

Einsteins theory of relativity

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Special relativity

- ▶ The ether and Maxwells equations
- ▶ Two postulates
 - ▶ Laws of physics are the same in all inertial reference frames
 - ▶ Speed of light is constant
- ▶ Ground-breaking new ideas
 - ▶ Space and time is not independent
 - ▶ Simultaneous depends upon reference frames
 - ▶ Synchronize if: $t_b - t_a = t_c - t_b$

Special relativity

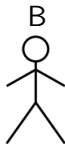
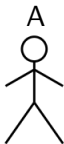
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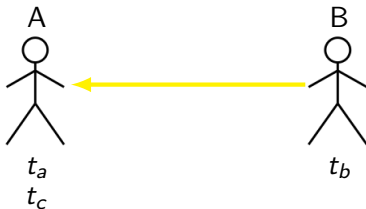
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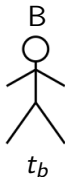
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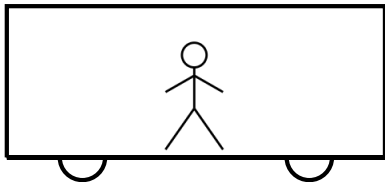
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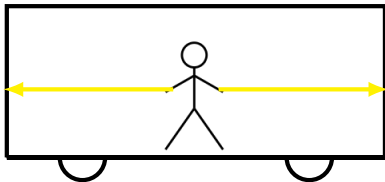
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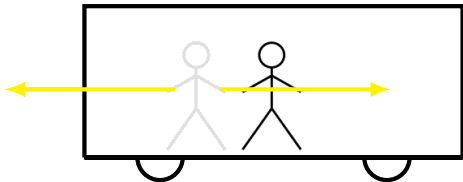
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Lorentz transformation

Assuming the two postulates, the transformations between two inertial reference frames moving with a velocity v relative to each other in the x -direction is given by the Lorentz transformations.

$$t' = \gamma(t - vx/c^2)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

where :

c = Speed of light

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Consequences of special relativity

$$v = 0.87c$$

$$\gamma = 2$$

x_1 = Back of the carpet

x_2 = Front of the carpet

t_1 = Start of flight

t_2 = End of flight

Flying on the
magic carpet
is the primed
reference frame



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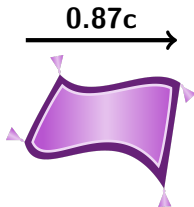
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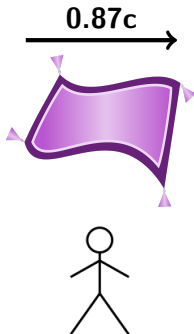
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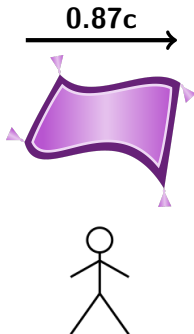
$$x'_2 - x'_1 = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1)$$

$$t_1 = t_2$$

$$\Delta x' = \gamma \Delta x$$

$$\Delta x = \frac{\Delta x'}{\gamma}$$

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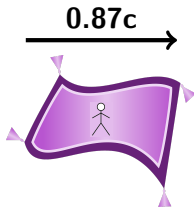
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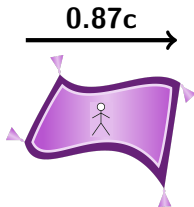
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$$x_1 = vt_1, \quad x_2 = vt_2$$

$$t'_2 - t'_1 = \gamma t_2(1 - \frac{v^2}{c^2}) - \gamma t_1(1 - \frac{v^2}{c^2})$$

$$\Delta t' = \frac{1}{\gamma} \Delta t$$



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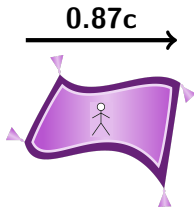
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My two hour
ride was only
one hour(:

In the real world

- ▶ Atomic clocks in flights
- ▶ Muons actually reach the earth
- ▶ Particle accelerators



Picture by Dave L. Jones

In the real world

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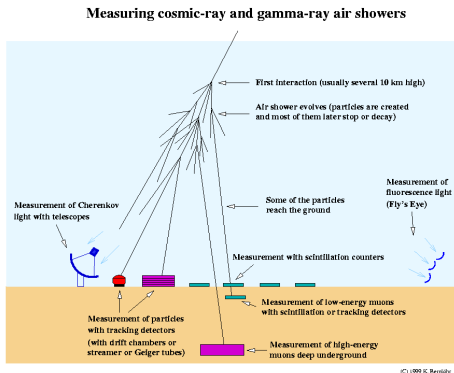


