Counting sort

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COUNTING-SORT(A, B, k)
 1  let C[0..k] be a new array
 2  for i = 0 to k
 3      C[i] = 0
 4  for j = 1 to A.length
 5      C[A[j]] = C[A[j]] + 1
 6  // C[i] now contains the number of elements equal to i.
 7  for i = 1 to k
 8      C[i] = C[i] + C[i − 1]
 9  // C[i] now contains the number of elements less than or equal to i.
10  for j = A.length downto 1
11      B[C[A[j]]] = A[j]
12      C[A[j]] = C[A[j]] − 1
```

Hvordan ser C ud efter vi har akkumuleret?

$A = [4, 3, 4, 5, 1, 1, 5, 1, 3]$, $k = 5$
Hvad er C i linje 9?

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```

$[0, 3, 0, 2, 2, 2]$ A

$[0, 3, 3, 5, 7, 9]$ B

$[0, 0, 3, 3, 5, 7]$ C

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$[0, 3, 3, 5, 9, 7]$ D

$[1, 3, 3, 5, 7, 9]$ E

Tidskompleksitet

$n = A.length$

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I alt: $\Theta(n + k)$.

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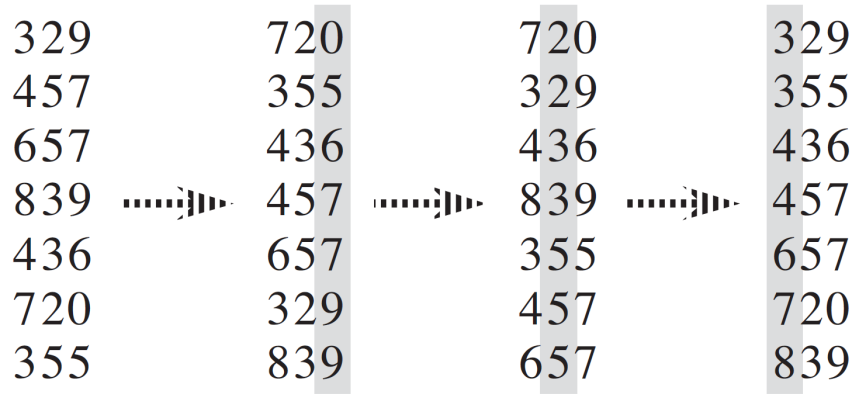
I alt: $\Theta(n + k)$.

Hvis $k = O(n)$: Sortering i $\Theta(n)$ tid!

Radix sort



RADIX-SORT(A, d)

1 **for** $i = 1$ **to** d

2 use a stable sort to sort array A on digit i

Radix sort

1001
1101
0100
1110
0101
0011

Hvad er rækkefølgende efter sortering af de to sidste cifre?

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1 **for** $i = 1$ **to** d

2 use a stable sort to sort array A on digit i

A
0100
1110
1001
1101
0101
0011

B
1001
1101
0101
1110
0011
0100

C
1001
0011
0100
1101
0101
1110

D
0100
1001
1101
0101
1110
0011

E
1001
1101
0100
1110
0101
0011

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Eksempel

Eksempel: Antag at vi vil sortere n binære tal fra mængden $\{0, 1, \ldots, n^4\}$.

