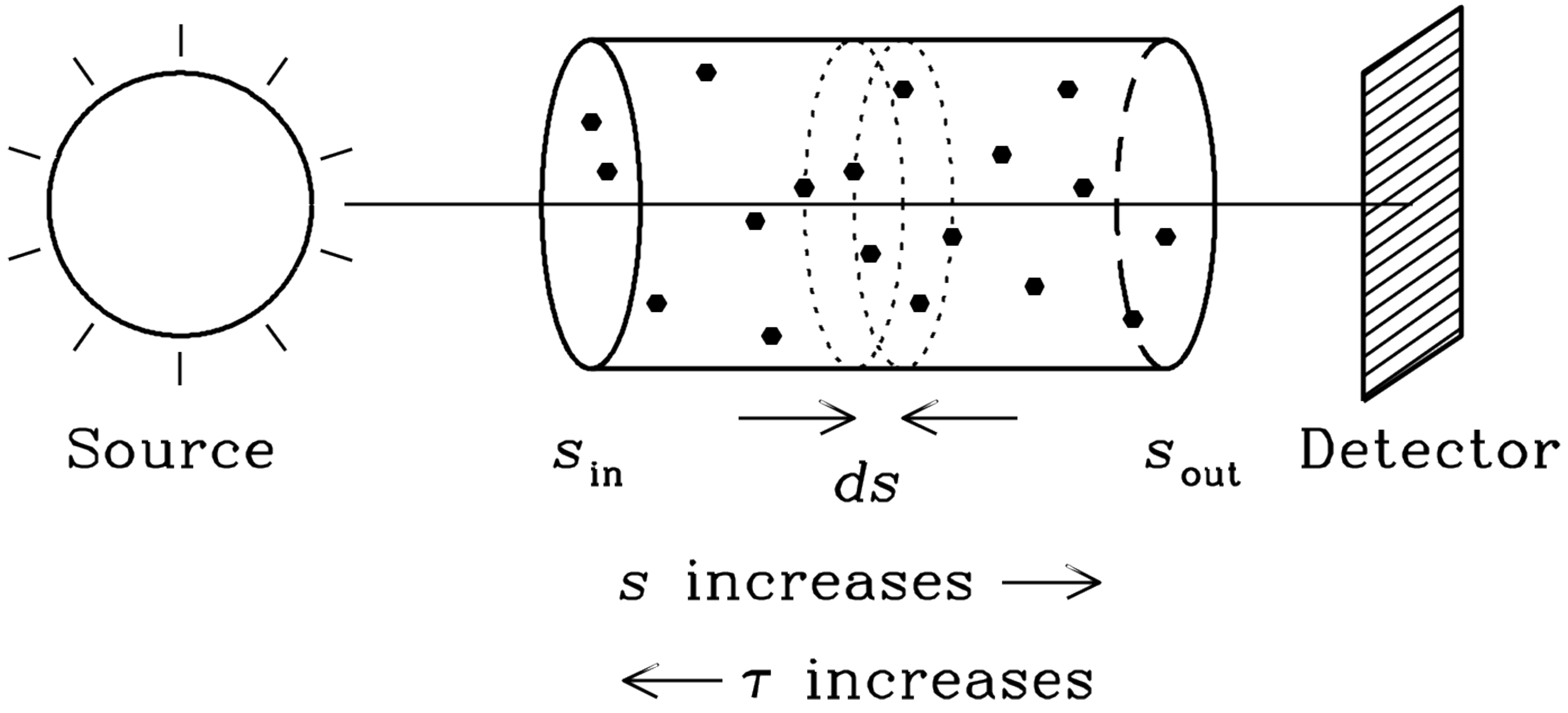

The infinitesimal probability dP of a photon being absorbed (e.g., by hitting an absorbing particle) in a thin slab of thickness ds is directly proportional to ds: $dP = \kappa ds$ , where the constant of proportionality

$$\boxed{\kappa \equiv \frac{dP}{ds}}$$

In the Astrophysics for Physicist book
And in the other lecture slides
this $\kappa$ is called $\alpha_\nu$

is called the linear **absorption coefficient**, and its dimension is inverse length.
A value $\varkappa = 1\text{m}{-}1$ means that over some small distance $\Delta s \ll \kappa^{-1}$ along the ray, $\Delta s = 10^{-3}$ m for example, the small fraction $\kappa \Delta s = 1\text{m}^{-1} \times 10^{-3}\text{m} = 10^{-3}$ of the photons will be absorbed.

# Absorption




The fraction of the specific intensity lost to absorption in the infinitesimal distance ds along the ray is

$$\frac{dI_\nu}{I_\nu} = -\kappa\, ds$$

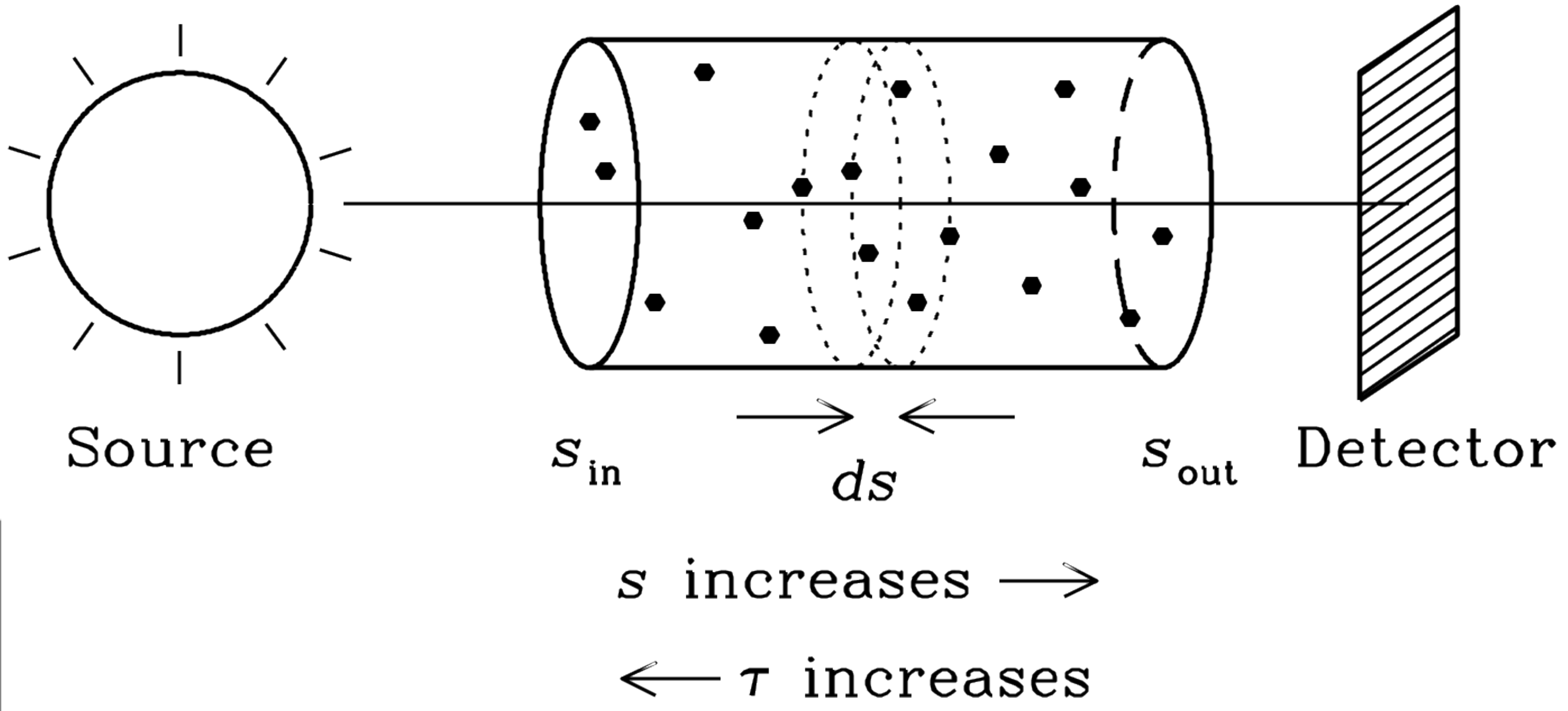
Integrating both sides of Equation along the absorbing path gives the output specific intensity as a fraction of the input specific intensity:

