# Beta Reputation Systems

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Feb 17, 2015

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*1.Gamma function and gamma distribution*

*2.Beta function and beta distribution*

# Gamma function

$$\Gamma(n) = (n-1)!$$

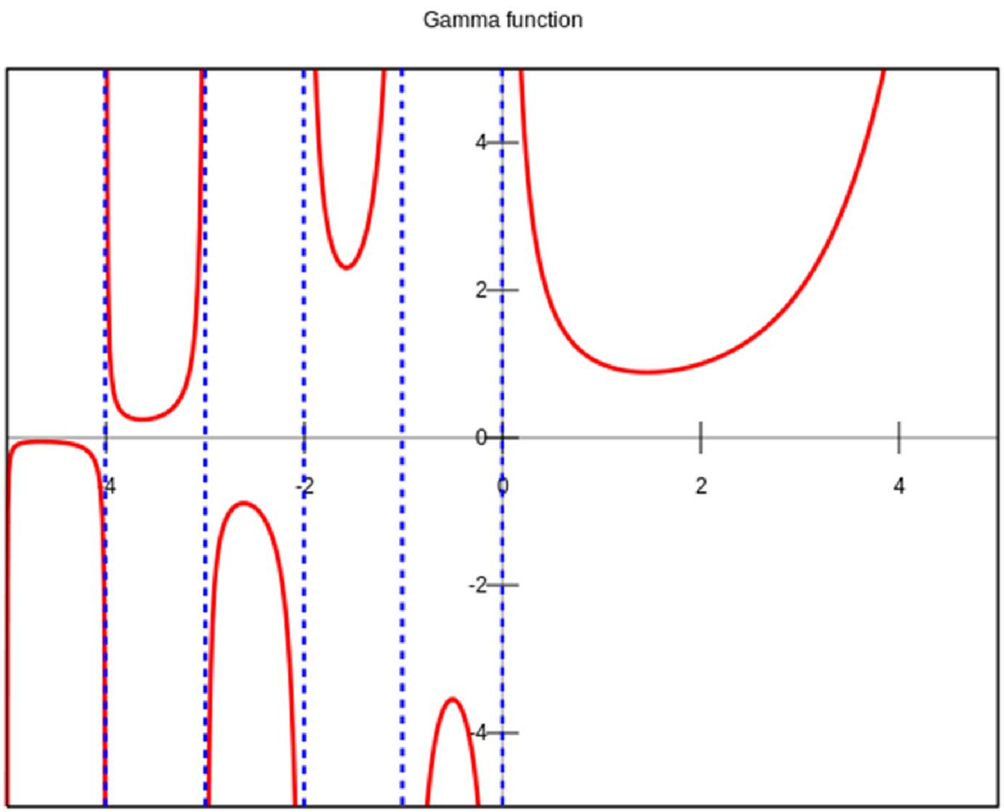
Extension of the factorial function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

$$n! = n \cdot (n-1)!$$

$$\Gamma(x+1) = x\Gamma(x)$$

Parts integration



# Gamma distribution

PDF of Gamma distribution:

$$f(\mathrm{x};\alpha,\beta)=\frac{\beta^{\alpha}x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}$$

**Shape parameter** $\alpha$
**Rate parameter** $\beta$

The bigger $\beta$ is, the bigger y-axis is

PDF of exponential distribution:

$$f(\mathrm{x})=\lambda\,\mathrm{e}^{-\lambda x}$$ ← When $\alpha$=1

Expectation and variance of gamma distribution:

$$E[f(\mathrm{x};\alpha,\beta)]=\frac{\alpha}{\beta} \qquad D[f(\mathrm{x};\alpha,\beta)]=\frac{\alpha}{\beta^{2}}$$