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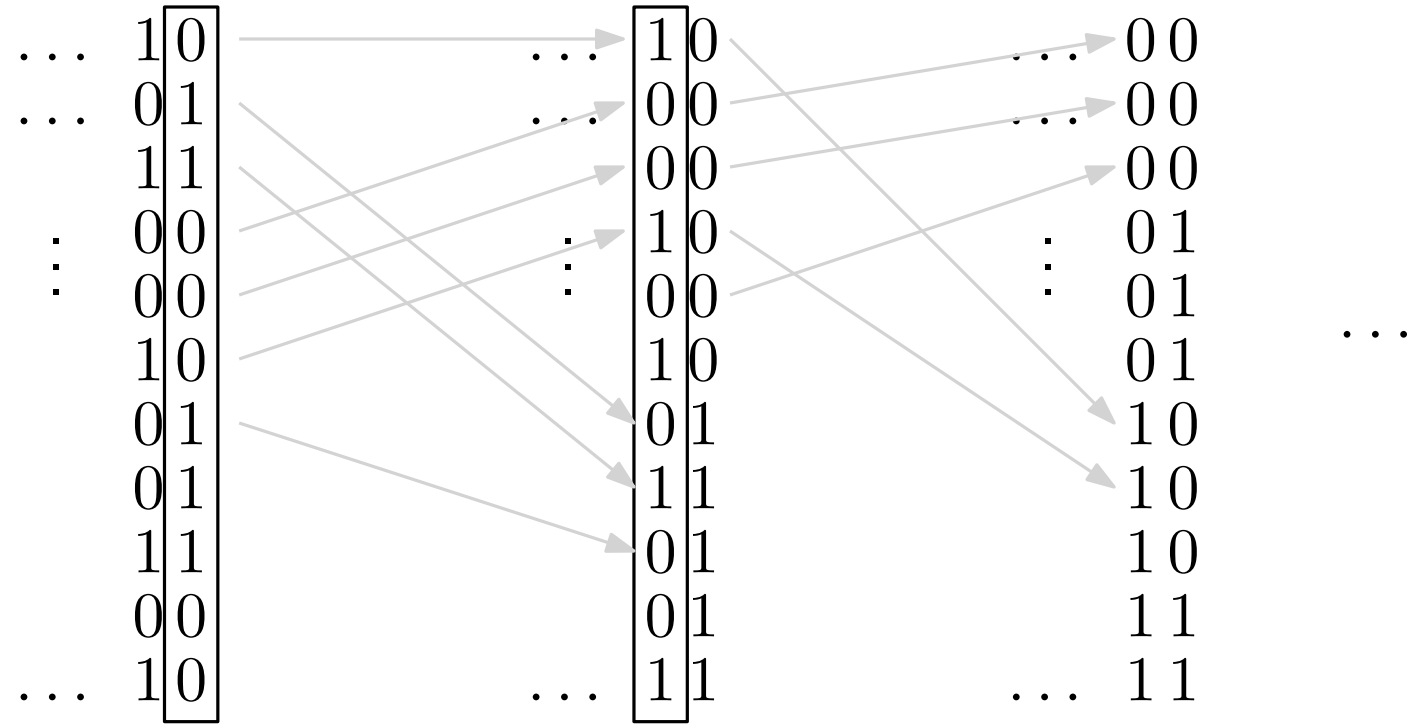

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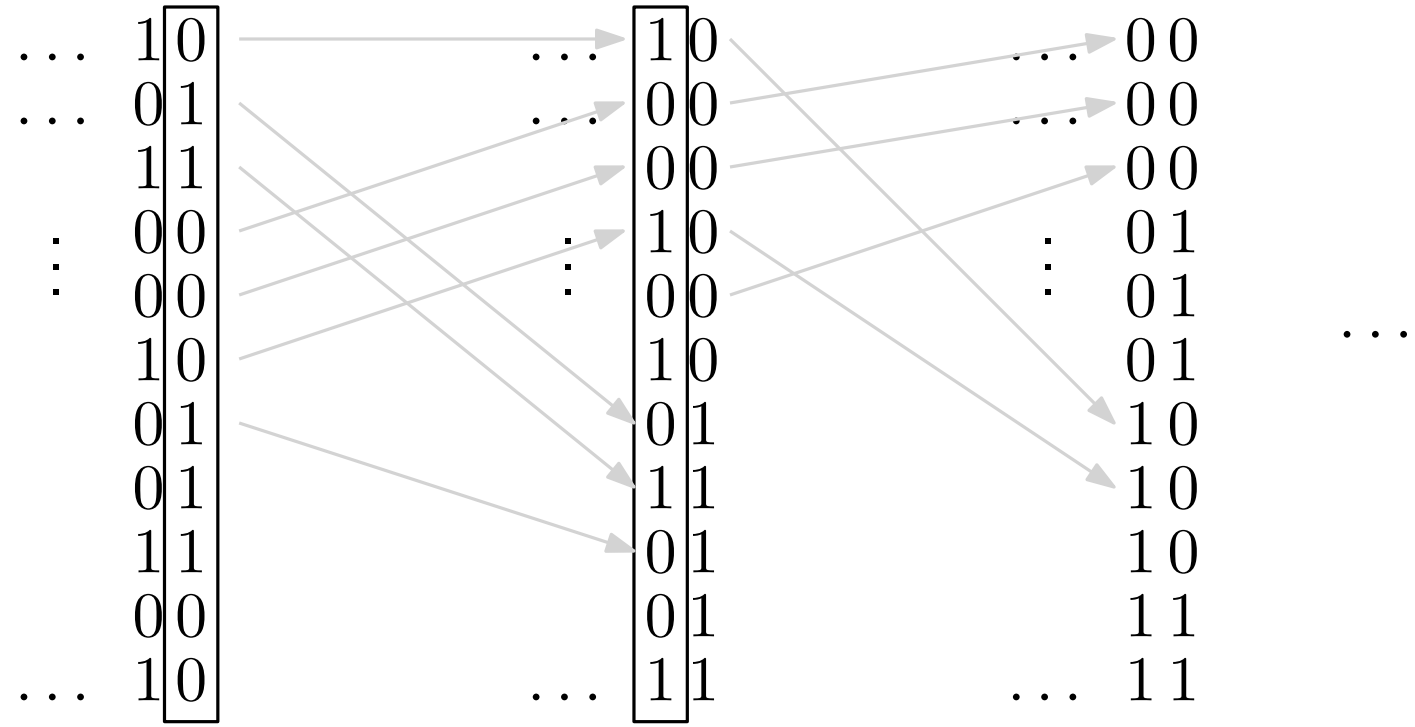

#bits $\approx \lg(n^4) = 4 \lg n = \Theta(\log n)$. Tid: $\Theta((n + 2) \log n) = \Theta(n \log n)$.

Korrekt formel: #bits i $x = \lceil \lg(x + 1) \rceil$.

Eksempel, fortsat