Figure by K. Bernikör

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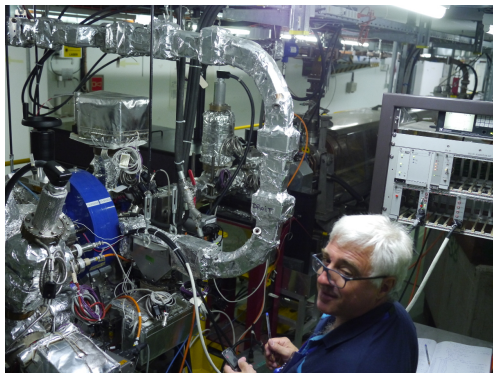


Photo by K.Sjøbæk

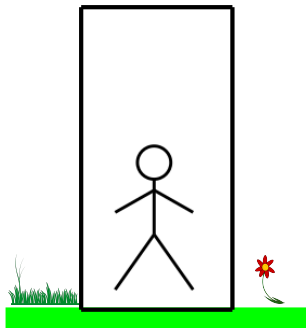
General relativity

Is gravity acceleration

- ▶ In 1907 Einstein had the “happiest thought of his life”
- ▶ There is no difference between being in an accelerated elevator in space or being in a gravitational field
- ▶ This was a continuation of the the universality of free fall

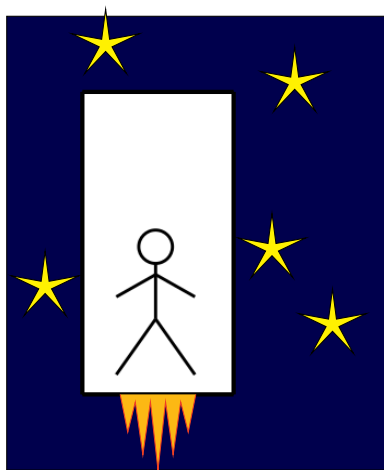
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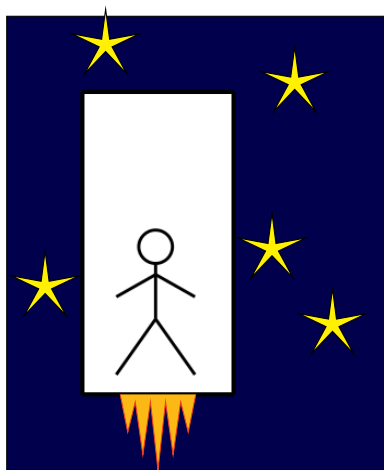
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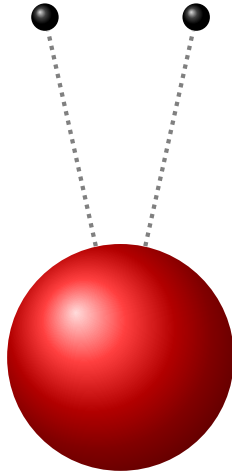


Is gravity a geometrical effect

- ▶ The equivalence principle is only valid locally
- ▶ Globally there is tidal forces
- ▶ Can gravity be a geometrical effect

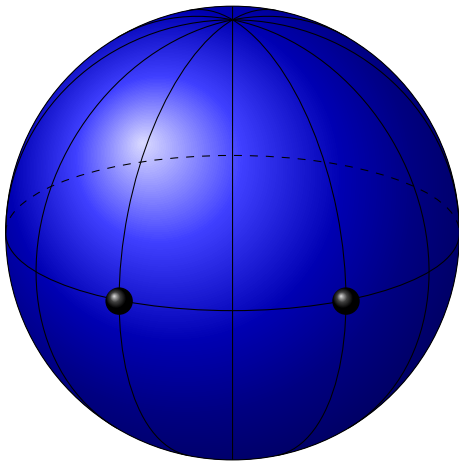
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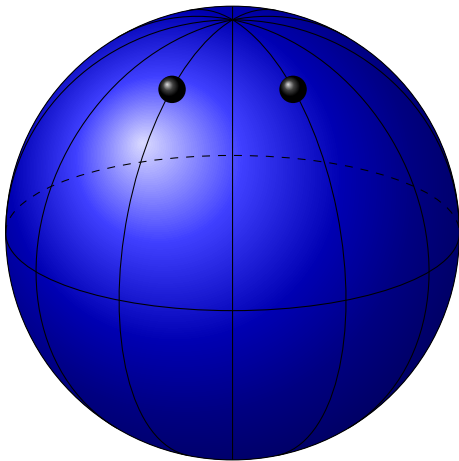
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A geometrical model for gravity

- ▶ General relativity describes spacetime as a manifold
- ▶ A manifold can describe a space that locally is flat, but is curved on larger scale
- ▶ The surface of a sphere:
 - ▶ Curved on larger scale
 - ▶ Locally flat
- ▶ Spacetime according to general relativity
 - ▶ Locally it is a free falling coordinate system, the weak equivalence principle holds
 - ▶ Curved spacetime on larger scale
- ▶ Describing spacetime as a manifold gave the theory a mathematical framework (tensor analysis)

