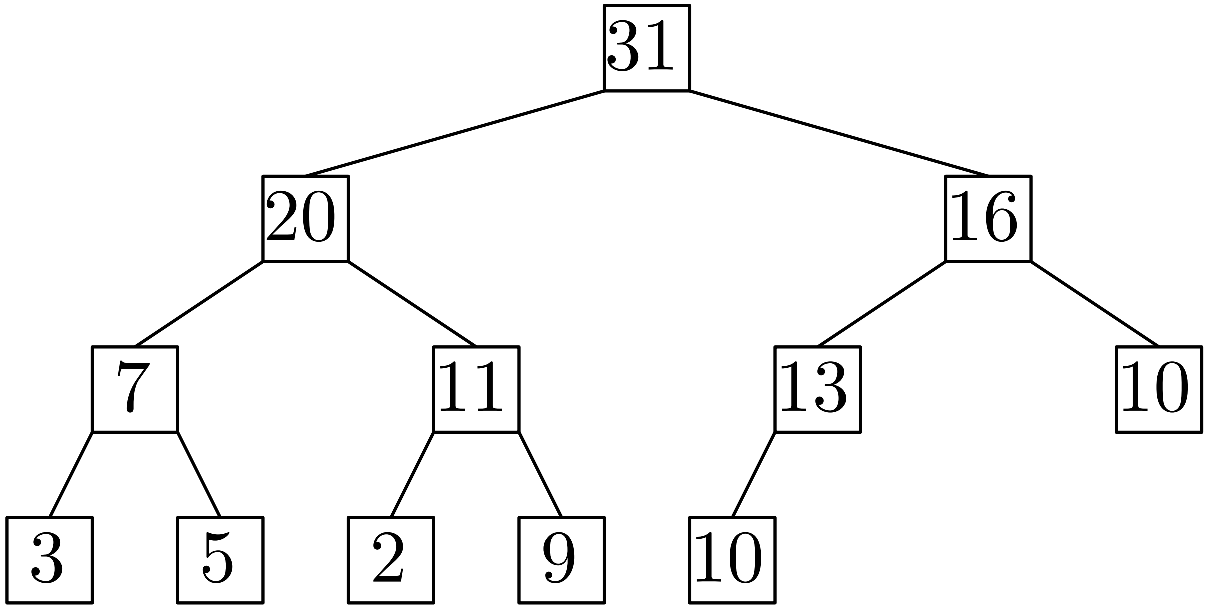
Prioritetskøer, hobe og heap sort

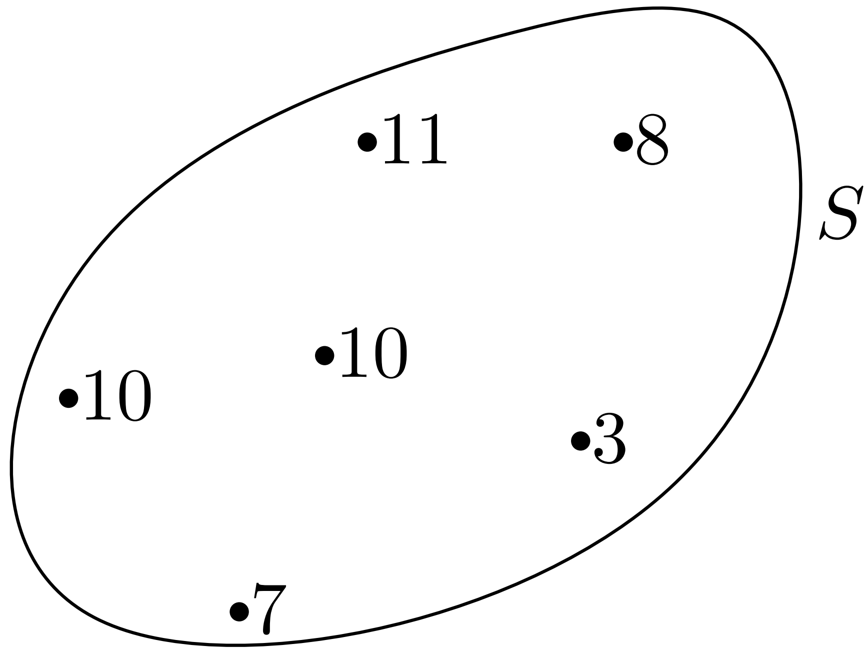


Mikkel Abrahamsen

Prioritetskø

Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

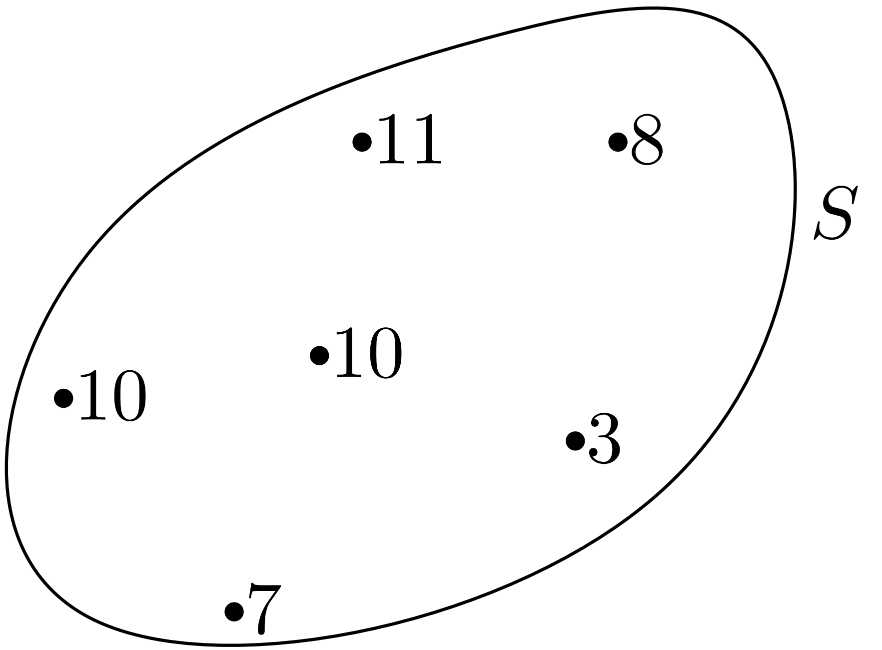


Prioritetskø

Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S)

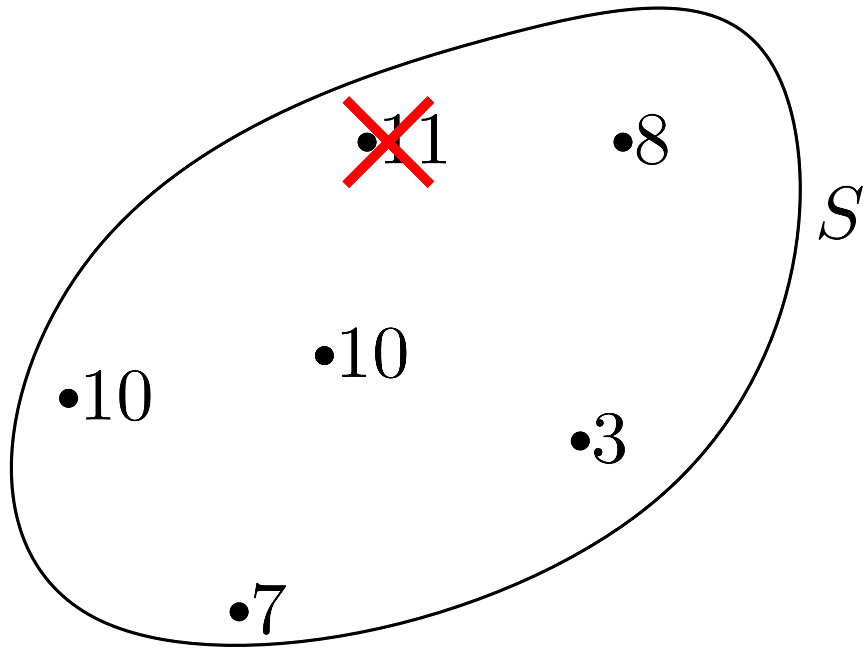


Prioritetskø

Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11



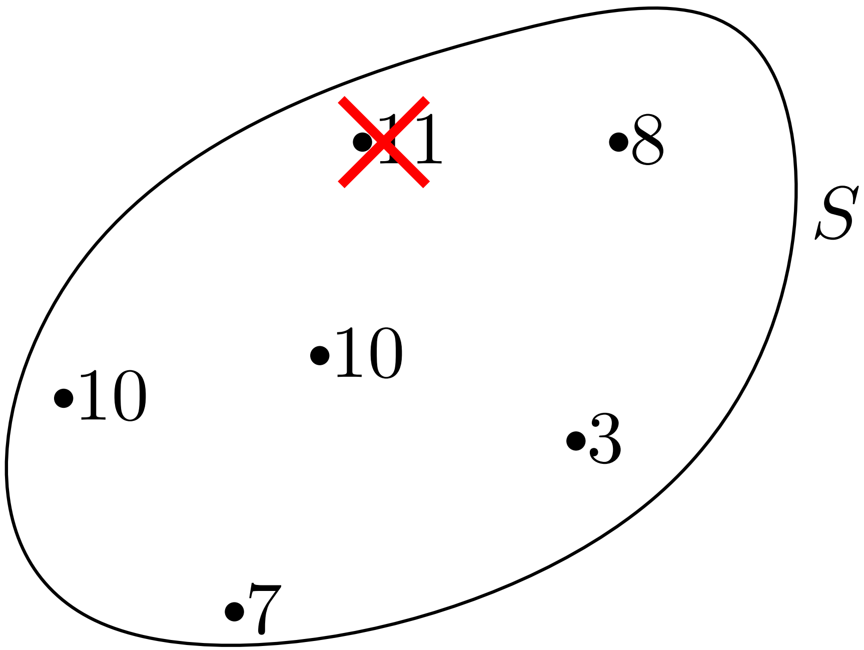
Prioritetskø

Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11

Extract-Max(S)



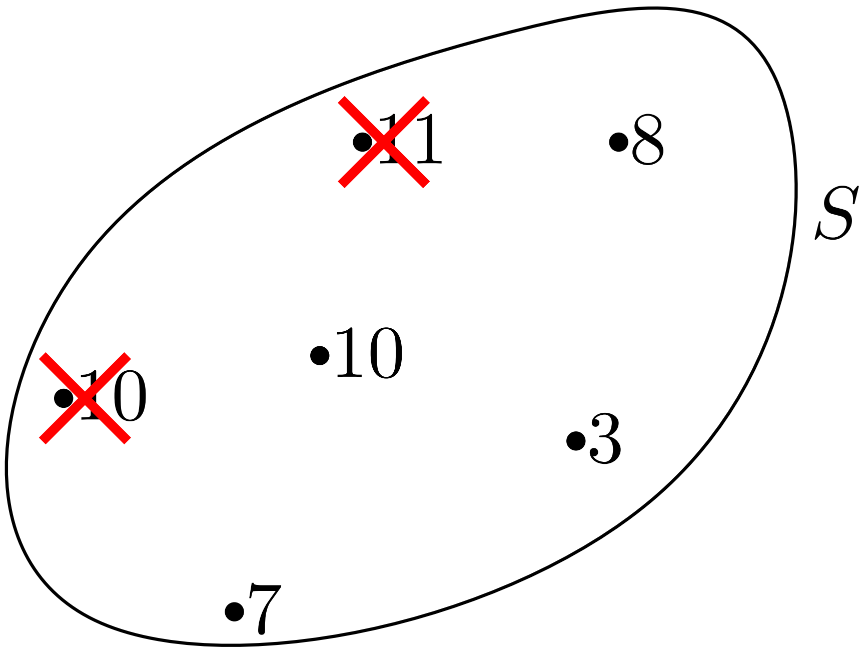
Prioritetskø

Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11

Extract-Max(S) returnér 10



Prioritetskø

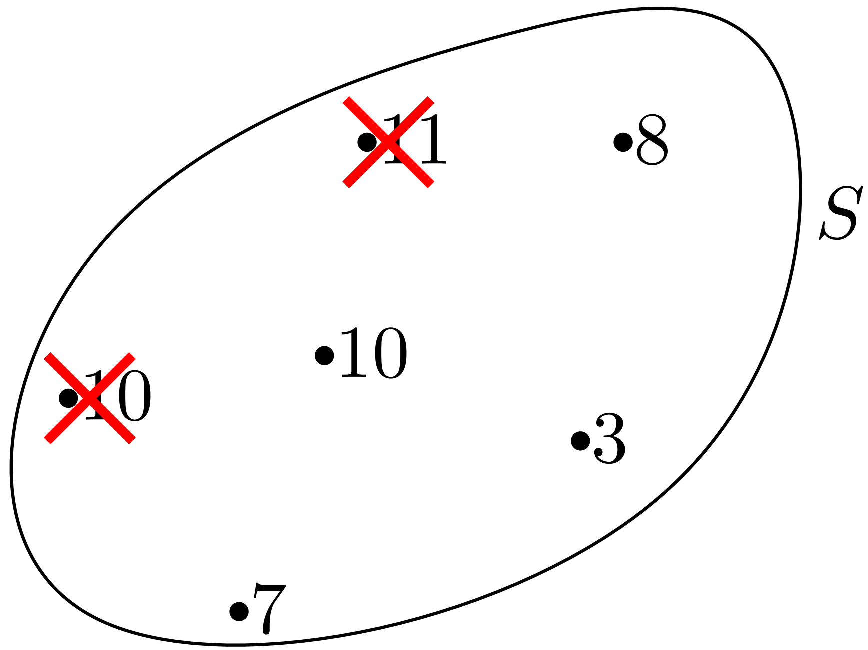
Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11

Extract-Max(S) returnér 10

Insert($S, 13$)



Prioritetskø

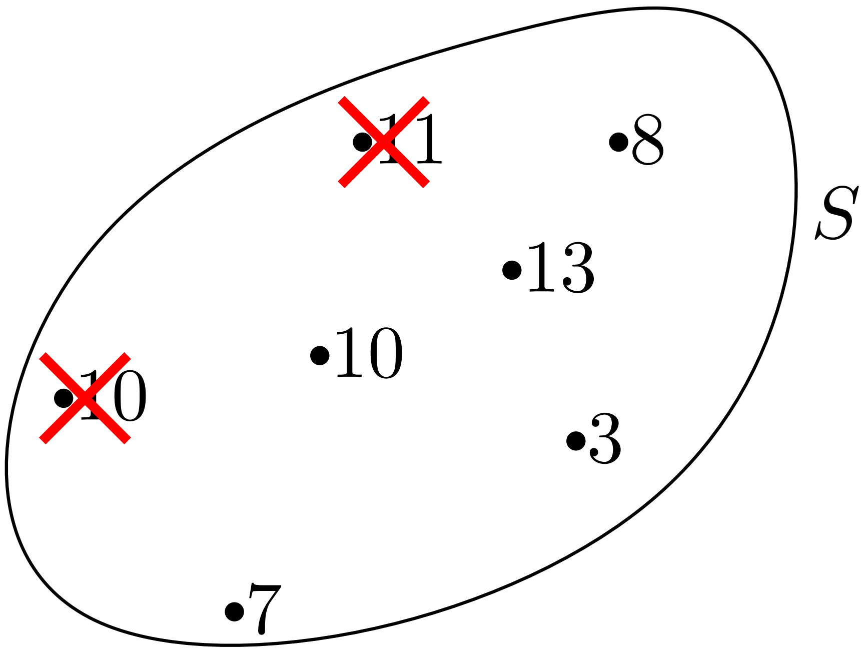
Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11

Extract-Max(S) returnér 10

Insert($S, 13$)



Prioritetskø

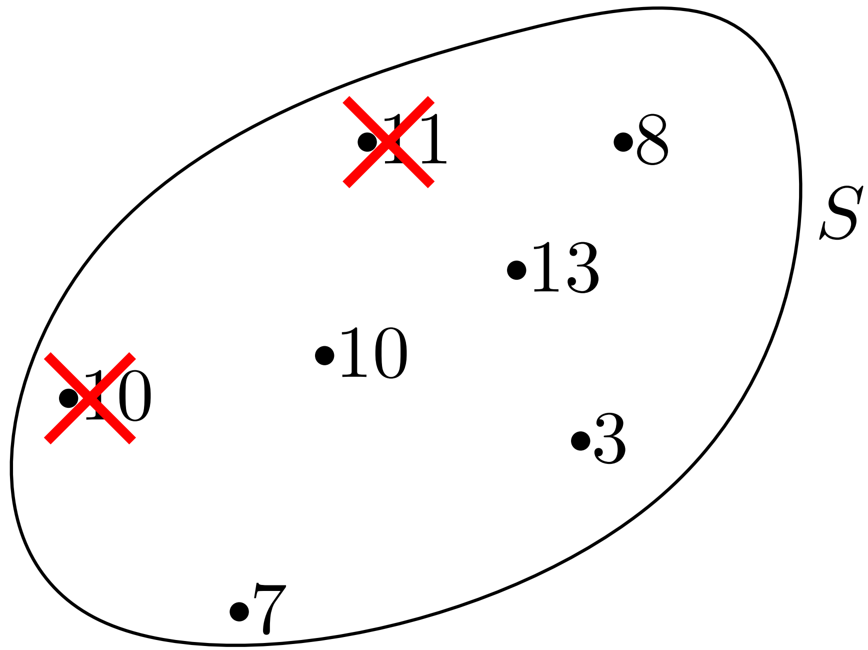
Dynamisk multi-mængde S af *nøgler*.
To (vigtigste) operationer:

- Extract-Max(S): fjern og returnér største nøgle.
- Insert(S, k): tilføj k til S.

Extract-Max(S) returnér 11

Extract-Max(S) returnér 10

Insert($S, 13$)



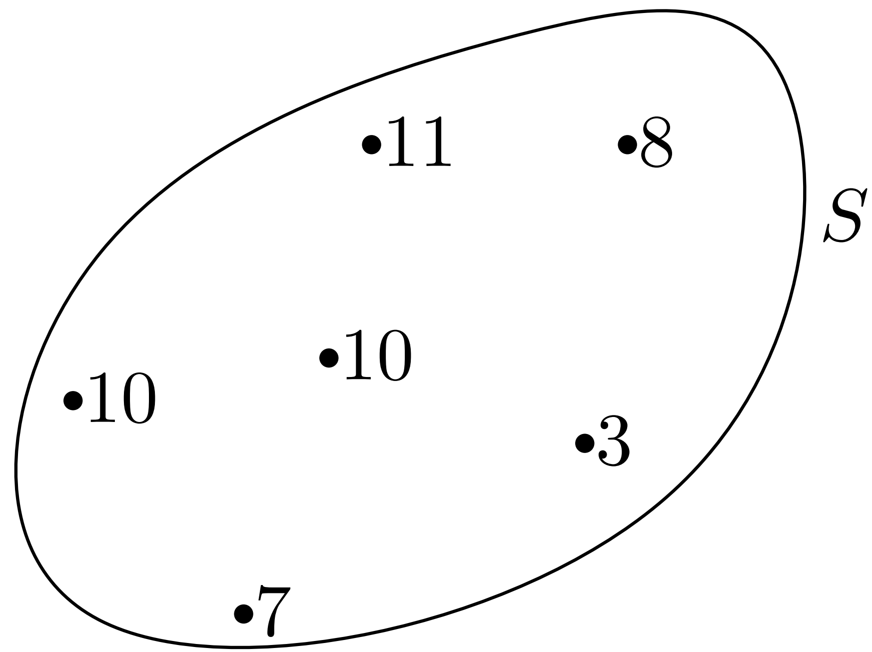
I praksis: Vi gemmer hver nøgle sammen med *sattelitdata*: $(k,$data$)$

Kendte teknikker

Array:

0	1	2	...					
3	10	7	11	8	10			

Extract-Max(S): $\Theta(n)$ tid.
Insert(S, k): $\Theta(1)$ tid.



Kendte teknikker

Array:

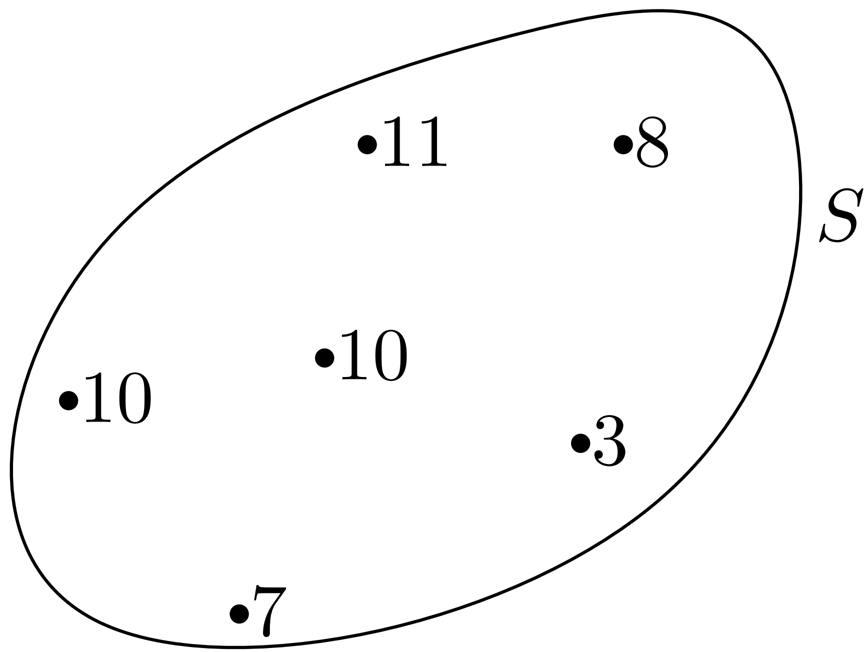
0	1	2	...					
3	10	7	11	8	10			

Extract-Max(S): $\Theta(n)$ tid.
Insert(S, k): $\Theta(1)$ tid.

Sorteret array:

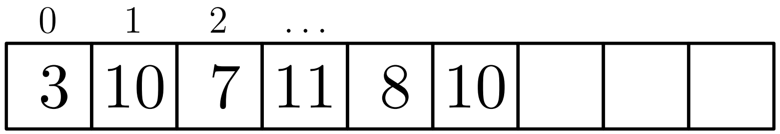
0	1	2	...					
3	7	8	10	10	11			

Extract-Max(S): $\Theta(1)$ tid.
Insert(S, k): $\Theta(n)$ tid.

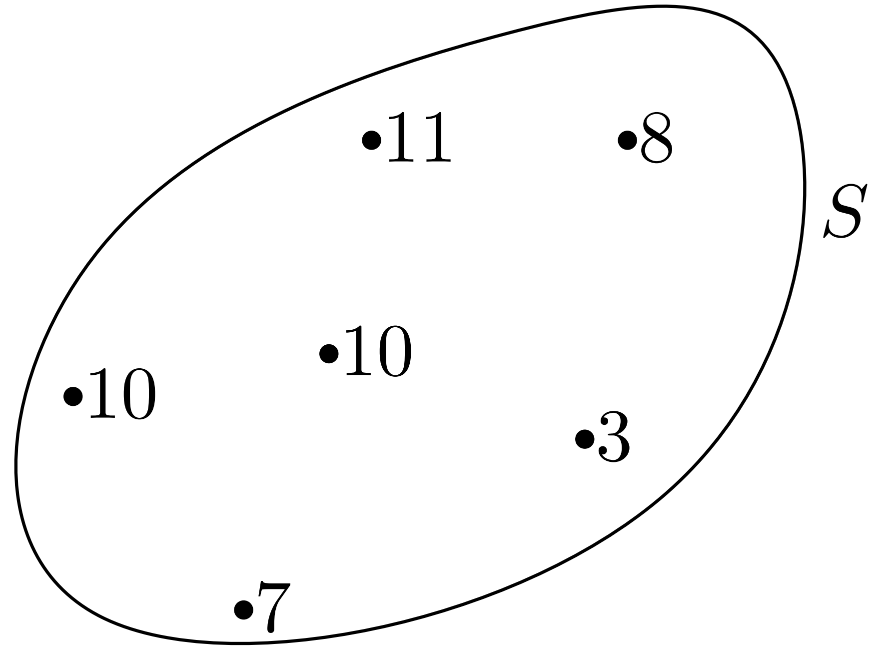


Kendte teknikker

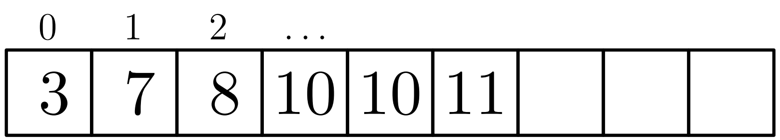
Array:



Extract-Max(S): $\Theta(n)$ tid.
Insert(S, k): $\Theta(1)$ tid.

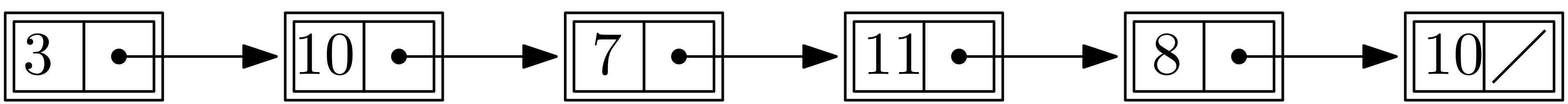


Sorteret array:



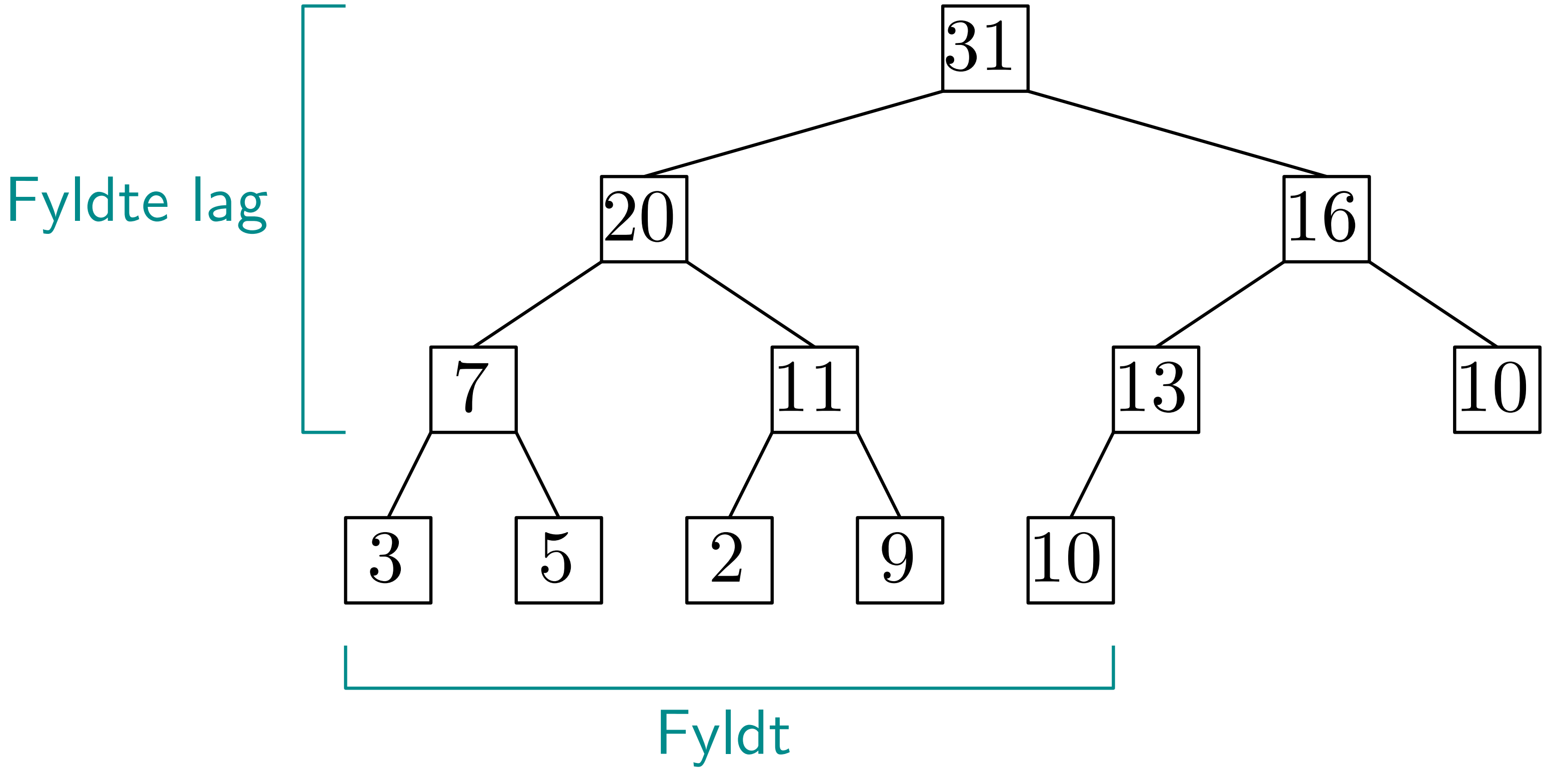
Extract-Max(S): $\Theta(1)$ tid.
Insert(S, k): $\Theta(n)$ tid.