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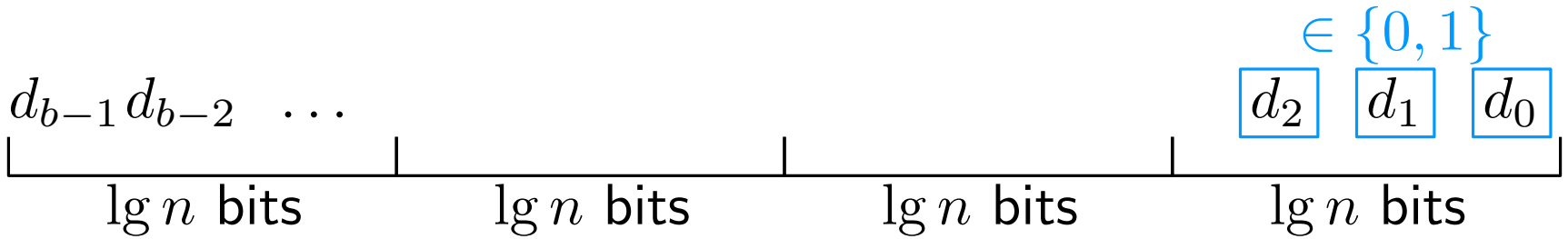
Tid: $\Theta(n + k) = \Theta(n^4)$.

Forsøg 2: Vi bruger radix sort på hver bit, dvs. $k = 2$. #bits $\approx 4 \lg n$.
Tid: $\Theta(n \log n)$.

Forsøg 3: Vi inddeler bits i blokke af $\lg n$ bits og bruger radix sort på hver blok.



Antal muligheder for hver blok: $2^{\lg n} = n$. Dvs. $k = n$.

Tid: $O((n + k) \cdot 4) = O(n)$.

