Hierophone Filter

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1 Introduction

Analysis of the Hierophone filter, a basic three-pole low-pass filter built from three cascaded OTA based integrator cells.

1.1 Notations

R_i is the value of the resistor through which the input signal is fed to the OTA

R_s is the value of the resistor at the voltage divider input of the OTA

R_f is the value of the resistor through which the integrator output is fed back to the OTA input

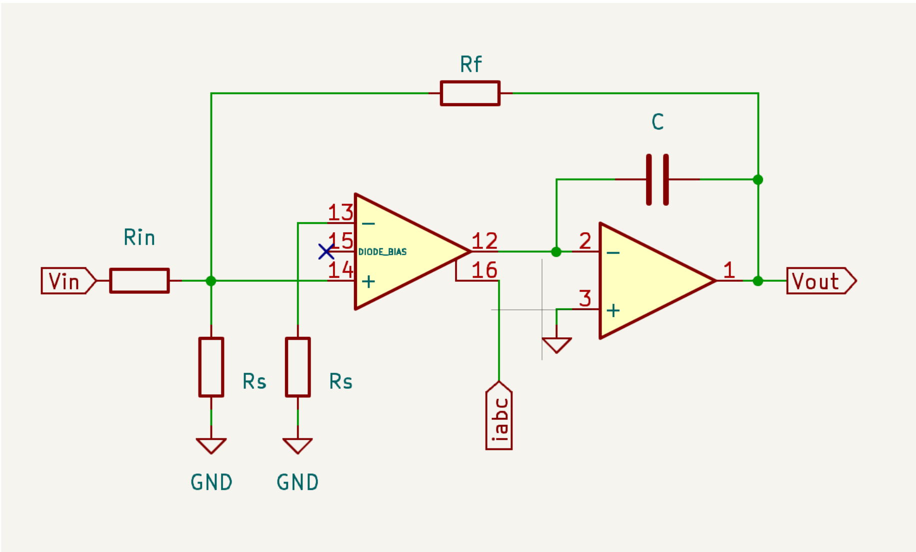
C is the value of the integrators' capacitor

v_{cv} is the cutoff frequency control voltage

$v_i(s)$ is the input voltage

$v_{out}(s)$ is the output voltage

1.2 OTA Low-Pass Filter



The transfer function of an OTA is:

$$i_o = g_m(v_+ - v_-)$$
$$i_o = 19.2 i_{cv}(v_+ - v_-)$$

If $R_f == R_i == R$ then Kirchoff in v_+ gives:

$$\frac{1}{R}(v_i(s) - v_+(s)) + \frac{1}{R}(v_{out}(s) - v_+(s)) = \frac{1}{R_s} v_+(s)$$
$$v_+(s) = \frac{R_s}{R + 2R_s}(v_i(s) + v_{out}(s))$$

Given that v_- is grounded, the current $i_c(s)$ at the output of the OTA is:

$$\begin{aligned} i_c(s) &= g_m(v_+(s) - v_-(s)) \\ &= 19.2 i_{cv} \frac{R_s}{R + 2R_s}(v_i(s) + v_{out}(s)) \end{aligned}$$

The voltage out of the OpAmp:

$$\begin{aligned} v_{out}(s) &= \frac{-i_c(s)}{Cs} \\ v_{out}(s) &= -19.2 i_{cv} \frac{R_s}{Cs(R + 2R_s)}(v_i(s) + v_{out}(s)) \end{aligned}$$

So the transfer function is:

