MASD 2022, Assignment 5

François Lauze, Stefan Sommer, Kasra Arnavaz

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Guidelines for the assignment.

- This is a group assignment. Hand-in in groups of 2 or 3 latest 12/10/2022 at 21:59. One submission per group and remember to include the name of all group members.
- The assignment report must be uploaded in PDF format, we strongly recommand the use of LaTeX to create the PDF.
- Please pay careful attention to the plagiarism rules, see `https://absalon.ku.dk/courses/61325/pages/course-information`.
- For all questions, you need to justify your answers with mathematical symbols and/or sentences rather than only reporting the final answer.

Exercise 1 (Forward Probability)

a. Two dice are rolled simultaneously. What is the probability that their sum is a prime number?

b. Five Danish books and three English books are randomly placed onto a shelf. What is the probability that all books of the same language are next to one another?

c. You know that there are 5 white mice and 6 black mice in a lab. Three mice chosen at random. What is the probability that **at least** one of them is white?

d. You know that there are 6 white balls and 4 black balls in an urn. Two balls are removed one after another without replacement. What is the probability that the second ball is white?

Exercise 2 (Inverse Probability)

a. A dice is selected at random from two twenty-faced dice on which the the symbols 1-10 are written with nonuniform frequency as follows.

Symbol	1	2	3	4	5	6	7	8	9	10
Number of faces of dice A	6	4	3	2	1	1	1	1	1	0
Number of faces of dice B	3	3	2	2	2	2	2	2	1	1

The randomly chosen die is rolled 7 times, with the following outcomes: 5, 3, 9, 3, 8, 4, 7.
What is the probability that the dice was dice A?

b. Assume that there is a third twenty-faced dice, dice C, on which the symbols 1-10 are written each twice. One of the three dice is selected at random and rolled 7 times, giving the exact same outcome as in part (a).

Does this information change your belief, compared to part (a), about the fact the selected dice was dice A? If so, are you more certain or less certain that the selected dice was dice A? Why? Does your answer depend on what dice C looks like?

In any case, compute the probability that the dice was dice A, dice B, dice C? Are your answers to the question above justified now that you have done the calculations?

Exercise 3 (Counting)

a. How many words can be made by swapping the order of letters **Y, A, M, A, H, A** such that every other letter is **A**?

b. How many odd 4-digit numbers are there which are the same when written backwards?

c. How many subsets of $\{1, 2, 3, 4, 5, 6\}$ are of size three which have 1 but not 2?

d. How many ways are there to distribute 5 unique books among 3 people such that each person has at least one book?