k-Nearest Neighbor Classification over Semantically Secure Encrypted Relational Data

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Main References

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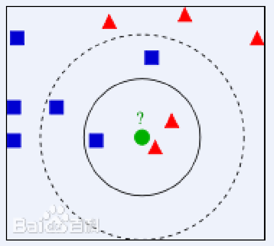
k-Nearest Neighbor algorithm



Problem Definition

Suppose Alice owns a database D of n records $t_1, \cdots, t_n$ and $m+1$ attributes. Let $t_{i,j}$ denote the j-th attribute value of record t_i. Initially, Alice encrypts her database attribute-wise, that is, she computes $E_{pk}(t_{i,j})$, for $1 \leq i \leq n$ and $1 \leq j \leq m+1$, where column $(m+1)$ contains the class labels. Let the encrypted database be denoted by D'.

Problem Definition

Let Bob be an authorized user who wants to classify his input record $q =< q_1, \cdots, q_m >$ by applying the k-NN classification method based on D'. We refer to such a process as privacy-preserving k-NN (PPkNN) classification over encrypted data in the cloud. Formally, we define the PPkNN protocol as:

$$PPkNN(D', q) \rightarrow c_q$$

where c_q denotes the class label for q after applying k-NN classification method on D and q.

Paillier Cryptosystem

a. Homomorphic Addition

$$E_{pk}(a + b) = E_{pk}(a) * E_{pk}(b) \mod N^2$$

b. Homomorphic Multiplication

$$E_{pk}(a * b) = E_{pk}(a)^b \mod N^2$$

Basic Privacy-Preserving Protocols

All of the below protocols are considered under two-party semi-honest setting. In particular, we assume the exist of two semi-honest parties P_1 and P_2 such that the Paillier's secret key sk is known only to P_2 whereas pk is treated as public.

1. Multiplication (SM) Protocol: This protocol considers P_1 with input $(E_{pk}(a), E_{pk}(b))$ and outputs $E_{pk}(a * b)$ to P_1, where a and b are not known to P_1 and P_2. During this process, no information regarding a and b is revealed to P_1 and P_2.

Basic Privacy-Preserving Protocols

Secure Squared Euclidean Distance (SSED) Protocol: In this protocol, P_1 with input $(E_{pk}(X), E_{pk}(Y))$ and P_2 with sk securely compute the encryption of squared Euclidean distance between vectors X and Y. Here X and Y are m dimensional vectors where $E_{pk}(X) =< E_{pk}(x1), \cdots, E_{pk}(x_m) >$ and $E_{pk}(Y) =< E_{pk}(y_1), \cdots, E_{pk}(y_m) >$. The output of the SSED protocol is $E_{pk}(|X - Y|^2)$ which is known only to P_1

Basic Privacy-Preserving Protocols

Secure Bit-Decomposition (SBD) Protocol: P_1 with input $E_{pk}(z)$ and P_2 securely compute the encryptions of the individual bits of z, where $0 \leq z < 2^l$. The output $[z] =< E_{pk}(z_1), \cdots, E_{pk}(z_l) >$ is known only to P_1. Here z_1 and z_l are the most and least significant bits of integer z, respectively

Basic Privacy-Preserving Protocols

Secure Minimum (SMIN) Protocol: In this protocol, P_1 holds private input (u', v') and P_2 holds sk, where $u' = ([u], E_{pk}(s_u))$ and $v = ([v], E_{pk}(s_v))$. Here s_u (resp., s_v) denotes the secret associated with u (resp., v). The goal of SMIN is for P_1 and P_2 to jointly compute the encryptions of the individual bits of minimum number between $u$and v. In addition, they compute $E_{pk}(smin(u, v))$. That is, the output is $([min(u, v)], E_{pk}(s_{min(u,v)}))$ which will be known only to P_1. During this protocol, no information regarding the contents of u, v, s_u, and s_v is revealed to P_1 and P_2.

Basic Privacy-Preserving Protocols

Secure Minimum out of n Numbers ($SMIN_n$) Protocol: In this protocol, we consider P_1 with n encrypted vectors $([d_1], \cdots, [d_n])$ along with their respective encrypted secrets and P_2 with sk. Here $[d_i] = E_{pk}(d_{i,1}), \cdots, E_{pk}(d_{i,l})$ where $d_{i,1}$ and $d_{i,l}$ are the most and least significant bits of integer d_i respectively, for $1 \leq i \leq n$. The secret of d_i is given by s_{d_i}. P_1 and P_2 jointly compute $[min(d_1, \cdots, d_n)]$. In addition, they compute $E_{pk}(s_{min(d_1,\cdots,d_n)})$. At the end of this protocol, the output $([min(d_1, \cdots, d_n)], E_{pk}(s_{min(d_1,\cdots,d_n)}))$ is known only to P_1. During the $SMIN_n$ protocol, no information regarding any of d_i's and their secrets is revealed to P_1 and P_2.