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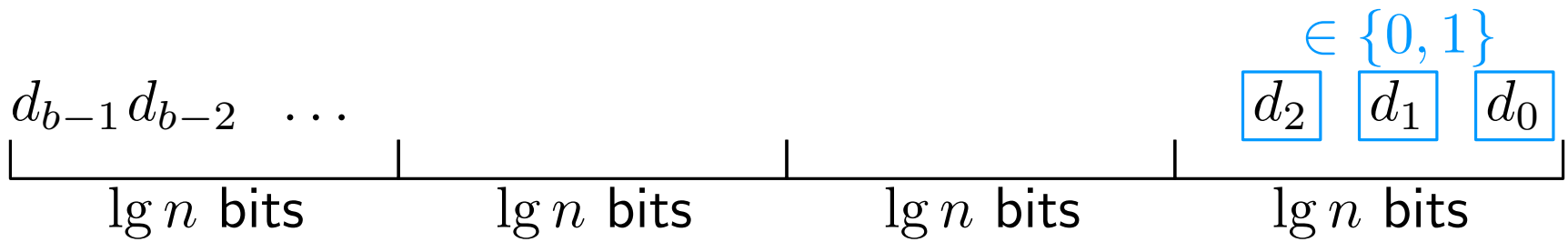

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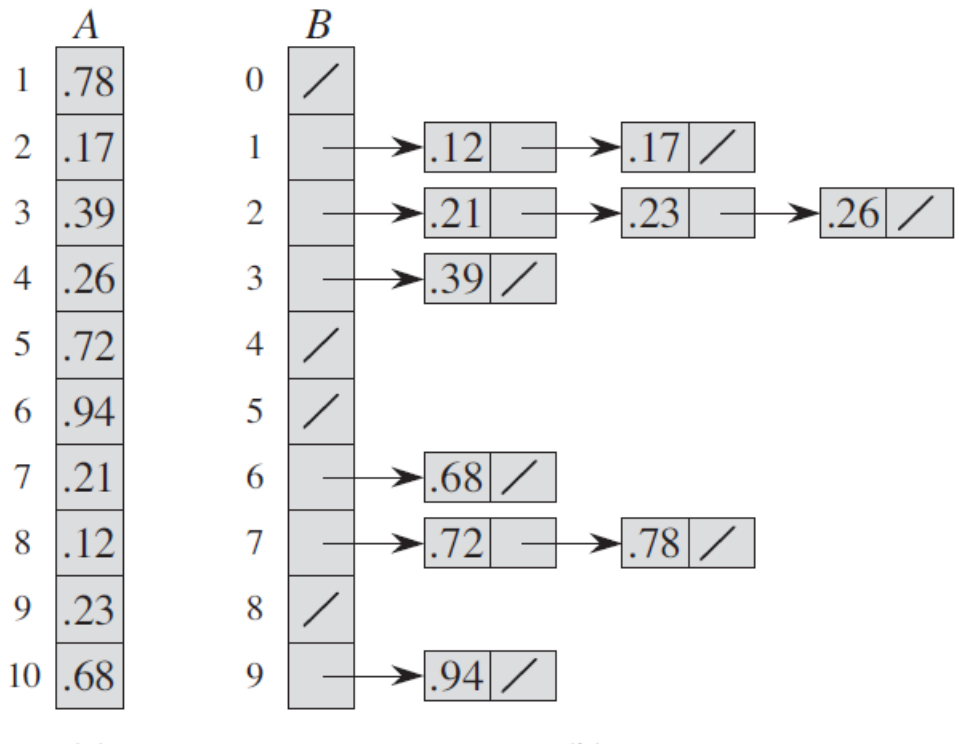
Lemma 8.4

Given n b-bit numbers and any positive integer $r \leq b$, RADIX-SORT correctly sorts these numbers in $\Theta((b/r)(n + 2^r))$ time if the stable sort it uses takes $\Theta(n + k)$ time for inputs in the range 0 to k.

