

1)

$$\underline{a} \in \mathbb{R}^n$$

$$\underline{a} = [a_1, a_2, a_3, \dots, a_n]$$

$$\underline{a}^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$\underline{b} \in \mathbb{R}^n$$

$$\underline{b} = [b_1, b_2, b_3, \dots, b_n]$$

$$\begin{matrix} i=1 \\ \underline{X} \\ M \end{matrix} \rightarrow \begin{bmatrix} X_{11} & X_{12} & X_{13} & \dots & X_{1N} \\ X_{21} & & & & \\ X_{31} & & & & \\ \vdots & & & & \\ X_{M1} & & & & \end{bmatrix} \begin{matrix} X_{1k} \\ \\ \\ \\ N \end{matrix}$$

$$\dim(\underline{X}) = M \times N$$

$$\sum X_{1k} \cdot a_k$$

$$\underline{y} = \underline{X} \cdot \underline{a}$$

fall

$$= \begin{pmatrix} X_{11} \cdot a_1 + X_{12} \cdot a_2 + \dots + \\ \vdots \\ X_{M1} \cdot a_1 + X_{M2} \cdot a_2 + \dots \end{pmatrix}$$

$$1b) y = \sum_{i=1}^N a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_N b_N = \underline{a} \cdot \underline{b}$$

1d) for $i = 1:M$
for $k = 1:N$

$$y_i += \underline{X}_{ik} \cdot a_k$$

$$1c) \quad \underline{a} \cdot \underline{b}$$

Numpy $a @ b$

$$\underline{X} \cdot \underline{b} \quad \text{OK}$$

$\begin{matrix} 3 \times 2 & 2 \times 1 \\ \nearrow & \searrow \end{matrix}$

$$1e) \quad \left. \begin{array}{l} \text{for } i=1:M \\ \text{for } j=1:N \\ y += X_{ik} a_k \end{array} \right\} \begin{array}{l} 1) y = X @ a \\ 2) y = y.sum() \end{array}$$

$$\underline{y} = \underline{X} \cdot \underline{a}$$

$$1f) \quad \underline{Z} = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1M} \\ \vdots & & & \\ z_{p1} & z_{p2} & \dots & z_{pM} \end{pmatrix}$$

$\underline{Z} \in \mathbb{R}^{p \times M}$

$$\underline{X} \in \mathbb{R}^{M \times N}$$

$$\underline{Z} \cdot \underline{X} = \underline{y}$$

$\begin{matrix} p \times M & M \times N & p \times N \\ \nwarrow & \nearrow & \\ & \text{OK} & \end{matrix}$

Exs:

$$3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix}^2$$

$$3 \times 2 \cdot 2 \times 2 = 3 \times 2$$

$$= \begin{pmatrix} \underbrace{1 \cdot 7 + 2 \cdot 9}_{y_{11}} & 0 \\ \underbrace{3 \cdot 7 + 4 \cdot 9}_{y_{21}} & 0 \\ 0 & \underbrace{5 \cdot 8 + 6 \cdot 10}_{y_{32}} \end{pmatrix}$$

Transpose

$$\underline{X} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$\underline{X}^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