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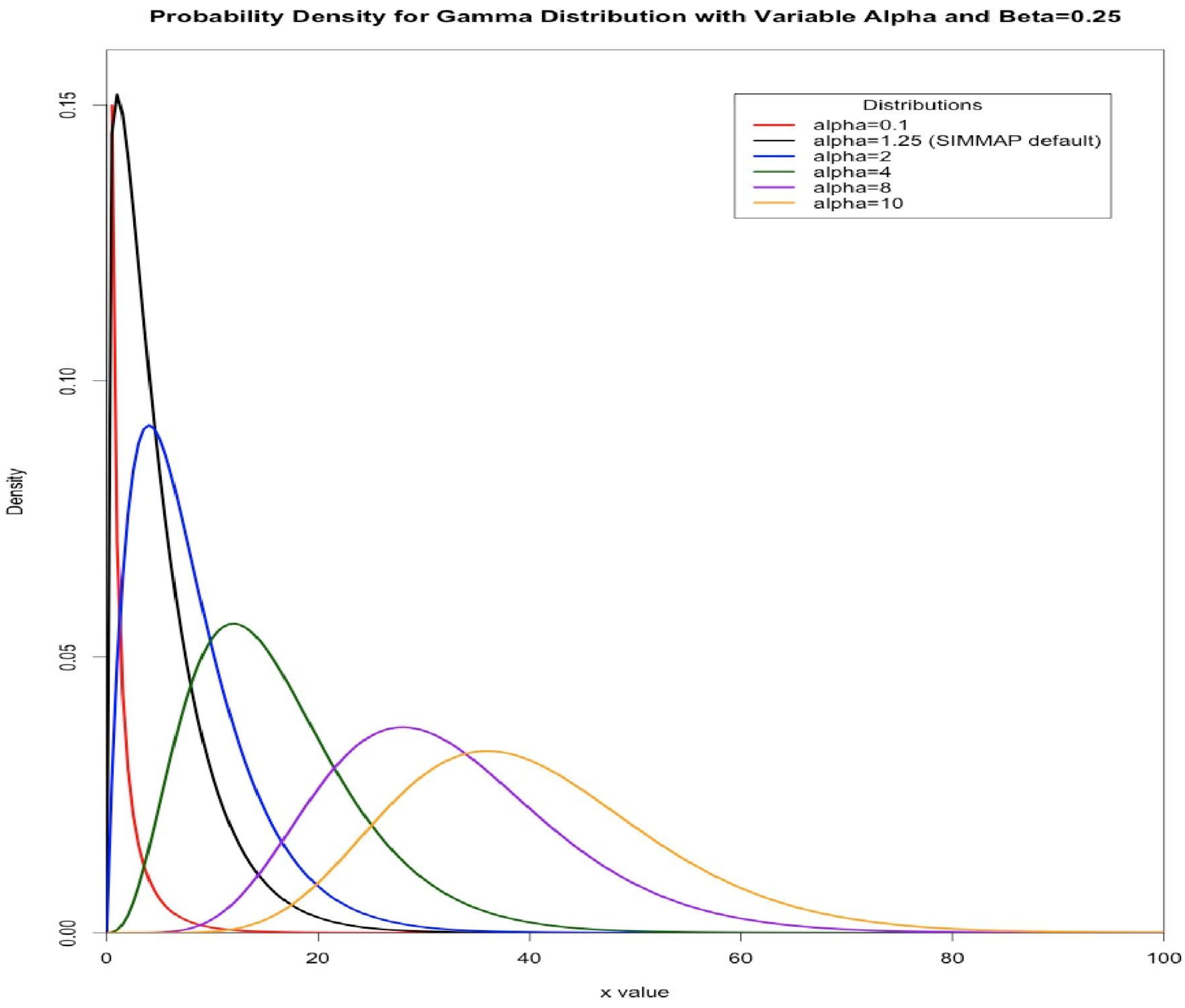