$$\int_{s_{\text{in}}}^{s_{\text{out}}} \frac{dI_\nu}{I_\nu} = -\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa(s')\, ds' = \ln I_\nu \Big|_{s_{\text{in}}}^{s_{\text{out}}},$$

$$\ln[I_\nu(s_{\text{out}})] - \ln[I_\nu(s_{\text{in}})] = -\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa(s')\, ds',$$

$$\frac{I_\nu(s_{\text{out}})}{I_\nu(s_{\text{in}})} = \exp\left[-\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa(s')\, ds'\right].$$

# Absorption




The dimensionless quantity:

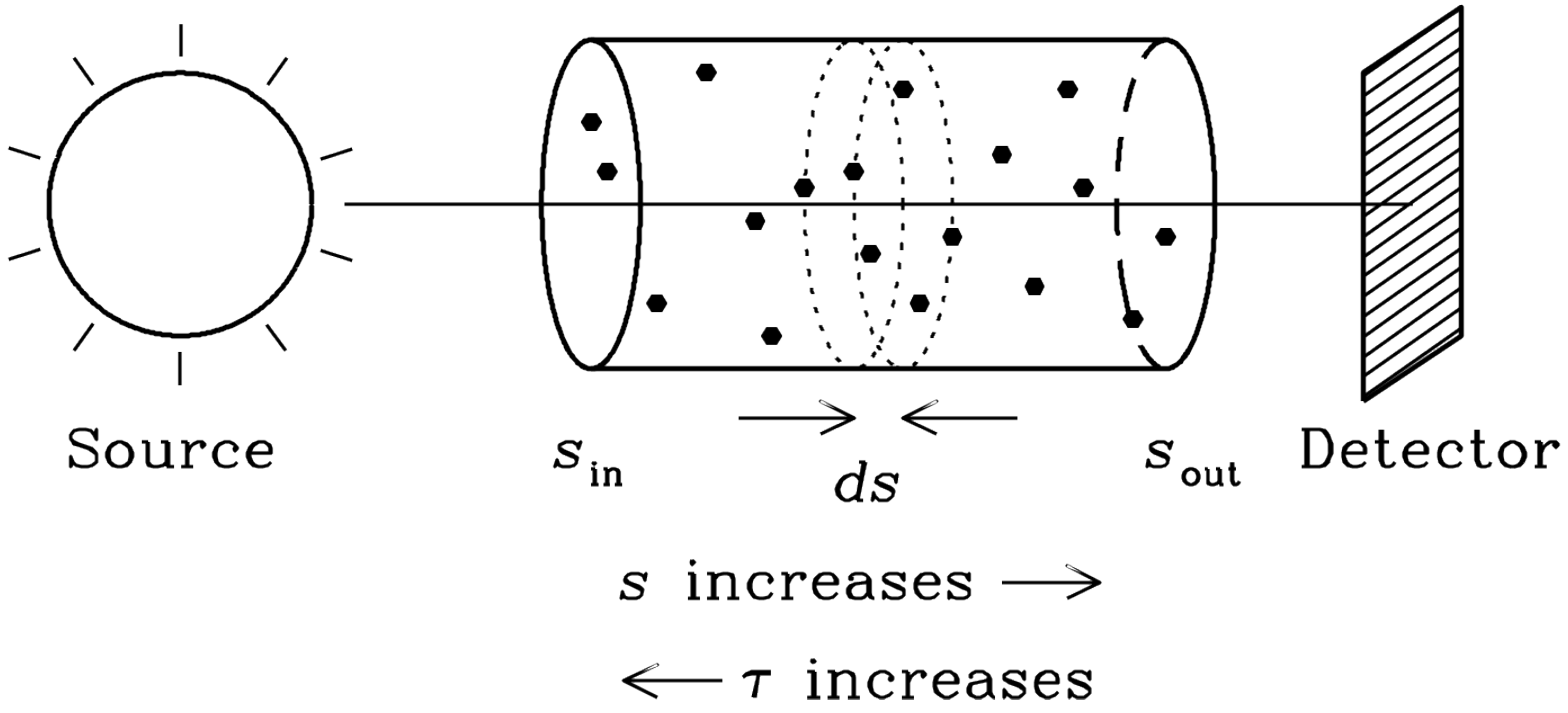
$$\boxed{\tau \equiv -\int_{s_{out}}^{s_{in}} \kappa(s')\, ds'}$$

is called the **optical depth** or opacity of the absorber.
Note that $d\tau = -\kappa ds$. The "backward" direction of the path integration along the line of sight was chosen to make $\tau > 0$ and increasing as you look deeper into an absorber. Thus

$$\frac{I_\nu(s_{out})}{I_\nu(s_{in})} = \exp(-\tau)$$

If $\tau \ll 1$, the absorber is said to be **optically thin;**
if $\tau \gg 1$, it is **optically thick.**

# Emission




The intervening medium may also emit photons, again by some unspecified microscopic process.