Basic Privacy-Preserving Protocols

Secure Bit-OR (SBOR) Protocol: P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 securely compute $E_{pk}(o_1 \vee o_2)$, where o_1 and o_2 are two bits. The output $E_{pk}(o_1 \vee o_2)$ is known only to P_1.

Secure Multiplication (SM)

$$a * b = (a + r_a) * (b + r_b) - a * r_a - b * r_b - r_a * r_b$$

Note that, for any given $x \in Z_N$, $N - x$ is equivalent to $-x$ under Z_N

Secure Multiplication (SM)

Algorithm 1 SM$(E_{pk}(a), E_{pk}(b)) \rightarrow E_{pk}(a * b)$

Require: P_1 has $E_{pk}(a)$ and $E_{pk}(b)$; P_2 has sk

1: P_1:
- (a). Pick two random numbers $r_a, r_b \in \mathbb{Z}_N$
- (b). $a' \leftarrow E_{pk}(a) * E_{pk}(r_a)$
- (c). $b' \leftarrow E_{pk}(b) * E_{pk}(r_b)$; send a', b' to P_2

2: P_2:
- (a). Receive a' and b' from P_1
- (b). $h_a \leftarrow D_{sk}(a')$; $h_b \leftarrow D_{sk}(b')$
- (c). $h \leftarrow h_a * h_b \bmod N$
- (d). $h' \leftarrow E_{pk}(h)$; send h' to P_1

3: P_1:
- (a). Receive h' from P_2
- (b). $s \leftarrow h' * E_{pk}(a)^{N-r_b}$
- (c). $s' \leftarrow s * E_{pk}(b)^{N-r_a}$
- (d). $E_{pk}(a * b) \leftarrow s' * E_{pk}(r_a * r_b)^{N-1}$

Secure Squared Euclidean Distance (SSED)

Algorithm 2 SSED$(E_{pk}(X), E_{pk}(Y)) \rightarrow E_{pk}(|X - Y|^2)$

Require: P_1 has $E_{pk}(X)$ and $E_{pk}(Y)$; P_2 has sk

1: P_1, **for** $1 \leq i \leq m$ **do**:

  (a). $E_{pk}(x_i - y_i) \leftarrow E_{pk}(x_i) * E_{pk}(y_i)^{N-1}$

2: P_1 and P_2, **for** $1 \leq i \leq m$ **do**:

  (a). Compute $E_{pk}((x_i - y_i)^2)$ using the SM protocol

3: P_1 computes $E_{pk}(|X - Y|^2) \leftarrow \prod_{i=1}^{m} E_{pk}((x_i - y_i)^2)$

Secure Bit-Decomposition (SBD)

We assume that P_1 has $E_{pk}(z)$ and P_2 has sk, where z is not known to both parties and $0 \leq z < 2^l$. Given $E_{pk}(z)$, the goal of the secure bit-decomposition (SBD) protocol is to compute the encryptions of the individual bits of binary representation of z. That is, the output is $[z] =< E_{pk}(z_1), \cdots, E_{pk}(z_l) >$, where z_1 and z_l denote the most and least significant bits of z respectively. At the end, the output $[z]$ is known only to P_1. During this process, neither the value of z nor any z_i's is revealed to P_1 and P_2.

Secure Minimum (SMIN)

the basic idea of the proposed SMIN protocol is for P_1 to randomly choose the functionality F (by flipping a coin), where F is either $u > v$ or $v > u$, and to obliviously execute F with P_2. Since F is randomly chosen and known only to P_1, the result of the functionality F is oblivious to P_2. Based on the comparison result and chosen F, P_1 computes $[min(u, v)]$ and $E_{pk}(s_{min(u,v)})$ locally using homomorphic properties.