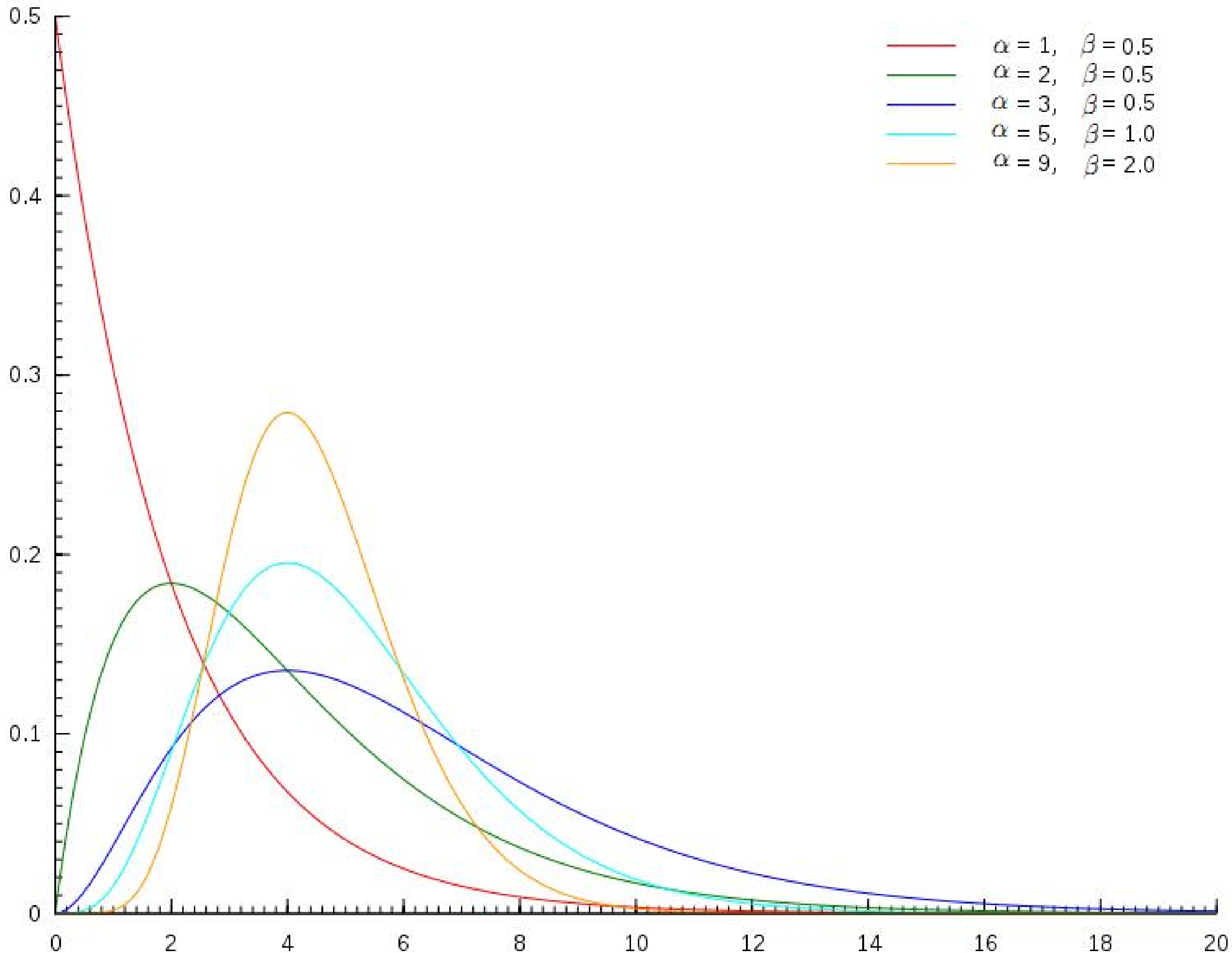
# Gamma distribution