In any infinitesimal volume (dsdσ) of thickness ds and cross section dσ, the probability per unit time that an isotropic source will emit a photon into the solid angle dΩ is directly proportional to the volume and solid angle:

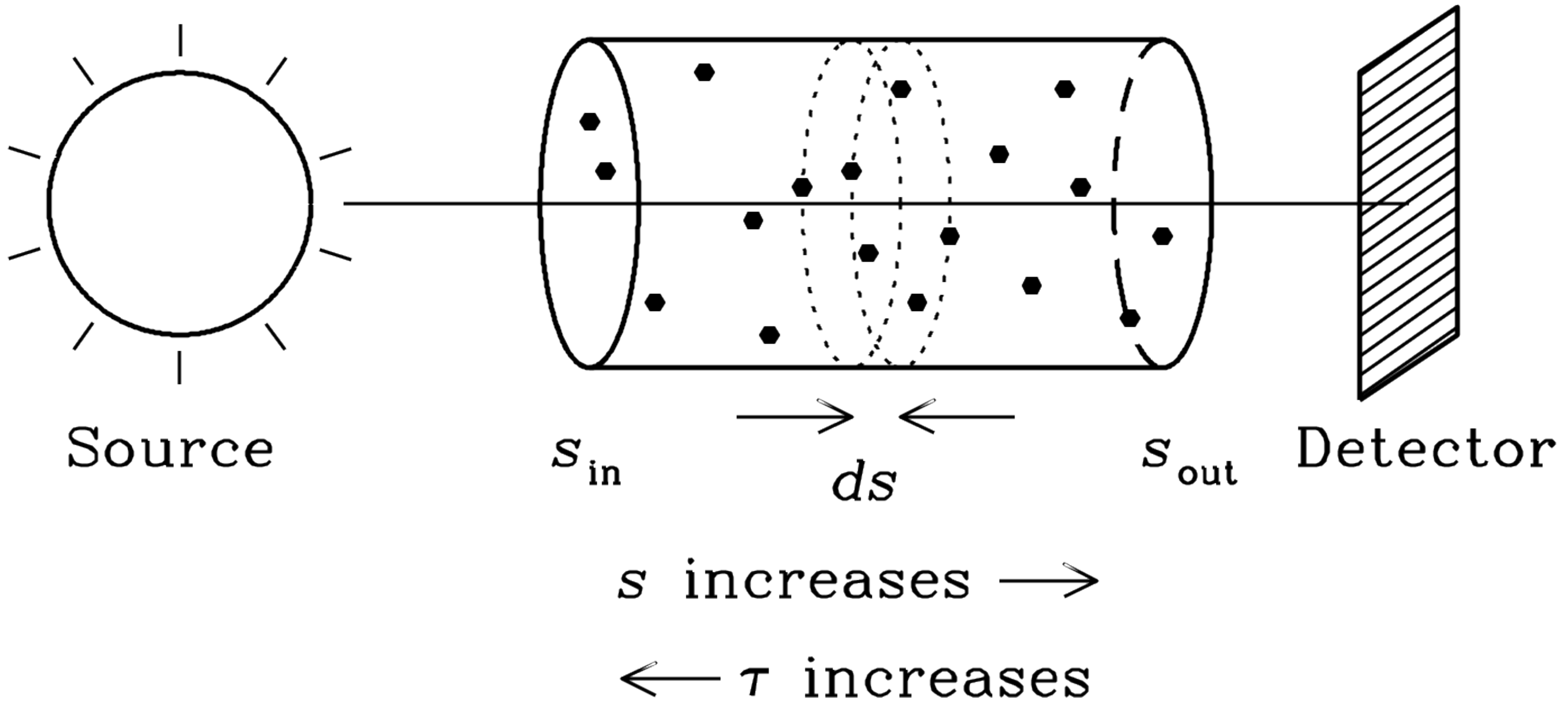
$$\dot{P}_{\text{em}} \propto ds\, d\sigma\, d\Omega.$$

The **emission coefficient** $j_\nu$ is defined so that:

$$\boxed{j_\nu \equiv \frac{dI_\nu}{ds}}$$

if there is no absorption. The dimensions of $j_\nu$ follow from its definition; they are power per unit volume per unit frequency per unit solid angle, and the units are $\mathrm{W m^{-3} Hz^{-1} sr^{-1}}$.

# Emission




Combining the effects of absorption and emission yields the **equation of radiative transfer:**

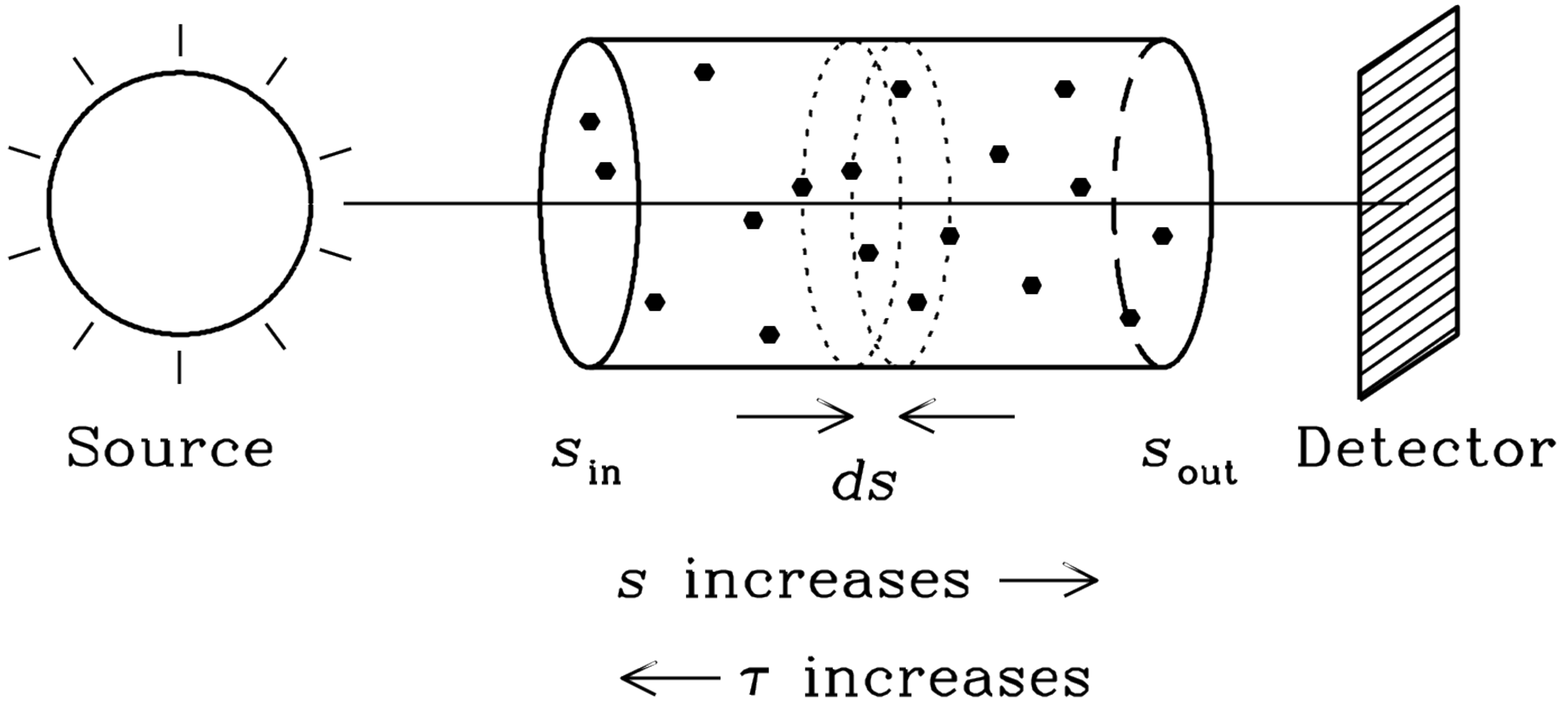
$$\boxed{\frac{dI_\nu}{ds} = -\kappa I_\nu + j_\nu.}$$

