DMA 2021

– Week 9 –

Work instructions

This week we focus on **matrices**. We can think of matrices (KBR 1.5) as tables of numbers like, e.g.,

$$\begin{bmatrix} 2 & 3 & 0 \\ -2 & 0 & 8 \\ 1 & 5 & 7 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Importantly, these can be manipulated with **operations** $+$ (sum) and $\cdot$ (product), which allows us to get new matrices from older ones. In DMA, we are particularly interested in **Boolean matrices** with entries that are always either 0 or 1, like the two matrices to the right in the list above. For such matrices, we also have operations $\vee$, $\wedge$, $\odot$, which are important for our later discussion of relations. We will not focus too much on the theory but rather ensure that you learn how to use the operations and that you develop a better sense of matrix multiplication. You will hear much more about matrices in the course *LinAlgDat*.

There are *no* scheduled lectures or exercise classes on Friday Nov. 12th.

Assigned reading

- KBR 1.5

Lecture plan

Monday Nov. 8th, 09:15-10:00

Matrices and their operations. Boolean matrices and their operations. (KBR 1.5)

Tuesday Nov. 9th 13:15-14:00

Applications of matrix multiplication.

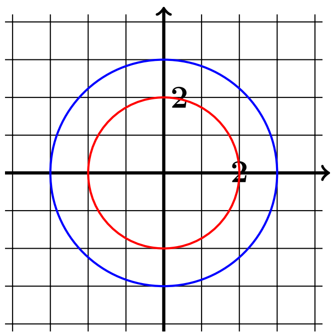
Exercise plan

Monday Nov. 8th 10:15-12:00

- Solve KBR exercises 1.5.1(a), 1.5.6, 1.5.7, 1.5.12, 1.5.21, 1.5.23, 1.5.30, 1.5.31.
- The instructor introduces the formula for inversion of 2×2 matrices.
- Solve KBR exercises 1.5.26, 1.5.27.
- Time permitting, instructor presents a sample solution for 1.5.21 and/or 1.5.23 (c).
- [*] An $n \times n$ matrix $M = [m_{ij}]$ is called *upper-triangular*, if $m_{ij} = 0$ for all $i > j$. Show that a product of two $n \times n$ upper-triangular matrices is an upper-triangular matrix.

Tuesday Nov. 9th, 15:15-17:00

- Find a 2×2 matrix T_1 that would reflect points in 2D (*i.e.* vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$) over the line $y = x$.
  Hint: Reflecting point (x, y) over the line $y = x$ is equivalent to mapping (x, y) to (y, x). So we need to find T_1 such that $T_1 \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = \left[\begin{smallmatrix} y \\ x \end{smallmatrix}\right]$
- Find a 2×2 matrix T_2 that would reflect points in 2D over the x-axis.
- Find a 2×2 matrix T_3 that would shrink (the points of) the blue circle to (the points of) the red circle.



- [**] Find a 2×2 matrix T_4 that would reflect points in 2D over the line $y = \sqrt{3}x$.
- Solve KBR exercises 1.5.16, 1.5.20, 1.5.22, 1.5.38 and anything left over from Monday.