Hægtet liste:

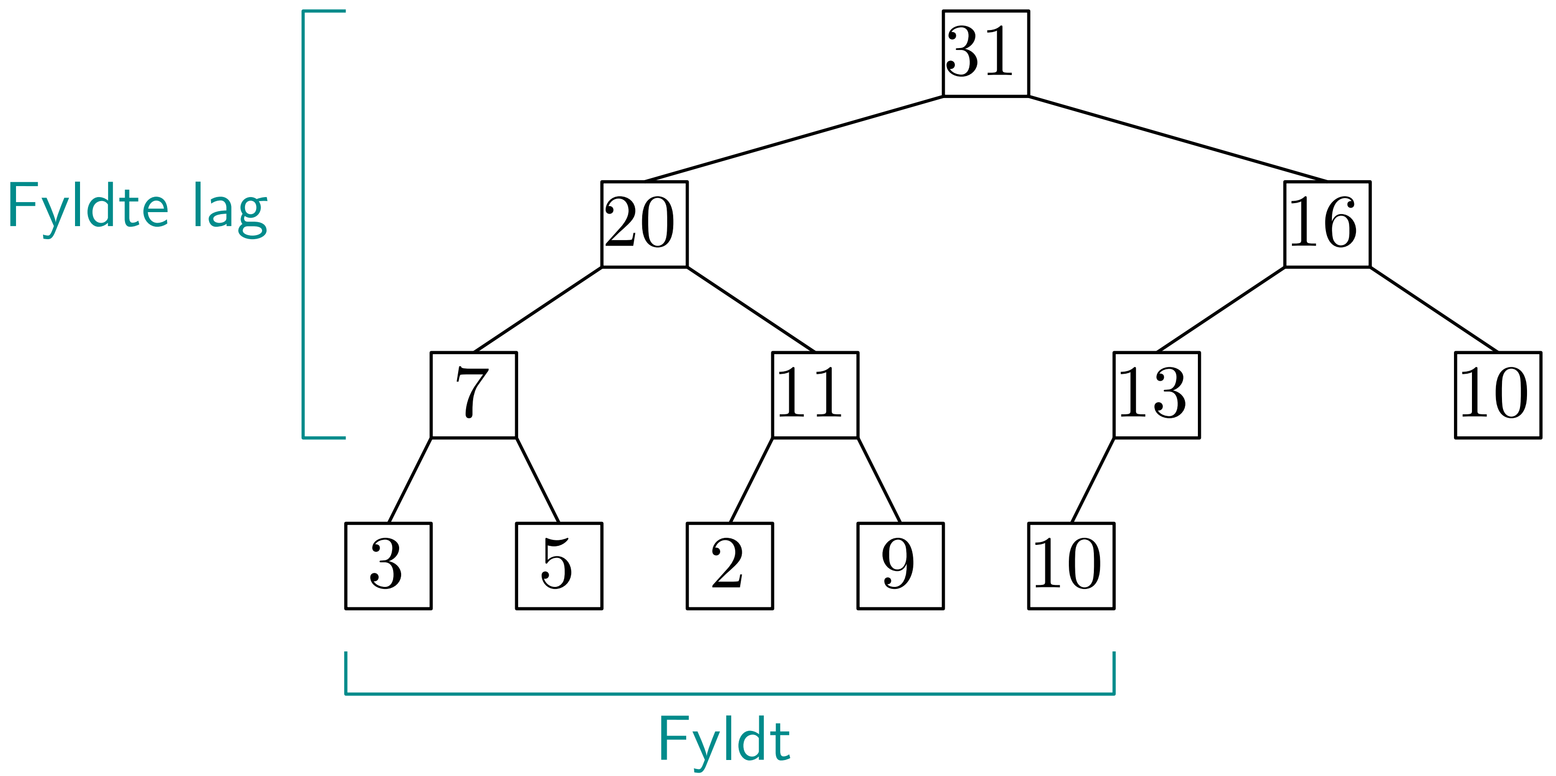


Extract-Max(S): $\Theta(n)$ tid.
Insert(S, k): $\Theta(1)$ tid.

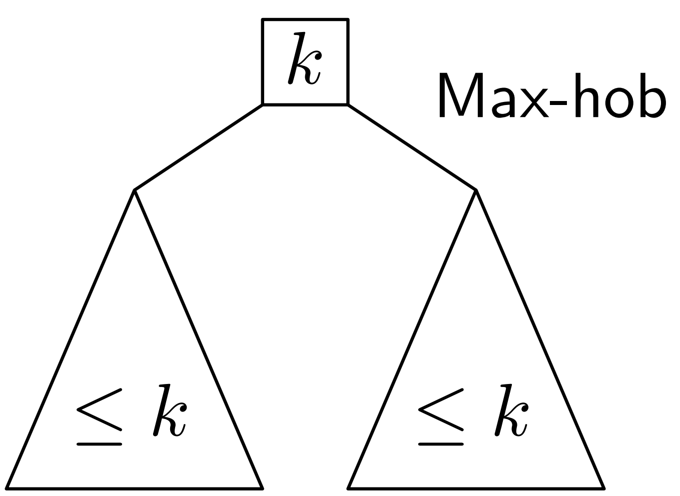
Hob



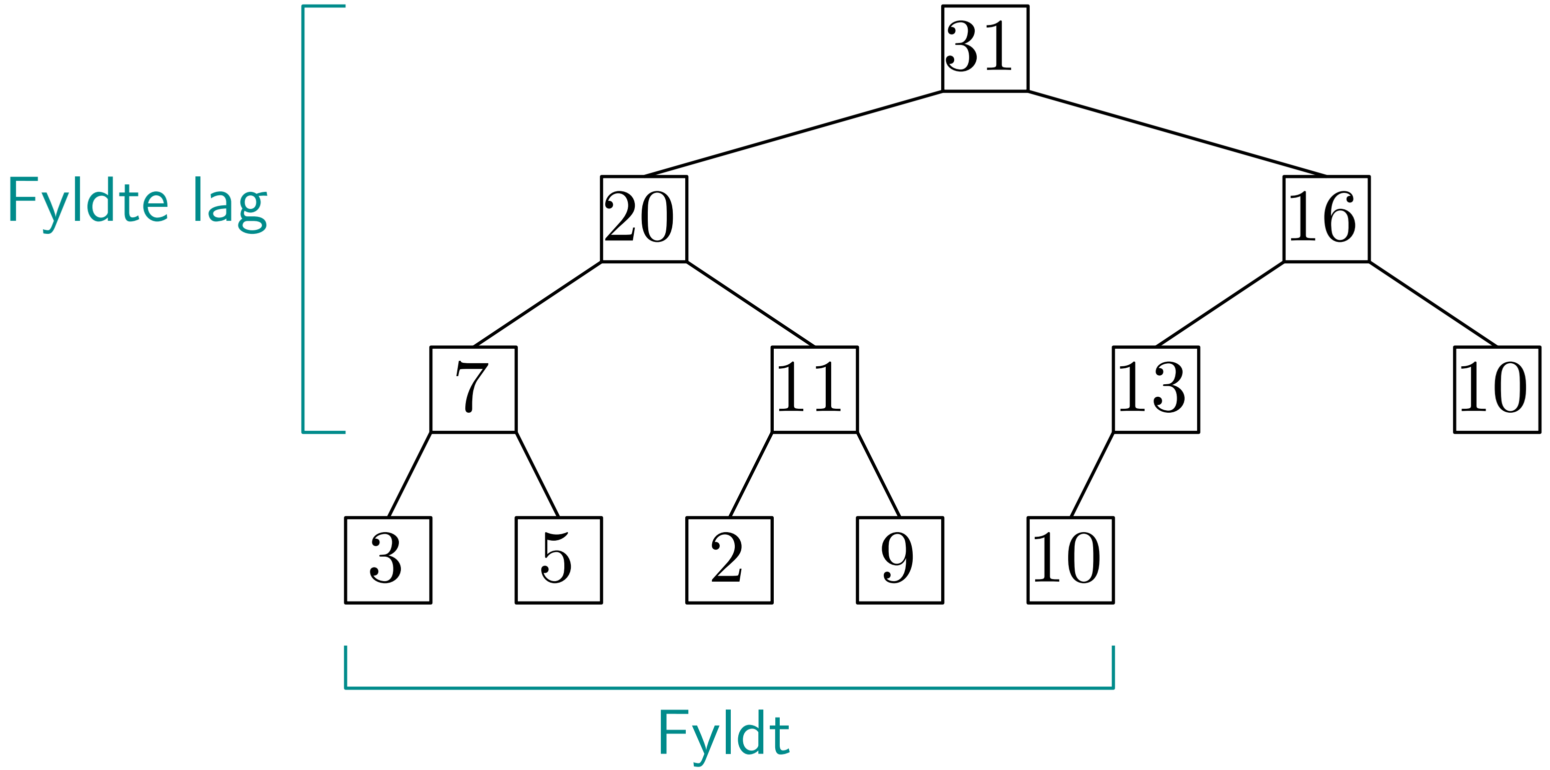
Hob



Hobeordenen:



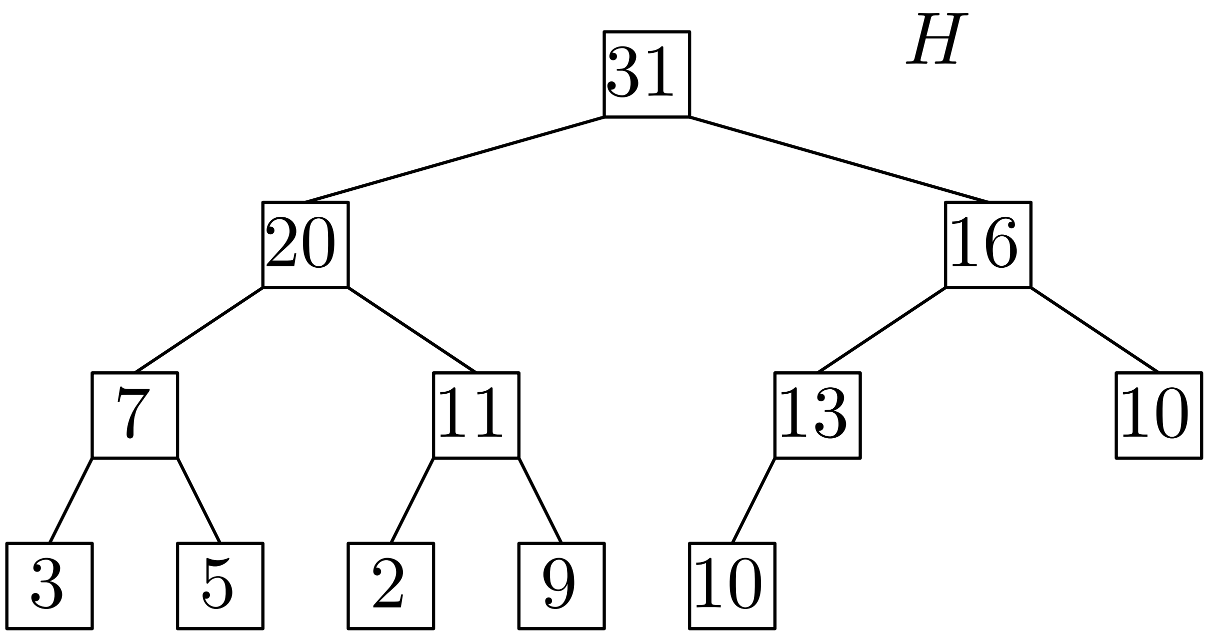
Hob



Hobeordenen:

Extract-Max

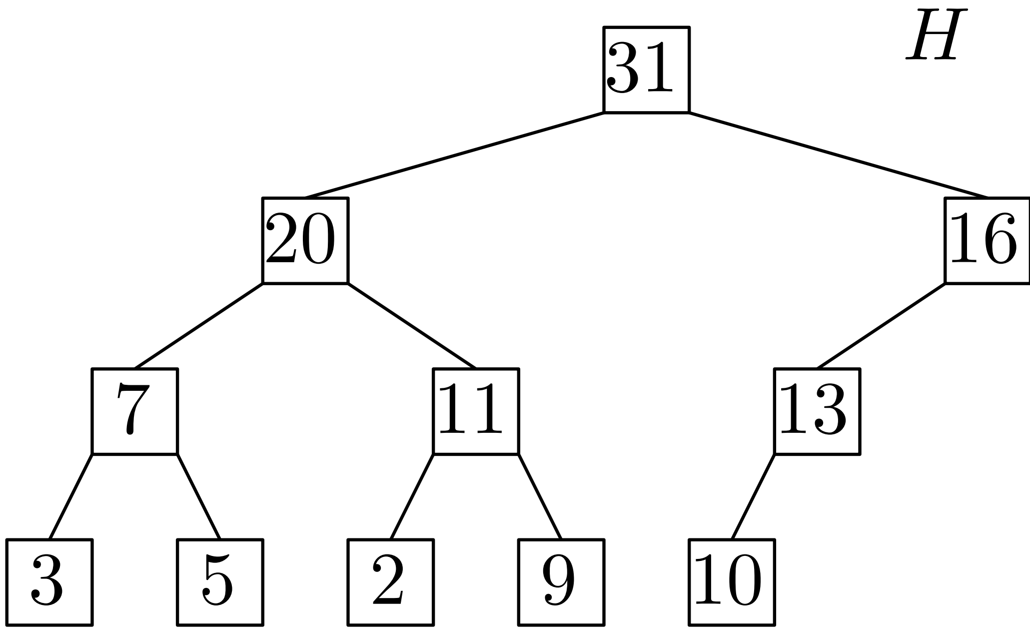
Extract-Max(H)



Extract-Max

Extract-Max(H)

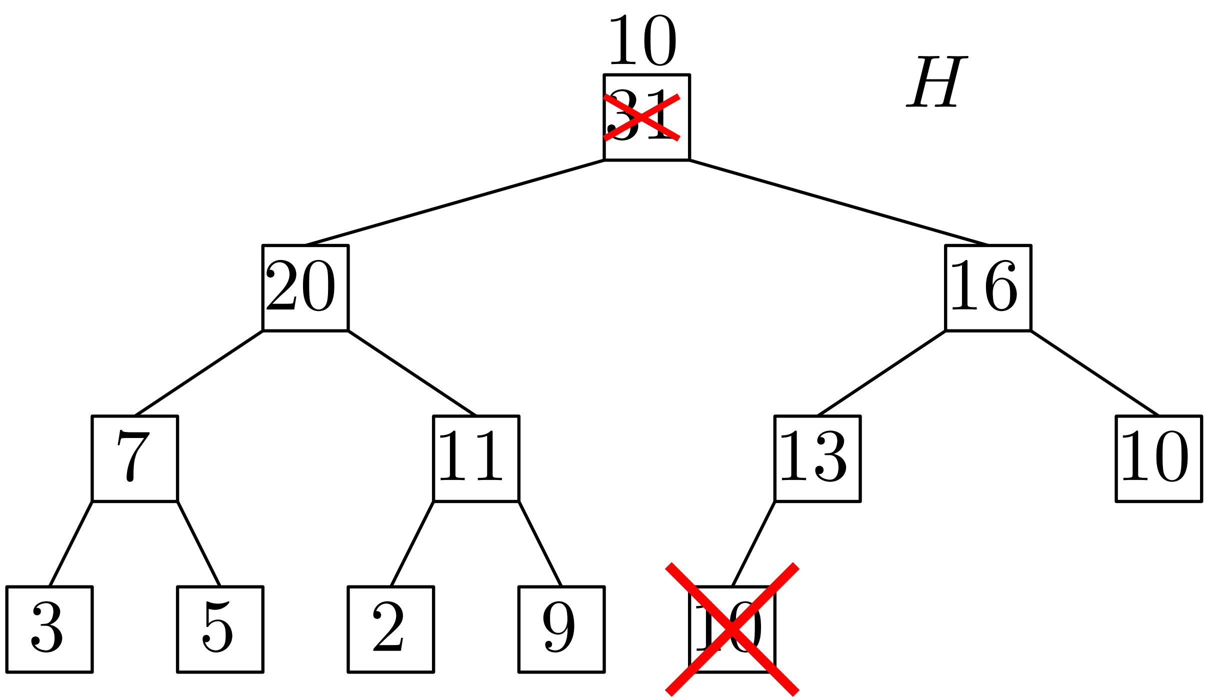
$max = 31$



Extract-Max

Extract-Max(H)

$max = 31$

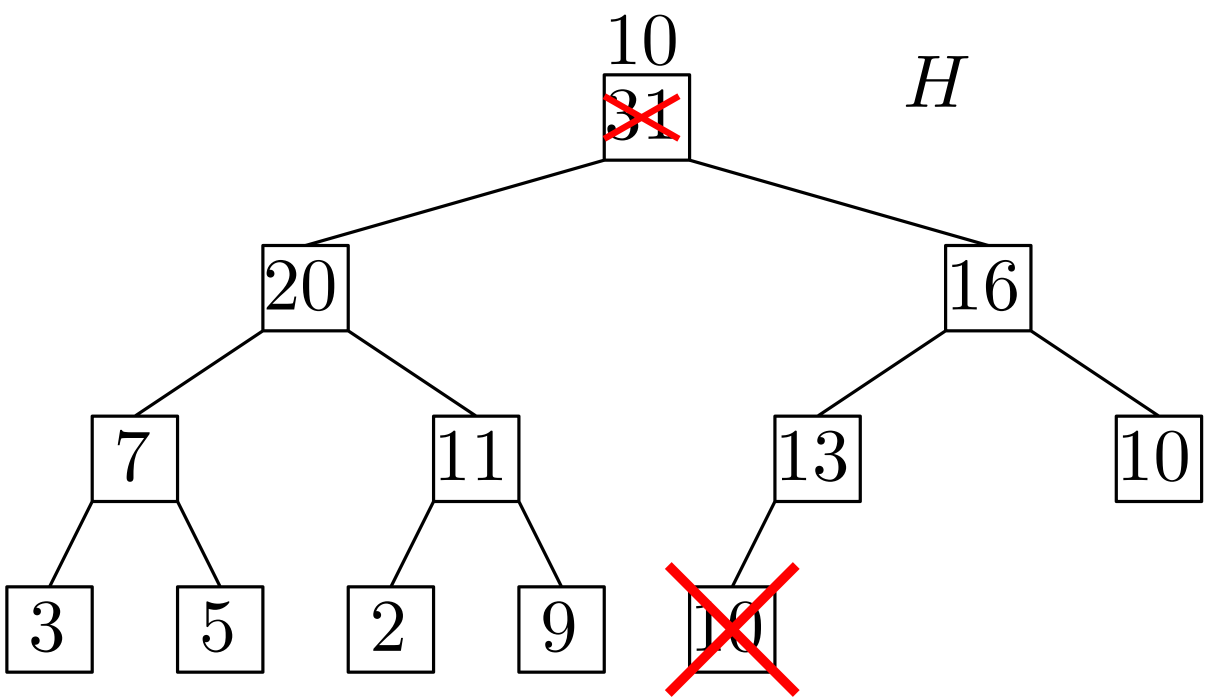


Extract-Max

Extract-Max(H)

$max = 31$

10 “bobler ned” (Max-Heapify)

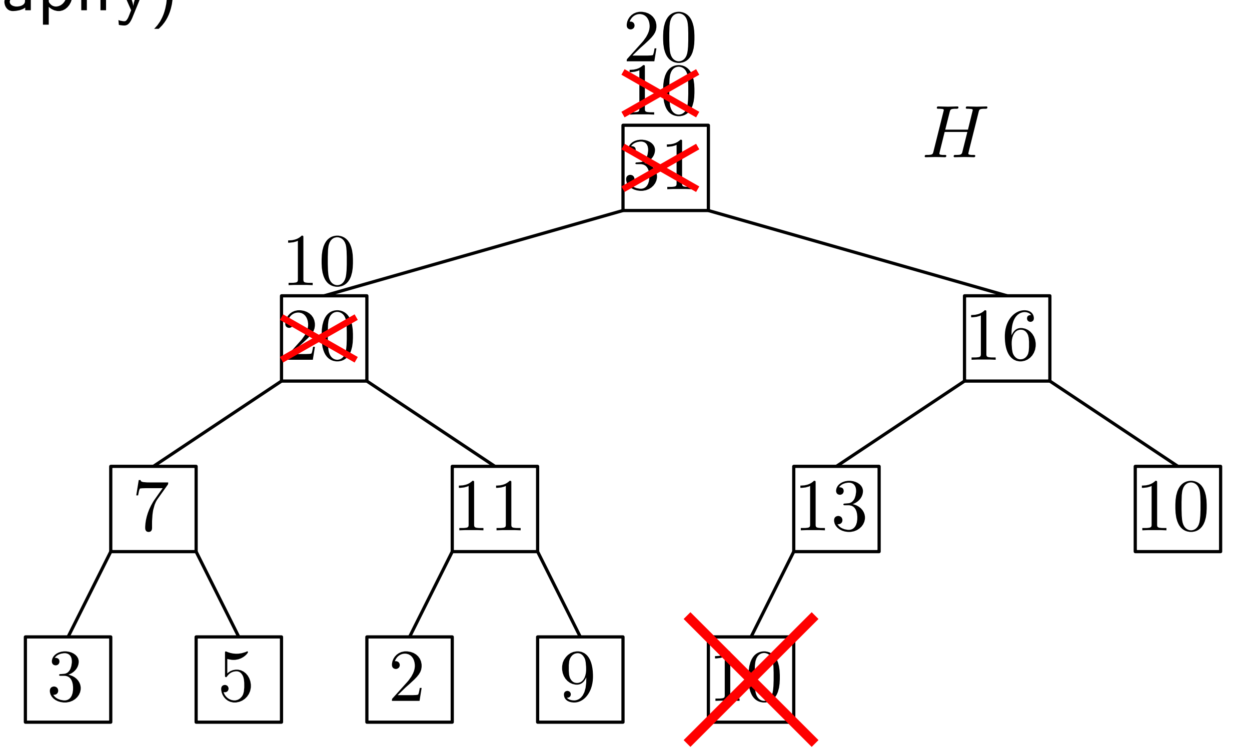


Extract-Max

Extract-Max(H)

$max = 31$

10 “bobler ned”
(Max-Heapify)

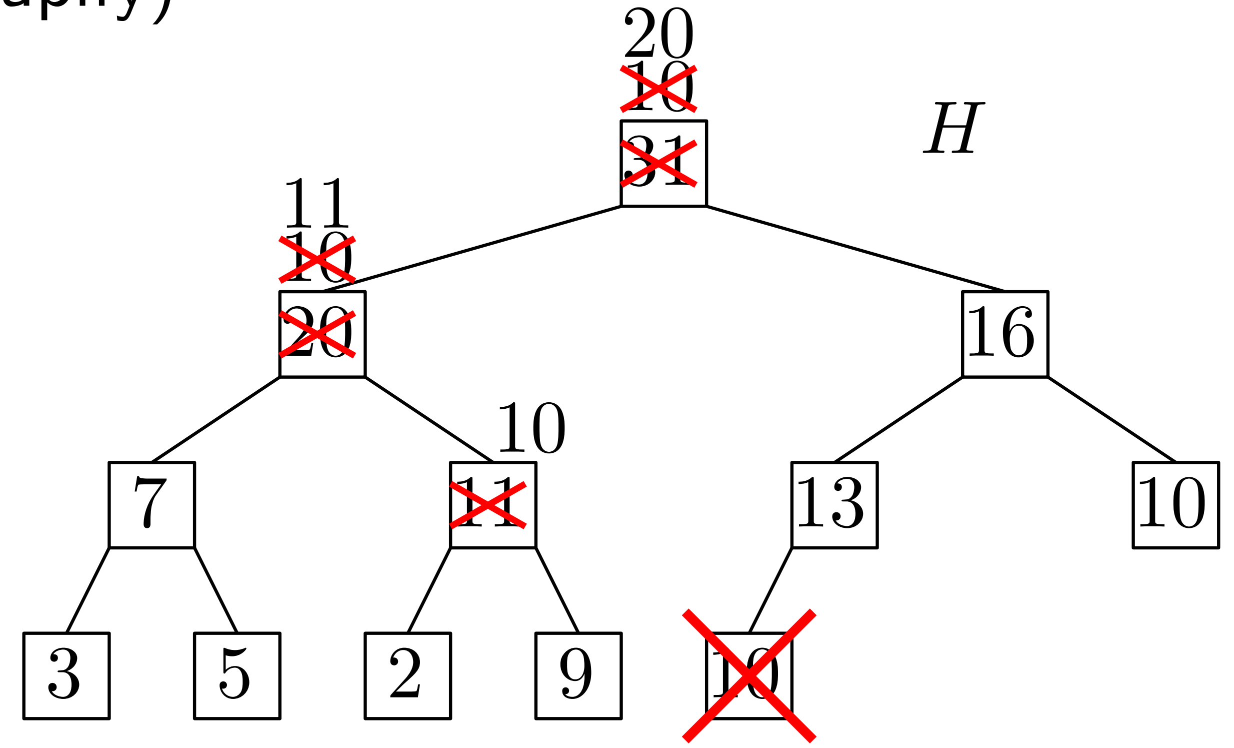


Extract-Max

Extract-Max(H)

$max = 31$

10 “bobler ned” (Max-Heapify)



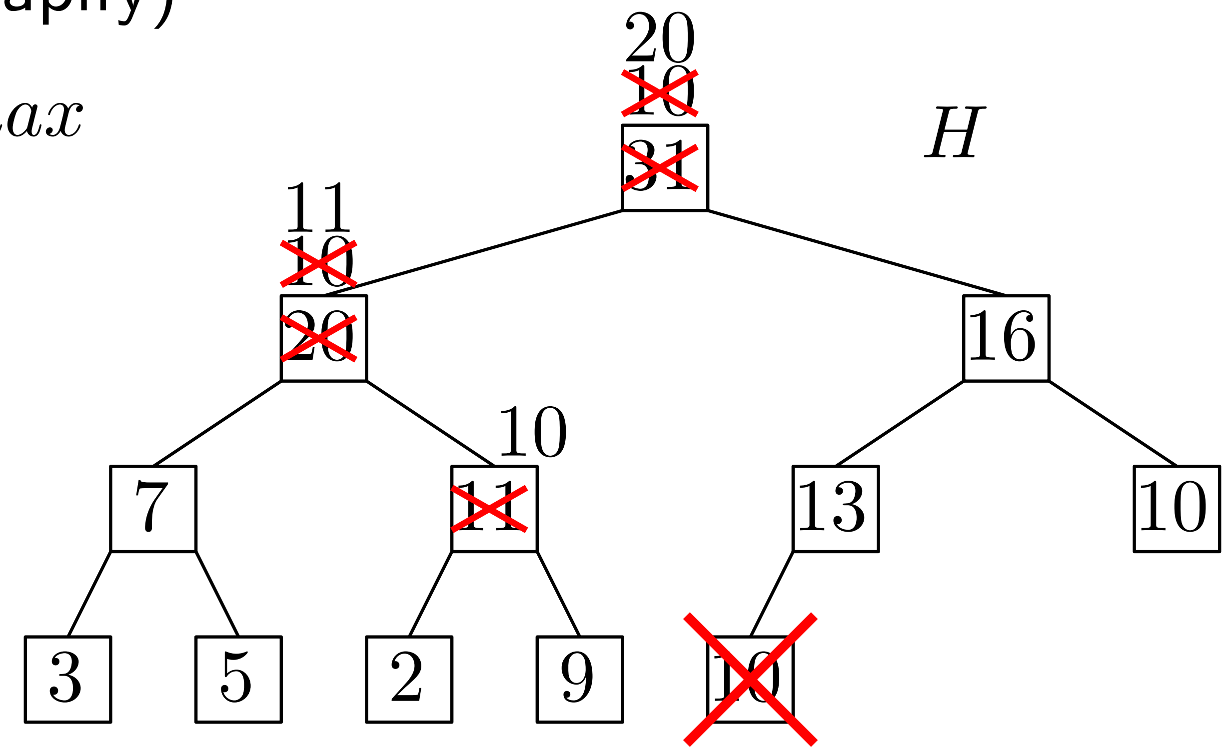
Extract-Max

Extract-Max(H)

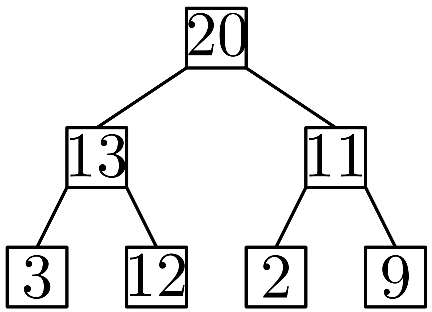
$max = 31$

10 "bobler ned"
(Max-Heapify)

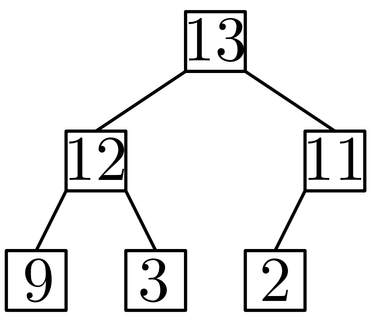
return max



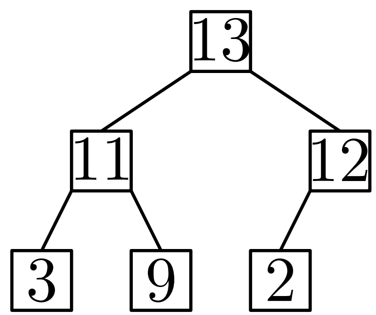
Hvordan ser hoben ud til sidst?



