

## DMA 2021

### – Week 8 –

## Work instructions

Starting from KBR 3.1–4, we are going to discuss **combinatorics**, which will allow us to answer questions of the form

- In how many ways can one draw 5 cards from a set of 52 different cards?
- What happens to the number if the order of the cards matter?

Starting from such combinatorial questions, we are going to introduce **probabilities** so that we can answer questions like

- What is the probability that two of the cards that are drawn are spades?

We will see that it is essential to differentiate between combinatoric questions where the elements are ordered and questions where the order doesn't matter.

We are also going to look at the **pigeonhole principle**, which simply says that if you put more than  $k$  objects in  $k$  containers then there will be more than one object in at least one of the containers.

## Assigned reading

- KBR 3.1–3.4.

## Lecture plan

### Monday Nov. 1st 09:00–09:45

Multiplication principle. Permutations & combinations. With and without repetitions. Ordered and unordered. (KBR 3.1–3.2).

### Tuesday Nov. 2nd, 13:00–14:45

- Examples of counting problems. The pigeonhole principle. Summary of combinatorics. (KBR 3.1–3.3).
- Basic probability theory (KBR 3.4)

## Friday Nov. 5th, 09:00–09:45

Basic probability theory, counting problems. Examples.

## Exercise plan

### Monday Nov. 1st, 10:15–12:00

- Solve KBR exercises 3.1.1, 3.1.4, 3.1.8, 3.1.9
- Solve KBR exercises 3.1.33 and 3.1.34
  - [\*] Write pseudocode for the procedure (algorithm) you came up with in 3.1.34 for counting the number of zeroes at the end of  $n!$
  - Analyze the runtime of your algorithm in terms of the input  $n$  and express it using the big- $O$  notation.
  - [\*\*] Can you find an algorithm that runs in time  $O(\log(n))$ ?
- Solve KBR exercises 3.2.1, 3.2.2, 3.2.6<sup>1</sup>, 3.2.25, 3.2.26, 3.2.33.

### Tuesday Nov. 2nd, 15:15–17:00

Solve KBR exercises

- 3.1.22, 3.1.31, 3.2.17<sup>2</sup>, 3.3.6, 3.3.23, 3.4.1, 3.4.12, 3.4.20, 3.4.33, 3.4.34.
- KBR 3.4.36 (part (c) is [\*\*])

### Friday Nov. 5th, 10:15–12:00

- Continue the discussion from the lecture regarding the fairness of the rules of poker by (calculating and) comparing the associated probabilities that we did not already compute in class. The hierarchy is as follows

(1) Straight flush	(4) Flush	(7) Two pairs
(2) Four of a kind	(5) Straight	(8) A pair
(3) Full house	(6) Three of a kind	

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<sup>1</sup>Assume that a person can be designated as the residential advisor for *at most* one floor. Note that it only matters which pairs have been chosen and it is irrelevant to which floors they have been assigned to.

<sup>2</sup>Assume that only one copy of each book can be chosen.

- Finish any leftover exercises from the previous exercise classes of this week.