In (full) thermodynamic equilibrium (TE) at temperature T:

$$\frac{dI_\nu}{ds} = 0 \quad \text{and} \quad I_\nu = B_\nu(T),$$

where $B_\nu(T)$ is the spectrum of equilibrium radiation (blackbody radiation) at temperature T.

# Emission




Rearranging the equation of radiative transfer:

$$\frac{dI_\nu}{ds} = 0 = -\kappa B_\nu(T) + j_\nu$$

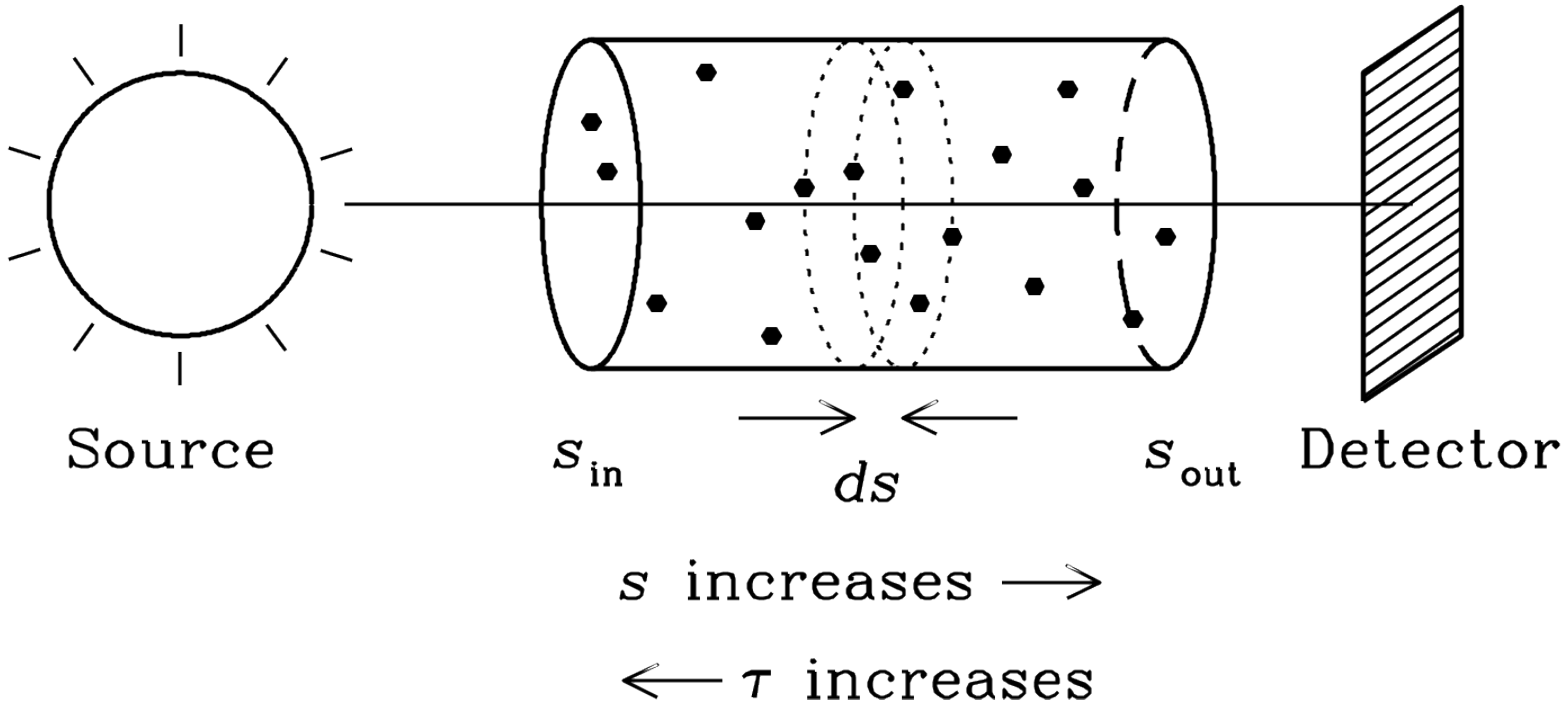
yields **Kirchhoff's law** for a system in TE:

$$\boxed{\frac{j_\nu(T)}{\kappa(T)} = B_\nu(T)\,,}$$

Although Kirchhoff's law was derived for a system in thermodynamic equilibrium, its applicability is not limited to radiation in full thermodynamic equilibrium with its material environment.

**Kirchhoff's law also applies whenever the radiating/absorbing material is in thermal equilibrium**, in any radiation field. If the emitting/absorbing material is in thermal equilibrium at a well-defined temperature T, it is said to be in **local thermodynamic equilibrium (LTE)** even if it is not in equilibrium with the radiation field.