# Gamma distribution



# Beta function

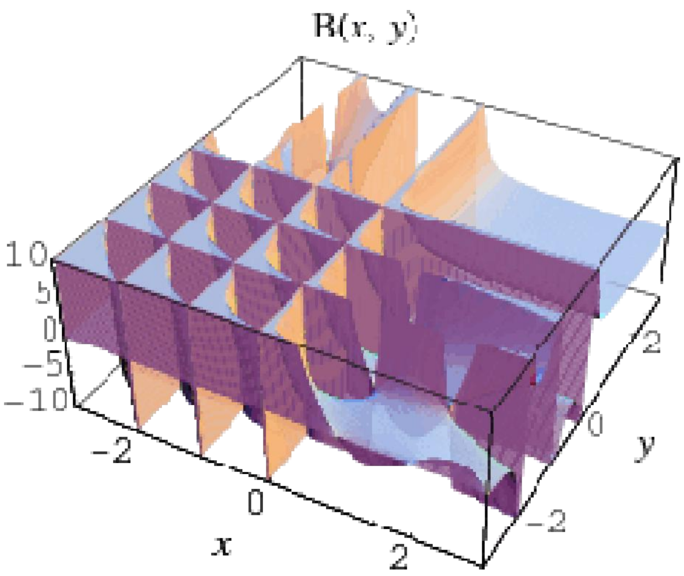
**Definition**:

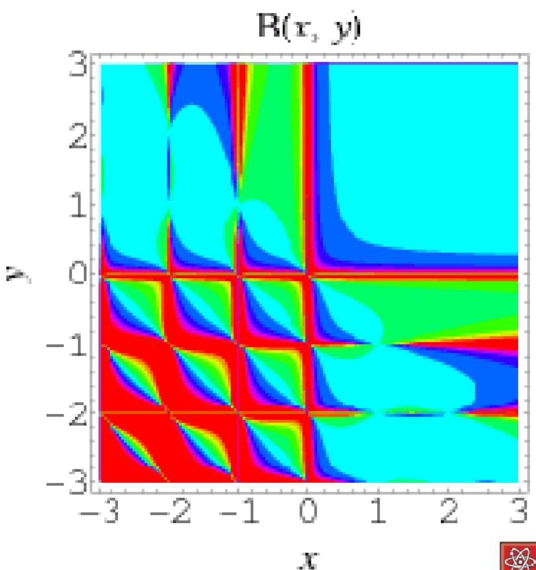
$$B(\mathrm{x},\mathrm{y}) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(\mathrm{x})\Gamma(\mathrm{y})}{\Gamma(\mathrm{x}+y)}$$

When x,y are real numbers

$$B(m,n) = \frac{(m-1)!(n-1)!}{(\mathrm{m}+n-1)!} = \frac{\Gamma(\mathrm{m})\Gamma(n)}{\Gamma(m+n)}$$