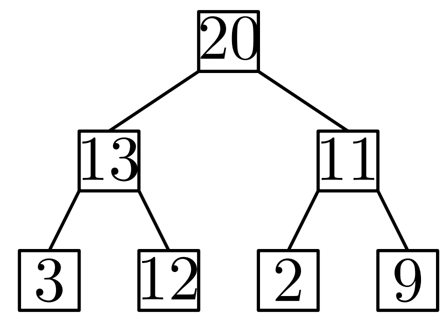
Extract-Max(H)



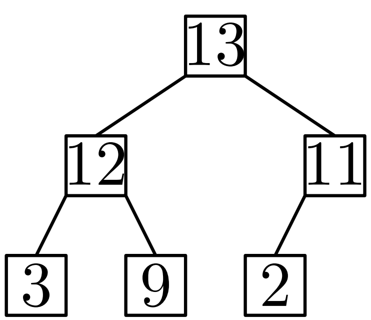
A



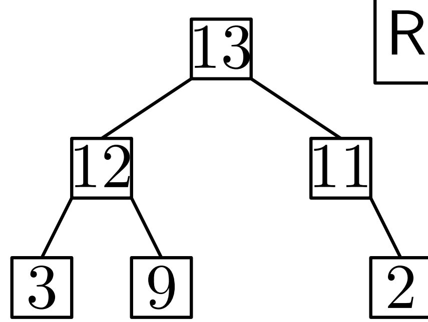
B



C



D



E

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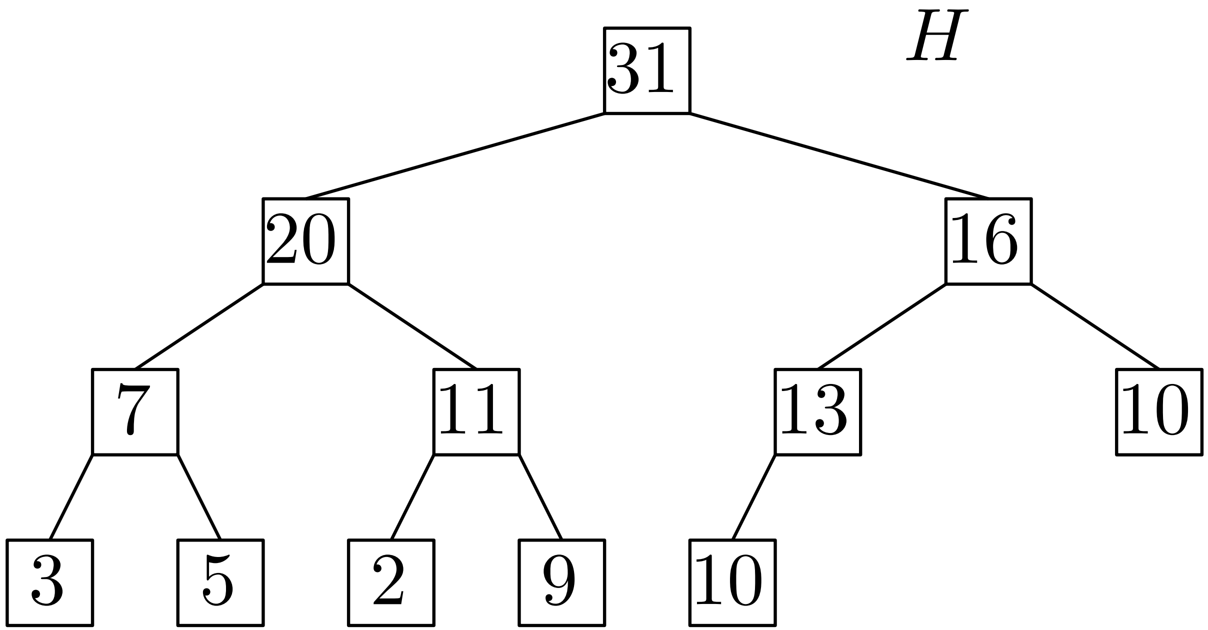
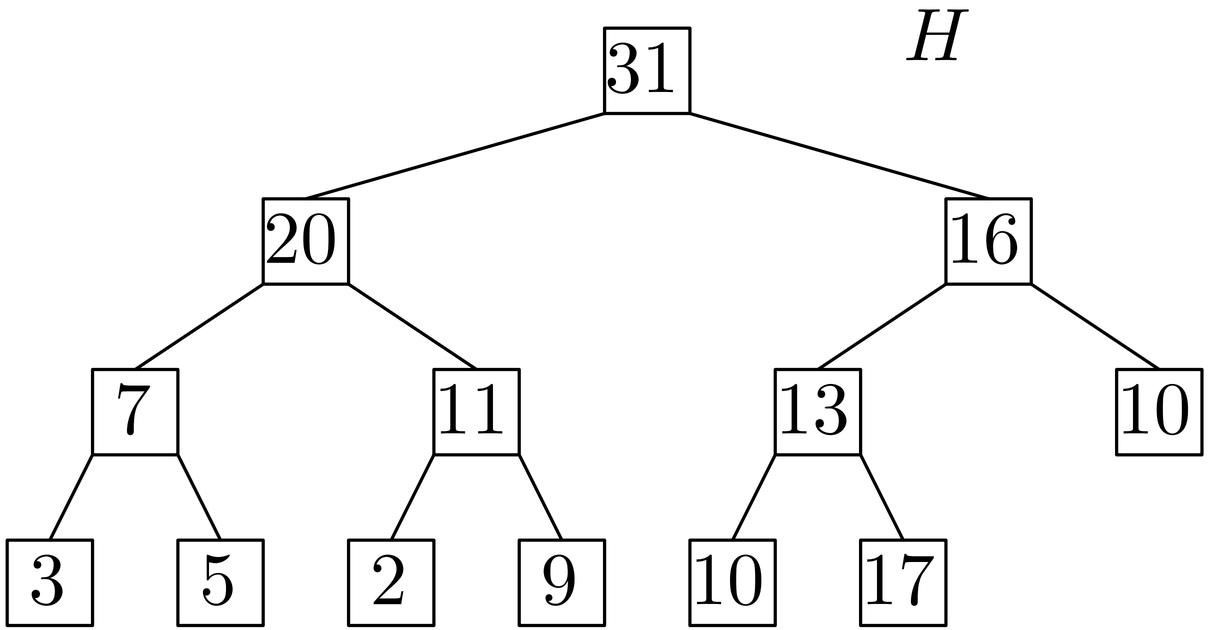
Insert

Insert$(H, 17)$



Insert

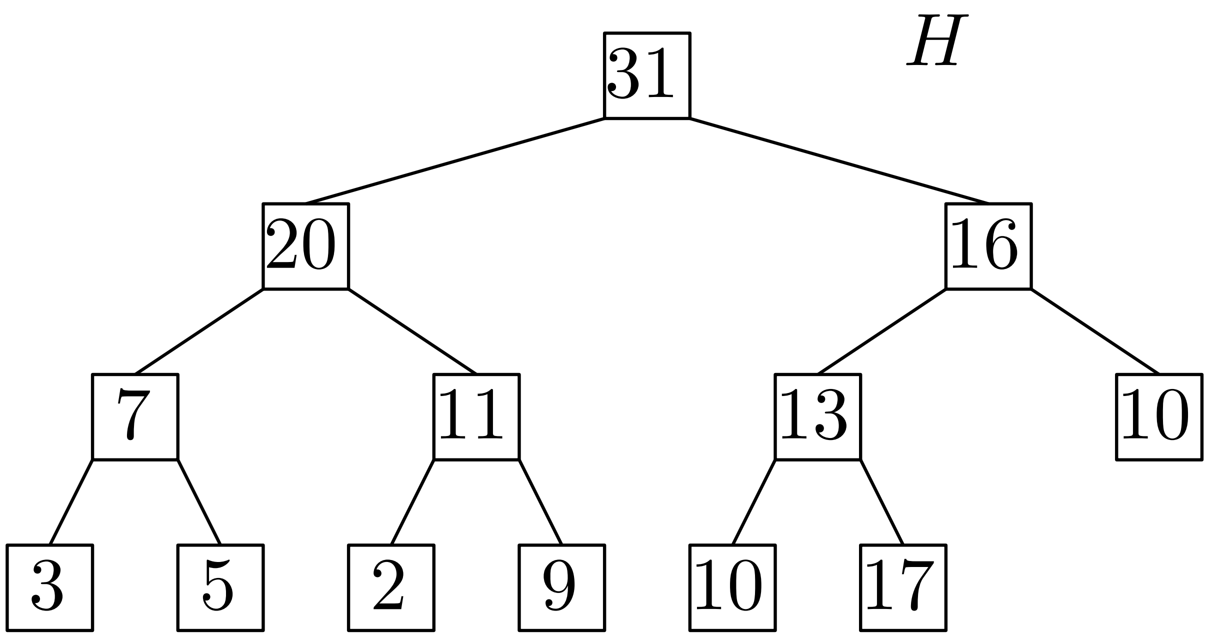
Insert$(H, 17)$



Insert

Insert$(H, 17)$

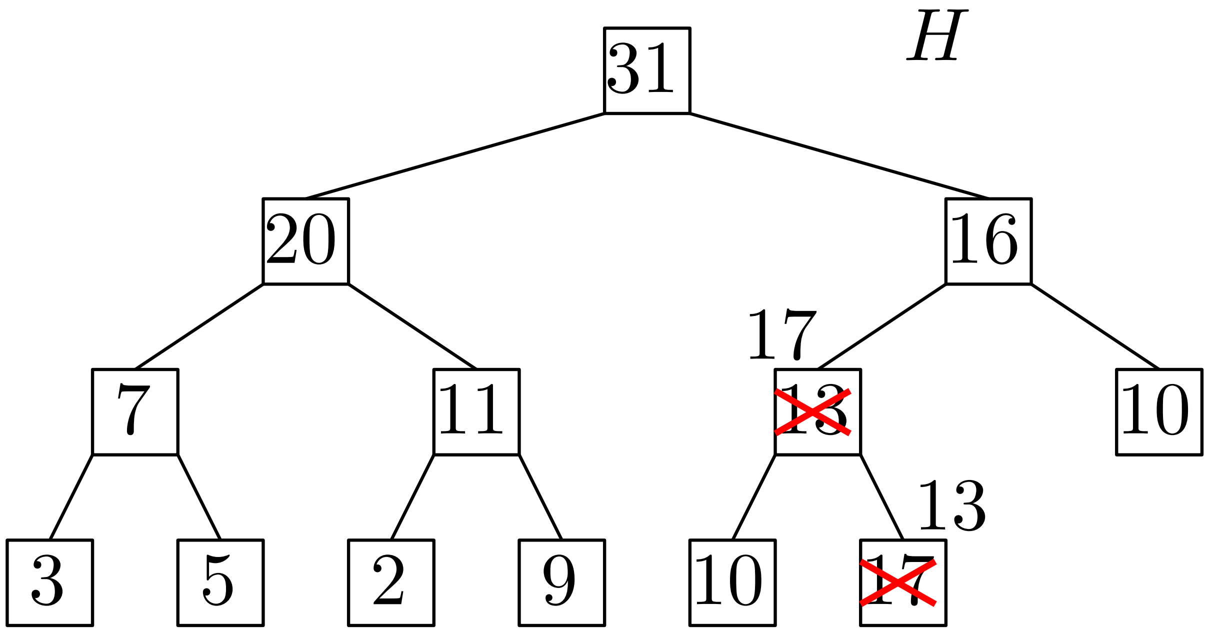
17 "bobler op"



Insert

Insert$(H, 17)$

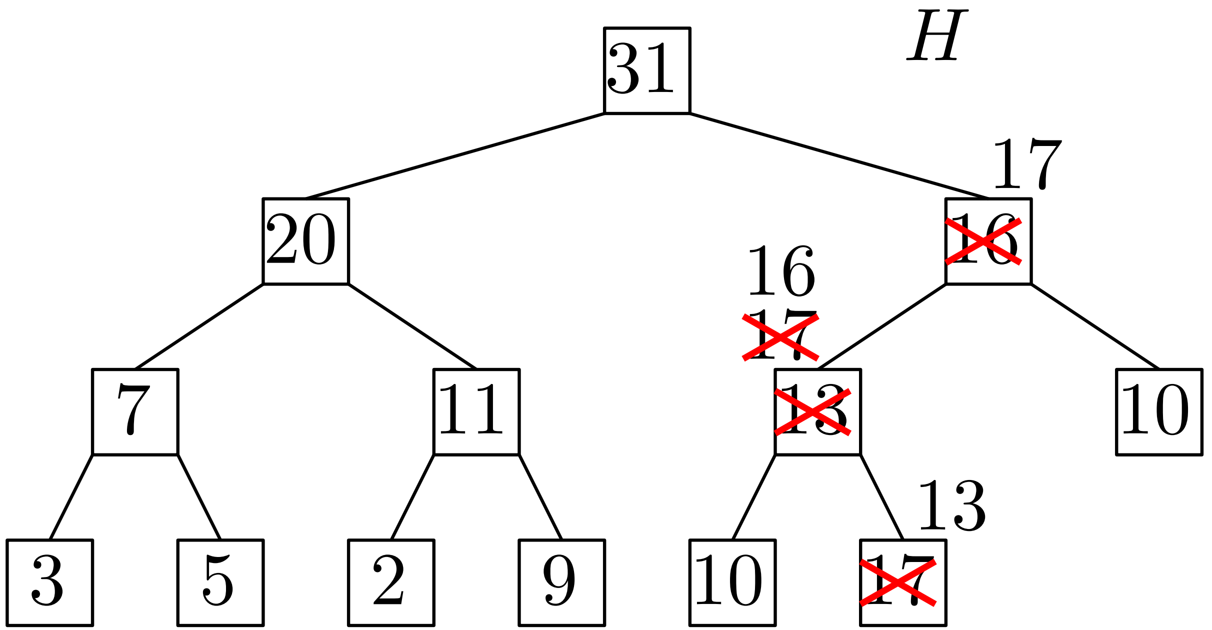
17 "bobler op"



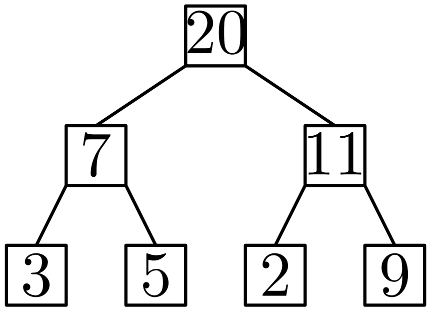
Insert

Insert$(H, 17)$

17 “bobler op”

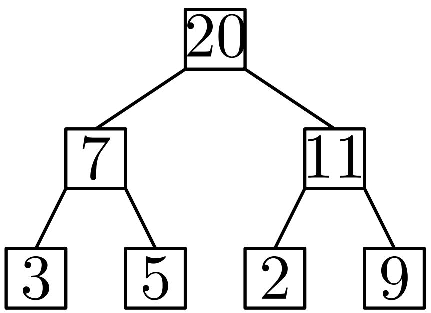


Hvordan ser hoben ud til sidst?



Insert$(H, 10)$

Extract-Max(H)



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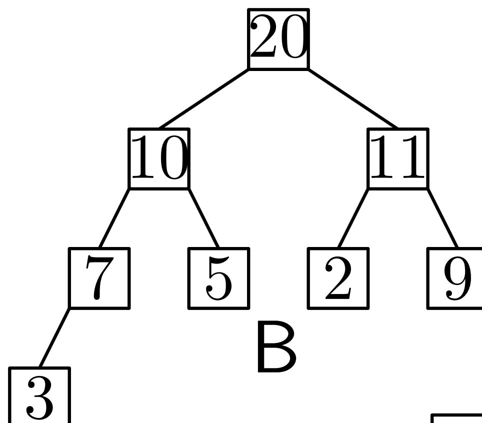
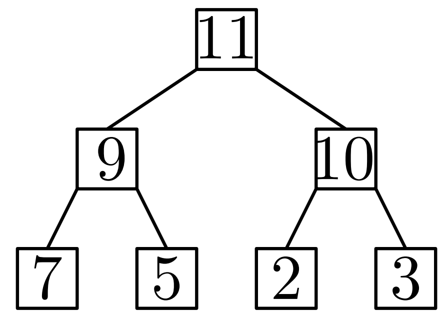


B



C

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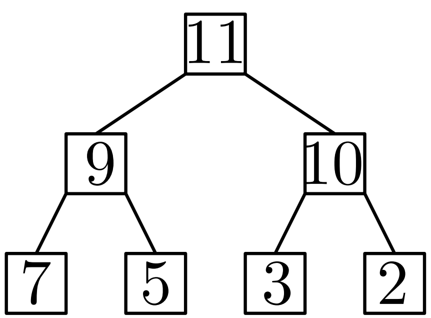
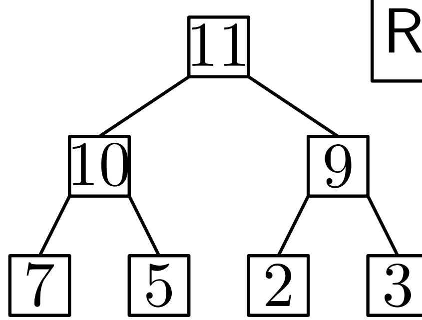


D



E

Køretider

Insert: Boble op

Extract-Max: Boble ned

Køretider

Insert: Boble op

Extract-Max: Boble ned

I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

Køretider

Insert: Boble op

Extract-Max: Boble ned

I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$$n = 2^0 = 1$$
$$h = 0$$

Køretider

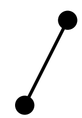
Insert: Boble op

Extract-Max: Boble ned

I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$
$h = 0$

$n = 2^1 = 2$
$h = 1$



Køretider

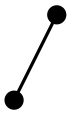
Insert: Boble op

Extract-Max: Boble ned

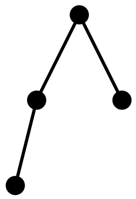
I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$
$h = 0$

$n = 2^1 = 2$
$h = 1$



$n = 2^2 = 4$
$h = 2$



Køretider

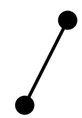
Insert: Boble op

Extract-Max: Boble ned

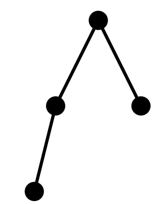
I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$
$h = 0$

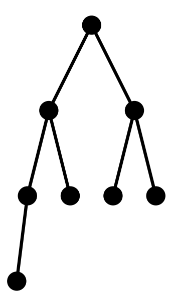
$n = 2^1 = 2$
$h = 1$



$n = 2^2 = 4$
$h = 2$



$n = 2^3 = 8$
$h = 3$



Køretider

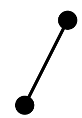
Insert: Boble op

Extract-Max: Boble ned

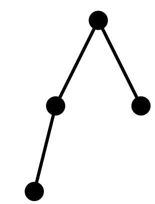
I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$
$h = 0$

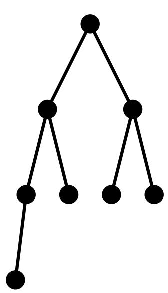
$n = 2^1 = 2$
$h = 1$



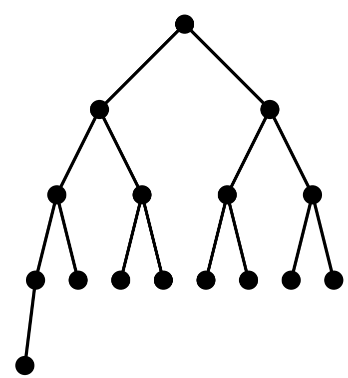
$n = 2^2 = 4$
$h = 2$



$n = 2^3 = 8$
$h = 3$



$n = 2^4 = 16$
$h = 4$



Køretider

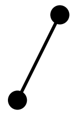
Insert: Boble op

Extract-Max: Boble ned

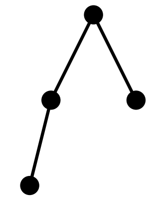
I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$
$h = 0$

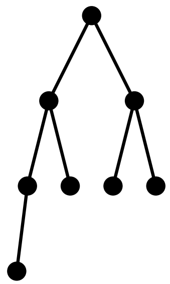
$n = 2^1 = 2$
$h = 1$



$n = 2^2 = 4$
$h = 2$

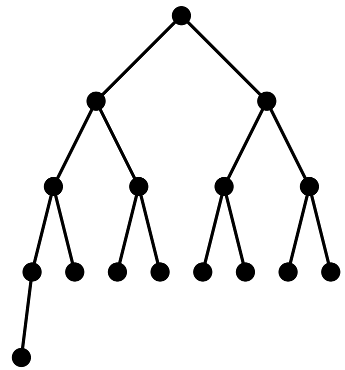


$n = 2^3 = 8$
$h = 3$



$1 + 1 + 2 + 4 + \ldots + 2^{h-1}$

$n = 2^4 = 16$
$h = 4$



Køretider

Insert: Boble op

Extract-Max: Boble ned

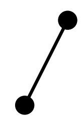
I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

$n = 2^0 = 1$

$h = 0$

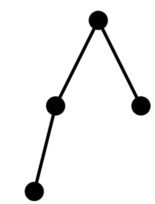
$n = 2^1 = 2$

$h = 1$



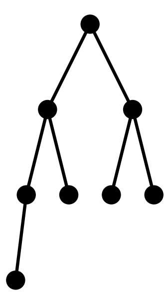
$n = 2^2 = 4$

$h = 2$



$n = 2^3 = 8$

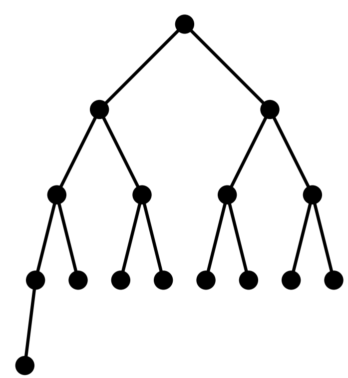
$h = 3$



$$\underbrace{1 + 1}_{2} + 2 + 4 + \ldots + 2^{h-1}$$

$n = 2^4 = 16$

$h = 4$



Køretider

Insert: Boble op

Extract-Max: Boble ned

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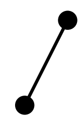

$n = 2^2 = 4$

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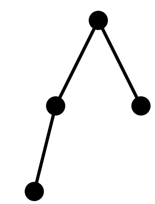

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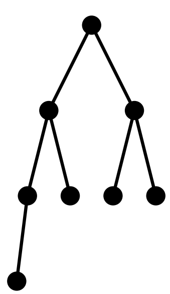

$$\underbrace{\underbrace{1+1}_{2}+2}_{4}+4+\ldots+2^{h-1}$$

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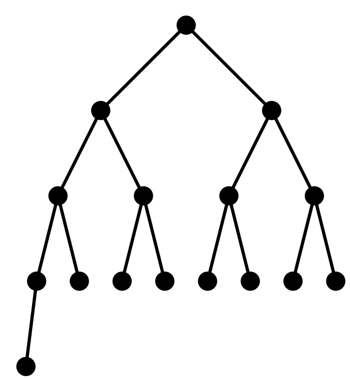
$h = 4$



Køretider

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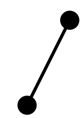

$n = 2^2 = 4$

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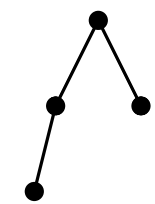

$n = 2^3 = 8$

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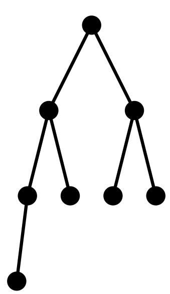

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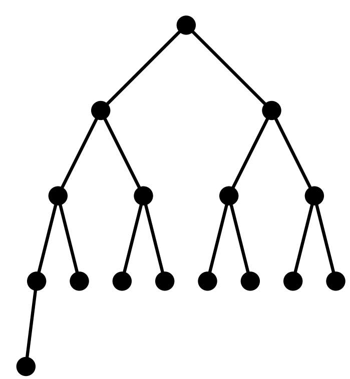
$h = 4$



$$\underbrace{\underbrace{\underbrace{1 + 1}_{2} + 2}_{4} + 4}_{8} + \ldots + 2^{h-1}$$

Køretider

Insert: Boble op

Extract-Max: Boble ned

I begge tilfælde er $T(n) = \Theta(h)$, hvor h er højden af hoben.