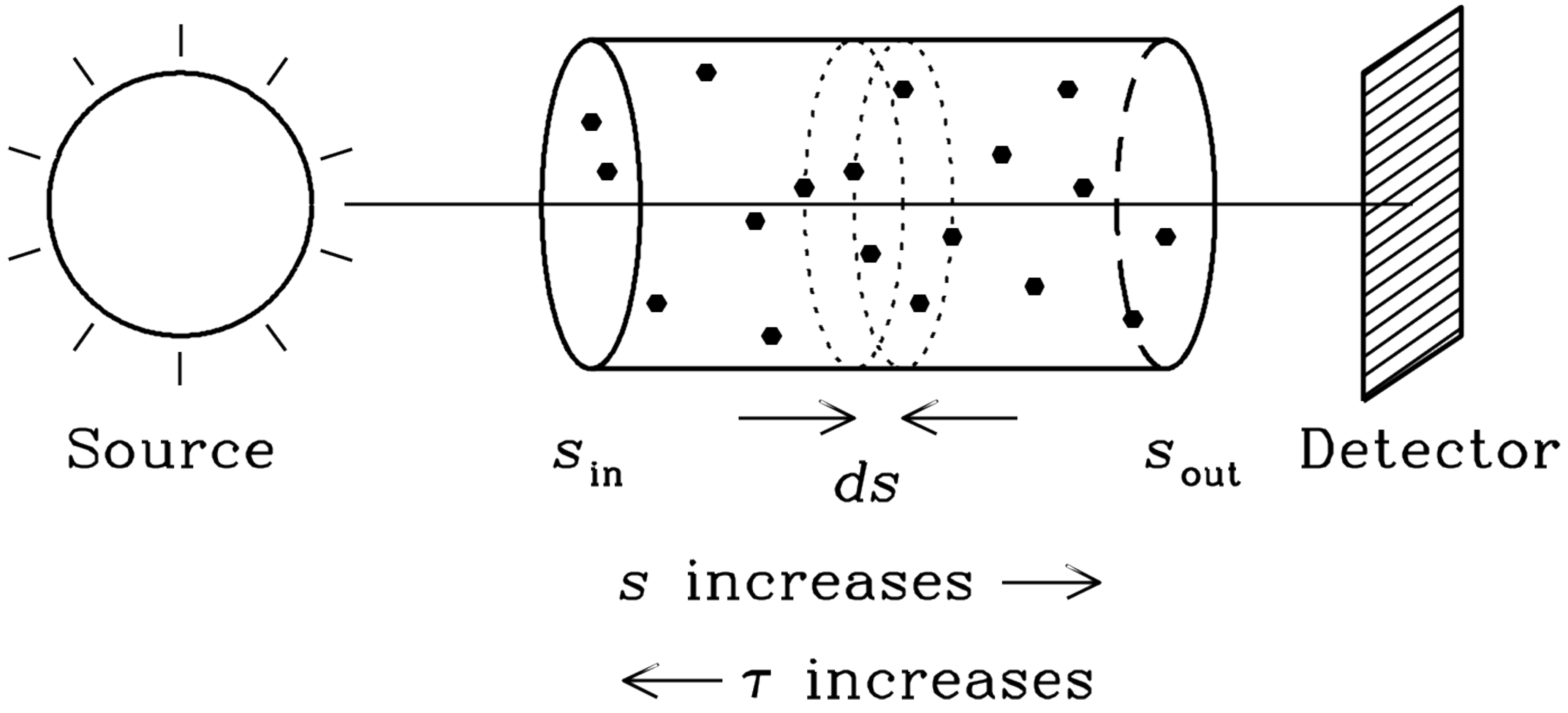
# Emission




**Kirchhoff's law applies in LTE as well as in TE.**

$B_\nu(T)$ is independent of the properties of the radiating/absorbing material. In contrast, both $j_\nu(T)$ and $\kappa(T)$ depend only on the materials in the cavity and on the temperature of that material; they do not depend on the ambient radiation field or its spectrum.

# Emission




Because $B_\nu$ is directly proportional to T in the Rayleigh–Jeans low-frequency approximation

$$B_\nu \approx \frac{2kT\nu^2}{c^2},$$

radio astronomers often find it convenient to specify the spectral brightness $I\nu$, even if $I_\nu \neq B_\nu$, in terms of the equivalent Rayleigh–Jeans **brightness temperature $T_b$** defined by the equation

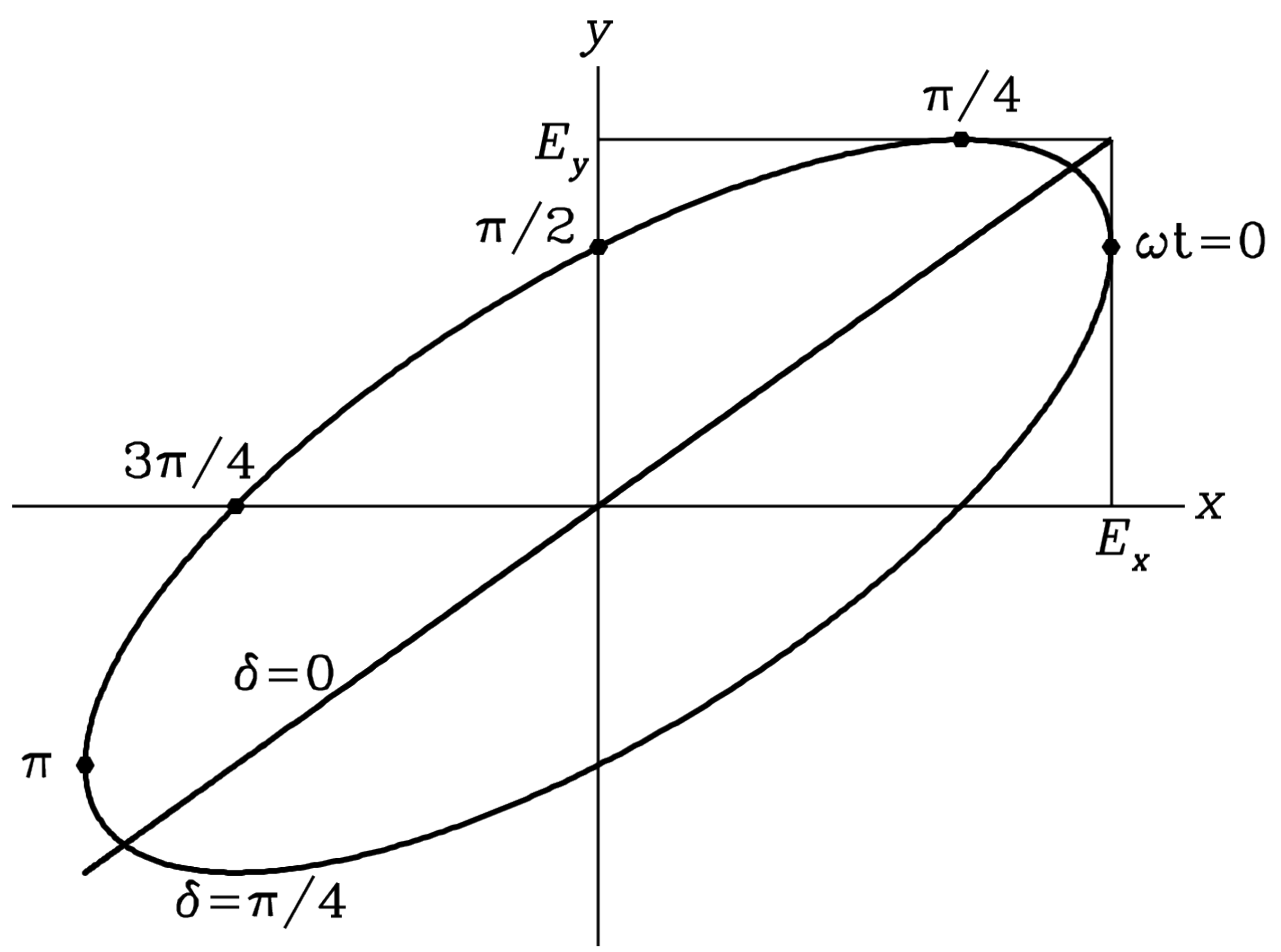
$$I_\nu = \frac{2kT_b\nu^2}{c^2}.$$

$$\boxed{T_b(\nu) \equiv \frac{I_\nu c^2}{2k\nu^2}.}$$

Tb can vary with frequency. Brightness temperature is just another way to specify **power per unit solid angle per unit bandwidth** in terms of the Rayleigh–Jeans approximation.