Bucket sort



BUCKET-SORT(A)

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2 $n = A.length$
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6 insert $A[i]$ into list $B[\lfloor n\,A[i] \rfloor]$
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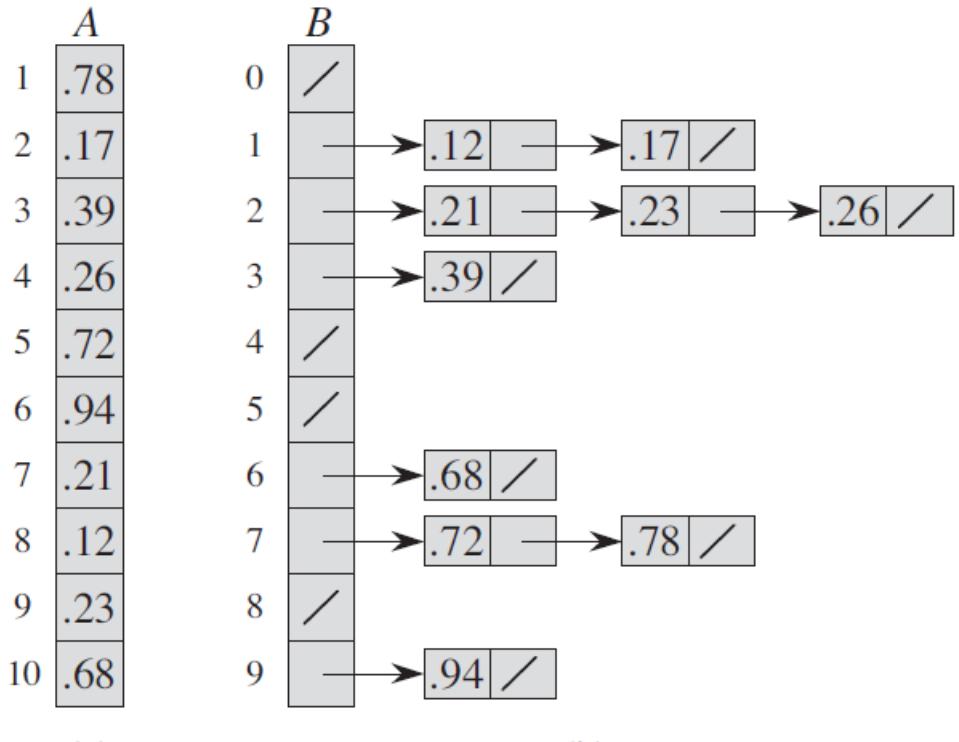
8 sort list $B[i]$ with insertion sort
9 concatenate the lists $B[0], B[1], \ldots, B[n-1]$ together in order

Bucket sort

Køretid:

$\Theta(n) + \Theta(n_0^2 + n_1^2 + \ldots + n_{n-1}^2)$



