DMA 2021

– Week 13 –

Work instructions

We continue our discussion of *relations* and consider two important types: **order relations** and **trees**. The former is relevant in computer science because this type of relation can, for example, be used to sort data, as we will see. The latter requires no further introduction since you have already worked with trees in connection to algorithms. But we will now treat them in a bit more systematic manner and also see how to describe them mathematically. Some algorithmic discussions regarding so called **spanning trees** are covered in KBR section 7.4 but we will only briefly cover these. You will read a similar section about spanning trees in CLRS in the new year.

Towards the end of the week, we will have a look at **undirected** graphs and relate them to what we have been discussing during the week.

Assigned reading

- KBR 6.1–6.2
- KBR 7.1 and 7.4
- KBR 8.1

Lecture plan

Monday Dec. 13th 09:00–09:45

Order relations. Posets and Hasse diagrams. Extremal elements and upper/lower bounds. (KBR 6.1–6.2).

Tuesday Dec. 14th, 13:00–14:45

Big-Θ as an equivalence relation. Big-O as an order relation. Topological sorting. Directed (oriented) and undirected trees as relations (KBR 7.1 and 7.4).

Friday Dec. 17th, 09:00–09:45

Graphs. Components. Valency. Subgraphs and quotient graphs. (KBR 8.1)

Exercise plan

Monday Dec. 13th, 10:15–12:00

- Solve KBR exercises 6.1.2, 6.1.6, 6.1.9, 6.1.13, 6.1.21, 6.1.22
- *(Partitioning a poset into linear orders)* Solve parts (e) and (d) of KBR 6.1.26–27
- *(Proof exercise)* Solve KBR 6.1.28
- Solve KBR exercises 6.2.1, 6.2.2, 6.2.9, 6.2.10, 6.2.13–14
- The instructor introduces the concept of "lexicographical order".
- Solve KBR exercises 6.1.19, 6.1.20.

Tuesday Dec. 14th, 15:15–17:00

- Solve KBR exercises 6.2.17–18, 6.2.36
- *(Proof exercise)* KBR 6.2.20
- Solve KBR exercises 7.1.1–4, 7.1.9–13, 7.2.1–2, 7.1.24–26
- *(Proof exercises)* Solve KBR 7.1.29–30, 7.1.31
- Discuss the following exercise in class:
  - Write a pseudo code that determines whether a relation R is a tree. The input relation R is given as a subset of $A \times A$.

Friday Dec. 17th, 10:15–12:00

- Solve KBR exercises 7.4.1, 7.4.2, 7.4.3.
- The instructor introduces Prim's algorithm for finding spanning trees.
- Solve KBR exercises 7.4.7, 7.4.8, 7.4.9
- Solve KBR exercises 8.1.1–2, 8.1.5, 8.1.17, 8.1.19, 8.1.20.