Beware that brightness temperature is **not the same as physical temperature.**

# Polarisation




The instantaneous transverse electric field $\vec{E}$ of a monochromatic electromagnetic wave traveling in the $\hat{z}$-direction can be projected onto orthogonal $\hat{x}$ - and $\hat{y}$ - (e.g., horizontal and vertical) directions:

$$\vec{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \; \exp[i\,(\vec{k} \cdot \hat{z} - \omega t)],$$

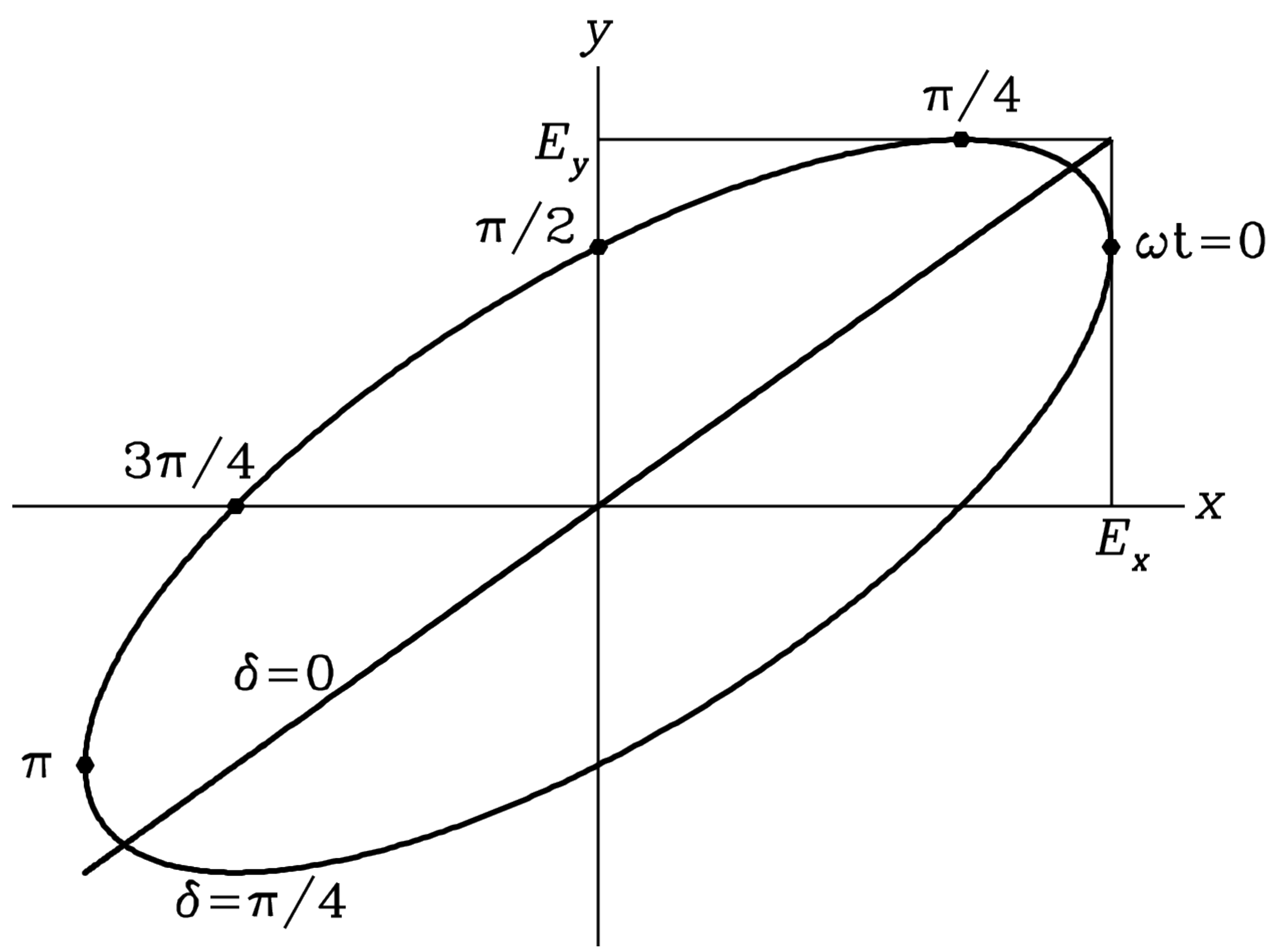
where $k \equiv 2\pi/\lambda$ is the magnitude of the wave vector $\vec{k}$ pointing in the direction of wave travel

$\omega \equiv 2\pi\nu$ Is the angular frequency

$\delta \equiv \phi_x - \phi_y$ is the phase difference between the orthogonal fields $E_x$ and $E_y$, and

