Superradiant instabilities in charged black holes

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We study the minimally coupled Klein-Gordon equation for a charged scalar field in the background of a Reissner-Nordström black hole.

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I. INTRODUCTION.

In classical relativistic gravity, black holes are observer independent space-time regions unable to communicate with their exterior [?]. Thus, within this description, information captured by black holes is trapped therein forever and cannot be recovered by exterior observers.

Given this picture it is intriguing, at first, to realise that there is a *classical* process through which energy can be extracted from a black hole: *superradiant scattering*. In one form, this process amounts to the amplification of waves impinging on a Kerr black hole, provided the frequency ω and azimuthal quantum number m of the wave modes obey the condition $\omega < m\Omega_+$, where Ω_+ is the angular velocity of the outer Kerr horizon [? ? ?]. The extraction of energy and consequent decrease of the black hole mass M is, however, necessarily accompanied by the extraction of angular momentum and consequent decrease of the black hole spin J. In fact, it was shown by Christodoulou [?] that the particle analogue of this process - the Penrose process [?] - is irreversible, subsequently realised to mean that the black hole area never decreases [?]. Finally, the identification between black hole area and entropy [? ?] made clear that it is only (rotational) energy that is being extracted from the black hole, not information.

In another form, superradiant scattering amounts to the amplification of *charged* waves impinging on a Reissner-Nordström (RN) black hole, provided the frequency ω and the charge q of the wave modes obey the condition $\omega < q\Phi_+$, where Φ_+ is the electric potential of the outer Reissner-Nordström horizon [?]. The extraction of (Coulomb) energy and consequent decrease of the black hole mass M is, in this case, necessarily accompanied by the extraction of charge and consequent decrease of the black hole charge Q, such that, again, the area/entropy of the RN black hole does not decrease.

The existence of superradiant modes can be converted into an *instability* of the background if a mechanism to trap these modes in a vicinity of the black hole is provided: heuristically, these modes are then recurrently scattered off the black hole and amplified, eventually producing a non-negligible background back-reaction. This possibility, antecipated by Zel'dovich [?], was named *black hole bomb* by Press and Teukolsky [?] and has been studied extensively in the Kerr case within the linear analysis [? ? ? ?].

The unstable states found in the Kerr case are localised in a potential well found outside the potential barrier of the effective potential. The growth of such states can be seen at linear level, but a fully non-linear study is required to address the end-point of this instability. This, however, has not yet been achieved, and will most certainly involve a numerical analysis [?].

Considerable less attention has been devoted to the charged case, perhaps due to the lack of astrophysical motivation. Moreover, the studies found in the literature [? ?] discard the possibility of an instability by performing an analysis of the effective potential and showing that no potential well is found for quasi-bound states compatible with the superradiance condition.

II. QUASI-BOUND STATES.

We shall consider a massive, charged scalar field, Φ, with mass μ and charge q, in the linear regime is described by the wave equation

$$\left[(D^\nu - iqA^\nu)(D_\nu - iqA_\nu) - \mu^2\right]\Phi = 0 \ , \qquad (1)$$

propagating in the background of a Reissner-Nordström black hole with charge Q and mass M. Written in terms of Boyer-Linquist type coordinates the line element is

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ , \qquad (2)$$

where

$$f(r) = \frac{(r-r_+)(r-r_-)}{r^2} \ , \quad r_\pm \equiv M \pm \sqrt{M^2 - Q^2} \ , \quad (3)$$

and $A = -Q/rdt$.

A. Properties of the Potential

In Fig. 1 we plot the effective potential for several values of the scalar charge while keeping the other parameters fixed. The asymptotic value of the potential at

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infinity is μ^2 and for $r \to r_+(r* \to -\infty)$ V_{eff} tends to a constant $\omega_c(2\omega - \omega_c) = qQ/r_+(2\omega - qQ/r_+)$. This depends on the value of the frequency, which is unknown, however we used an arbitrary value to make clear the behaviour of the potential close the horizon.