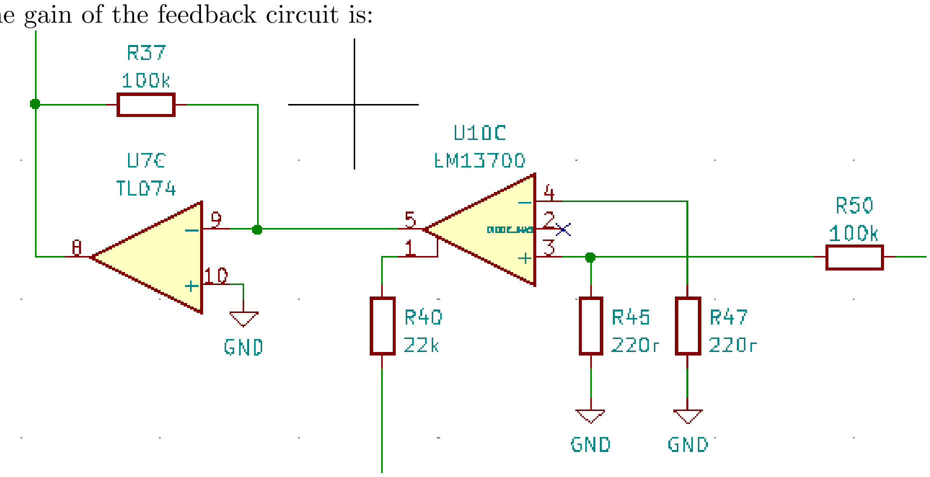
$$\alpha(s) = \frac{-1}{1 + \frac{R+2R_s}{R_s} Cs \frac{1}{19.2 i_{cv}}} \tag{1}$$

The pole is at the frequency f so that:

$$\begin{aligned} 0 &= 1 + \frac{R + 2R_s}{R_s} C 2\pi f \frac{1}{19.2 i_{cv}} \\ f &= -\frac{19.2 i_{cv}}{2\pi \frac{R+2R_s}{R_s} C} \end{aligned}$$

1.3 Feedback Control

The gain of the feedback circuit is:



$$
\begin{aligned}
v_+ &= v_i \frac{R_s}{R + R_s} \\
i_o &= 19.2 i_q (v_+ - v_-) \\
i_o &= 19.2 i_q (v_+ - 0) \\
i_o &= 19.2 i_q v_i \frac{R_s}{R + R_s} \\
i_o &= R_f (v_- - v_o) \\
i_o &= -R_f v_o \\
-R_f v_o &= 19.2 i_q v_i \frac{R_s}{R + R_s} \\
-R_f v_o &= 19.2 i_q v_i \frac{R_s}{R + R_s} \\
v_o &= -19.2 i_q v_i \frac{R_f R_s}{R_i + R_s} \\
\beta &= -19.2 i_q \frac{R_f R_s}{R_i + R_s}
\end{aligned}
$$

1.4 Filter

$$
\begin{aligned}
\frac{v_i - v_-}{R_i} + \frac{v_{hp} - v_-}{R_g} + \frac{v_{lp} - v_-}{R_g} + \frac{v_{bp}\beta - v_-}{R_q} &= i_- = 0 \\
\frac{v_i}{R_i} + \frac{v_{hp}}{R_g} + \frac{v_{lp}}{R_g} + \frac{v_{bp}\beta}{R_q} &= 0
\end{aligned}
$$

It is important to note that $v_{bp}(s) = v_{hp}(s)\alpha(s)$, and $v_{lp}(s) = v_{hp}(s)\alpha(s^2)$.

1.4.1 High-pass

$$
\begin{aligned}
\frac{v_{hp}(s)}{R_g} &= -\frac{v_i(s)}{R_i} - \frac{v_{lp}(s)}{R_g} - \frac{v_{bp}(s)\beta}{R_q} \\
\frac{v_{hp}(s)}{R_g} &= -\frac{v_i(s)}{R_i} - \frac{v_{hp}(s)\alpha(s^2)}{R_g} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
\frac{v_i(s)}{R_i} &= -\frac{v_{hp}(s)}{R_g} - \frac{v_{hp}(s)\alpha(s^2)}{R_g} - \frac{v_{hp}(s)\alpha(s)\beta}{R_q} \\
H_{hp}(s) &= \frac{v_{hp}(s)}{v_i(s)} \\
\frac{v_i(s)}{R_i} &= -v_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{1}{R_i} &= -H_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
\frac{-1}{R_i} &= H_{hp}(s)(\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}) \\
H_{hp}(s) &= \frac{-1/R_i}{\frac{1}{R_g} + \frac{\alpha(s^2)}{R_g} + \frac{\alpha(s)\beta}{R_q}} \\
&= \frac{-R_g/R_i}{1 + \frac{R_g\alpha(s)\beta}{R_q} + \alpha(s^2)}
\end{aligned}
$$