Secure Minimum (SMIN)

Algorithm 3 SMIN$(u', v') \rightarrow ([\min(u,v)], E_{pk}(s_{\min(u,v)}))$

Require: P_1 has $u' = ([u], E_{pk}(s_u))$ and $v' = ([v], E_{pk}(s_v))$, where $0 \leq u, v < 2^l$; P_2 has sk

1: P_1:

(a). Randomly choose the functionality F

(b). **for** $i = 1$ to l **do**:

- $E_{pk}(u_i * v_i) \leftarrow \text{SM}(E_{pk}(u_i), E_{pk}(v_i))$
- $T_i \leftarrow E_{pk}(u_i \oplus v_i)$
- $H_i \leftarrow H_{i-1}^{r_i} * T_i$; $r_i \in_R \mathbb{Z}_N$ and $H_0 = E_{pk}(0)$
- $\Phi_i \leftarrow E_{pk}(-1) * H_i$
- **if** $F : u > v$ **then** $W_i \leftarrow E_{pk}(u_i) * E_{pk}(u_i * v_i)^{N-1}$ and $\Gamma_i \leftarrow E_{pk}(v_i - u_i) * E_{pk}(\hat{r}_i)$; $\hat{r}_i \in_R \mathbb{Z}_N$
  else $W_i \leftarrow E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-1}$ and $\Gamma_i \leftarrow E_{pk}(u_i - v_i) * E_{pk}(\hat{r}_i)$; $\hat{r}_i \in_R \mathbb{Z}_N$
- $L_i \leftarrow W_i * \Phi_i^{r'_i}$; $r'_i \in_R \mathbb{Z}_N$

(c). **if** $F : u > v$ **then**: $\delta \leftarrow E_{pk}(s_v - s_u) * E_{pk}(\bar{r})$
else $\delta \leftarrow E_{pk}(s_u - s_v) * E_{pk}(\bar{r})$, where $\bar{r} \in_R \mathbb{Z}_N$

(d). $\Gamma' \leftarrow \pi_1(\Gamma)$ and $L' \leftarrow \pi_2(L)$

(e). Send δ, Γ' and L' to P_2

Secure Minimum (SMIN)

2: P_2:

(a). Decryption: $M_i \leftarrow D_{sk}(L'_i)$, for $1 \leq i \leq l$

(b). **if** $\exists\, j$ such that $M_j = 1$ **then** $\alpha \leftarrow 1$
else $\alpha \leftarrow 0$

(c). **if** $\alpha = 0$ **then**:

- $M'_i \leftarrow E_{pk}(0)$, for $1 \leq i \leq l$
- $\delta' \leftarrow E_{pk}(0)$

else

- $M'_i \leftarrow \Gamma'_i * r^N$, where $r \in_R \mathbb{Z}_N$ and is different for $1 \leq i \leq l$
- $\delta' \leftarrow \delta * r_\delta^N$, where $r_\delta \in_R \mathbb{Z}_N$

(d). Send $M', E_{pk}(\alpha)$ and δ' to P_1

Secure Minimum (SMIN)

3: P_1:

(a). $\widetilde{M} \leftarrow \pi_1^{-1}(M')$ and $\theta \leftarrow \delta' * E_{pk}(\alpha)^{N-\bar{r}}$

(b). $\lambda_i \leftarrow \widetilde{M}_i * E_{pk}(\alpha)^{N-\hat{r}_i}$, for $1 \leq i \leq l$

(c). **if** $F : u > v$ **then**:

- $E_{pk}(s_{\min(u,v)}) \leftarrow E_{pk}(s_u) * \theta$
- $E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(u_i) * \lambda_i$, for $1 \leq i \leq l$

else

- $E_{pk}(s_{\min(u,v)}) \leftarrow E_{pk}(s_v) * \theta$
- $E_{pk}(\min(u,v)_i) \leftarrow E_{pk}(v_i) * \lambda_i$, for $1 \leq i \leq l$

Secure Minimum (SMIN)

- Compute the encrypted bit-wise XOR between the bits u_i and v_i as $T_i = E_{pk}(u_i \oplus v_i)$ using the below formulation[1]:

$$T_i = E_{pk}(u_i) * E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-2}$$

- Compute an encrypted vector H by preserving the first occurrence of $E_{pk}(1)$ (if there exists one) in T by initializing $H_0 = E_{pk}(0)$. The rest of the entries of H are computed as $H_i = H_{i-1}^{r_i} * T_i$. We emphasize that at most one of the entry in H is $E_{pk}(1)$ and the remaining entries are encryptions of either 0 or a random number.
- Then, P_1 computes $\Phi_i = E_{pk}(-1) * H_i$. Note that "$-1$" is equivalent to "$N - 1$" under $\mathbb{Z}_N$. From the above discussions, it is clear that $\Phi_i = E_{pk}(0)$ at most once since H_i is equal to $E_{pk}(1)$ at most once. Also, if $\Phi_j = E_{pk}(0)$, then index j is the position at which the bits of u and v differ first (starting from the most significant bit position).