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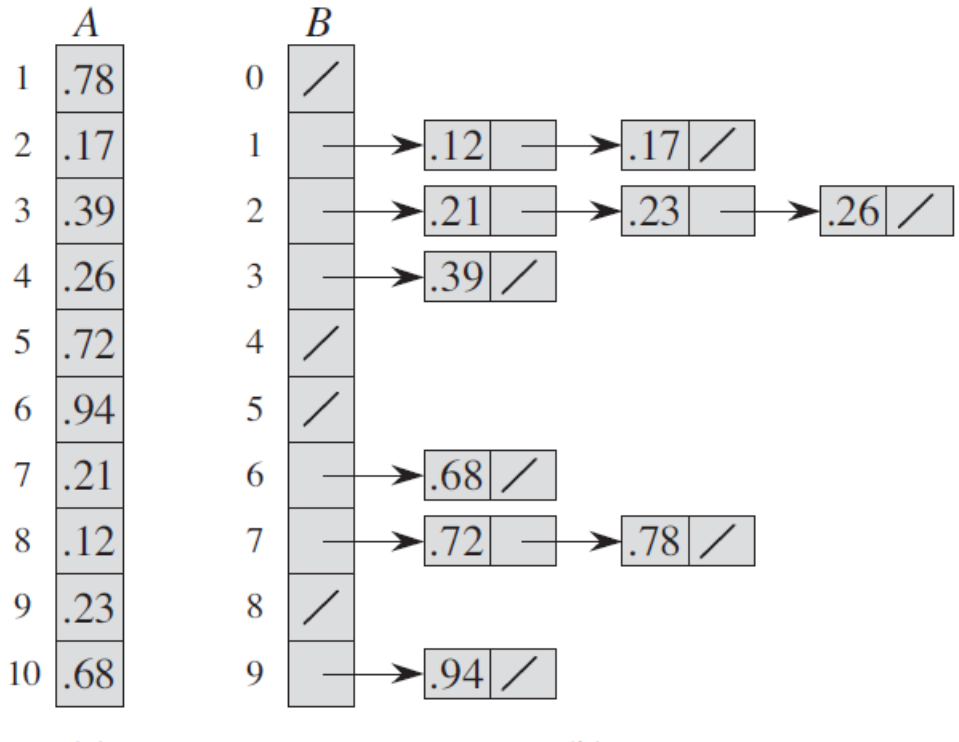

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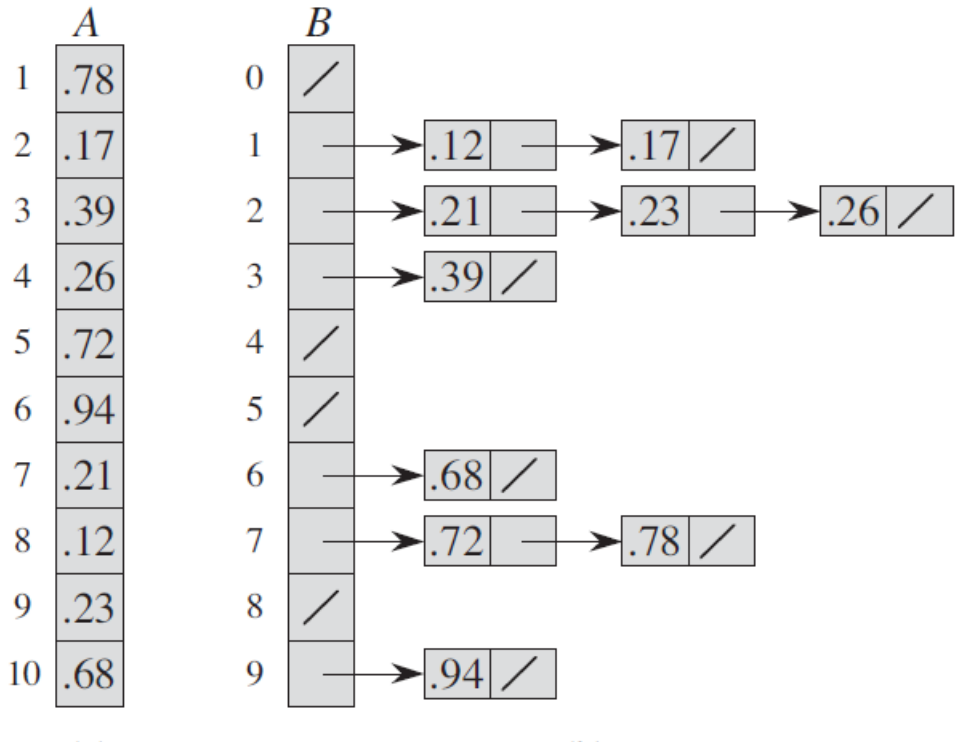

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Tal i A uafhængige og uniformt tilfældige i $[0, 1)$:
$\mathrm{E}[n_i^2] = 2 - 1/n$

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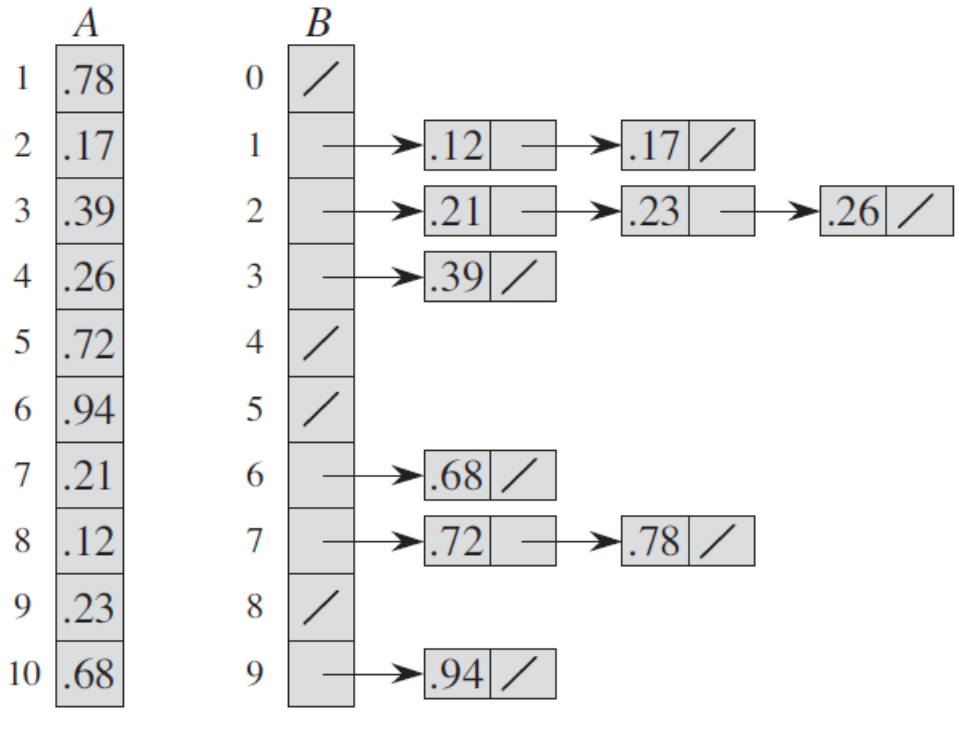

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$\Theta(n) + \Theta(n \cdot (2 - 1/n)) = \Theta(n)$.