1.4.2 Low-Pass

$$
\begin{aligned}
H_{lp}(s) &= \frac{v_{lp}(s)}{v_i(s)} \\
&= \frac{v_{hp}(s)}{v_i(s)}\alpha^2(s) \\
&= \frac{-R_g/R_i}{\frac{1}{\alpha^2(s)} + \frac{R_g\beta}{R_q\alpha(s)} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{\alpha^2(s)} + \beta\frac{R_g}{R_q}\frac{1}{\alpha(s)} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{(-19.2i_{cv}\frac{R_s}{Cs(R+2R_s)})^2} + (-19.2i_q\frac{R_f R_s}{R_i+R_s})\frac{R_g}{R_q}\frac{1}{-19.2i_{cv}\frac{R_s}{Cs(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{(19.2i_{cv}\frac{R_s}{Cs(R+2R_s)})^2} + 19.2i_q\frac{R_f R_s}{R_i+R_s}\frac{R_g}{R_q}\frac{1}{19.2i_{cv}\frac{R_s}{Cs(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{1}{\frac{1}{s^2}(19.2i_{cv}\frac{R_s}{C(R+2R_s)})^2} + 19.2i_q\frac{R_f R_s}{R_i+R_s}\frac{R_g}{R_q}\frac{1}{19.2i_{cv}\frac{1}{s}\frac{R_s}{C(R+2R_s)}} + 1} \\
&= \frac{-R_g/R_i}{\frac{s^2}{(19.2i_{cv}\frac{R_s}{C(R+2R_s)})^2} + 19.2i_q\frac{R_f R_s}{R_i+R_s}\frac{R_g}{R_q}\frac{s}{19.2i_{cv}\frac{R_s}{C(R+2R_s)}} + 1}
\end{aligned}
$$

One simplification worth making is to say that $R_q = R_g$.

By comparing with the standard form of a second order filter transfer function we can work out the following.

Pass-band gain, $-\frac{R_g}{R_i}$

Cutoff frequency, $19.2i_{cv}\frac{R_s}{C2\pi(R+2R_s)}$

Quality factor, $\frac{1}{19.2i_q\frac{R_f R_s}{R_i+R_s}}$

1.5 Calculating cutoff frequencies

Given the calculation for frequency, and picking some standard values we can calculate cutoff for different i_{cv} values.

R is 100k

R_s is 220r

C is 220pF

$$f = 19.2i_{cv}\frac{220}{220 * 10^{-12} * 2\pi(100000 + 2(220))} \tag{2}$$

for i_{cv} of 0.5ma $f = 15212Hz$

for i_{cv} of 0.3ma $f = 9127Hz$

for i_{cv} of 0.1ma $f = 3042Hz$

for i_{cv} of 0.05ma $f = 1521Hz$

for i_{cv} of 0.01ma $f = 304Hz$

1.6 Calculating resonance

A quality factor of 1/2 gives no resonance, whilst the resonance (and likelihood of self oscillating) increases as Q goes to infinity.

R_f is 100k

R_i is 100k

R_s is 220r

C is 220pF

$$q = \frac{1}{19.2 i_q \frac{100000*220}{100000+220}} \tag{3}$$

for i_q of 0.5ma $q = 0.47$

for i_q of 0.3ma $q = 0.79$

for i_q of 0.1ma $q = 2.37$

for i_q of 0.05ma $q = 4.75$

for i_q of 0.01ma $q = 23.7$