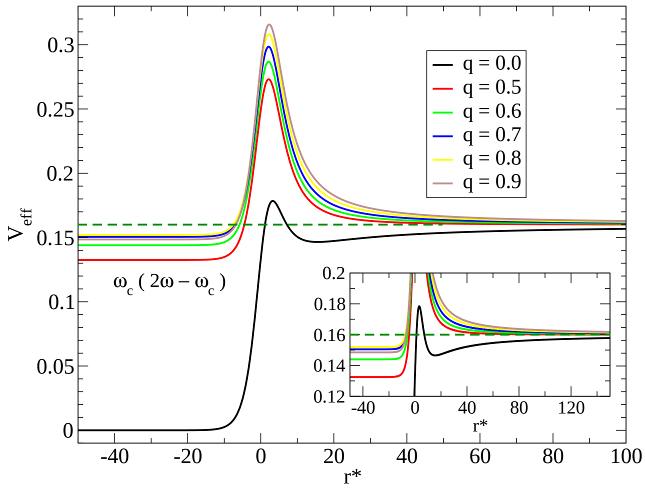

FIG. 1: Effective potential for some scalar charges. The values of the other parameters are: $M = 1$, $Q = 0.9$, $\mu = 0.4$, $\omega = 0.39$, $\ell = 1$. The asymptotic value of the potential close the outer horizon is given by $\omega_c(2\omega - \omega_c)$. The height of the centrifugal barrier increases with the charge of the field; the constant value of the potential near the outer horizon also increases with the charge of the field but only up to some maximum; then it starts decreasing, in accordance with the quadratic behaviour in q present in $\omega_c(2\omega - \omega_c)$.

Figure 2 shows the dependence of the potential on the background charge Q. The value of the peak increase with the charge. Only for small enough Q a potential well is seen.

Figure 3 shows the variation of V_{eff} with the mass of the field for fixed Q, q and ω. For a massless scalar there is no well because the potential tends to zero asymptotically. As the mass increases a potential well appears but above a threshold, it vanishes. This behaviour of the potential is qualitatively similar to that of the potential for an uncharged massive scalar field on a Schwarzschild background [**?**].

Finally, the variation of the potential with ω is shown in Fig. 4. The trend is that both the height of the centrifugal barrier and the constant value near the outer horizon increase with increasing frequency.

Let us close this section with two remarks concerning this effective potential, both of which originate from the fact that it depends on ω. Firstly, since the frequency is unknown, and only a discrete set of complex frequencies will be solutions of the wave equation, the plots of the effective potential should be taken just as guide to understand the physical problem. Secondly, the fact that V_{eff} depends on ω makes it unorthodox as compared to a standard proper potential in a Schrödinger-like equation. In particular, the intuition that a potential well is necessary for the existence of quasi-bound states (or bound states) is questionable, and indeed the results we exhibit show that such states may exist even in the absence of a potential well for this type of effective potential.

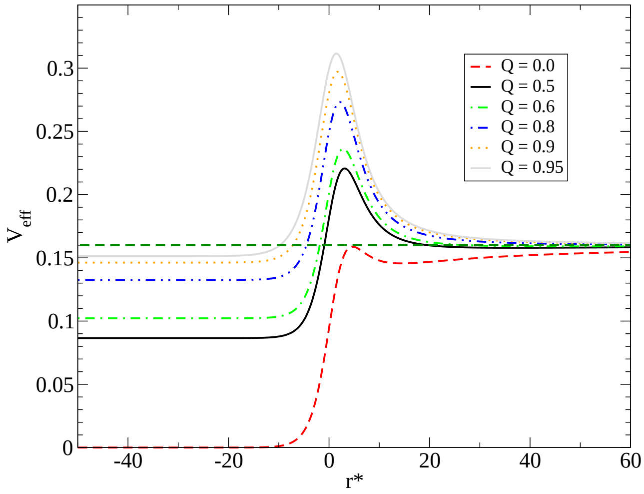

FIG. 2: Variation of the potential with the charge of the black hole for $\ell = 1.$ $q = 0.5$, $\mu = 0.4$, and $\omega = 0.39$. The notable feature is that for small enough background charge a well is present; but it ceases to exist when Q is increased.

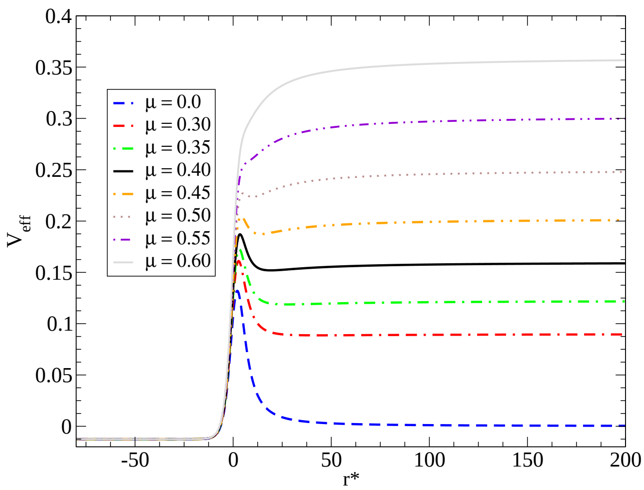

FIG. 3: Variation of the effective potential with mass of the field for $\ell = 1$, $q = 0.5$, $Q = 0.8$ $\omega = 0.1$. The asymptotic value at infinity is μ^2. A potential well appears above a certain minimum mass but it disappears above a certain maximum mass.