Secure Minimum (SMIN)

- If $F : u > v$, compute

$$
\begin{aligned}
W_i &= E_{pk}(u_i) * E_{pk}(u_i * v_i)^{N-1} \\
&= E_{pk}(u_i * (1 - v_i)) \\
\Gamma_i &= E_{pk}(v_i - u_i) * E_{pk}(\hat{r}_i) \\
&= E_{pk}(v_i - u_i + \hat{r}_i)
\end{aligned}
$$

- If $F : v > u$, compute:

$$
\begin{aligned}
W_i &= E_{pk}(v_i) * E_{pk}(u_i * v_i)^{N-1} \\
&= E_{pk}(v_i * (1 - u_i)) \\
\Gamma_i &= E_{pk}(u_i - v_i) * E_{pk}(\hat{r}_i) \\
&= E_{pk}(u_i - v_i + \hat{r}_i)
\end{aligned}
$$

Secure Minimum out of n Numbers ($SMIN_n$)

Algorithm 4 $\mathrm{SMIN}_n([d_1], \ldots, [d_n]) \rightarrow [d_{\min}]$

Require: P_1 has $([d_1], \ldots, [d_n])$; P_2 has sk

1: P_1:

(a). $[d'_i] \leftarrow [d_i]$, for $1 \leq i \leq n$, and $num \leftarrow n$

2: P_1 and P_2, **for** $i = 1$ to $\lceil \log_2 n \rceil$:

(a). **for** $1 \leq j \leq \left\lfloor \frac{num}{2} \right\rfloor$:

- **if** $i = 1$ **then**:
  - $[d'_{2j-1}] \leftarrow \mathrm{SMIN}([d'_{2j-1}], [d'_{2j}])$
  - $[d'_{2j}] \leftarrow 0$

  else
  - $[d'_{2i(j-1)+1}] \leftarrow \mathrm{SMIN}([d'_{2i(j-1)+1}], [d'_{2ij-1}])$
  - $[d'_{2ij-1}] \leftarrow 0$

(b). $num \leftarrow \left\lceil \frac{num}{2} \right\rceil$

3: P_1 sets $[d_{\min}]$ to $[d'_1]$

Secure Bit-OR (SBOR)

Suppose P_1 holds $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 holds sk, where o_1 and o_2 are two bits not known to both parties. The goal of the SBOR protocol is to securely compute $E_{pk}(o_1 \vee o_2)$. At the end of this protocol, only P1 knows $E_{pk}(o_1 \vee o_2)$. During this process, no information related to o_1 and o_2 is revealed to P_1 and P_2. Given the secure multiplication (SM) protocol, P_1 can compute $E_{pk}(o_1 \vee o_2)$ as follows:

P_1 with input $(E_{pk}(o_1), E_{pk}(o_2))$ and P_2 involve in the SM protocol. At the end of this step, the output $E_{pk}(o1 * o2)$ is known only to P_1. Note that, since o_1 and o_2 are bits,
$Epk(o1 * o2) = Epk(o1 \wedge o2)$.
$Epk(o1 \vee o2) = Epk(o1 + o2) * Epk(o1 \wedge o2)^{N-1}$

Basic scheme

Algorithm 5 $\mathrm{S}k\mathrm{NN_b}(E_{pk}(T), Q) \rightarrow \langle t'_1, \ldots, t'_k \rangle$

Require: C_1 has $E_{pk}(T)$; C_2 has sk; Bob has Q

1: Bob:
- (a). Compute $E_{pk}(q_j)$, for $1 \leq j \leq m$
- (b). Send $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ to C_1

2: C_1 and C_2:
- (a). C_1 receives $E_{pk}(Q)$ from Bob
- (b). **for** $i = 1$ to n **do**:
  - $E_{pk}(d_i) \leftarrow \mathrm{SSED}(E_{pk}(Q), E_{pk}(t_i))$
- (c). Send $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ to C_2