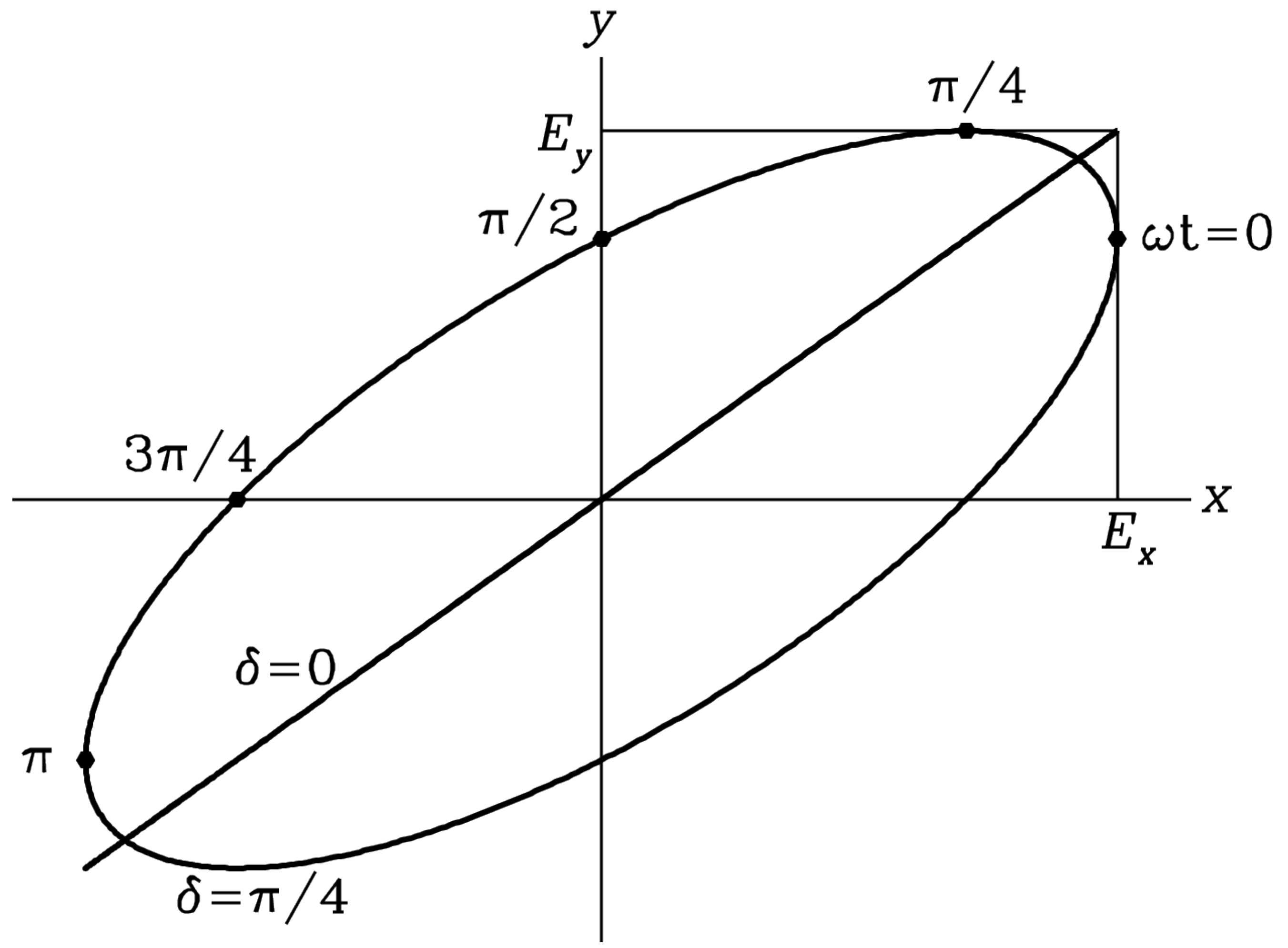
$$E^2 = |\vec{E}|^2 = E_x^2 + E_y^2$$

# Polarisation




- Any time-independent combination of phases and amplitudes yields an **elliptically polarized wave** (Figure) whose electric field vector traces out an ellipse in the (x,y) plane.
- If the phase difference $\delta$ is zero, the electric field vector does not rotate and the wave is **linearly polarized**.
- If $E_x=E_y$ and $|\delta|=\pi/2$, the electric field vector rotates with angular frequency $\omega$ and traces out a circle; such radiation is said to be circularly polarized.
- The sign convention for circular polarization used by the Institute of Electrical and Electronics Engineers (IEEE) and by the International Astronomical Union (IAU) is to call the polarization **right-handed or left-handed** depending on whether the rotation is clockwise ($\delta>0$) or counterclockwise ($\delta<0$), respectively, as viewed from the source toward the observer. An observer looking toward the source sees the electric-field vector from right-handed polarization rotating counterclockwise, as shown by the different time samples $\omega t=0,\pi/4,\pi/2,\ldots$ in the Figure.
- Beware that some optics textbooks use the opposite sign convention.

# Polarisation



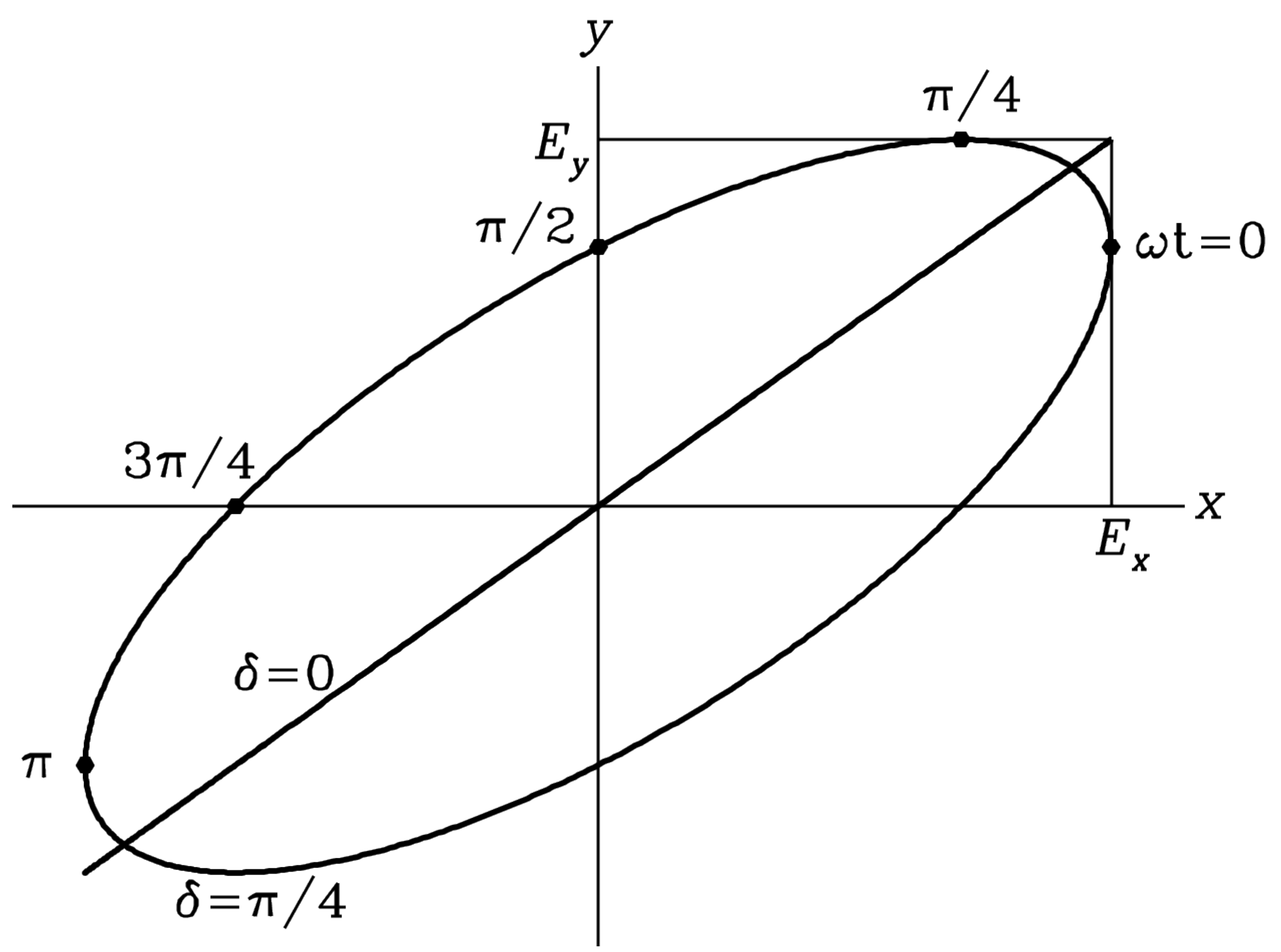

The radiation from astronomical sources is wideband noise whose electric field vector varies rapidly and randomly in amplitude and direction.
If radiation in a unit frequency range $\Delta\nu=\Delta\omega/(2\pi)$ is averaged over timescales $\tau \gg (\Delta\omega)-1$, its average polarization can be characterized by the four **Stokes parameters,**

