Bayesisk sannsynlighetsteori i maskinlæring

Klassifiserer: Gitt data (for trening, og for inferens), designer et sample til en viss klasse.

Decision rule: Binært tilfelle:

Gitt at

Data

$$X \to C_1 \quad \text{dersom} \quad p(C_1 \mid X = x) > p(C_2 \mid X = x)$$

Klasse

$$\text{ellers} \quad x \to C_2$$

Posterior = Likelihood × Prior

Hvor stor andel i data av de forskj. klassene.

$$p(C_i \mid x) = p(x \mid C_i) \cdot P(C_i)$$

F.eks. norm. fordelinga

Tre varianter

Uni → σ_u, μ_u

Vgs → σ_v, μ_v

Grunnskole → σ_g, μ_g

Normalfordelinge

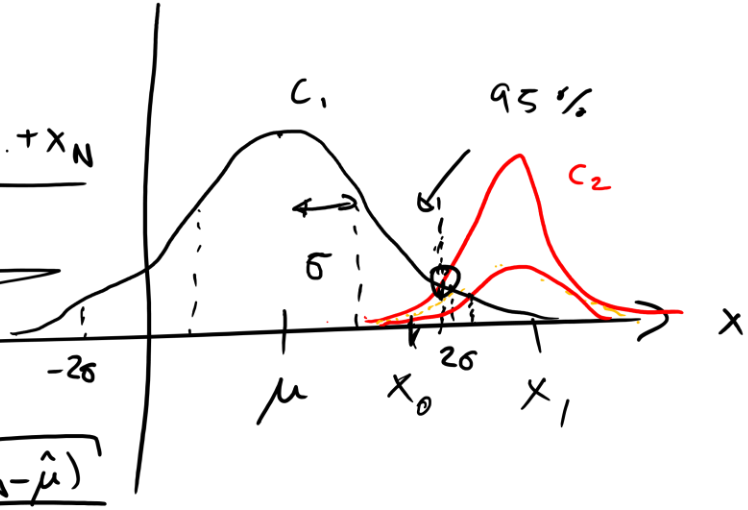
$$N(\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ : Middelverdien ESTIMERT

$$\hat{\mu} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

σ : St. avvik ESTIMERT

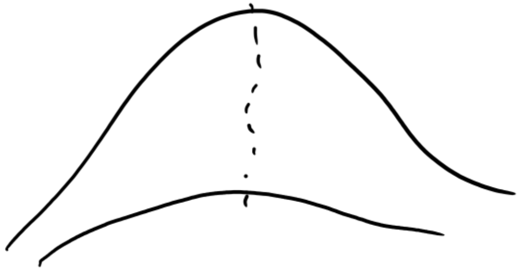
$$\hat{\sigma} = \frac{\sqrt{(x_1-\hat{\mu})^2 + (x_2-\hat{\mu}) + \dots (x_N-\hat{\mu})}}{N}$$



$$P(C_2) = \frac{N_2}{N_{tot}}$$

$$N_2 = \frac{1}{4} N_1$$

$$N_{tot} = N_2 + N_1$$



Kalles også MLE : "Maximum Likelihood Estimation"

Likelihood : $\mathcal{X} = \{x_1, x_2, x_3, \dots, x_N\} \rightarrow \mu, \sigma$

$$l(x) = p(x_1)\cdot p(x_2)\cdot p(x_3)\cdots p(x_N)$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2} \cdot \frac{1}{\sqrt{2\pi}\,\sigma}\cdot e^{-\frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2} \cdot \dots\dots \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x_N-\mu}{\sigma}\right)^2}$$

(se s. 70 i Alpaydin)

4 a) Spam-filter baseres på BOW-modell

Ser på d ord som kan opptre. Opptrer ord #m i epost, så settes feature #m til 1 ($x_m = 1$), $m = 1, \dots, d$

$$\underset{\text{En e-post}}{\underline{X}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \begin{matrix} \sim \text{Bernoulli}(\pi_1) \\ \\ \vdots \\ \sim \text{Bernoulli}(\pi_d) \end{matrix}$$

$$p_{\pi_m}(x) = \begin{cases} \pi_m & \text{om } x_m = 1 \\ (1-\pi_m) & \text{om } x_m = 0 \end{cases}$$

π_m: pmssh for at ord nr. m opptrer

$$= \pi_m^{x}(1-\pi_m)^{1-x}$$

for $x \in \{0,1\}$

Så for d ord i en epost vil vi ha følgende ssh-fordeling (likelihood)

$$p(x \mid C_i) = p_1(x) \cdot p_2(x) \cdot \ldots \cdot p_d(x)$$

$$= \pi_1^{x_1}(1-\pi_1)^{1-x_1} \cdot \pi_2^{x_2}(1-\pi_2)^{1-x_2} \cdot \ldots \cdot \pi_d^{x_d}(1-\pi_d)^{1-x_d}$$

$$= \prod_{m=1}^{d} \pi_m^{x_m}(1-\pi_m)^{1-x_m}$$

$\{\pi_m\}^{+/\div}$ må estimeres (for + og ÷ kategori)

Hvordan : Gitt N spam og M ikke-spam, altså tot. N + M samples

$$\hat{\pi}_m^{+} = \frac{1}{N}\sum_{j=1}^{N} x_m^{+j} = \frac{1}{N}\left(1+1+1+0+0+1+1+1 \ldots +\right)$$

$$\hat{\pi}_m^{\div} = \frac{1}{M}\sum_{j=1}^{M} x_m^{\div j}$$

Prior

Diskriminant-funksjoner :

$$g_+(\underline{x}^{test}) = \prod_{m=1}^{d} \left(\hat{\pi}_m^{+}\right)^{x_m^{test}} \left(1-\hat{\pi}_m^{+}\right)^{(1-x_m^{test})} \frac{N}{N+M}$$

$$g_\div(\underline{x}^{test}) = \prod_{m=1}^{d} \left(\hat{\pi}_m^{\div}\right)^{x_m^{test}} \left(1-\hat{\pi}_m^{\div}\right)^{(1-x_m^{test})} \frac{M}{N+M}$$