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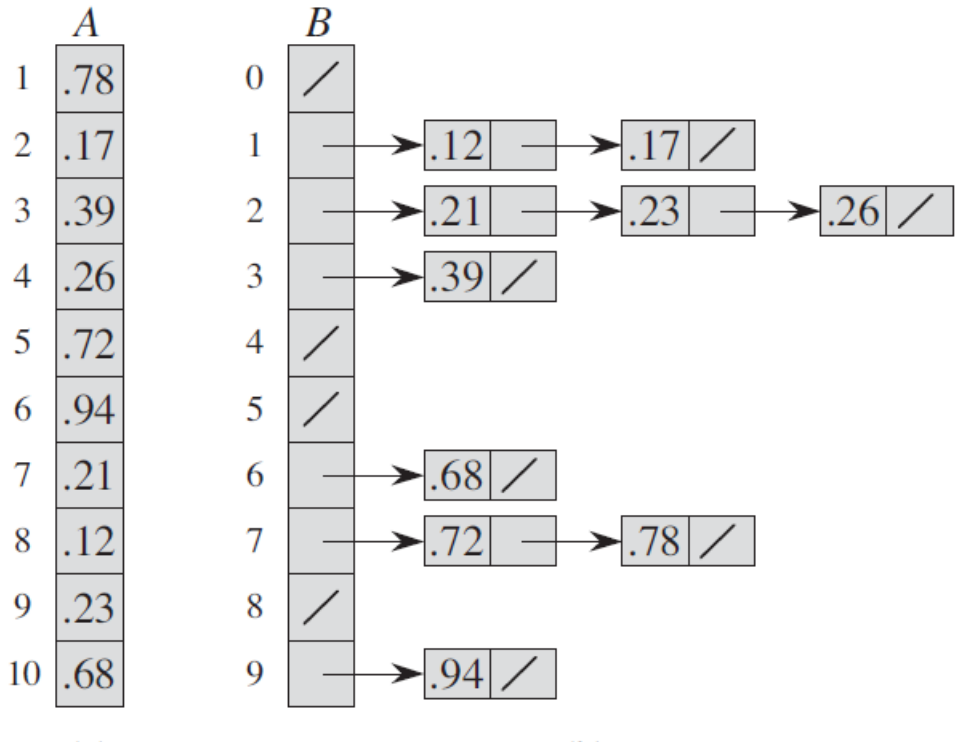

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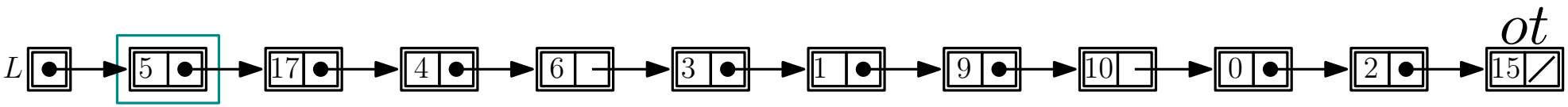
Hvorfor insertion sort?
Hvorfor hægtede lister?