$$\begin{aligned} I &= \langle E_x^2 + E_y^2 \rangle / R_0, \\ Q &= \langle E_x^2 - E_y^2 \rangle / R_0, \\ U &= \langle 2E_xE_y \cos\delta \rangle / R_0, \\ V &= \langle 2E_xE_y \sin\delta \rangle / R_0, \end{aligned}$$

where the brackets indicate time averages, $R_0$ is the radiation resistance of free space, and I is the total flux density, regardless of polarization.

$$R_0 = \frac{4\pi}{c} = \frac{4\pi}{3\times10^{10}\ \text{cm s}^{-1}} = 4.19\times10^{-10}\ \text{s cm}^{-1}.$$

# Polarisation




The **polarized flux density** is

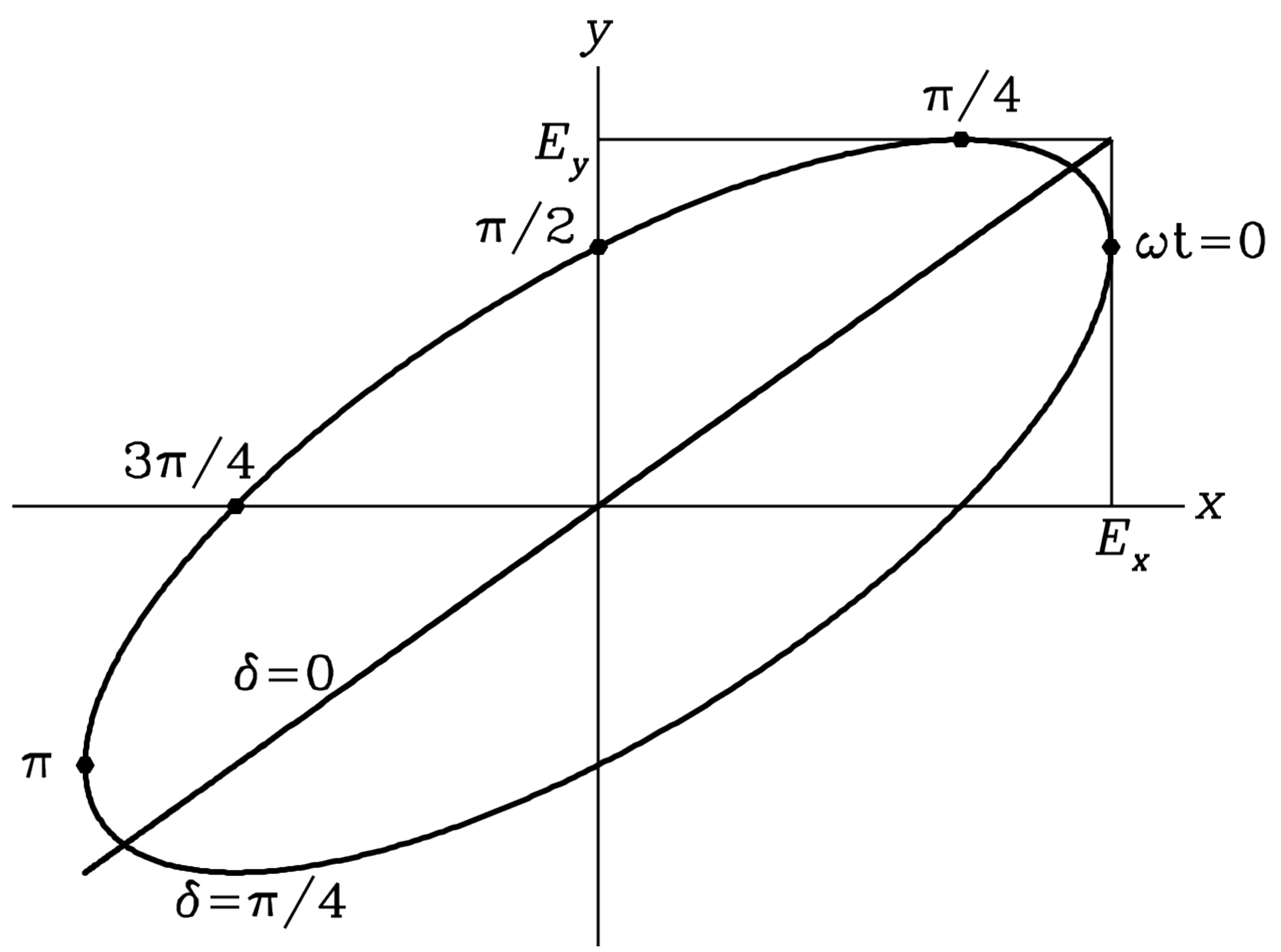
$$I_{\mathrm{p}} = (Q^2 + U^2 + V^2)^{1/2}$$

and the **degree of polarization** is defined as

$$p \equiv \frac{I_{\mathrm{p}}}{I}.$$

- If $\langle E_x \rangle$ and $\langle E_y \rangle$ have equal amplitudes and their phases are completely uncorrelated, then Q=U=V=0, Ip=0, p=0, and the wave is said to be unpolarized.
- For example, blackbody radiation is unpolarized.
- An antenna sensitive only to one polarization (e.g., a dipole antenna oriented so it is sensitive only to the $\hat{x}$ component of linear polarization or a helical antenna sensitive only to the left-handed component of circular polarization) will detect only half the power radiated by an unpolarized source.
- **Two orthogonally polarized antennas** (e.g., dipoles aligned in the $\hat{x}$ - and $\hat{y}$ -directions or left-handed and right-handed helical antennas) **are needed to collect all of the unpolarized power.**

# Polarisation




- **Many astronomical sources are partially polarized** with $0<p<1$.
- The quantity $(Q^2+U^2)^{1/2}$ measures the **linearly polarized component of flux**, and the ratio Q/U depends on the **linear polarization position angle.**
- The **circularly polarized flux** is given by |V|, with V>0 indicating right-handed and V<0 indicating left-handed circular polarization.