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Einsteins field equations

Field equations

- ▶ Left side describes the curvature of space
- ▶ Right side describes the content of space
- ▶ The metric, $g_{\mu\nu}$, is what we want to solve for
- ▶ The metric is the input to the geodesic equation
- ▶ Only solvable in special cases, for instance the Schwarzschild solution

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$R_{\mu\nu}$ = Ricci curvature tensor($f(g_{\mu\nu})$)

$g_{\mu\nu}$ = The metric

Λ = Cosmological constant

$T_{\mu\nu}$ = Stress-energy tensor

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The geodesic equation

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\beta\alpha} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

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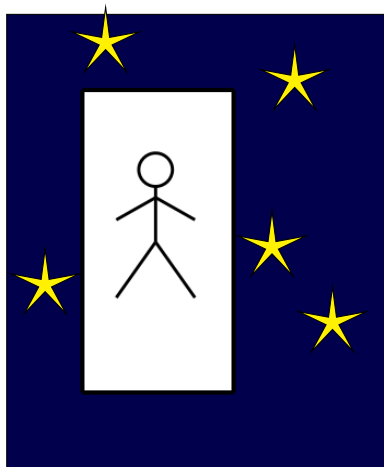
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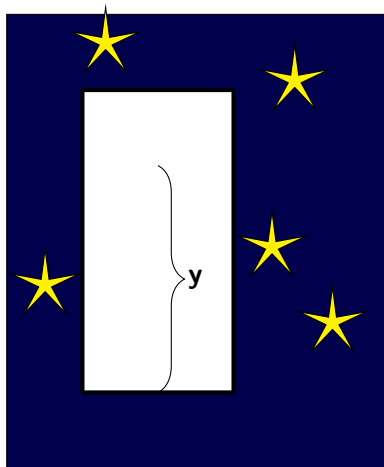
Gravitational redshift

- ▶ Line segment λ
- ▶ $y'(y + \lambda) = y + \lambda + \frac{g}{2} \left(\frac{y+\lambda}{c} \right) - y + \frac{g}{2} \left(\frac{y}{c} \right) = \lambda + \frac{gy\lambda}{c^2} + \frac{1g\lambda^2}{2c^2} \approx \lambda + \frac{gy\lambda}{c^2}$
- ▶ As seen the the wavelength increases as a function of y , this is the gravitational redshift
- ▶ The frequency of the lights can not be changed, because then the light would be fill up the elevator, more lightwaves in than out
- ▶ In order to get all the waves out of the elevator the time also has to move faster at



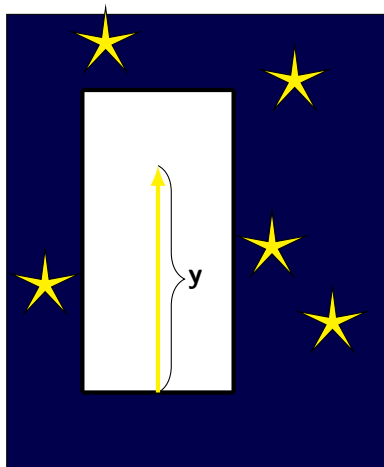
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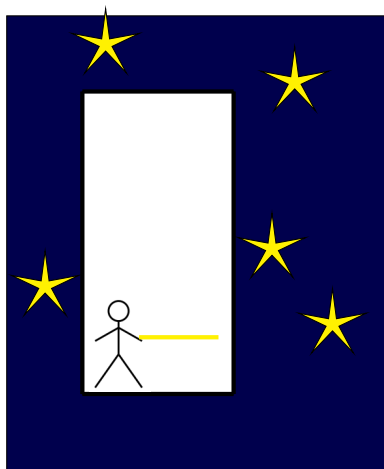
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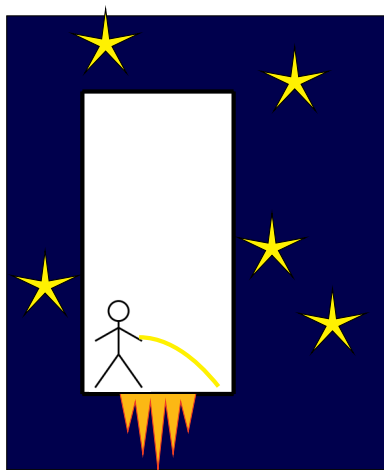
Gravity and light

- ▶ The equivalence principle predicts that light is being bent by gravitational fields
- ▶ What happens to position of stars
- ▶ This can be tested, when there is a solar eclipse



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Solar eclipse

- ▶ Light is bent by the sun
- ▶ Wrong position of stars
- ▶ Only possible to see during a solar eclipse
- ▶ Solar eclipse 1919
- ▶ No bending, Newton bending or general relativity bending
- ▶ GR bending predicted by the Schwarzschild solution
- ▶ Turned out that GR was right