Insertion sort af enkelthægtet liste



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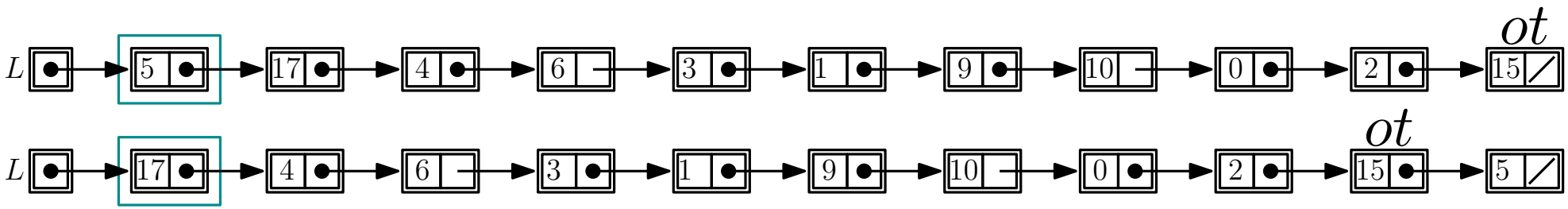

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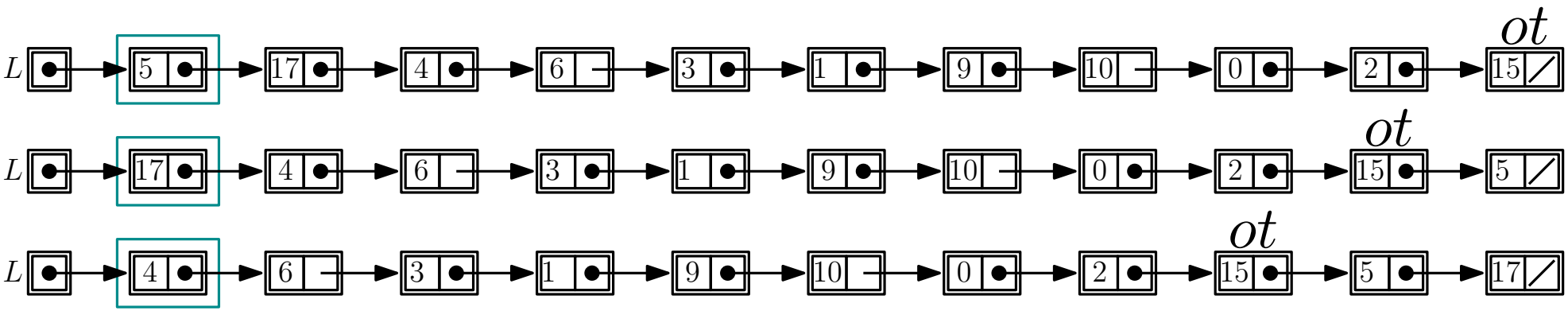

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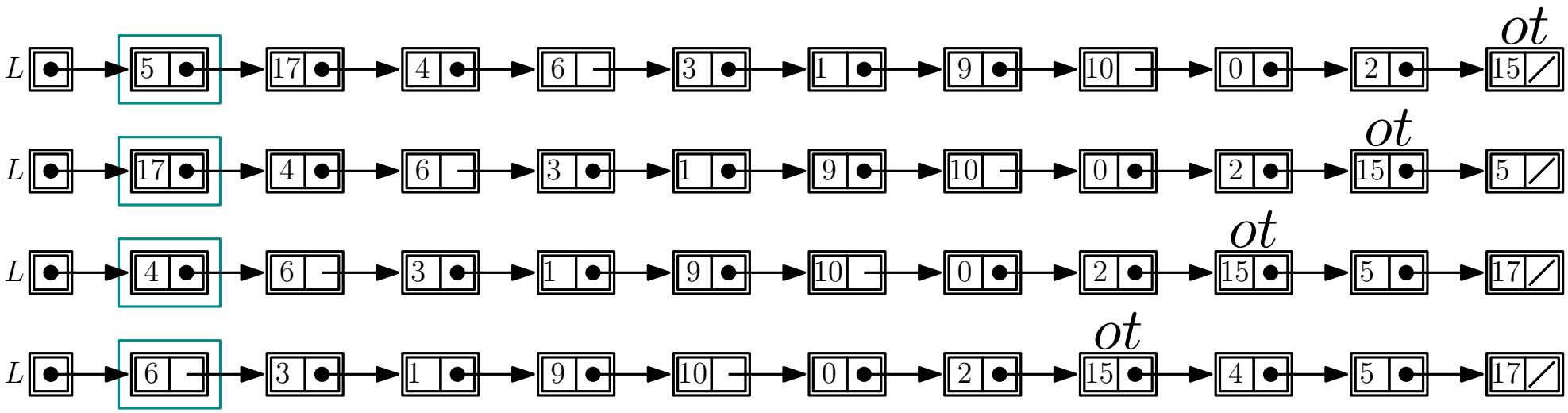

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