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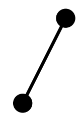

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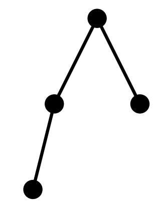

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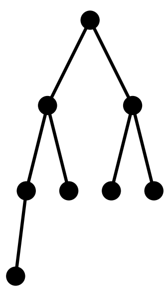
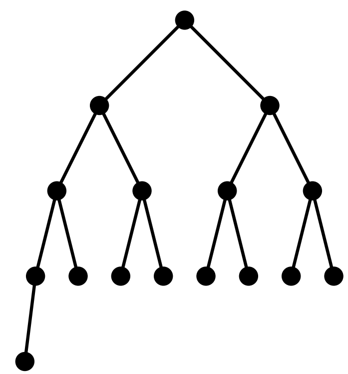
$h = 3$



$$\underbrace{\underbrace{\underbrace{\underbrace{1 + 1}_{2} + 2}_{4} + 4}_{8} + \ldots + 2^{h-1}}_{2^h}$$

$n = 2^4 = 16$

$h = 4$



Køretider

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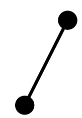

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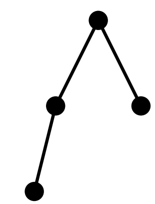

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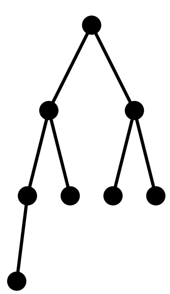
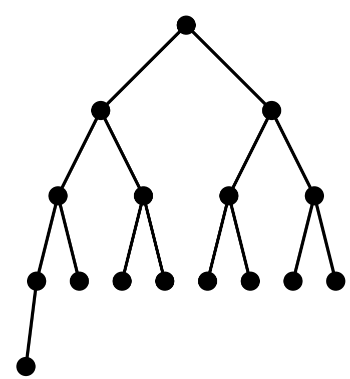


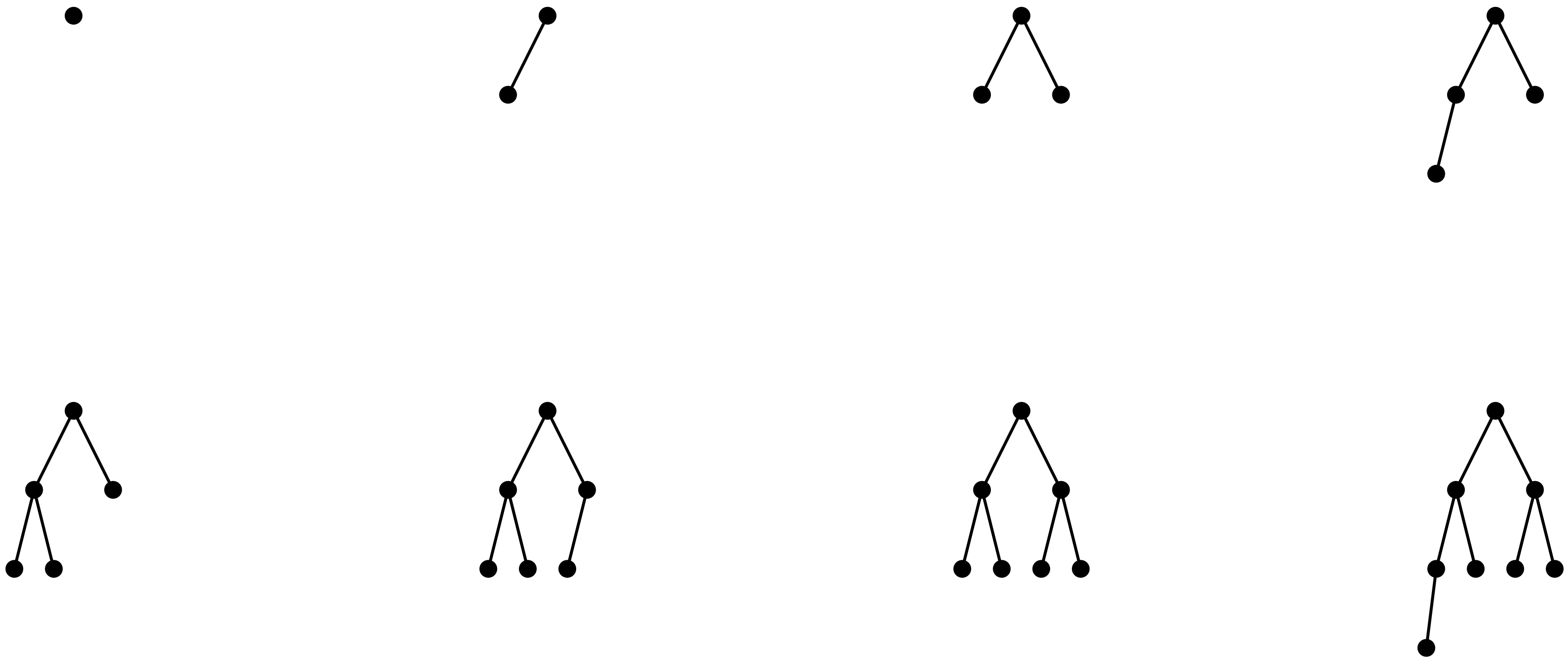
$$\underbrace{\underbrace{\underbrace{\underbrace{1 + 1}_{2} + 2}_{4} + 4}_{8} + \ldots + 2^{h-1}}_{2^h}$$

$n = 2^4 = 16$
$h = 4$



$h = \lfloor \lg n \rfloor$, så
$T(n) = \Theta(\log n)$.

Hvor mange blade er der?



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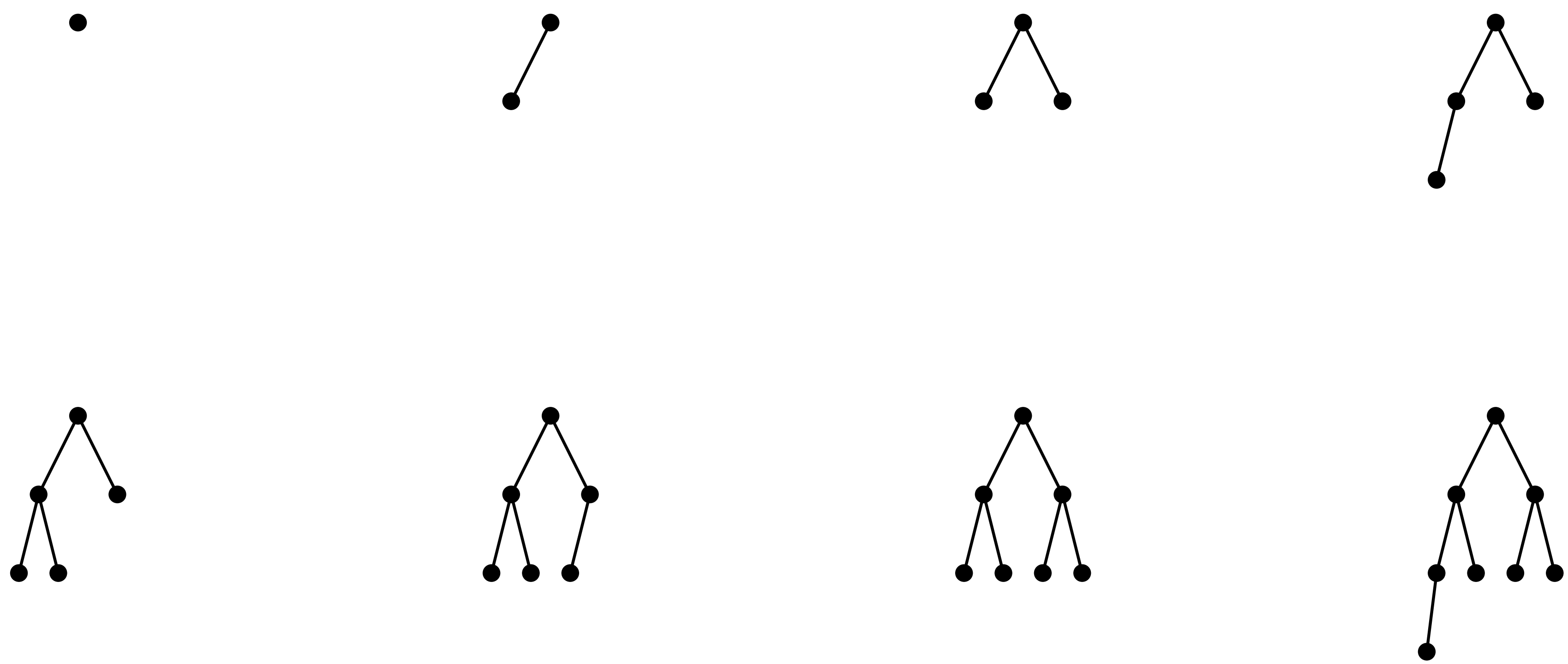
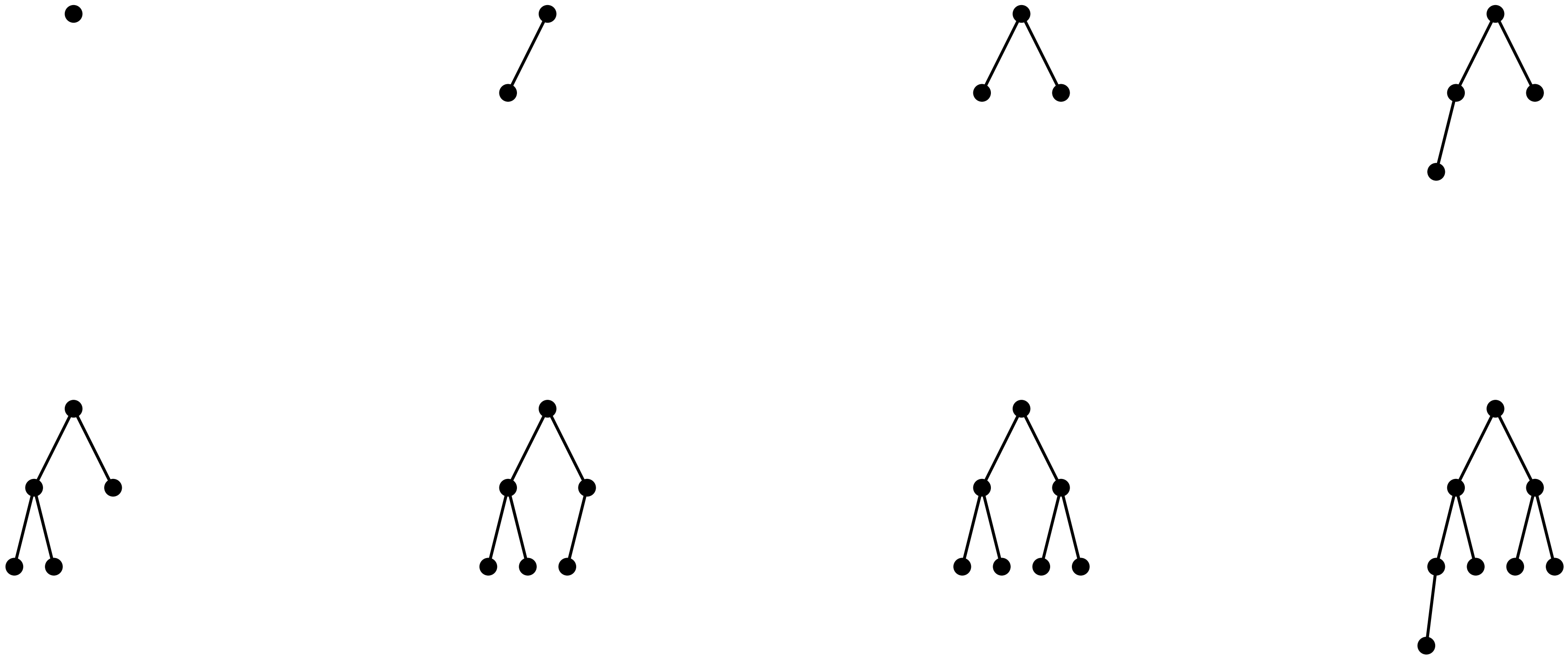


Hver anden gang vi tilføjer en knude er antallet uændret, de andre gange vokser det med én. Derfor: $\#\text{blade} = \lceil \frac{n}{2} \rceil$.

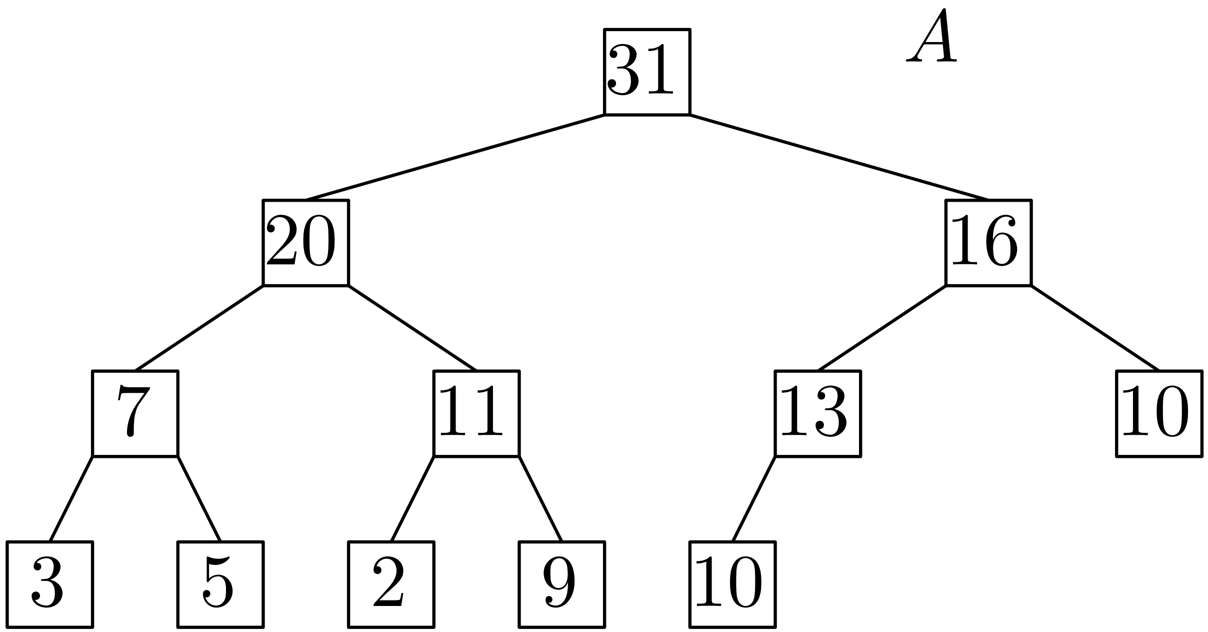
Hvor mange blade er der?



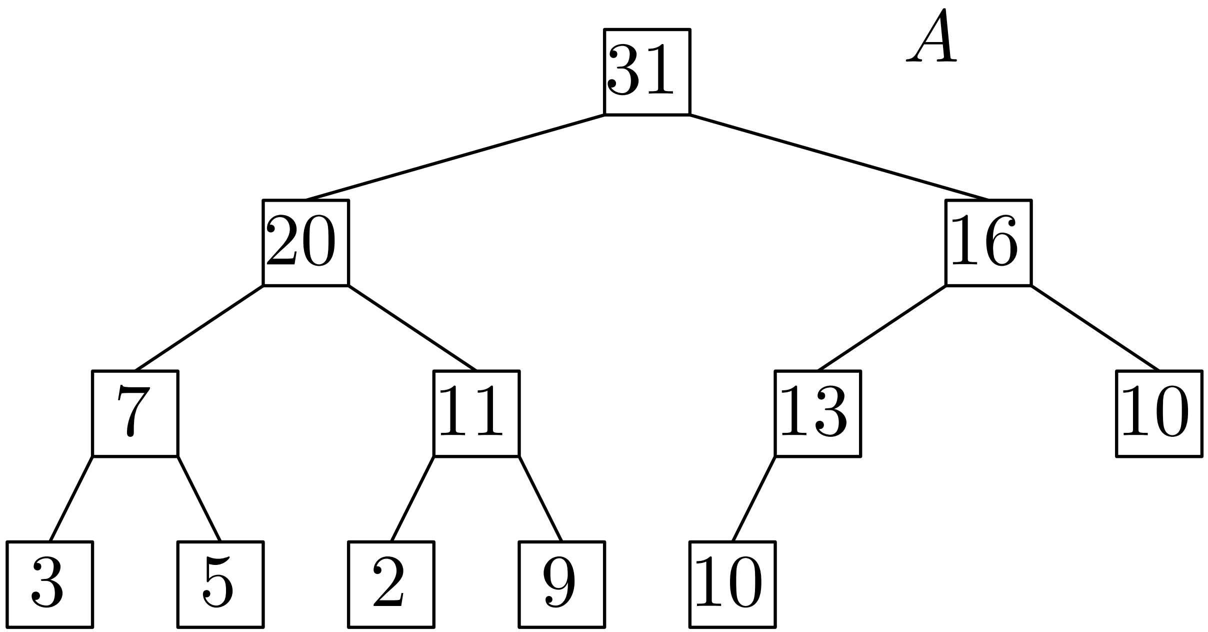
Hver anden gang vi tilføjer en knude er antallet uændret, de andre gange vokser det med én. Derfor: #blade $= \lceil \frac{n}{2} \rceil$.

$$\boxed{\begin{aligned}&\text{Højde} = \lfloor \lg n \rfloor\\ &\text{Blade} = \lceil \tfrac{n}{2} \rceil\end{aligned}}$$

Repræsentation med array

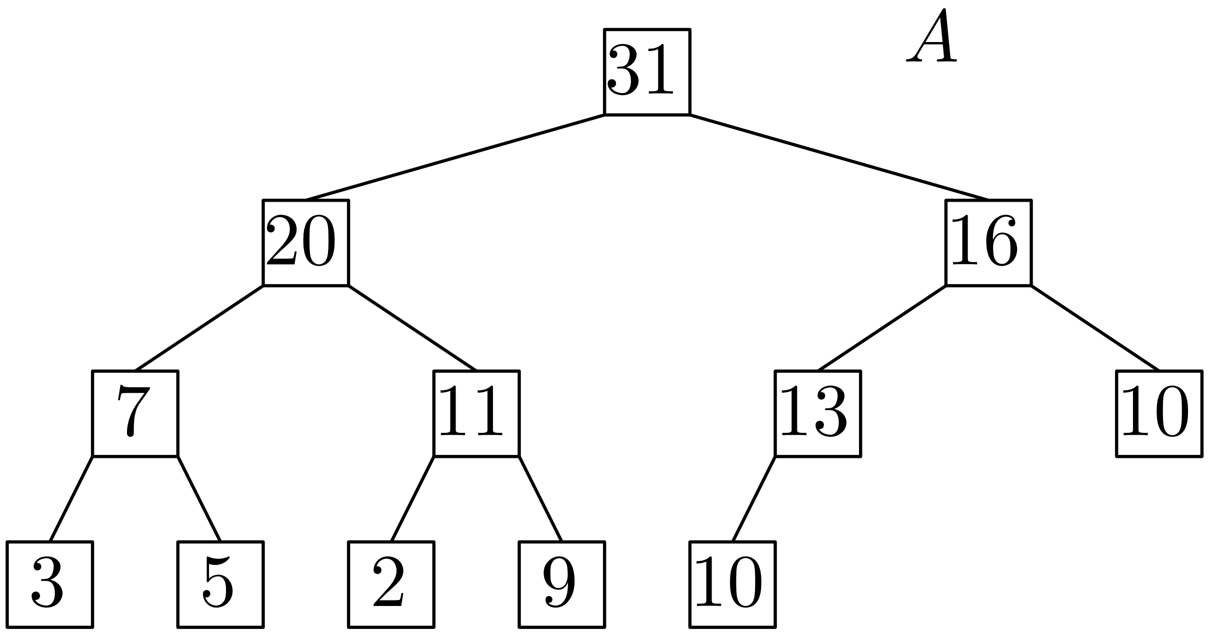


Repræsentation med array



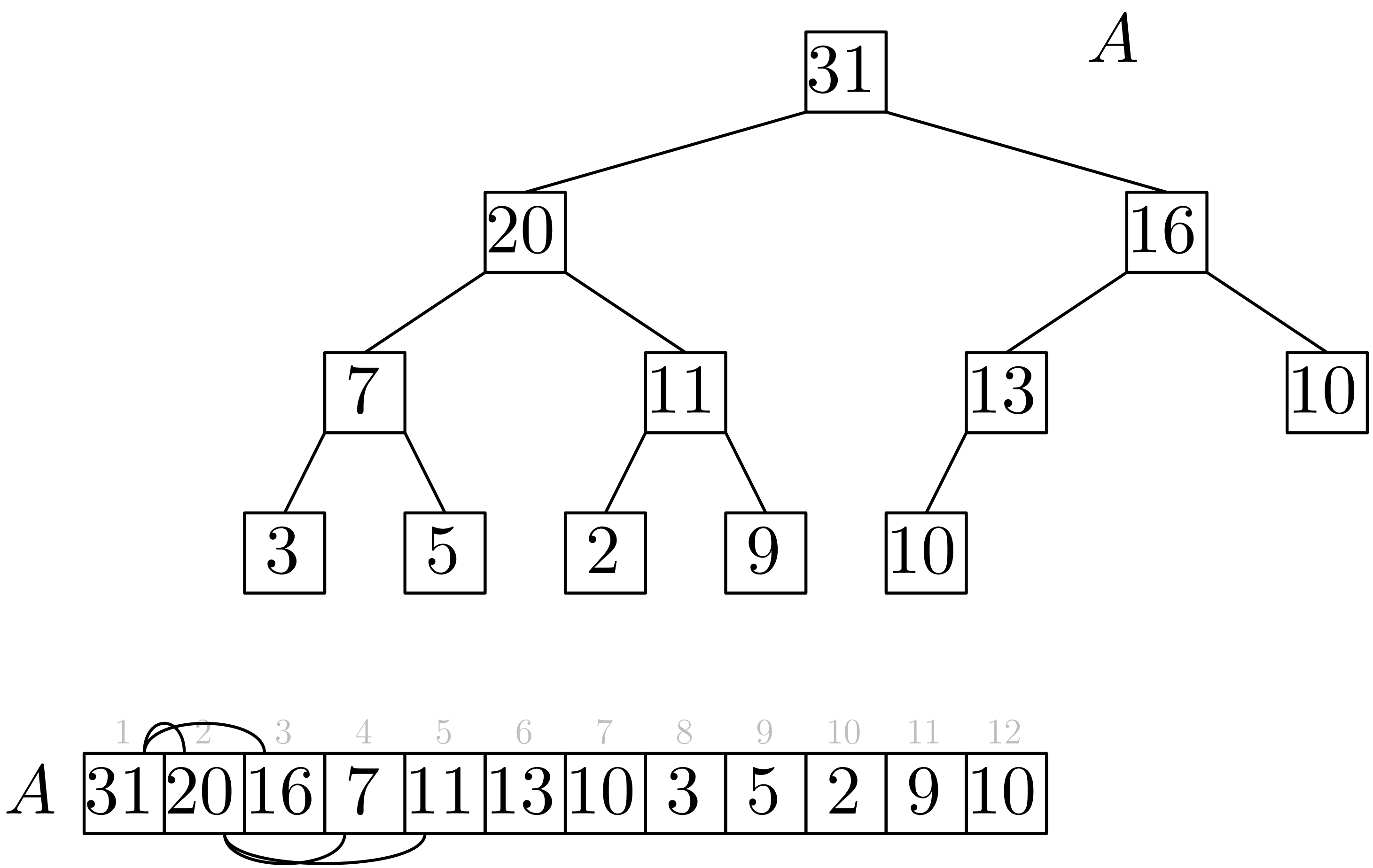
	1	2	3	4	5	6	7	8	9	10	11	12
A	31	20	16	7	11	13	10	3	5	2	9	10