When m,n are positive integer(m>=1, n>=1) numbers

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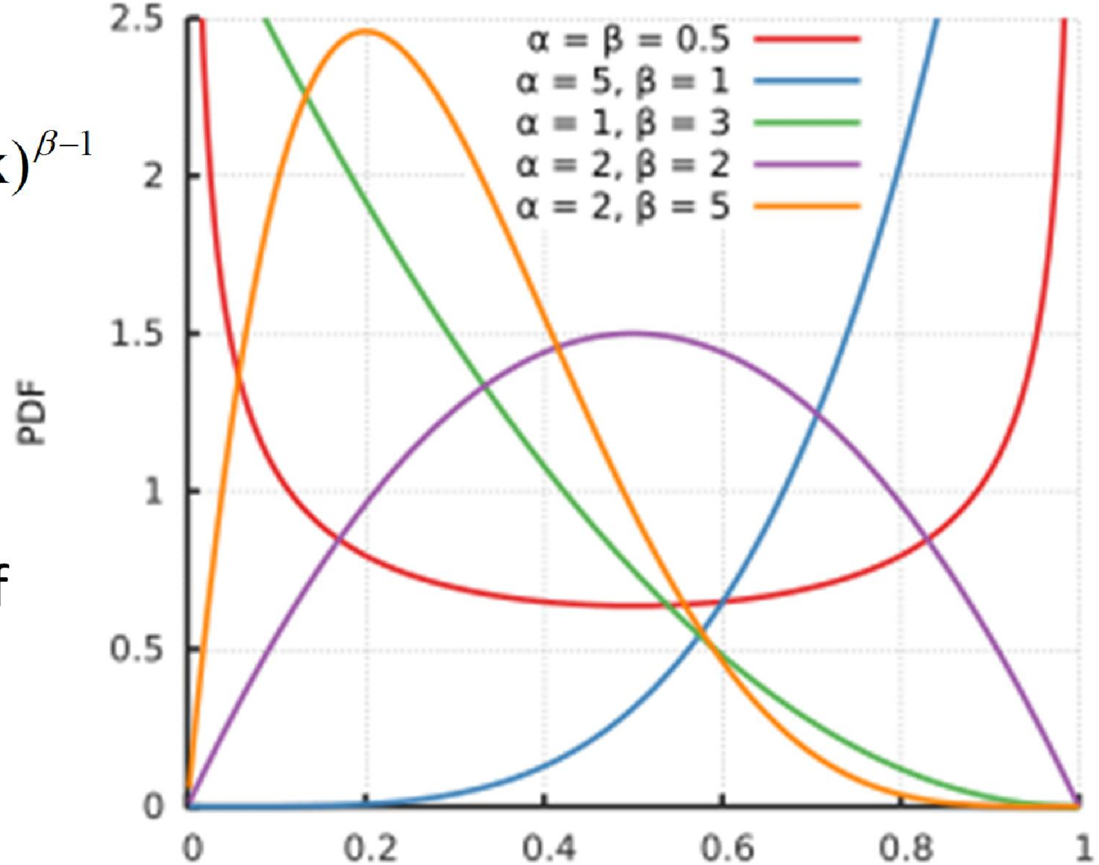
# Beta distribution

**Definition**:

$$f(\mathrm{x};\alpha,\beta)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-\mathrm{x})^{\beta-1}$$

$(0<x<1);(\alpha,\beta>0)$




Expectation and variance of beta distribution:

$$E[f(\mathrm{x};\alpha,\beta)]=\frac{\alpha}{\alpha+\beta}$$

$$\mathrm{D}[f(\mathrm{x};\alpha,\beta)]=\frac{\alpha\beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)}$$

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# The reputation function

**Definition**:

$$f(\mathrm{x};\alpha,\beta)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-\mathrm{x})^{\beta-1}$$

Binary rating system $\rho=\begin{bmatrix} r \\ s \end{bmatrix}$

(0<x<1);$(\alpha,\beta>0)$

$\rho^{X}{}_{Z,t_R}$

$\alpha=r+1;\beta=s+1$

It can be read as X's rating of Z at time $t_R$.

The aggregate rating of Z, taking into account rating by the entire agent community C, can be calculated

$$\rho^{t}(\mathrm{X},\mathrm{Z})=\sum\rho^{X,t}{}_{Z,t_R}$$

For each pair of agents (X,Z), an aggregate rating can be calculated that reflects X's overall opinion of Z at time t.

$$\rho^{t}(\mathrm{Z})=\sum_{X\in C}\rho^{t}(\mathrm{X},\mathrm{Z})$$

# Reputation Score

Let $\rho^t(\mathrm{Z})$ represent the target Z's aggregate rating at time t. Then the function $R^t(\mathrm{Z})$ defined by:

$$R^t(\mathrm{Z}) = \mathrm{E}[\mathrm{beta}(\rho^t(\mathrm{Z}))] = \frac{r+1}{r+s+2}$$

# HAPPY NEW YEAR!


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