MASD 2022, Assignment 4

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Guidelines for the assignment.

- This is a group assignment. Hand-in in groups of 2 or 3 latest 05.10.2022 at 21.59. One submission per group and remember to include the name of all group members.
- The assignment report must be uploaded in PDF format, we strongly recommand the use of LaTeX to create the PDF.
- Please pay careful attention to the plagiarism rules, see `https://absalon.ku.dk/courses/61325/pages/course-information`.

Exercise 1 (**Practicing integration).** Solve the following definite and indefinite integrals. Remember possible constants when solving indefinite integrals. Include (and explain) intermediate steps.

a) $\int 2x^3 dx$

b) $\int_{-1}^{1} 2x^{100} dx$

c) $\int_1^8 \sqrt[3]{x} dx$

d) $\int_0^1 (5 + x\sqrt{x}) dx$

e) $\int xe^{-x^2} dx$ using substitution $u = -x^2$

f) $\int \sqrt{x} \ln(x) dx$ using integration by parts with $u = \ln(x), dv = \sqrt{x} dx$

g) $\int (4x - 1)^{20} dx$

h) $\int x^2 e^{x^3} dx$

i) $\int 3s2^s ds$

j) $\int (\ln x)^2 dx$

Deliverables. Solutions and intermediate steps.

Exercise 2 (**More integration).** For any $\lambda > 0$, define $f_\lambda : \mathbb{R} \to \mathbb{R}$ by

$$f_\lambda(x) = \begin{cases} \lambda \exp(-\lambda x) & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Solve the following integrals. Include (and explain) intermediate steps. You may use that $\lim_{s\to\infty} s \exp(-\lambda s) = 0$.

a) $\int_{-\infty}^{\infty} f_\lambda(x) dx$

b) $\int_{-\infty}^{\infty} x f_\lambda(x) dx$

c) $\int_{-\infty}^{\infty} x^2 f_\lambda(x) dx$

Deliverables. Solutions and intermediate steps.

Exercise 3 (**Double integrals and numerical integration).** Consider the following Python code:

a) Solve the integral

$$\iint_R 3x^2 y dA$$

on the rectable $R = \{(x, y)|a \leq x \leq b, c \leq y \leq d\}$. Include and explain intermediate steps.

b) Compute the volume between 0 and $3x^2y$ on the domain between $x = -1$ and $x = 1$, and $y = 0$ and $y = 2$.

c) Consider the following Python code:

```
def g(xint,yint,nx,ny):
    dx = (xint[1]-xint[0])/nx
    dy = (yint[1]-yint[0])/ny
    xs = np.linspace(xint[0],xint[1],nx,endpoint=False)
    ys = np.linspace(yint[0],yint[1],ny,endpoint=False)
    v = 0.
    for ix in range(nx):
        for iy in range(ny):
            v += 3*xs[ix]**2*ys[iy]*dx*dy
    return v
```

Explain each line of the code. What does the the call `g([-1,1],[0,2],100,100)` approximate?

d) Explain the option `endpoint=False` to `np.linspace` (look up the documentation for the function if necessary). If we change it to `endpoint=True`, how should the lines

```
dx = (xint[1]-xint[0])/nx
dy = (yint[1]-yint[0])/ny
```

be changed to keep the code consistent? Explain why the change is necessary.

Exercise 4 (**Numerical integration).** See Exercise4.ipynb. (Please submit the code as a separate file and as part of the latex document.)