Repræsentation med array

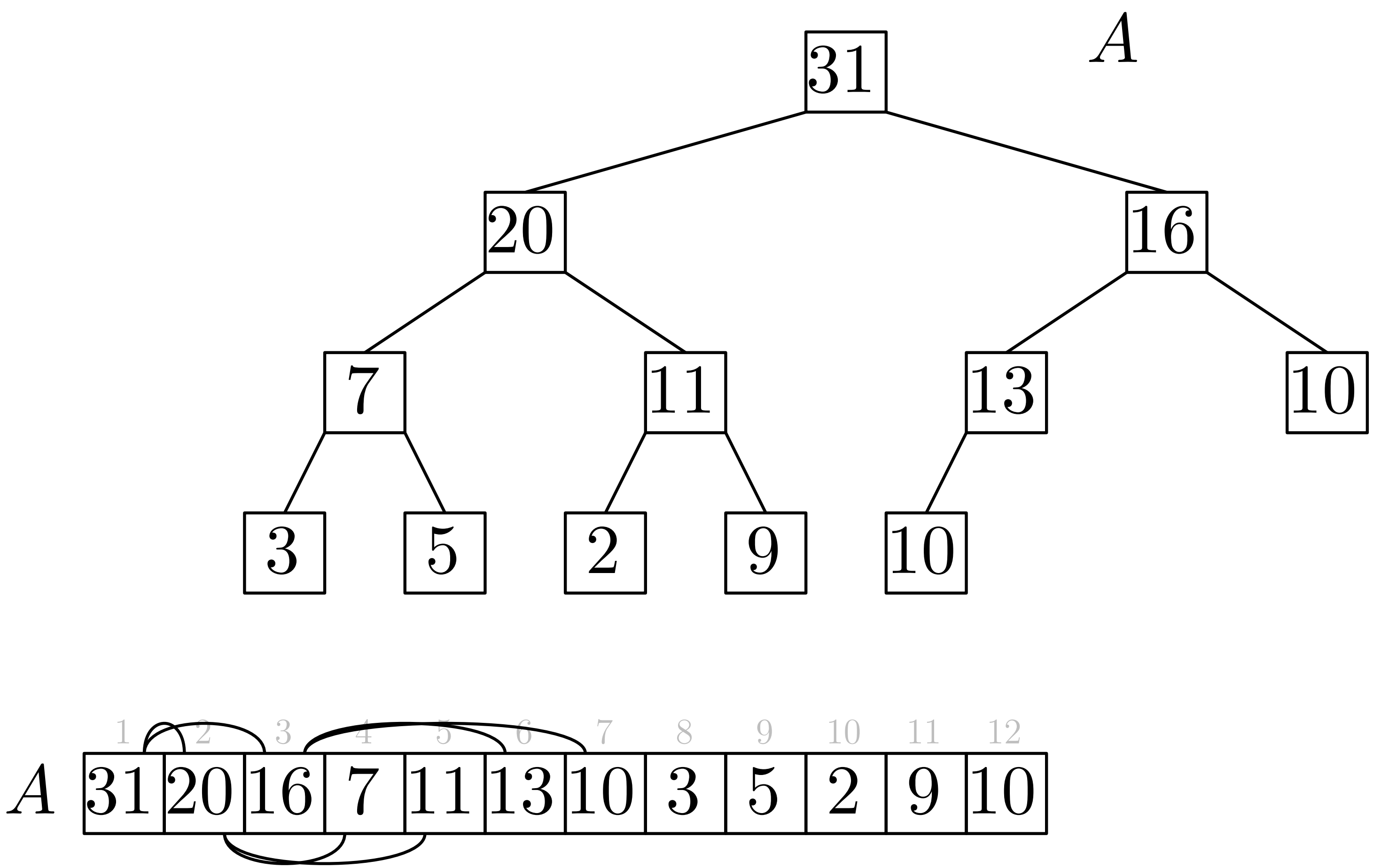


	1	2	3	4	5	6	7	8	9	10	11	12
A	31	20	16	7	11	13	10	3	5	2	9	10

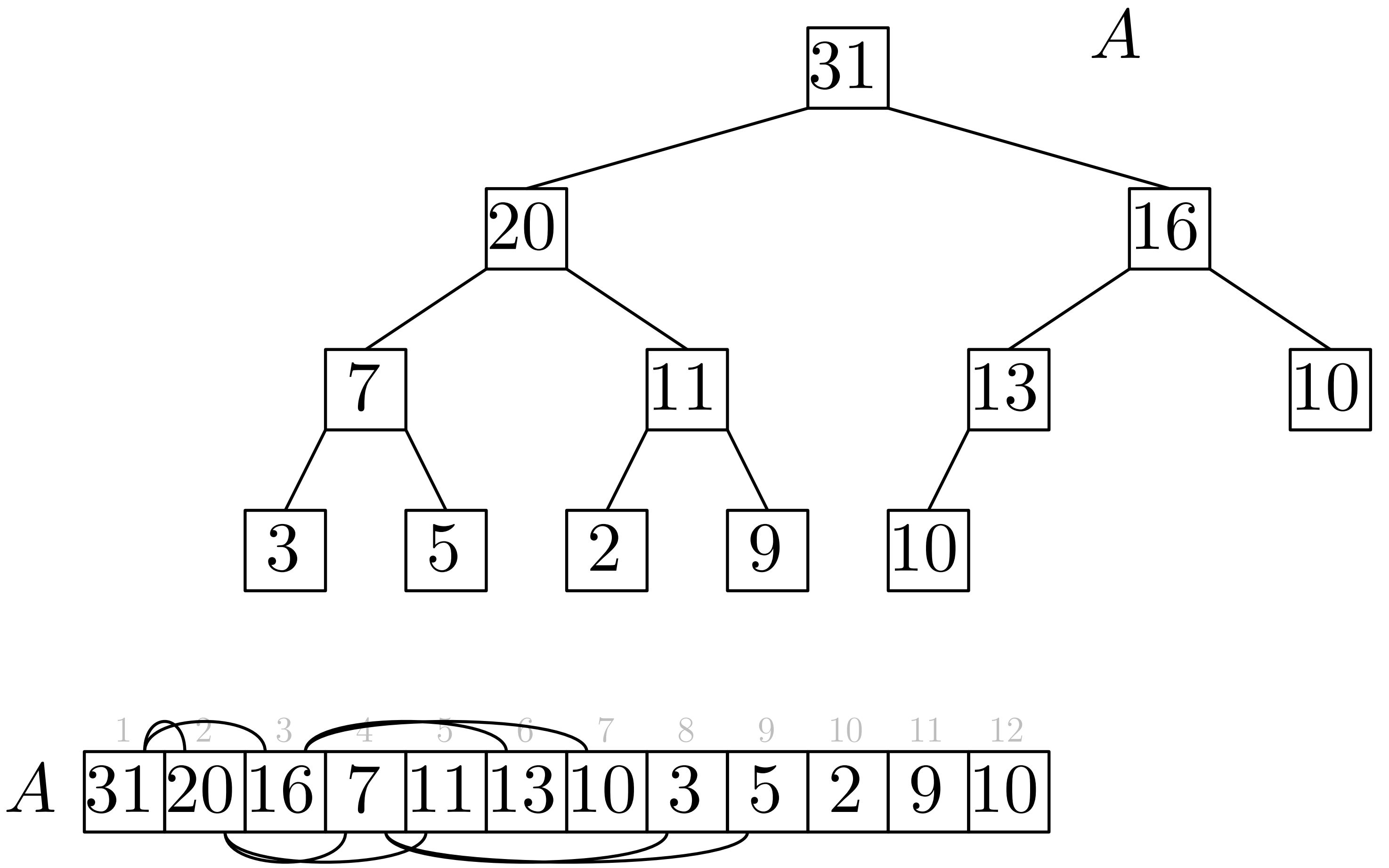
Repræsentation med array



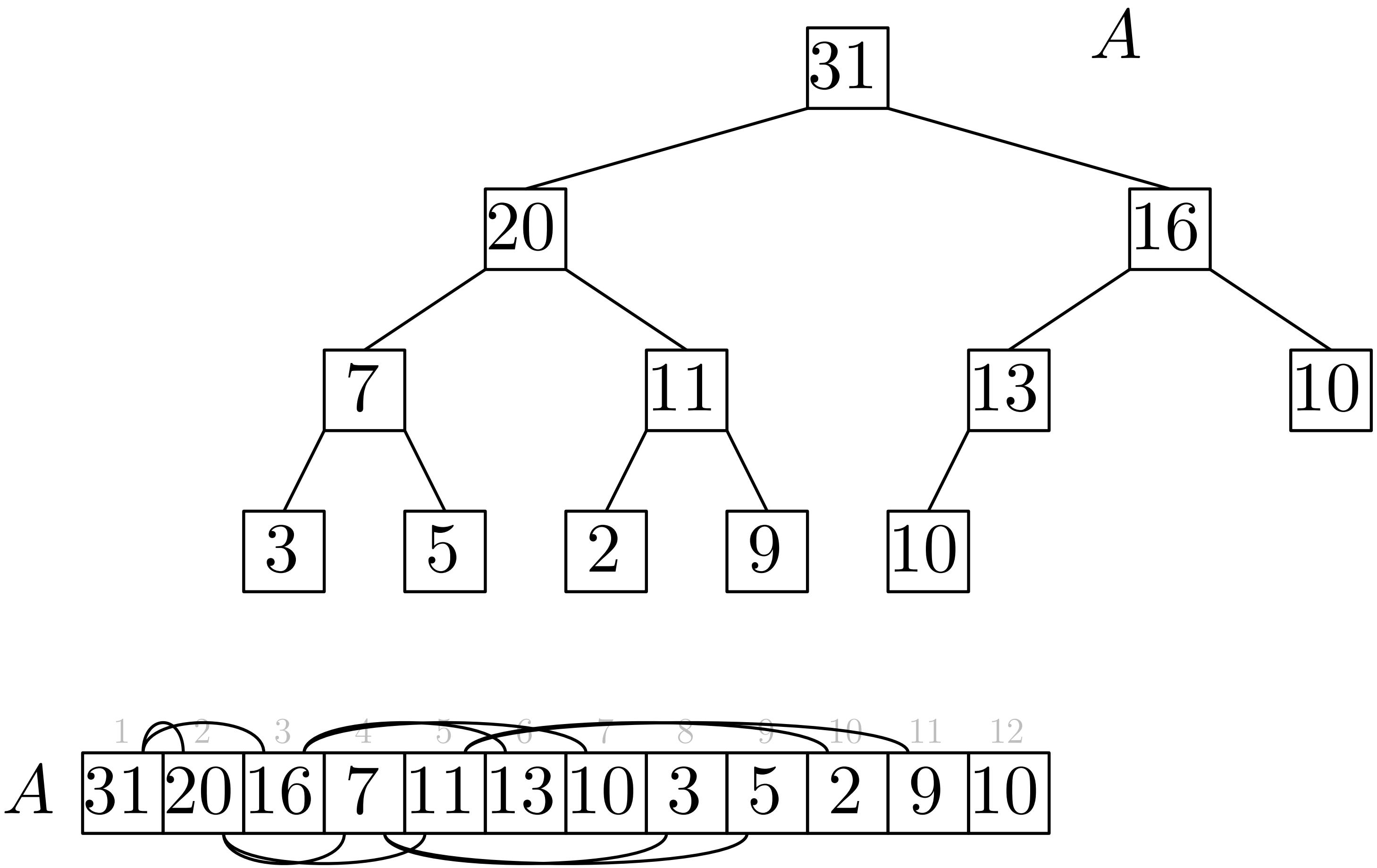
Repræsentation med array



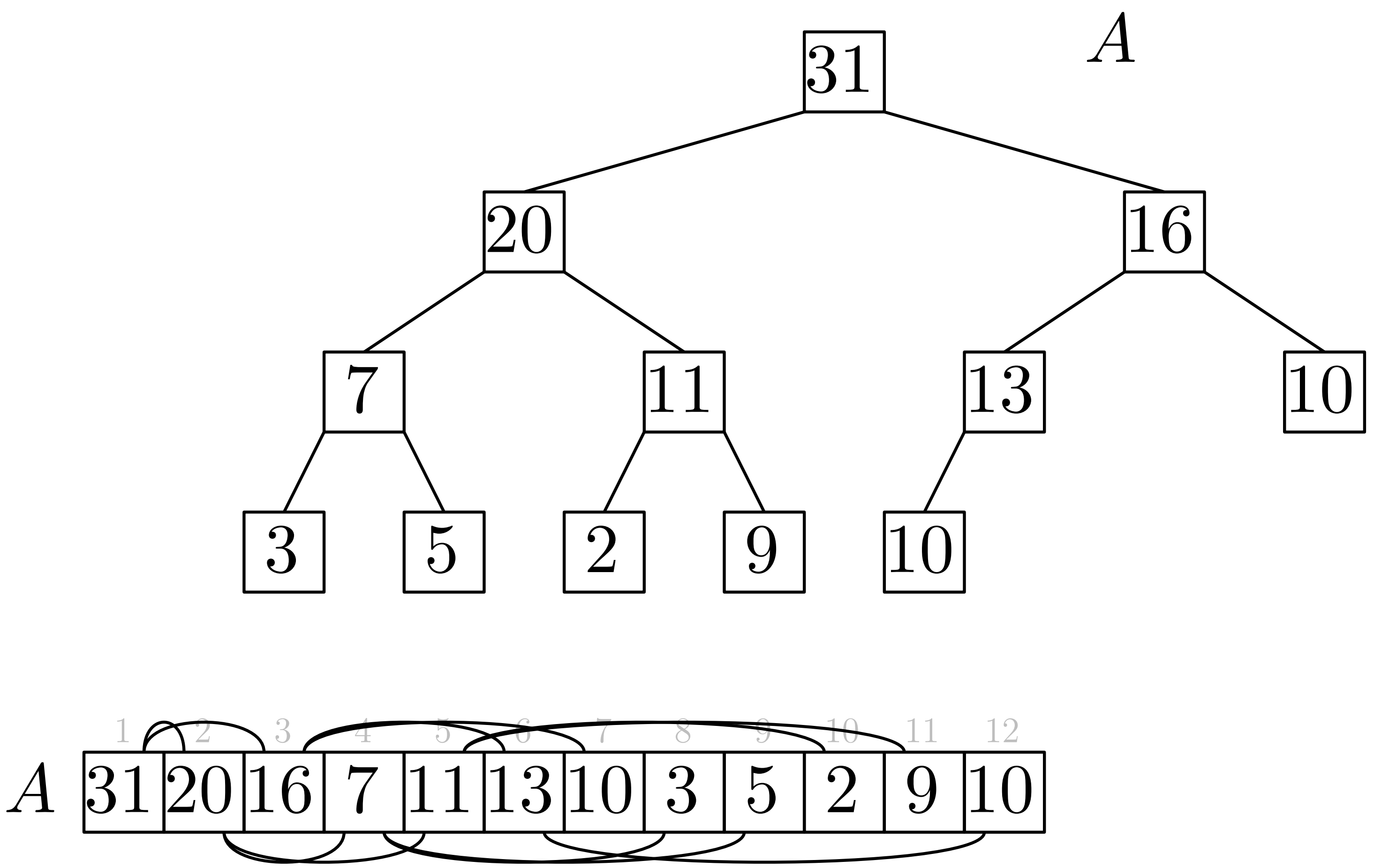
Repræsentation med array



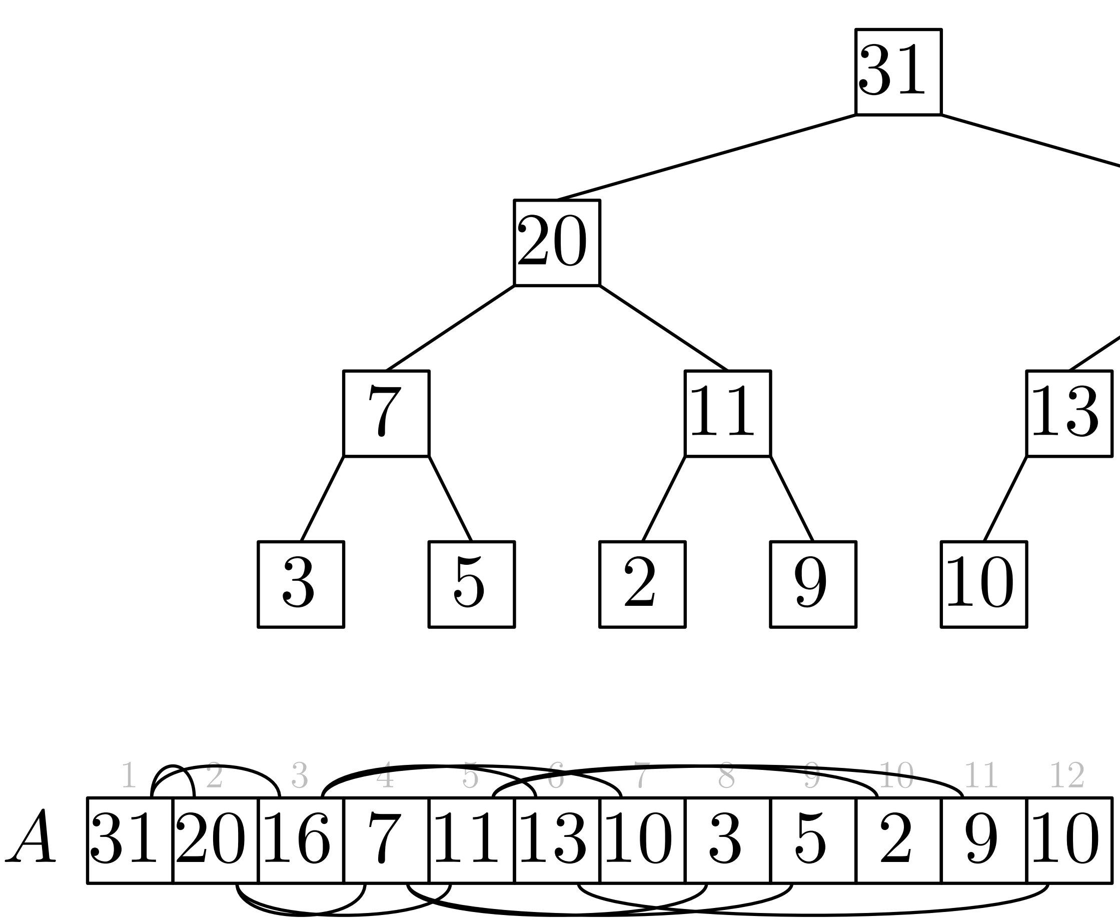
Repræsentation med array



Repræsentation med array



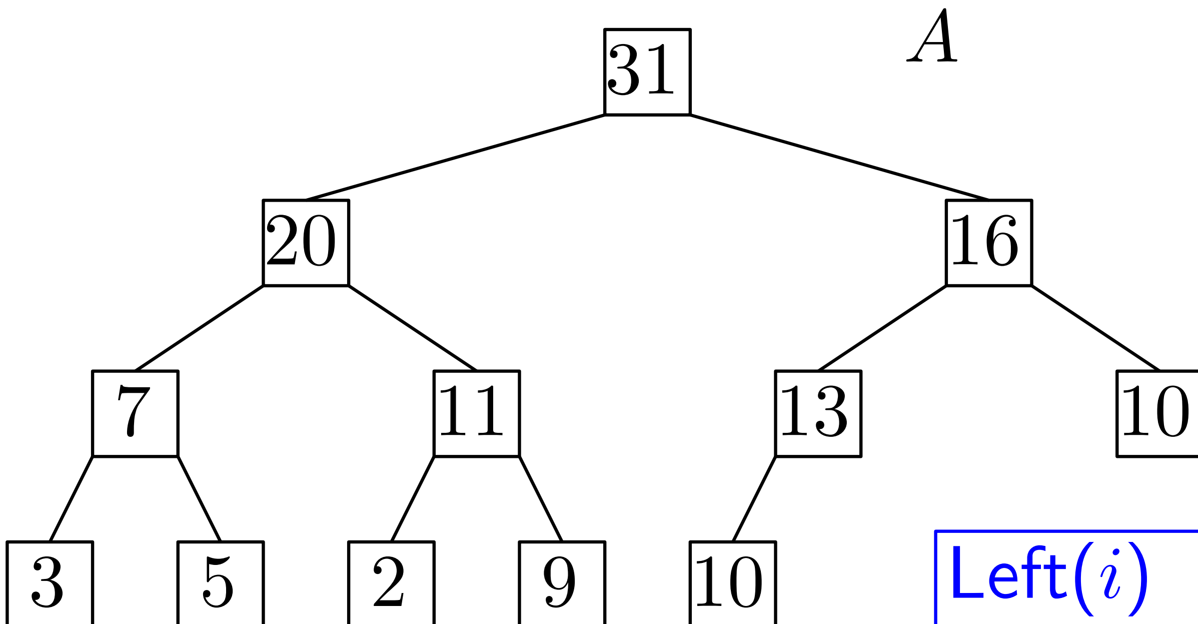
Repræsentation med array

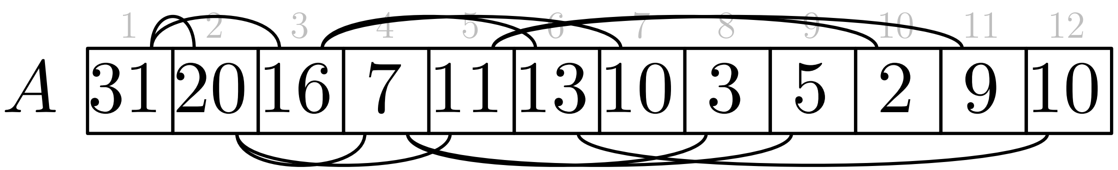


Left(i)
    return $2i$

Right(i)
    return $2i + 1$

Repræsentation med array





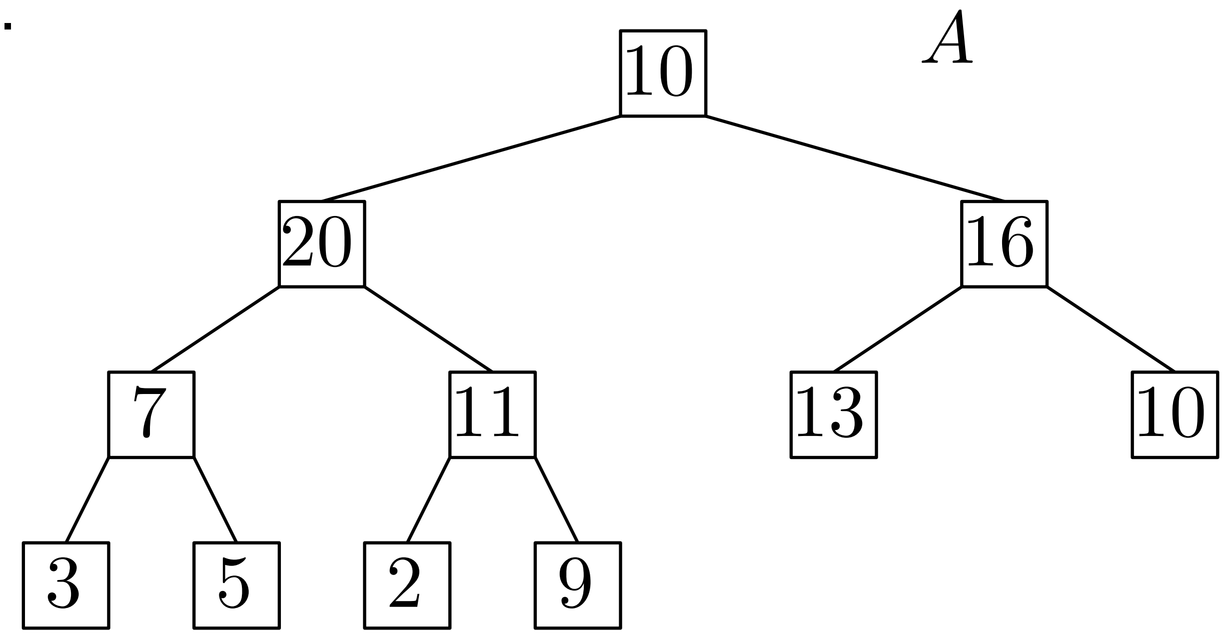
Left(i)
    return $2i$

Right(i)
    return $2i + 1$

Parent(i)
    return $\lfloor \frac{i}{2} \rfloor$

Max-Heapify

Vi lader 10 "boble ned".
Max-Heapify$(A, 1)$



Max-Heapify(A, i)
  $l =$ Left(i)
  $r =$ Right(i)
  $largest = i$
  if $l \leq A.heap\text{-}size$ and $A[l] > A[largest]$
    $largest = l$
  if $r \leq A.heap\text{-}size$ and $A[r] > A[largest]$
    $largest = r$
  if $largest \neq i$
    swap $A[i]$ and $A[largest]$
    Max-Heapify$(A, largest)$

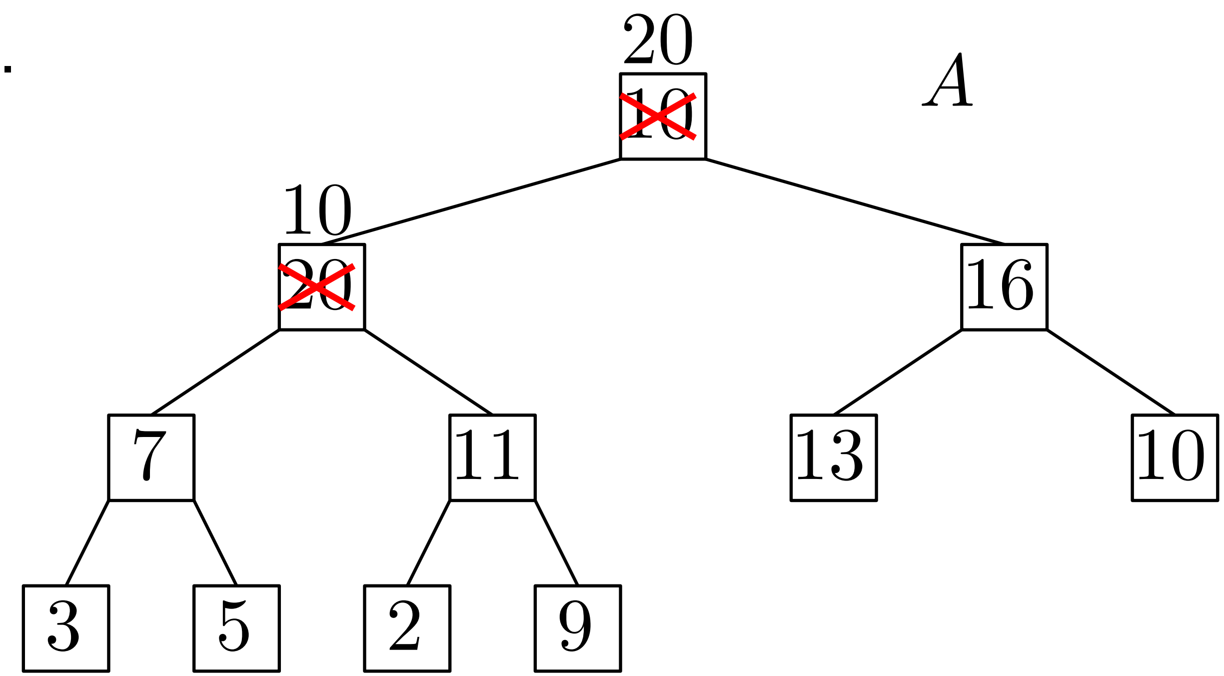
Left(i)
  return $2i$

Right(i)
  return $2i + 1$

Max-Heapify

Vi lader 10 "boble ned".

Max-Heapify$(A, 1)$



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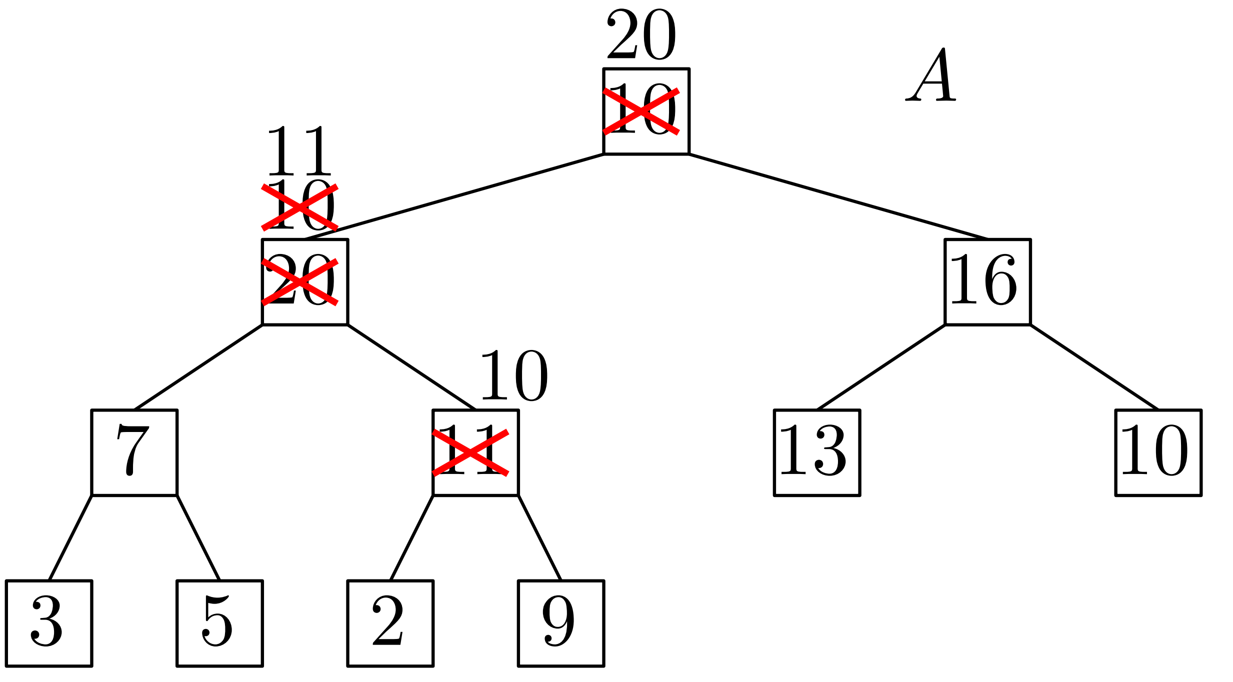

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    swap $A[i]$ and $A[largest]$
    Max-Heapify$(A, largest)$

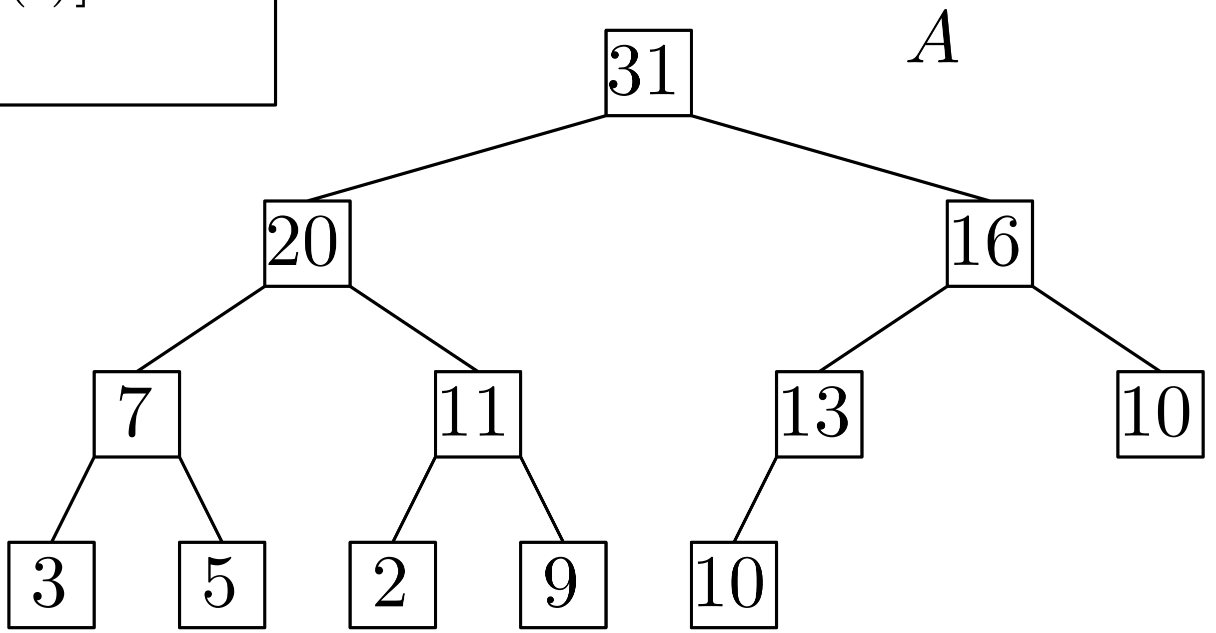
Left(i)
  return $2i$

Right(i)
  return $2i + 1$

Insert

Insert(A, k)
  $A.heap\text{-}size = A.heap\text{-}size + 1$
  $i = A.heap\text{-}size$
  $A[i] = k$
  while $i > 1$ and $A[\text{Parent}(i)] < A[i]$
    swap $A[i]$ and $A[\text{Parent}(i)]$
    $i = \text{Parent}(i)$

Parent(i)
  return $\lfloor \frac{i}{2} \rfloor$



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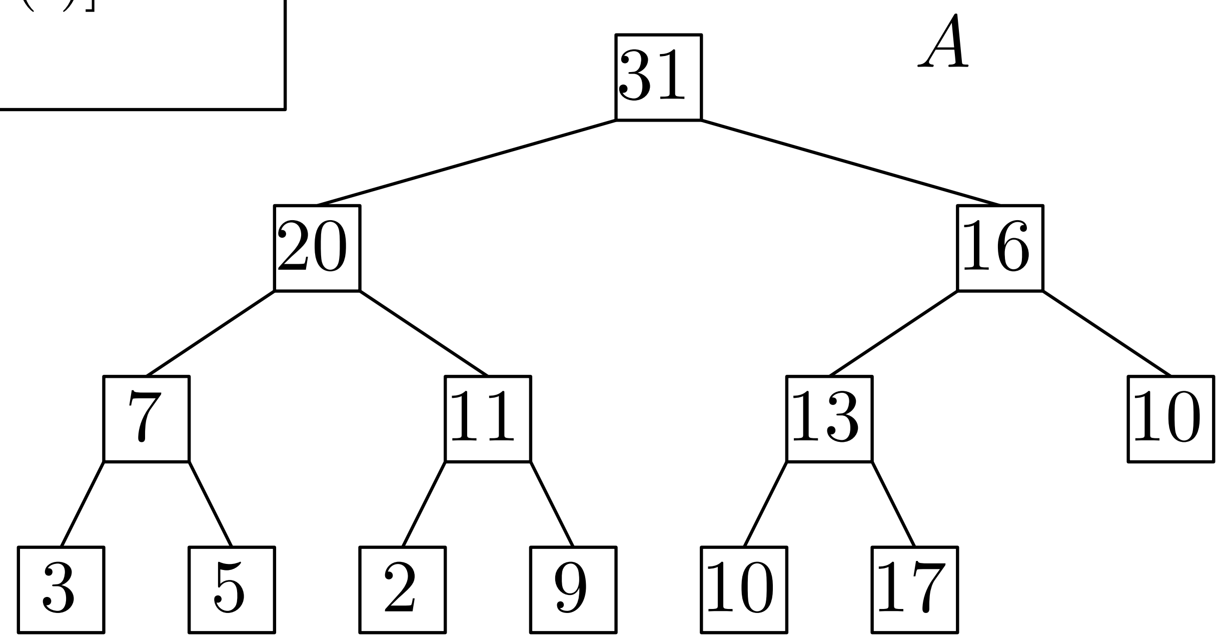

