Local Sensitivity and Smooth Sensitivity

Presenter: Guoming Wang

27 Jan, 2015

Materials

1. S. Raskhodnikova et al.
    1. Smooth Sensitivity and Sampling

2. A. Machanavajjhala
    1. Smooth Sensitivity and Sampling

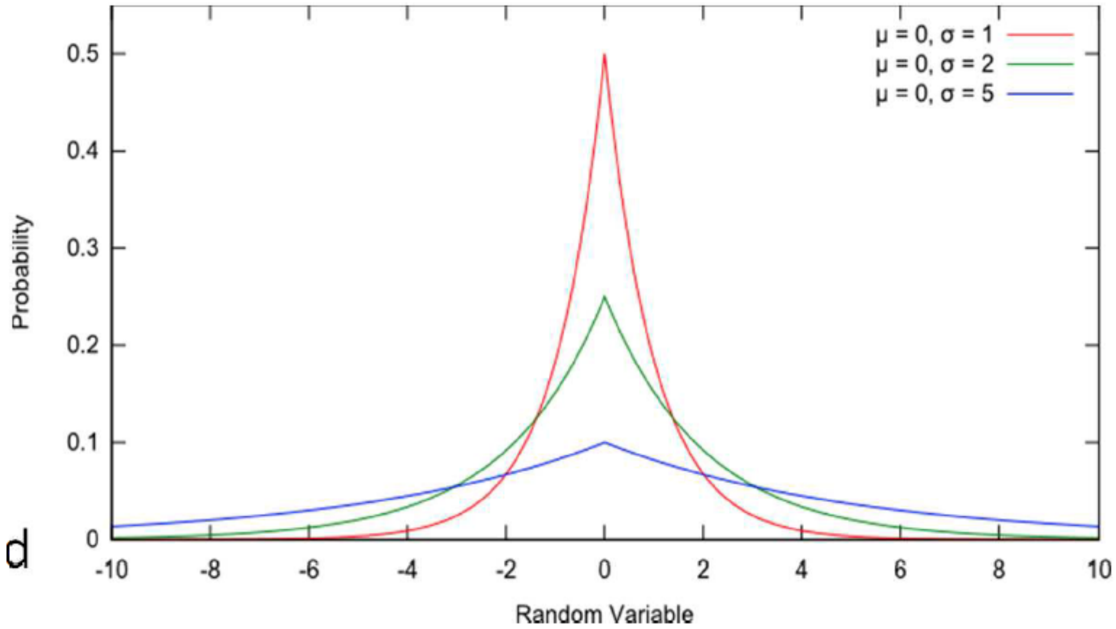
Outline

- **Local Sensitivity**
- **Smooth Sensitivity**

Laplacian Noise

In order for two worst-case neighboring data sets to produce a similar distribution of privatized answers, we need to add noise to span the sensitivity gap.

Adding laplacian Noise is not the only way, but it’s easy.



$$Prob(R = x \mid D \text{ is the true world}) = \frac{\varepsilon}{2\Delta F} e^{-\frac{|x - F(D)|\varepsilon}{\Delta F}}$$

Global Sensitivity

How to privatize a series of FIVE overlapping counts across a data set? (ie, “How many people in the data set are female?”, “How many like biber?”, “How many are between age 12-16”, etc)

$$\Delta F = \max_{\{D1,D2\}} \left|\left|F(D1) - F(D2)\right|\right|_{L1}$$

Add laplacian noise calibrated to ΔF = 5, to each count

Local Sensitivity

Example: median of x_1, ……, x_n [0, 1]

X = 0000111   x' = 0001111

Too much noise

Median(x) = 0,   median(x') = 1

$Gs_{median} = 1$

Local Sensitivity: $LS_f(x) = \max_{y:d(x,\, y)=1} \|\, f(x) - f(y) \,\|$

Global Sensitivity: $GS_f = \max_{x,y:d(x,y)=1} \|f(x) - f(y)\|$

$GS_f = \max_x LS_f(x)$

Local Sensitivity

Add noise proportional to $LS_f(x)$ instead of GS_f?

Not good idea, because it reveals information.