3: C_2:
- (a). Receive $\{\langle 1, E_{pk}(d_1) \rangle, \ldots, \langle n, E_{pk}(d_n) \rangle\}$ from C_1
- (b). $d_i \leftarrow D_{sk}(E_{pk}(d_i))$, for $1 \leq i \leq n$
- (c). Generate $\delta \leftarrow \langle i_1, \ldots, i_k \rangle$, such that $\langle d_{i_1}, \ldots, d_{i_k} \rangle$ are the top k smallest distances among $\langle d_1, \ldots, d_n \rangle$
- (d). Send δ to C_1

Basic scheme

4: C_1:

(a). Receive δ from C_2

(b). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:

- $\gamma_{j,h} \leftarrow E_{pk}(t_{i_j,h}) * E_{pk}(r_{j,h})$, where $r_{j,h} \in_R \mathbb{Z}_N$
- Send $\gamma_{j,h}$ to C_2 and $r_{j,h}$ to Bob

5: C_2:

(a). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:

- Receive $\gamma_{j,h}$ from C_1
- $\gamma'_{j,h} \leftarrow D_{sk}(\gamma_{j,h})$; send $\gamma'_{j,h}$ to Bob

6: Bob:

(a). **for** $1 \leq j \leq k$ and $1 \leq h \leq m$ **do**:

- Receive $r_{j,h}$ from C_1 and $\gamma'_{j,h}$ from C_2
- $t'_{j,h} \leftarrow \gamma'_{j,h} - r_{j,h} \mod N$

Fully Secure kNN Protocol

Algorithm 6 $\mathrm{S}k\mathrm{NN_m}(E_{pk}(T), Q) \rightarrow \langle t'_1, \ldots, t'_k \rangle$

Require: C_1 has $E_{pk}(T)$ and π; C_2 has sk; Bob has Q

1: Bob sends $E_{pk}(Q) = \langle E_{pk}(q_1), \ldots, E_{pk}(q_m) \rangle$ to C_1

2: C_1 and C_2:

- (a). C_1 receives $E_{pk}(Q)$ from Bob
- (b). **for** $i = 1$ to n **do:**
  - $E_{pk}(d_i) \leftarrow \mathrm{SSED}(E_{pk}(Q), E_{pk}(t_i))$
  - $[d_i] \leftarrow \mathrm{SBD}(E_{pk}(d_i))$

3: **for** $s = 1$ to k **do:**

- (a). C_1 and C_2:
  - $[d_{\min}] \leftarrow \mathrm{SMIN}_n([d_1], \ldots, [d_n])$
- (b). C_1:
  - $E_{pk}(d_{\min}) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{\min,\gamma+1})^{2^{l-\gamma-1}}$
  - **if** $s \neq 1$ **then**, for $1 \leq i \leq n$
    - $E_{pk}(d_i) \leftarrow \prod_{\gamma=0}^{l-1} E_{pk}(d_{i,\gamma+1})^{2^{l-\gamma-1}}$
  - **for** $i = 1$ to n **do:**
    - $\tau_i \leftarrow E_{pk}(d_{\min}) * E_{pk}(d_i)^{N-1}$
    - $\tau'_i \leftarrow \tau_i^{r_i}$, where $r_i \in_R \mathbb{Z}_N$
  - $\beta \leftarrow \pi(\tau')$; send β to C_2

Fully Secure kNN Protocol

(c). C_2:

- Receive β from C_1
- $\beta'_i \leftarrow D_{sk}(\beta_i)$, for $1 \leq i \leq n$
- Compute U, for $1 \leq i \leq n$:
  - **if** $\beta'_i = 0$ **then** $U_i = E_{pk}(1)$
  - **else** $U_i = E_{pk}(0)$
- Send U to C_1

(d). C_1:

- Receive U from C_2 and compute $V \leftarrow \pi^{-1}(U)$
- $V'_{i,j} \leftarrow \mathrm{SM}(V_i, E_{pk}(t_{i,j}))$, for $1 \leq i \leq n$ and $1 \leq j \leq m$
- $E_{pk}(t'_{s,j}) \leftarrow \prod_{i=1}^{n} V'_{i,j}$, for $1 \leq j \leq m$
- $E_{pk}(t'_s) = \langle E_{pk}(t'_{s,1}), \ldots, E_{pk}(t'_{s,m}) \rangle$

(e). C_1 and C_2, for $1 \leq i \leq n$:

- $E_{pk}(d_{i,\gamma}) \leftarrow \mathrm{SBOR}(V_i, E_{pk}(d_{i,\gamma}))$, for $1 \leq \gamma \leq l$

The rest of the steps are similar to steps 4-6 of SkNN$_\mathrm{b}$

Thank you

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