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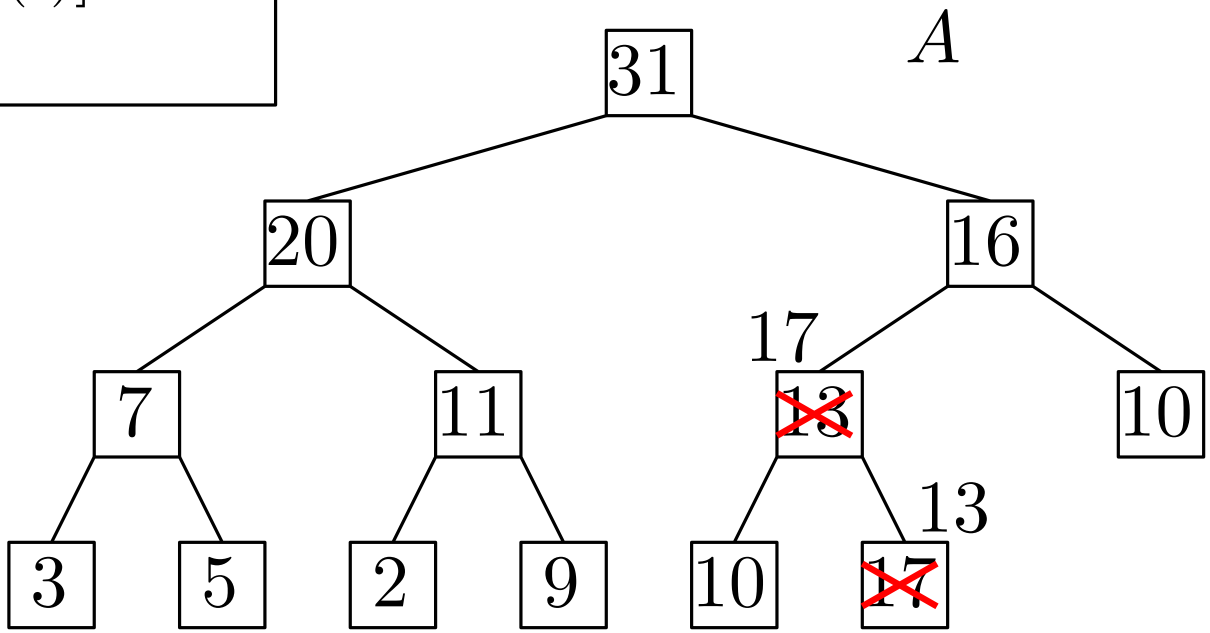

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    $i = \text{Parent}(i)$

Parent(i)
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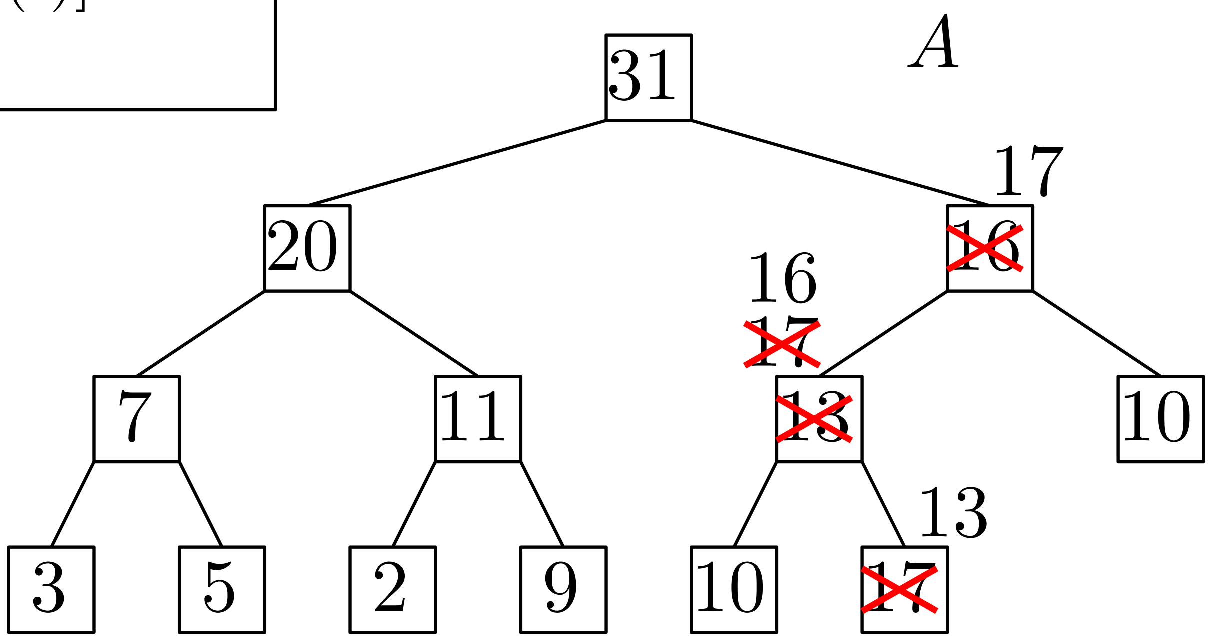
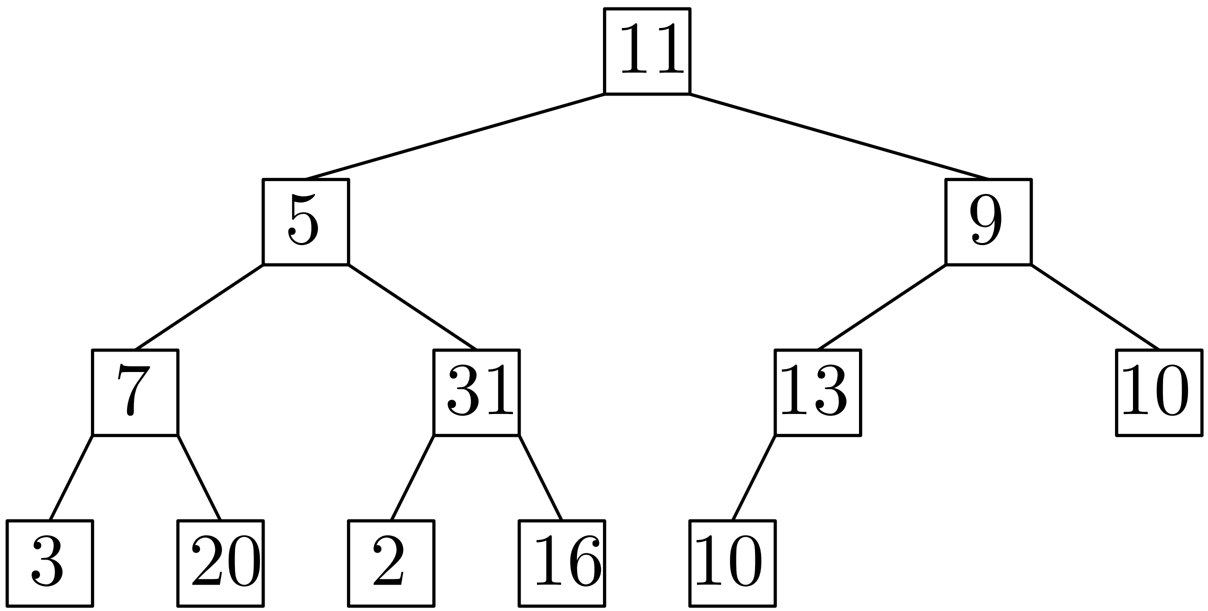


Konstruktion af hob

Build-Max-Heap(A)
  $A.heap\text{-}size = A.length$
  for $i = \lfloor \frac{A.length}{2} \rfloor$ downto 1
    Max-Heapify(A, i) // Lad $A[i]$ boble ned



11	5	9	7	31	13	10	3	20	2	16	10

Konstruktion af hob

Build-Max-Heap(A)
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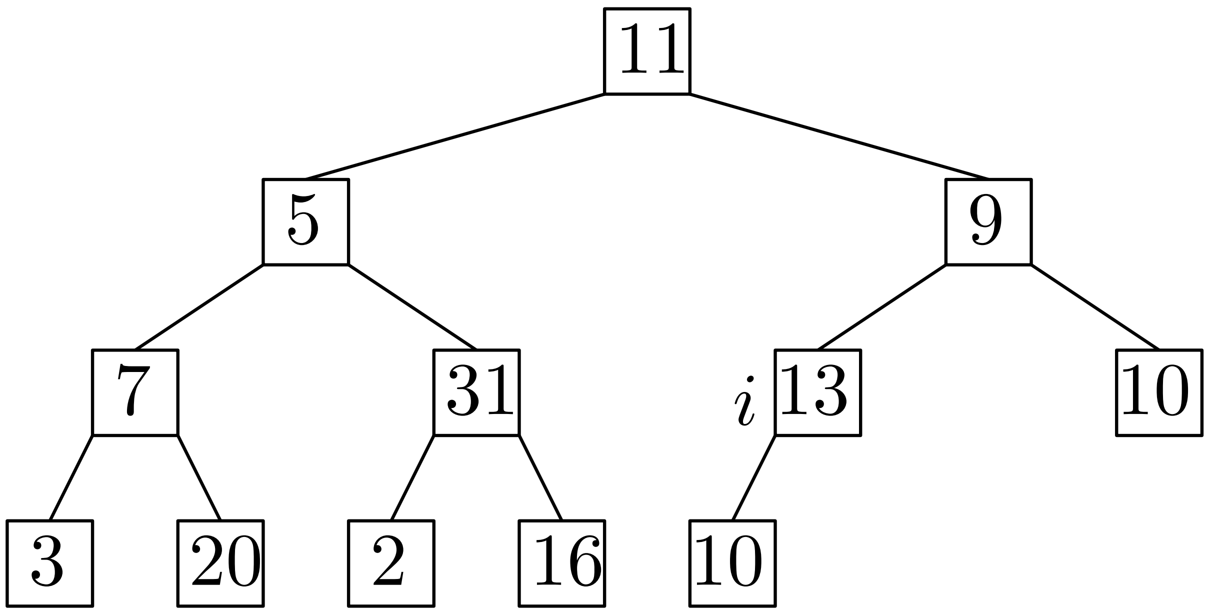
    Max-Heapify(A, i) // Lad $A[i]$ boble ned

$= A.length - \lceil \frac{A.length}{2} \rceil$
Sidste knude med et barn



11	5	9	7	31	13	10	3	20	2	16	10
					i						

Konstruktion af hob

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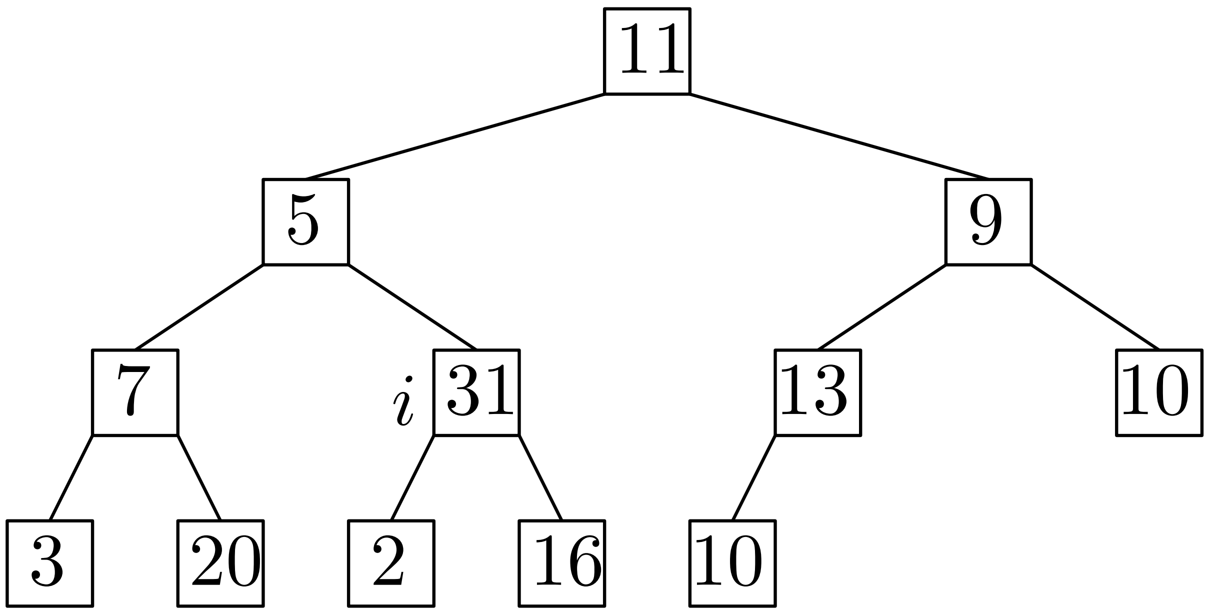

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				i							

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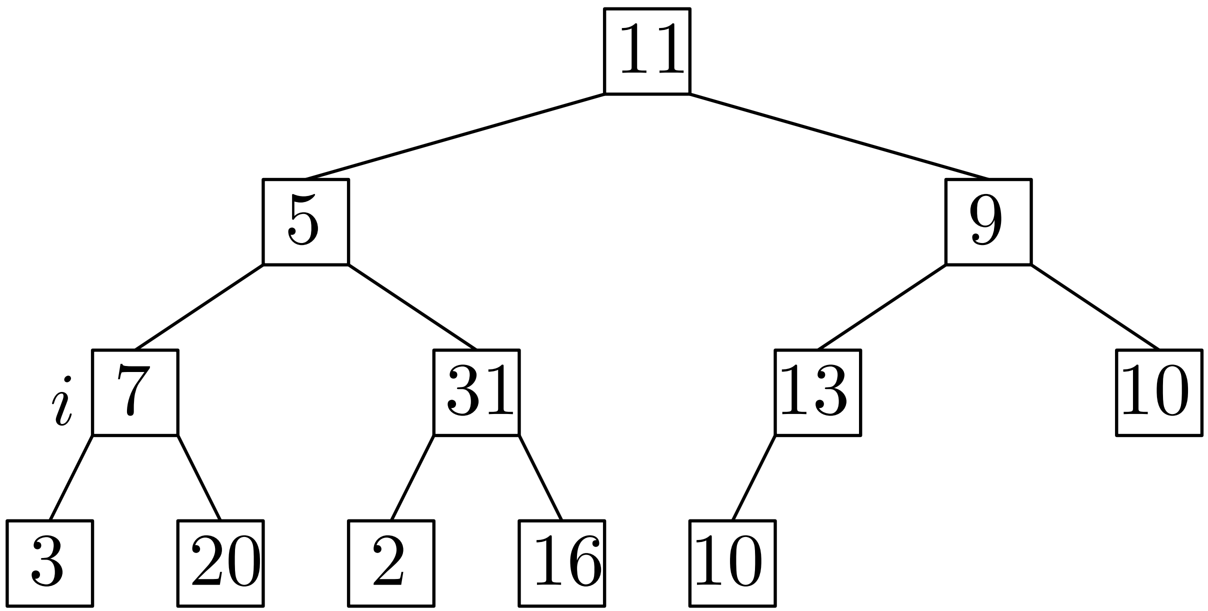

11	5	9	7	31	13	10	3	20	2	16	10
			i								

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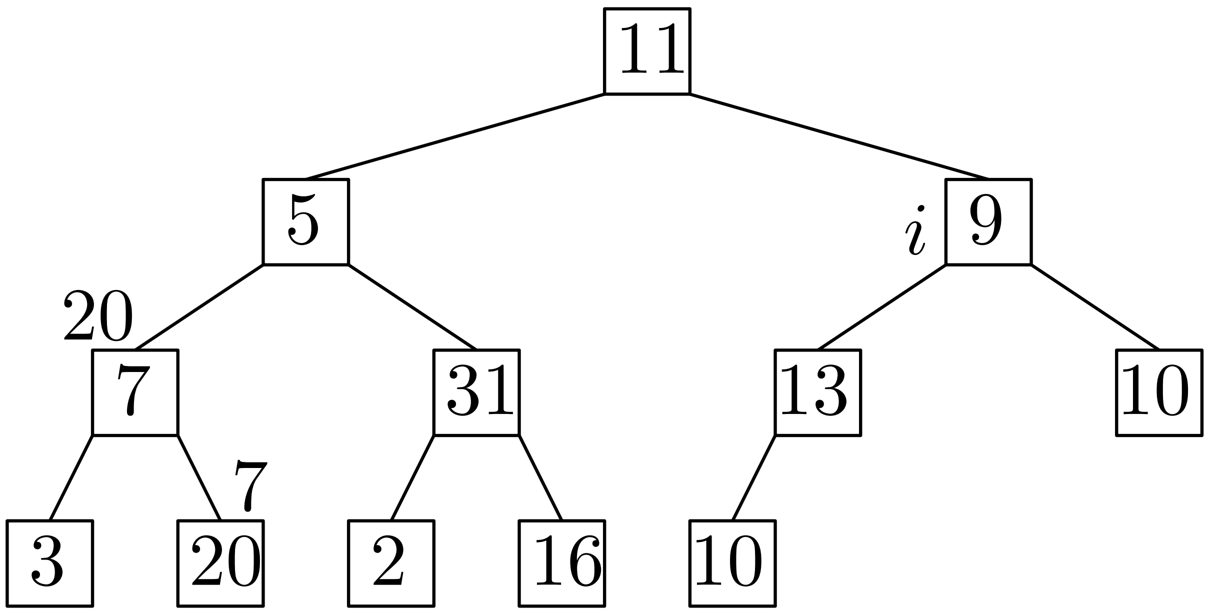
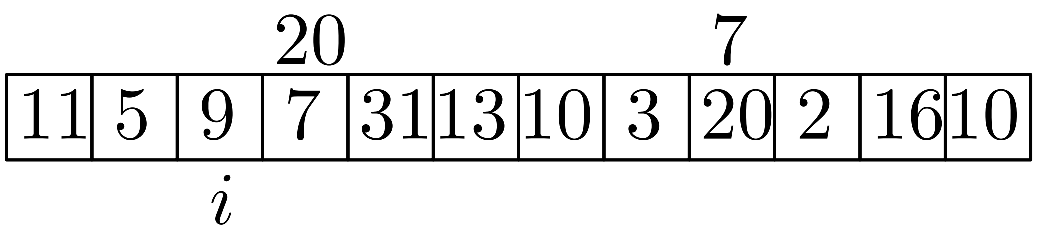
    Max-Heapify(A, i) // Lad $A[i]$ boble ned

$= A.length - \lceil \frac{A.length}{2} \rceil$
Sidste knude med et barn





Konstruktion af hob

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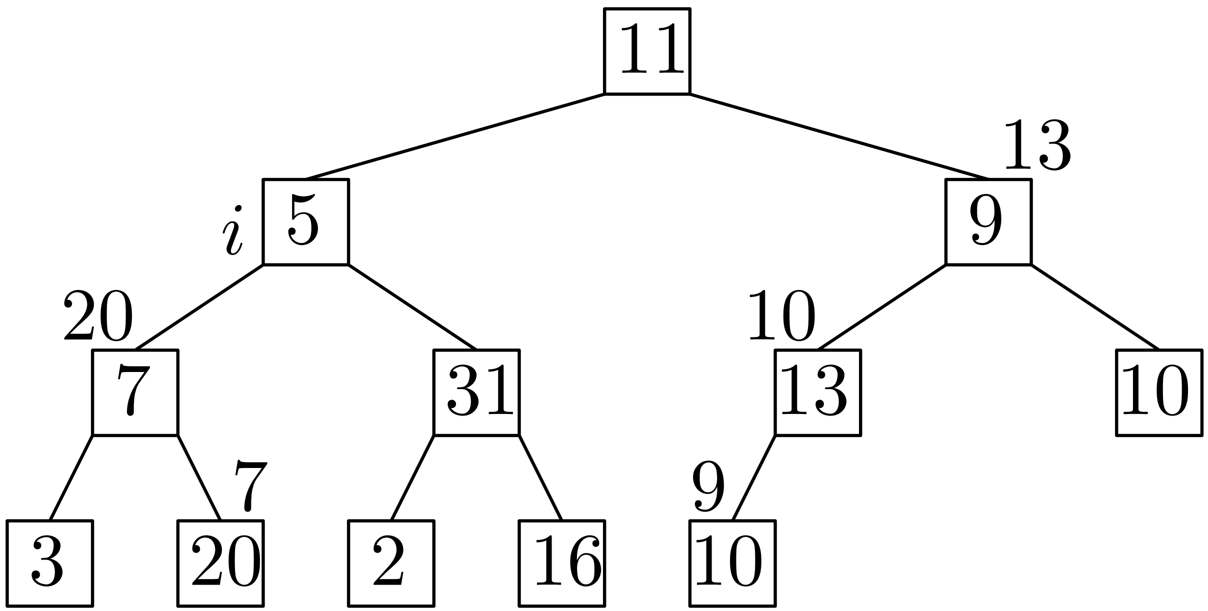

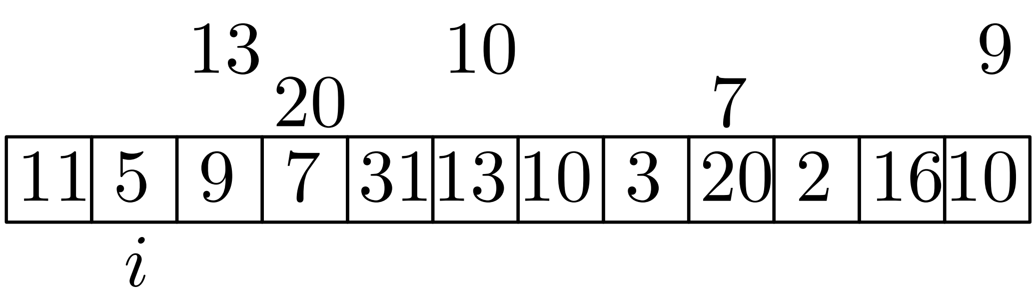

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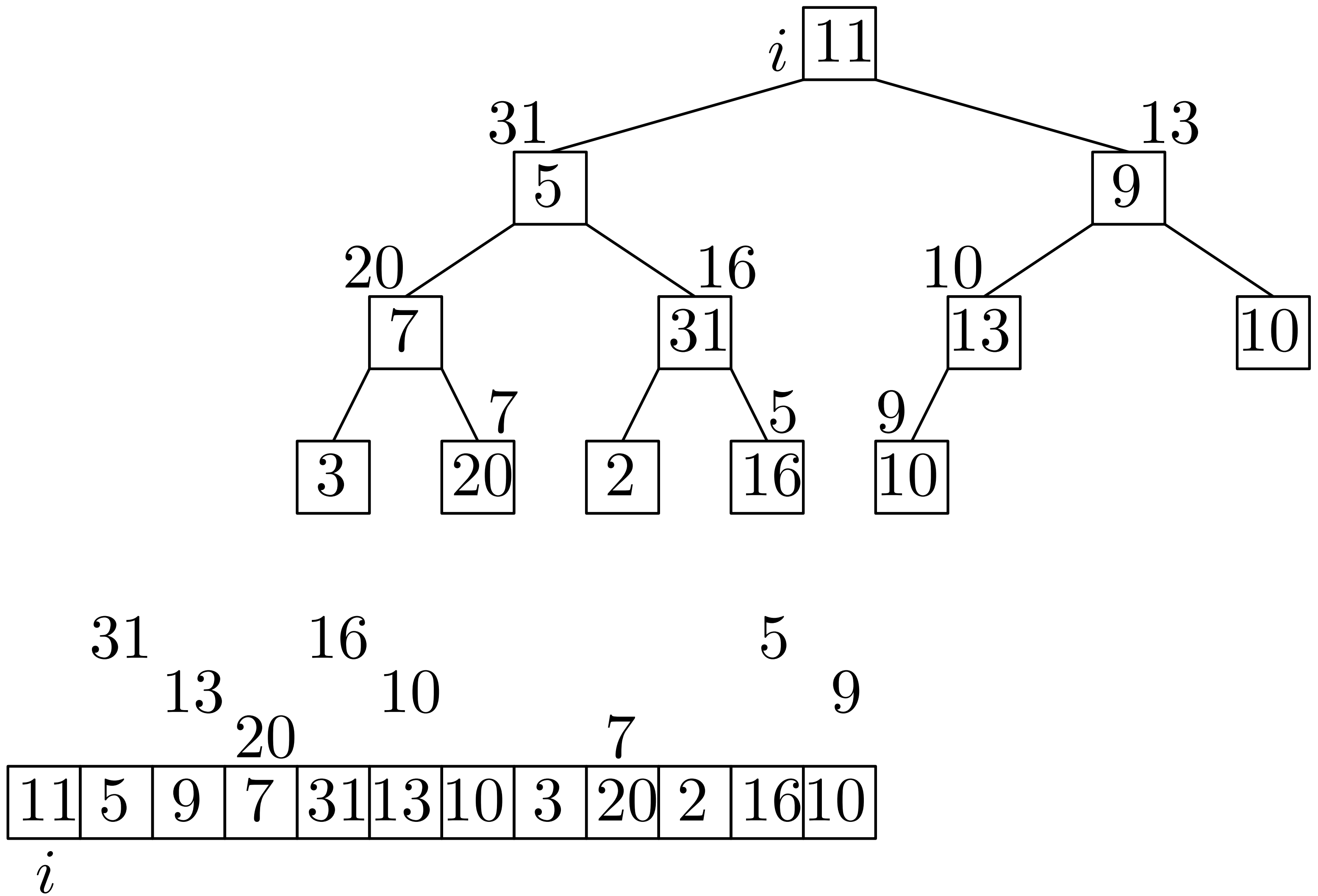

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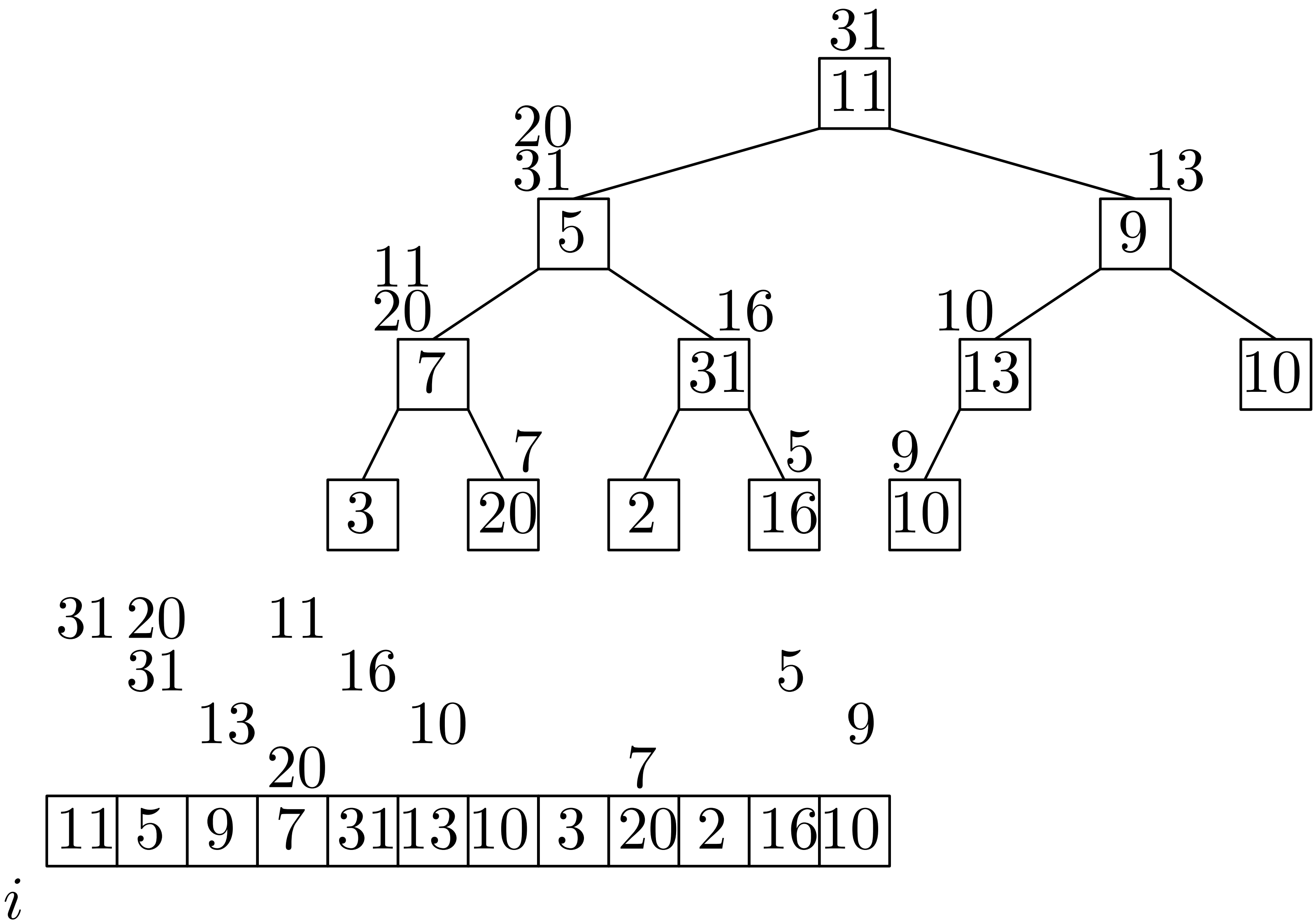

