

What does Exp(n) return?

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1  If (n = 0) then
2      return 1
3  Elseif (n % 2) = 0 then      (n is even)
4      x ← Exp (n/2)
5      return x*x
6  Else                        (n is odd)
7      x ← Exp ((n-1) / 2)
8      return 3*x*x
```

DMA: Strong Induction

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Corona guidelines



Welcome back to classes and auditorium lectures!
We must follow the following guidelines:



- Don't attend if you have symptoms of COVID-19 – including mild symptoms
- Cough/sneeze into your sleeve



- Keep a minimum distance of one metre
- Avoid using lifts. If you have to use a lift, keep a distance of two metres



- Disinfect your hands when you enter the building
- Disinfect your hands when you enter the classroom/auditorium
- Ensure good hand hygiene – wash and sanitise thoroughly and often



- Upon entering a room; sit to avoid close passage of others
- Pay attention to where you are allowed to sit/not to sit



- Sit at least one metre apart
- Avoid walking around during classes/lectures



- Help clean seats after classes/lectures, and remember to clean spray bottles after use
- Don't share equipment with others



- Empty the room near the exit first
- Leave to room in a calmly manner
- Maintain distance to each other



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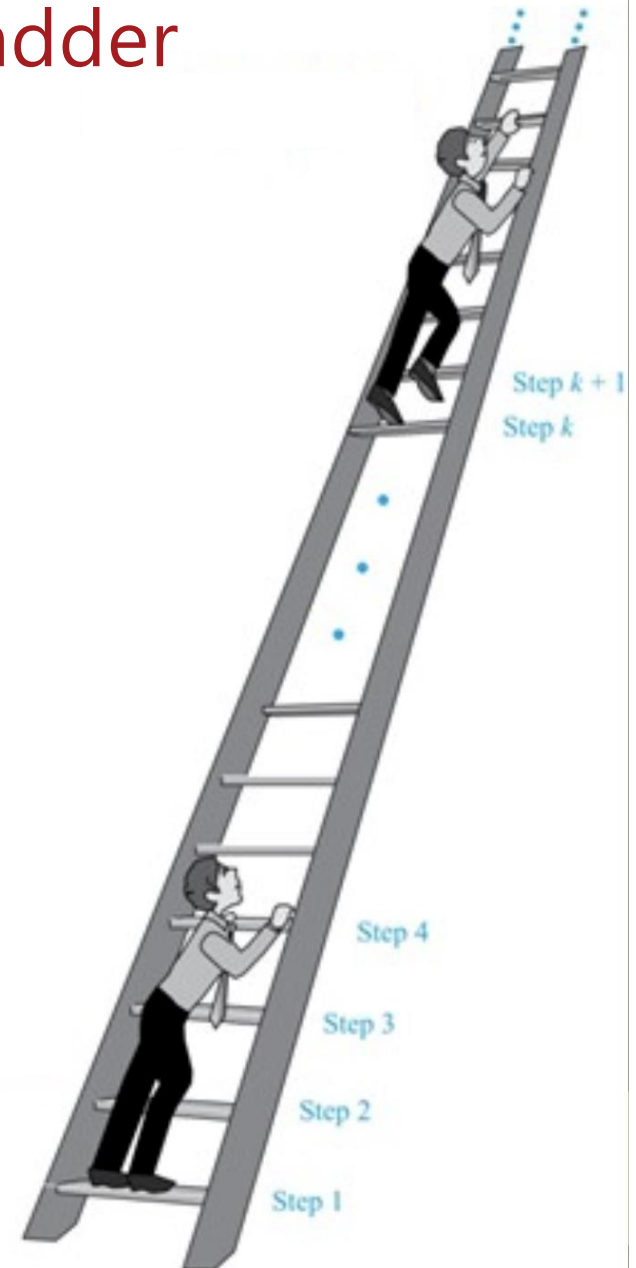
Plan for today

- Recap of the principle of induction
- Strong induction
- Examples

Reading: Notes for Week 5 (on Absalon)

Analogy: climbing an infinite ladder

- Suppose you can reach the first rung
- If you are on a particular rung k you can get on the next rung $k + 1$



The principle of mathematical induction

Let $P(n)$ be a predicate (statement) defined for integers $\{n_0, n_0 + 1, \dots\}$. If

a) [Base case] $P(n_0)$ is true and

b) [Induction step] For any $n \geq n_0$, we have that $P(n)$ being true implies that $P(n + 1)$ is true

Then $P(n)$ is true for all integers $n \geq n_0$.

We used induction to show that

- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{Z}^+$
- $5|(6^n - 5n + 4)$ for all $n \in \mathbb{Z}^+$
- A diagonally moving robot cannot reach position (1,0) when starting from (0,0).

Strong induction

The principle of strong induction

Let $P(n)$ be a predicate (statement) defined for integers $\{n_0, n_0 + 1, \dots\}$. If

1. **[Base case]** $P(n_0)$ is true and
 2. **[Induction step]** for any $n \geq n_0$, we have that all of $P(n_0), \dots, P(n)$ being true implies that $P(n + 1)$ is true
- then
- $P(n)$ is true for all integers $n \geq n_0$.

The Strong of Inductia: making change

- Inductia's currency is Strong (Sg)
- They only have 3- and 5- Strong coins



- Which of these amounts can Inductians make:

4, 5, 6, 7, 8, 9, 10, 11?



Claim. Inductians can make any integer amount $n \geq 8$.

Proof by strong induction.

Let $P(n)$ be the statement that Inductians can make n using their coins. Take $n_0 = 8$.



The Strong of Inductia: proof continued



- Base case: $P(8)$ holds since $8=3+5$
- Induction step.
 - Take $n = 8$. If $P(8)$ holds then $P(9)$ holds.
 - Take $n = 9$. If $P(8)$ and $P(9)$ hold then $P(10)$ holds.
 - Assume that all of $P(8), P(9), \dots, P(n)$ hold for some $n \geq 10$.
(**Strong Induction Hypothesis**)
Need to show that $P(n + 1)$ holds.

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Computing 3^n with repeated squaring

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```

Claim. Exp (n) returns 3^n for any $n \in \mathbb{Z}^+$.

Let $P(n)$ be the statement that Exp (n) returns 3^n .

Detour: All horses are the same color
("induction example")

All horses have the same color

$P(n)$: in any set of n horses, all horses have the same color.



Base case: $P(1)$ is true since in a set with only one horse, all (one) horses are of the same color.

Inductive step: Suppose $P(n)$ is true for some $n \geq 1$ **(IH)**
So in any set of n horses all horses have the same color.

Let us argue that $P(n + 1)$ is true. Let
$$\{h_1, h_2, \dots, h_{n+1}\}$$

be a set of $n + 1$ horses.

Have we shown that all horses have the same color?

Corona – guidelines

Rengøring mellem undervisning

Cleaning between lectures



- Rengør bordflader, stolerygge og andre kontaktpunkter
- Spray på og tør af med papir efter ca. 30 sek
- Husk at rengøre sprayflasken efter brug
- Sprit eller vask dine hænder
- *Clean tables, the backs of chairs and other contact surfaces in the room*
- *Spray and wipe with paper after approx. 30 secs*
- *Remember to clean the spray bottle after use*
- *Disinfect or wash your hands*



Tak for hjælpen

Thanks for helping out