III. MIRROR QUASI-BOUND STATES.

For a given q/μ, if r_m increases the real part of frequencies decreases, this is the same behaviour than in

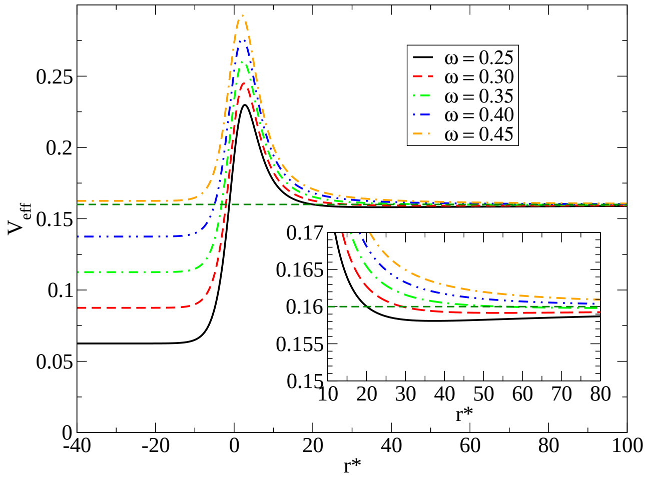

FIG. 4: Variation of the effective potential with the frequency for $M = 1$, $q = 0.5$, $\mu = 0.4$, $Q = 0.8$, $\ell = 1$.

the non charged case. **a plot of this**

The real part of the frequency increases with Q this is also consistent with the picture of the rotating case, however the growing rate in the later the rate is very small ??. For a charged BH the effect is larger. **the plots we want to show are: for a set of q/μ, the imaginary and real part of the frequencies as a function of r_m Given the usuals $Q's$ For which value of q/μ (if any) the imaginary part of ω stops to be positive as function of r_m ?**