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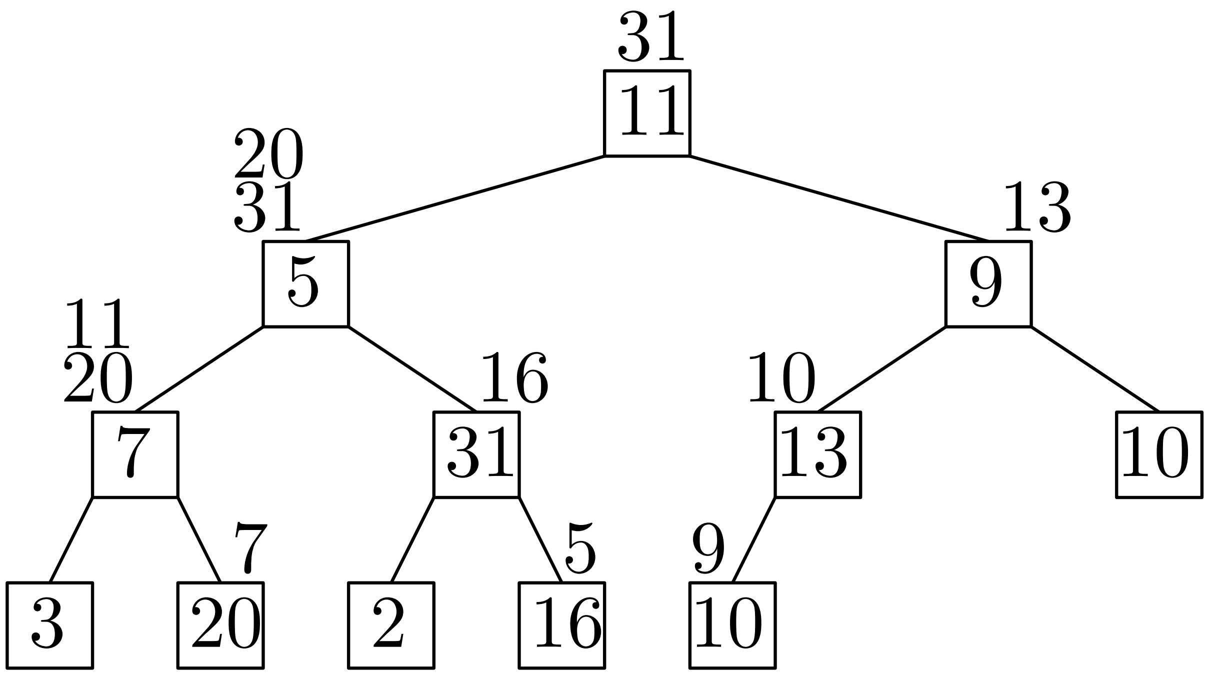
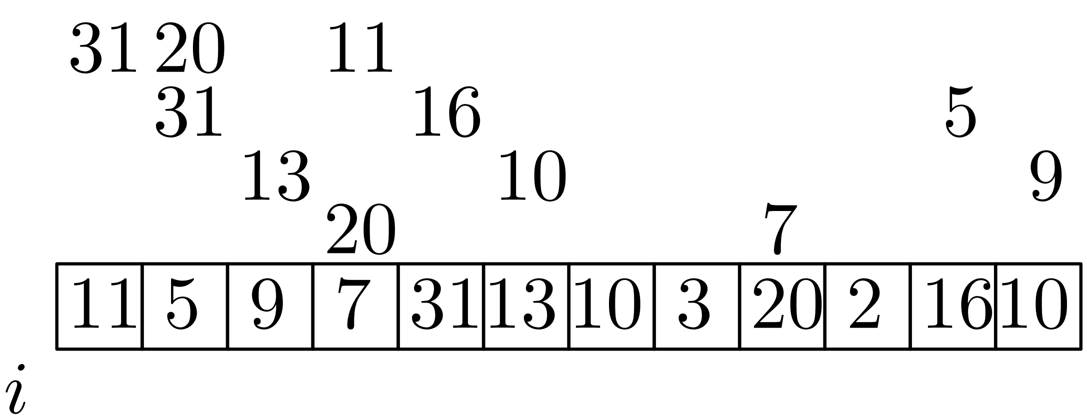
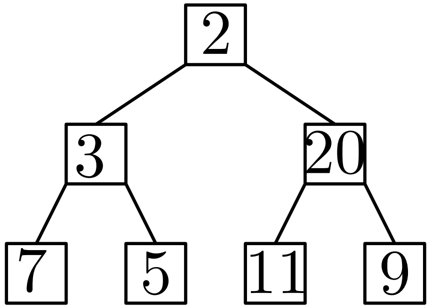


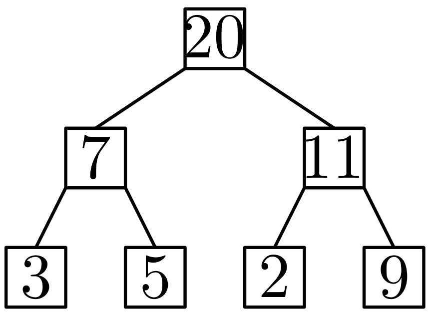


Køretid: Max-Heapify tager $O(\log n)$ tid. I alt $O(n \log n)$.
Bedre analyse: $\Theta(n)$.

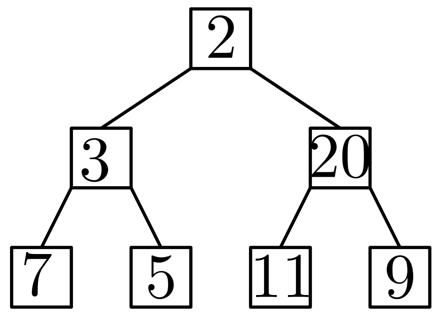
Hvordan ser hoben ud til sidst?



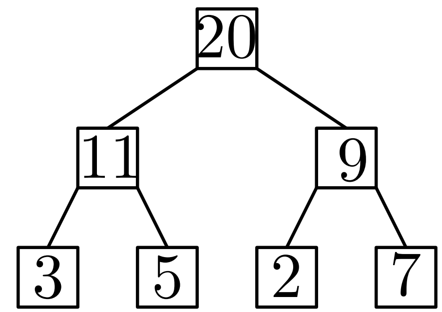
Build-Max-Heap(A)



A

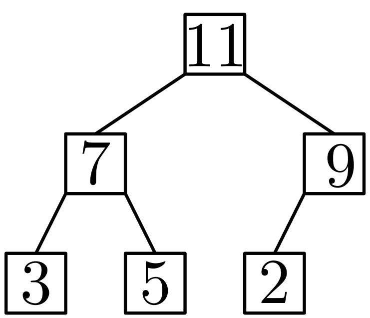


B

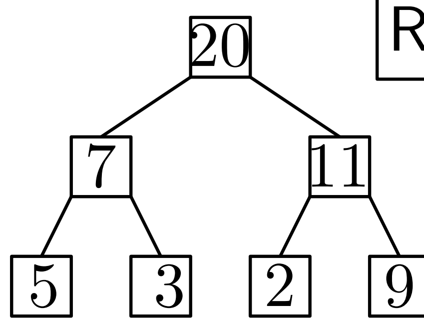


C

socrative.com $\rightarrow$ Student login,
Room name: ABRAHAMSEN3464

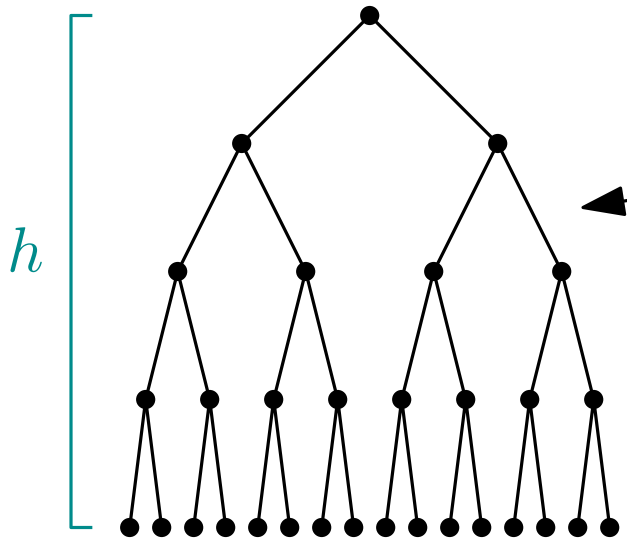


D



E

Bedre analyse af Build-Max-Heap



$$n = 2^{h+1} - 1$$

Her: $h = 4$, $n = 2^5 - 1 = 31$.

$$B = 2^{h-1} \cdot 1 + 2^{h-2} \cdot 2 + 2^{h-3} \cdot 3 + \ldots + 2^0 \cdot h$$

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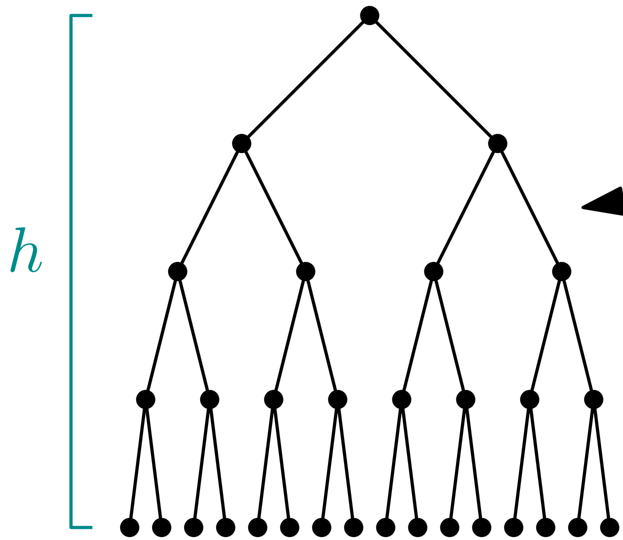

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$$B/2 = B - B/2$$

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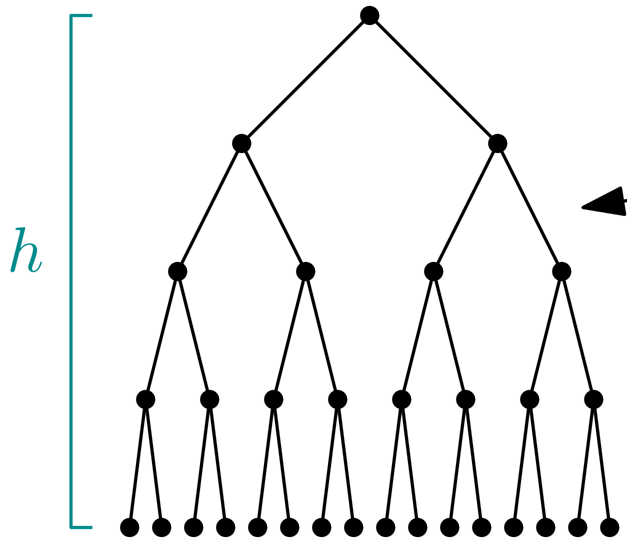

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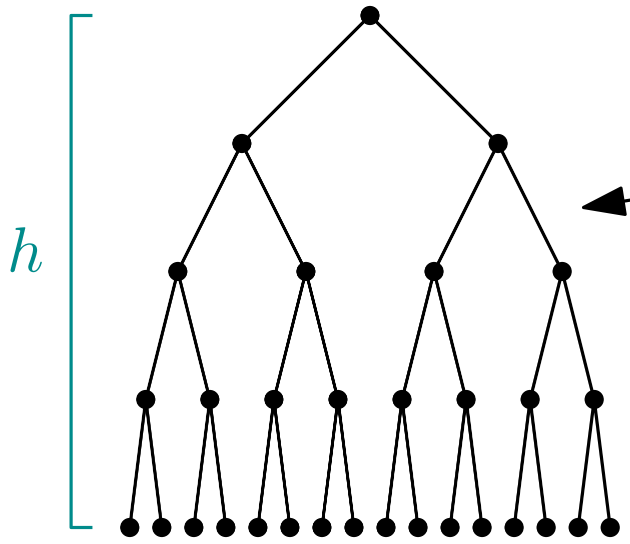

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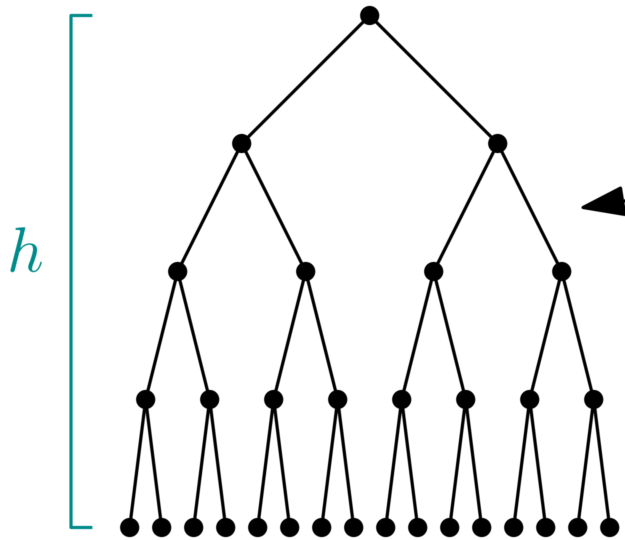

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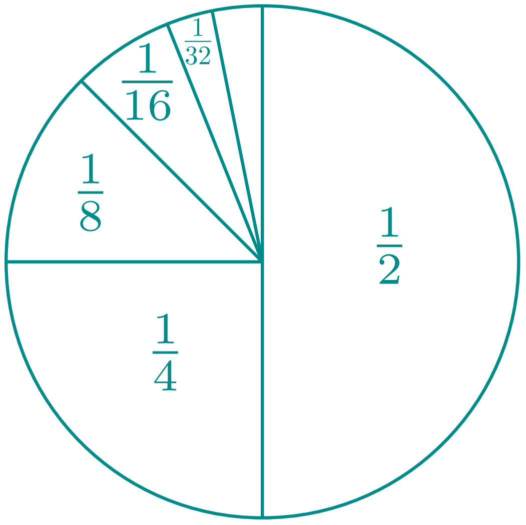

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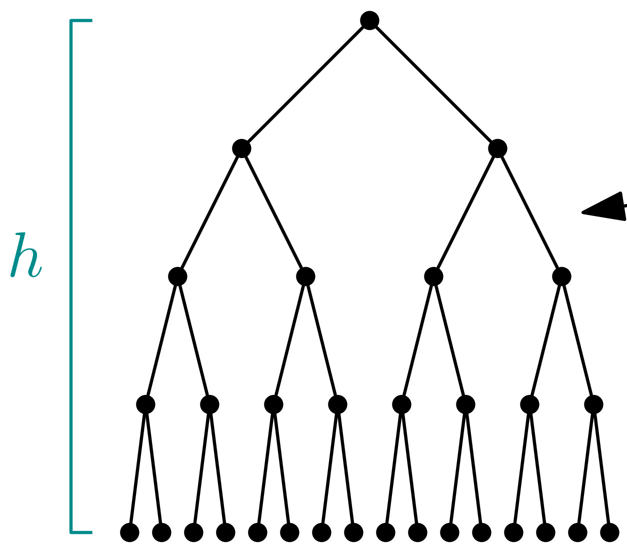

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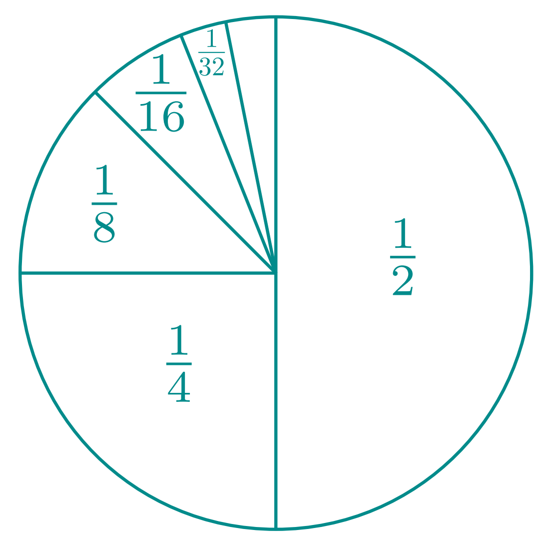

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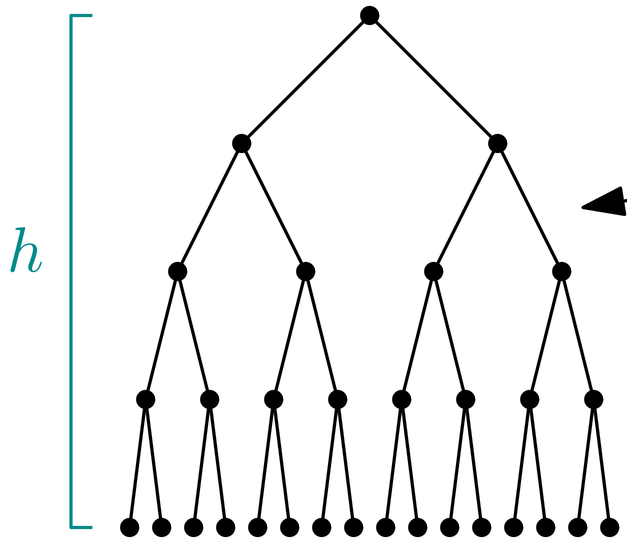

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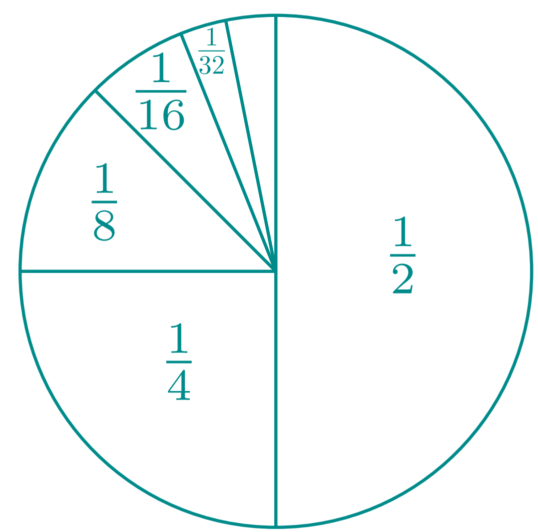
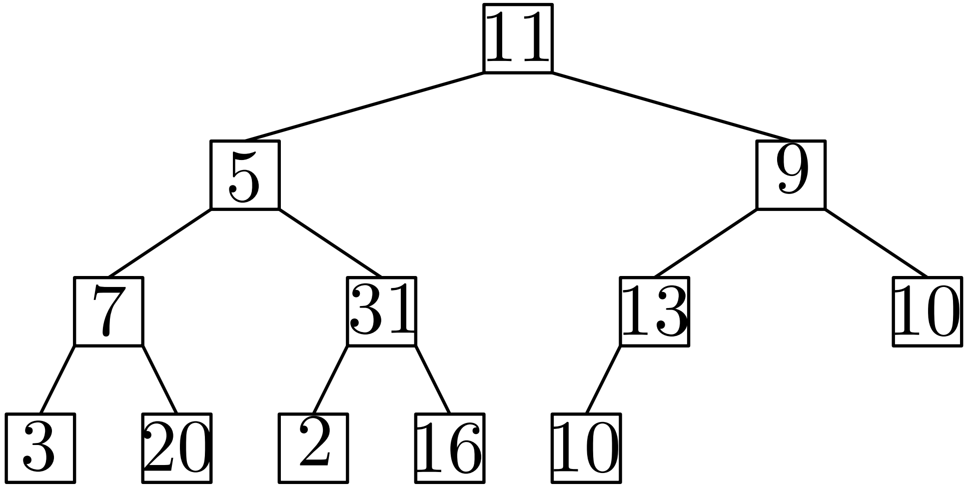


Konklusion: $B < 2^{h+1}$, så $B \leq 2^{h+1} - 1 = n$.

Køretid for Build-Max-Heap: $\Theta(n)$.

Heapsort

Heapsort(A)
  Build-Max-Heap(A)
  for $i = A.length$ downto 2
    swap $A[1]$ and $A[i]$
    $A.heap\text{-}size = A.heap\text{-}size - 1$
    Max-Heapify($A, 1$)



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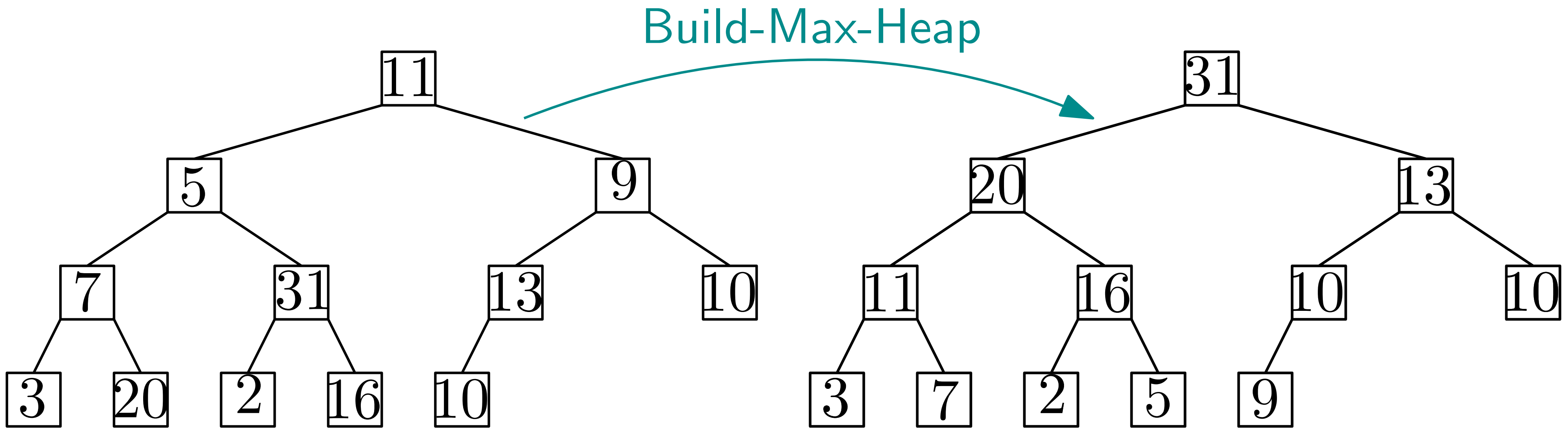

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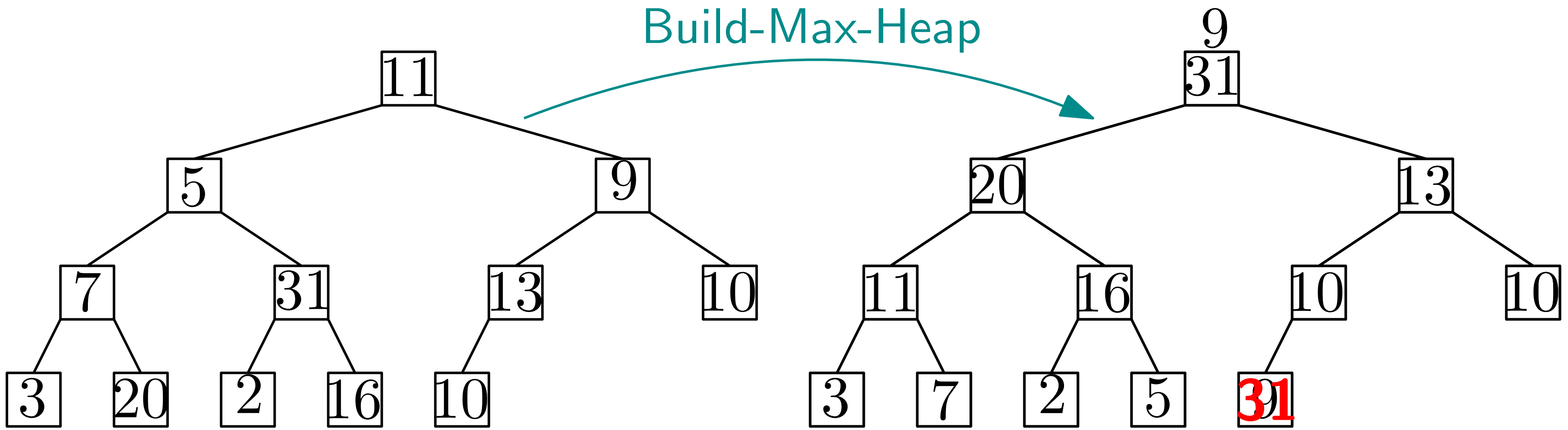

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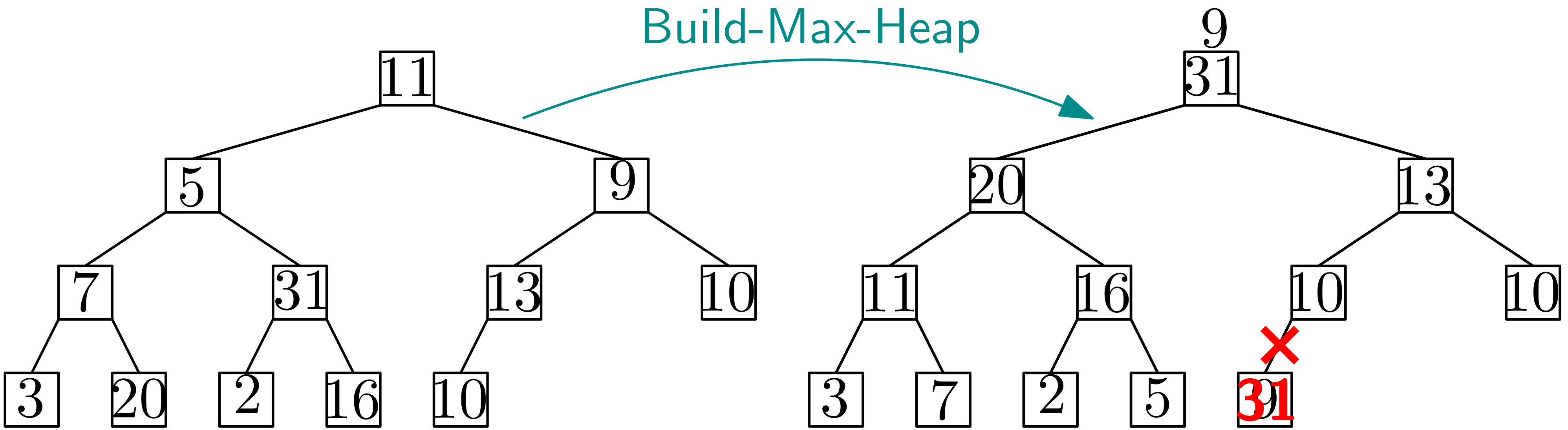