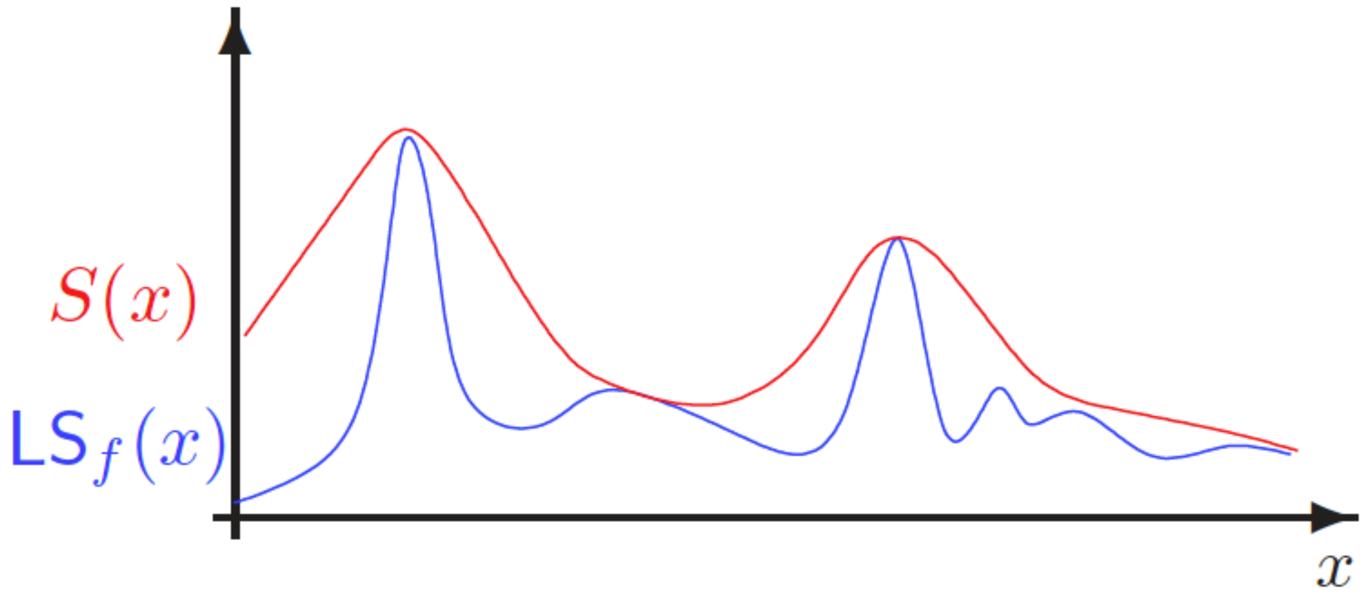
D_1 = {0 0 0 0 0 0 ^ ^ ^ ^}
$Q_{med}(D1) = 0$
$LS_{qmed}(D1) = 0$ Noise sampled from Lap(0)

D_2 = {0 0 0 0 0 ^ ^ ^ ^ ^}
$Q_{med}(D2) = 0$
$LS_{qmed}(D2)$ = ^ Noise sampled from Lap(^/ε)

Smooth Sensitivity

$S(x)$ is an ε-smooth upper bound on $\mathsf{LS}_f(x)$ if:

- for all x: $\quad S(x) \geq \mathsf{LS}_f(x)$
- for all neighbors x, x' : $\quad S(x) \leq e^{\varepsilon} S(x')$



Smooth Sensitivity

$$S^*_q(d) = \max_{d'} \left(LS_q(d') \exp(-m\beta) \right)$$

where d and d' differ in **m** entries.

d	x_1	x_2	x_3	x_4	$\boxed{x_5}$	x_6	x_7	x_8	x_9	x_{10}
d'	Λ	Λ	Λ	x_4	$\boxed{x_5}$	x_6	x_7	0	0	0

- $x_{5-k} \leq q_{med}(d') \leq x_{5+k}$
- $LS(d') = \max(x_{med+1} - x_{med}, x_{med} - x_{med-1})$

$$S^*_{qmed}(d) = \max_k \left(\exp(-k\beta) \times \max_{5-k \leq med \leq 5+k}(x_{med+1} - x_{med}, x_{med} - x_{med-1})\right)$$

Smooth Sensitivity

For instance, $\Lambda = 1000$, $\beta = 2$.

d	1	2	3	4	5	6	7	8	9	10

$$\mathbf{S^*_{qmed}(d)} = \max\left(\max_{0\le k\le 4}(\exp(-\beta\cdot k)\cdot 1), \max_{5\le k\le 10}(\exp(-\beta\cdot k)\cdot \Lambda) \right)$$

$$= 1$$

Thank you – Enjoy the rest of your night