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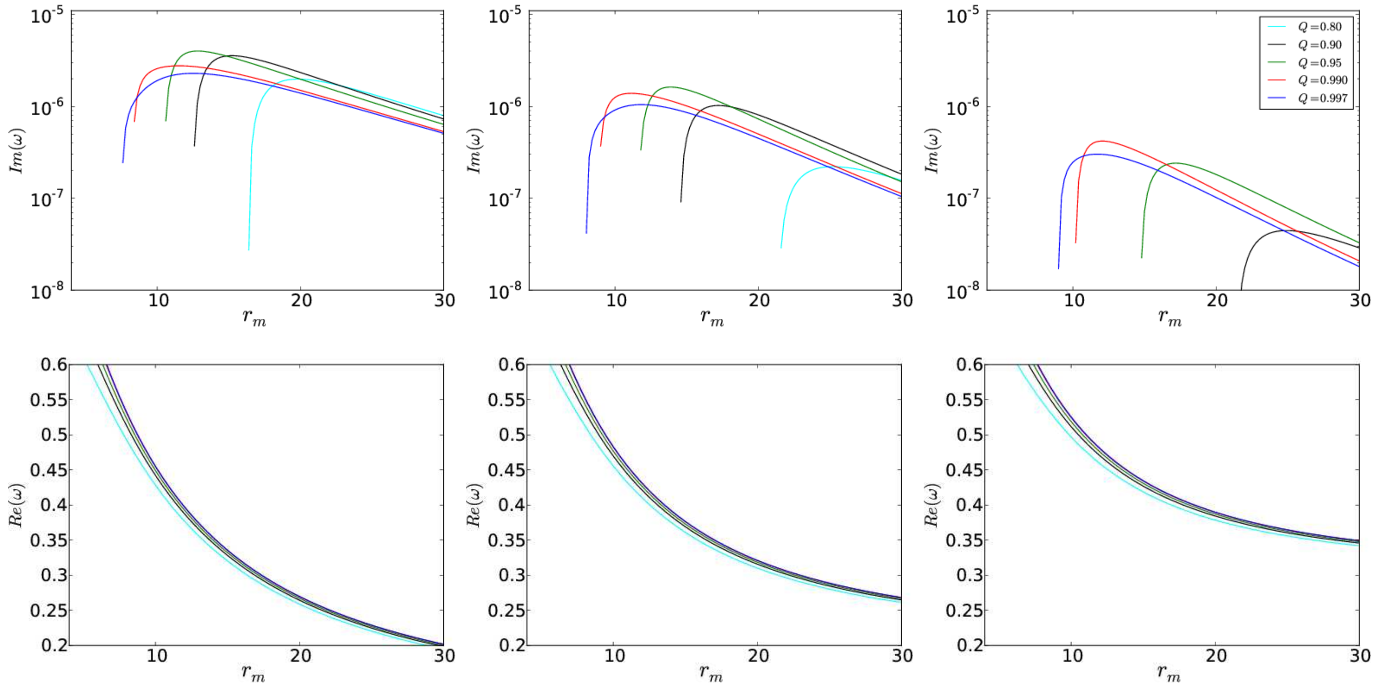
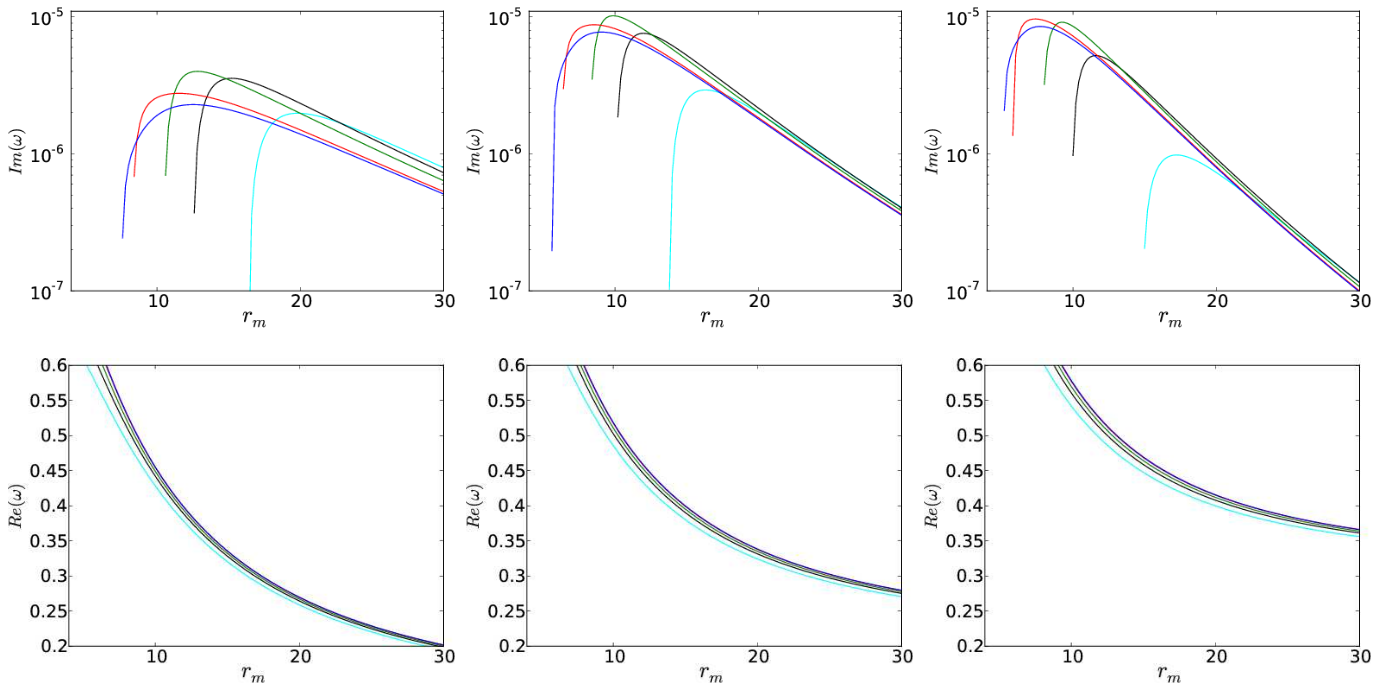


(a) $\mu = 0.1$ (b) $\mu = 0.2$ (c) $\mu = 0.3$

FIG. 5: The imaginary and real part of ω drawn as a function of the mirror radius r_m for various values of the black hole charge, Q, and the scalar mass, μ. The charge of the scalar field is $q = 0.6$.



(a) $\mu = 0.1$, $q = 0.6$ (b) $\mu = 0.2$, $q = 0.8$ (c) $\mu = 0.3$, $q = 0.9$

FIG. 6: The imaginary and real part of ω drawn as a function of the mirror radius r_m for various values of the black hole charge, Q, the scalar charge, q, and the scalar mass, μ.