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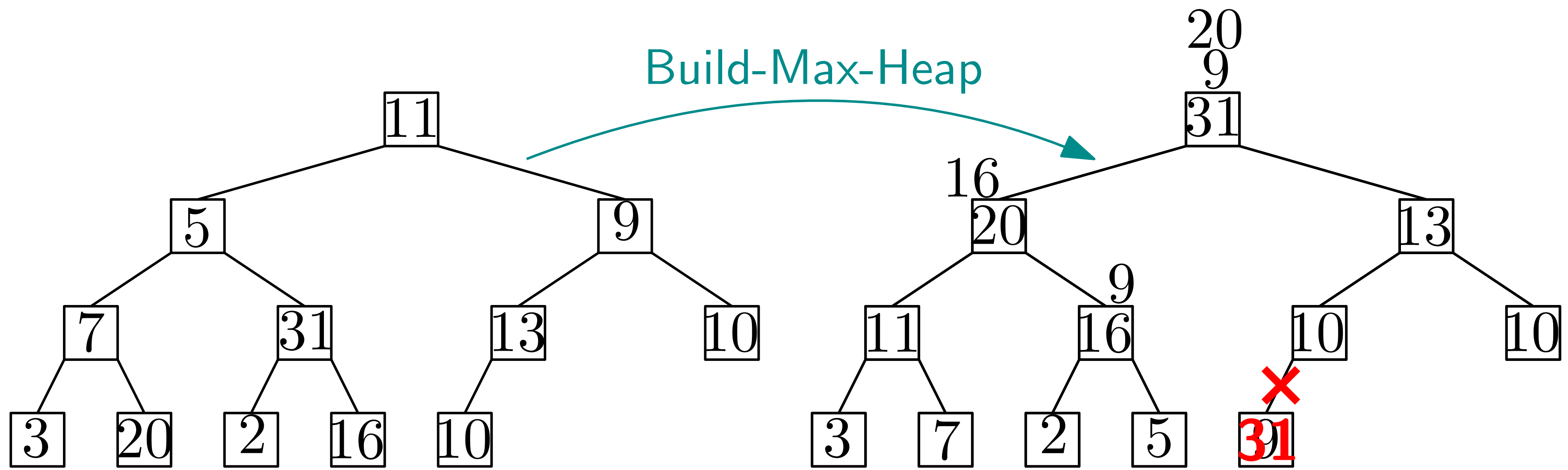

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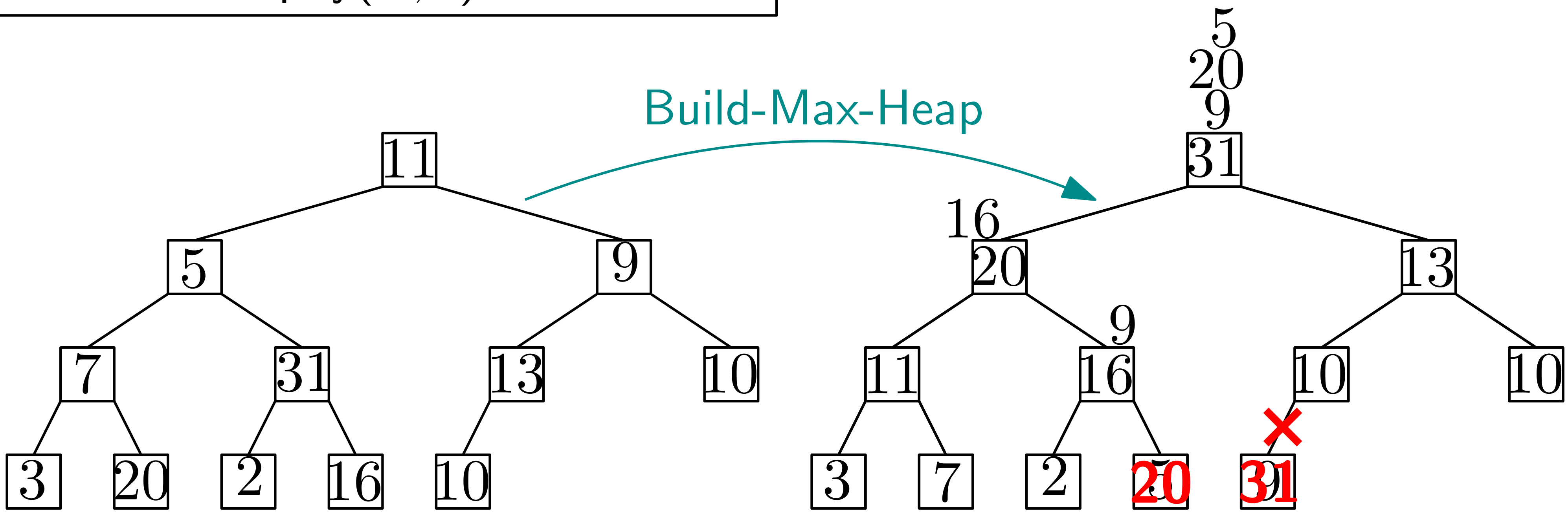

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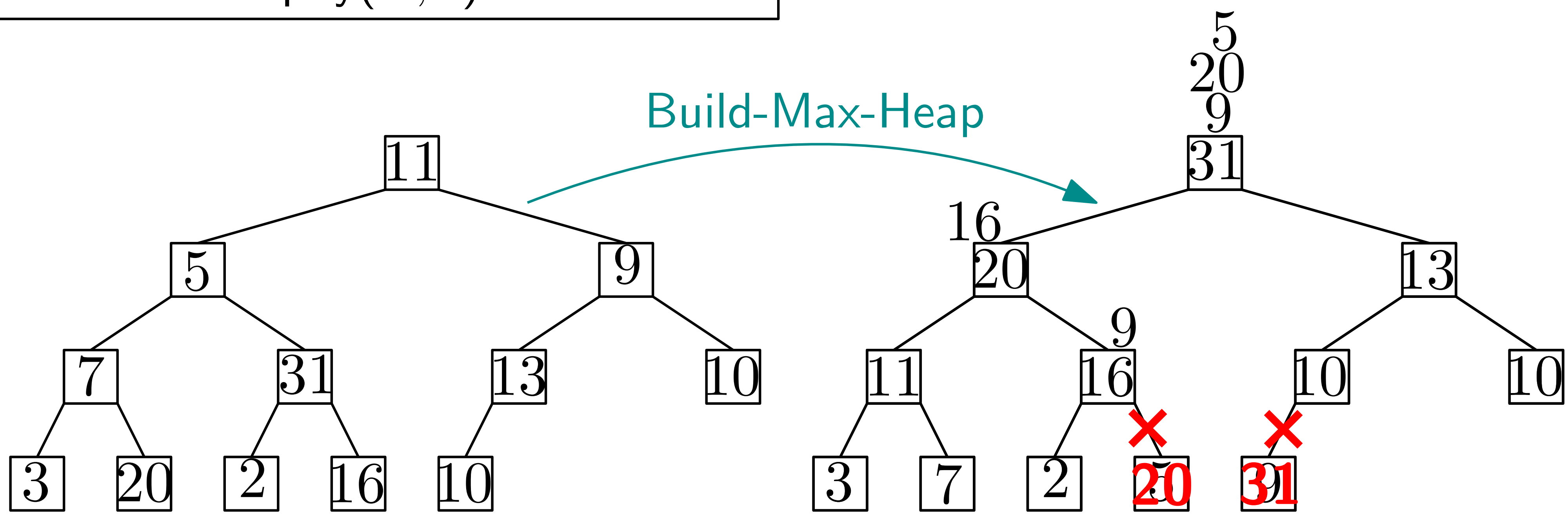

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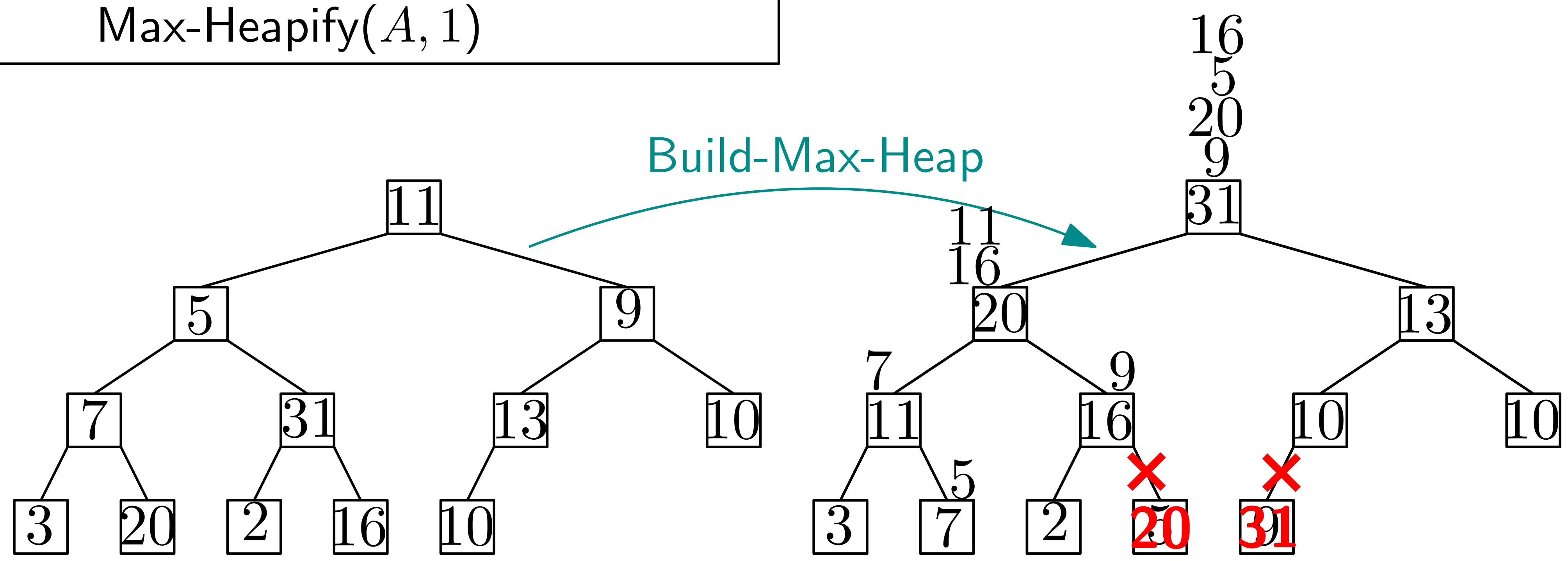

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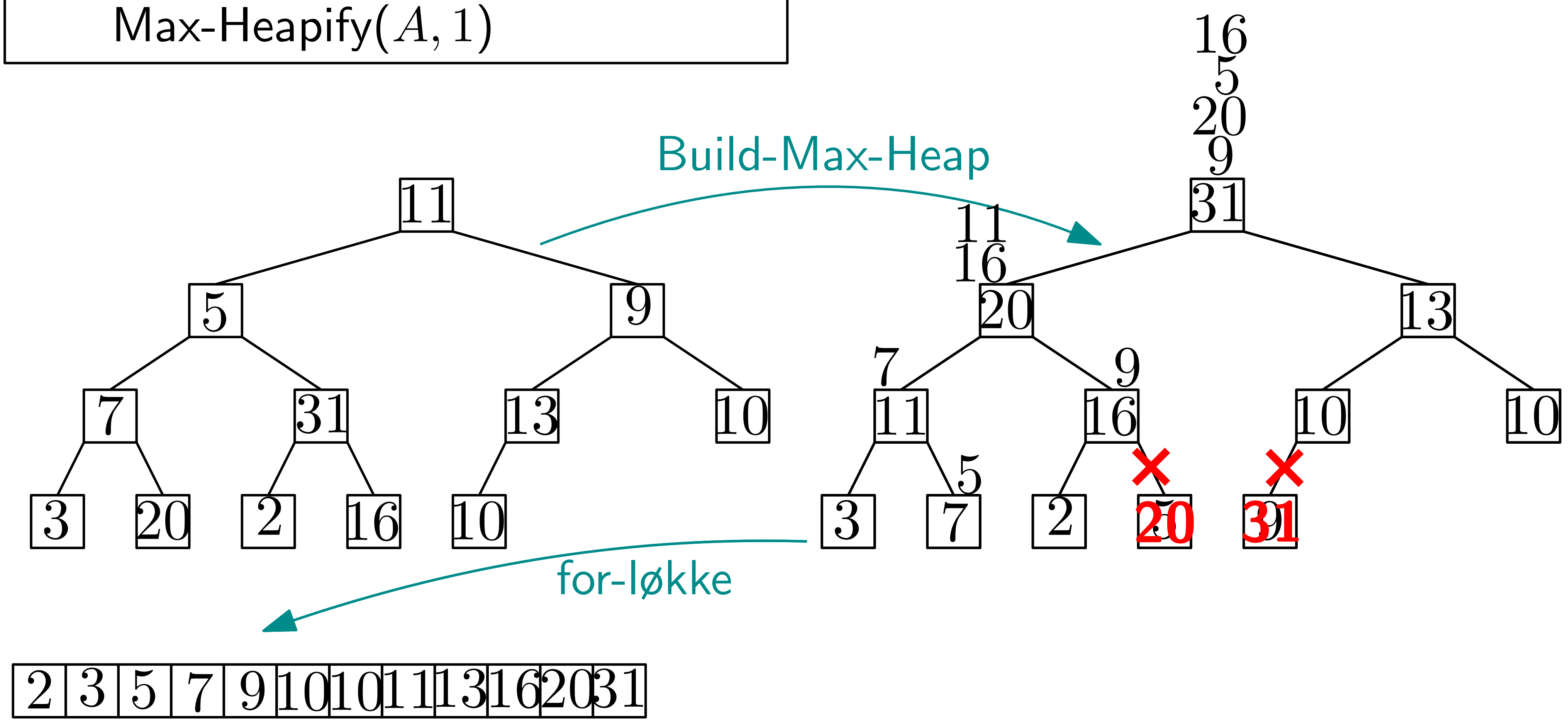

Køretid

	Tid	Gange
Heapsort(A)		
Build-Max-Heap(A)	$\Theta(n)$	1
for $i = A.length$ downto 2 swap $A[1]$ and $A[i]$ $A.heap\text{-}size = A.heap\text{-}size - 1$	$\Theta(1)$	n
Max-Heapify($A, 1$)	$O(\log n)$	n

Køretid

Heapsort(A)	Tid	Gange
Build-Max-Heap(A)	$\Theta(n)$	1
for $i = A.length$ downto 2 swap $A[1]$ and $A[i]$ $A.heap\text{-}size = A.heap\text{-}size - 1$	$\Theta(1)$	n
Max-Heapify($A, 1$)	$O(\log n)$	n

Køretid: $T(n) = O(n \cdot 1) + O(1 \cdot n) + O(\log n \cdot n) = O(n \log n)$.

Der gælder også $T(n) = \Omega(n \log n)$, så $T(n) = \Theta(n \log n)$.

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Heapsort(A)	Tid	Gange
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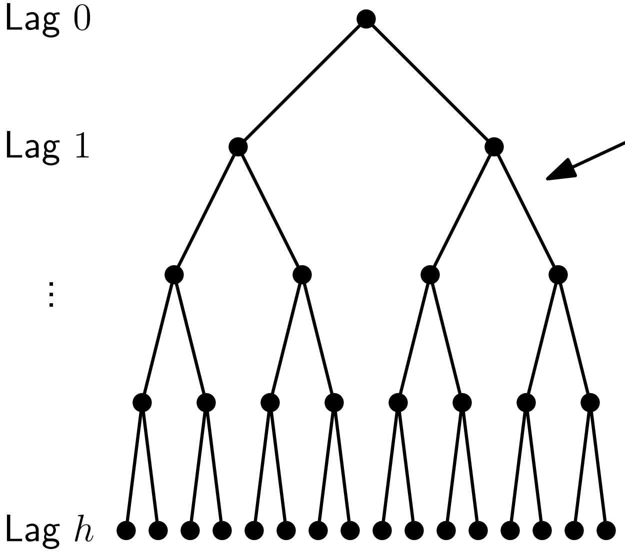
Ekstra plads: $\Theta(1)$.
Bemærk: Merge-Sort bruger $\Theta(n)$ ekstra plads!

Nedre grænse for køretid



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Nøgleværdier: $\{1, 2, \ldots, 2^{h+1} - 1\}$, én af hver.

Knuder med de 2^h største værdier kaldes *store*, dvs. værdi $\geq 2^h - 1$.

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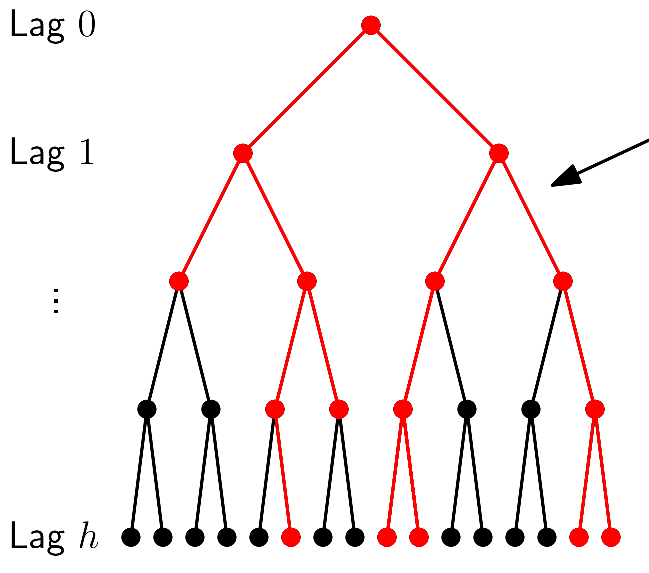

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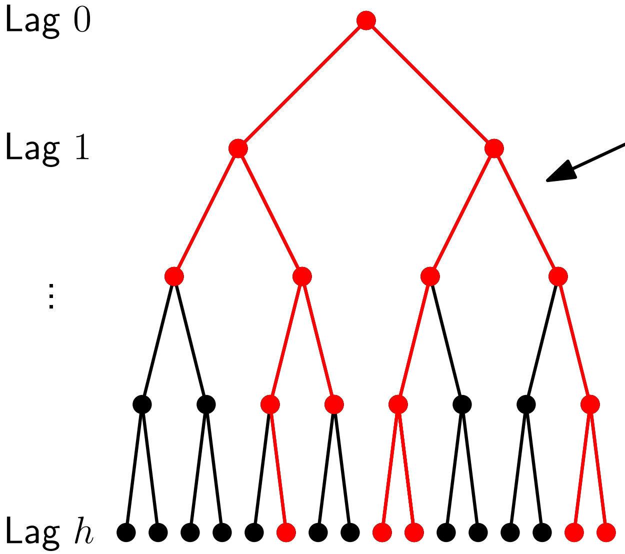

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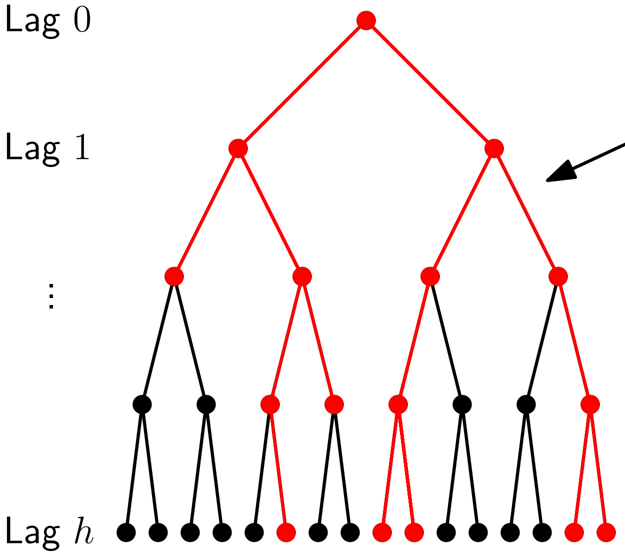

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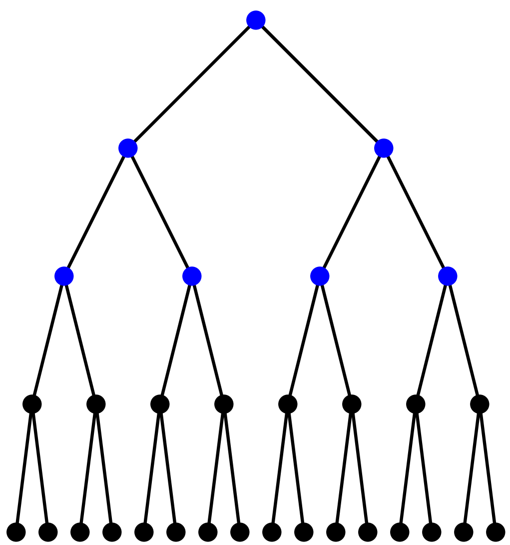
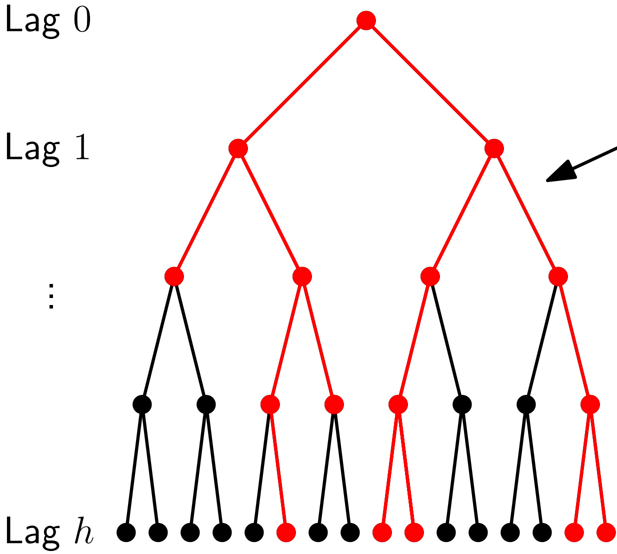
Observation 3: Knuder i lag $1, \ldots, h-2$:

$1 + 2 + \ldots + 2^{h-2} < 2^{h-1}$.



Nedre grænse for køretid



$n = 2^{h+1} - 1$

Her: $h = 4$, $n = 2^5 - 1 = 31$.

Nøgleværdier: $\{1, 2, \ldots, 2^{h+1} - 1\}$, én af hver.

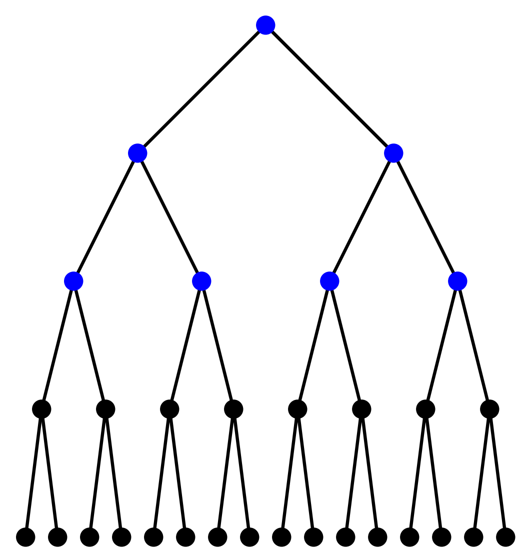
Knuder med de 2^h største værdier kaldes *store*, dvs. værdi $\geq 2^h - 1$.

Observation 1: Efter Build-Max-Heap danner de store knuder et (sammenhængende) træ.

Observation 2: Enhver stor knude der starter i lag $h-1$, bevæger sig til roden og bliver fjernet i løbet af 2^h Extract-Max. Dvs. $\Omega(\log n)$ skridt op.

Observation 3: Knuder i lag $1, \ldots, h-2$:
$1 + 2 + \ldots + 2^{h-2} < 2^{h-1}$.

Observation 4: Mindst 2^{h-1} store knuder i lag $h-1, h$.



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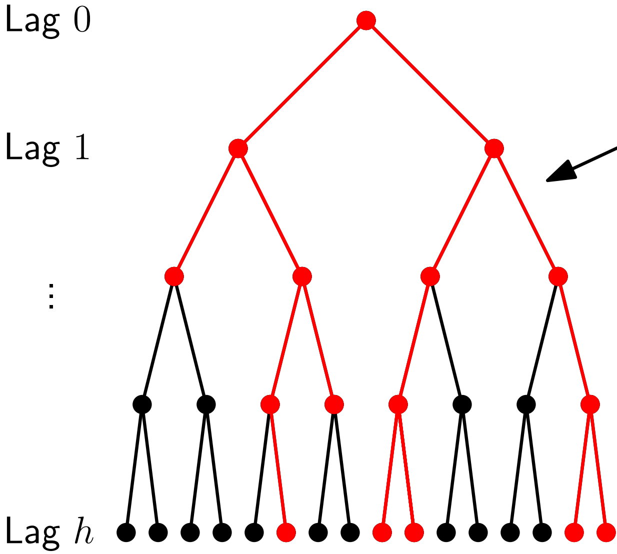

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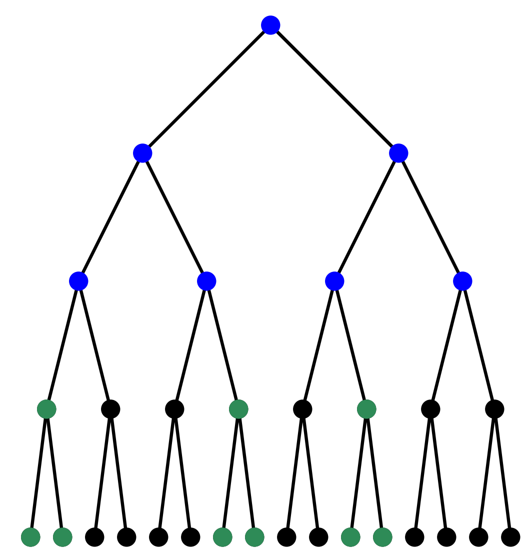
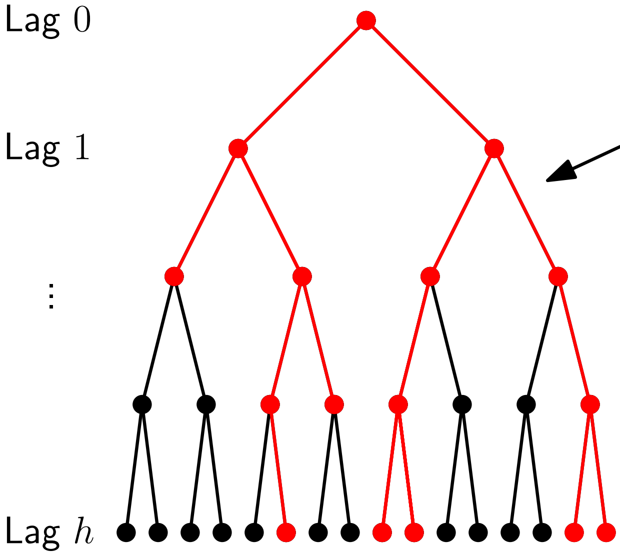
Observation 5: x knuder i lag $h-1 \Rightarrow$ højst $3x$ knuder i lag $h-1, h$. Derfor
$3x \geq 2^{h-1} \Rightarrow x \geq \frac{2^{h-1}}{3} = \frac{2^{h+1}}{12} = \frac{n+1}{12} = \Omega(n)$.



Nedre grænse for køretid



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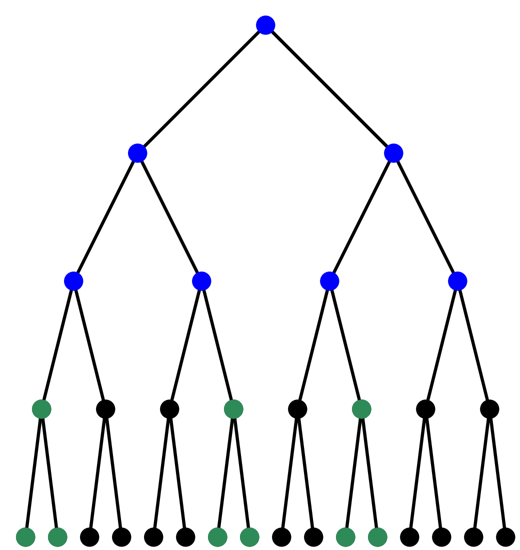

Konklusion: Observation 2+5 giver $\Omega(n \log n)$ tid.

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Det hele.
A

Det meste.
B

Noget.
C

Kun en lille smule.
D

Ingenting.
E