Entanglement-Guided QEC: From Stabilizer TEE to Correlation-Aware Decoding with holographic models

Overview. This summary note describes briefly the general pipeline and background of the current, ongoing and unpublished project I am working on. We are developing a pipeline that links two complementary views of quantum error correction (QEC):

- The state-level structure of entanglement in CSS/topological codes which is measured via von Neumann entanglement entropy and topological entanglement entropy (TEE), and furthermore, generalized to holographic models via Ryu-Takayangi conjecture [1].
- The syndrome-level correlations used by practical decoders. The repos in my personal github page (not the group's page) include a stabilizer-entropy engine that computes entanglement entropy $S(A)$ and Kitaev-Preskill topological entanglement entropy (TEE) estimate, also the stim and pyMatching pipeline that samples 1) detection events 2) extracts mutual information and 3) performs minimum-weight perfect matching.

Background info: TEE and its relation to QEC

Area law of EE and the topological constant Gapped phases (meaning the mass gap) with topological order typically obey an *area law* for bipartite entanglement entropy $S(A) = \alpha|\partial A| - \gamma + \cdots$, where γ is the (TEE) [2, 3]. For paradigmatic stabilizer models (like toric or surface codes), $\gamma = \log D$ with D the total quantum dimension (for toric code $D = 2$ so $\gamma = \log 2$ in bits). In stabilizer states, we can compute $S(A)$ efficiently by symplectic linear algebra over $\mathbb{F}_2$ [4]. The included outline of the code implements this and forms the basis for a TEE estimate via the Kitaev–Preskill combination.

Code design implications In surface or toric codes, logical operators are supported on non-contractible curves. Correspondingly, when aregion A grows to span the code in given direction its $S(A)$ reflects the onset of logical support. Checking $S(A)$ across shapes and boundaries helps select patch geometries and surgery partitions that keep long-range structure controlled. During repeated stabilizer measurements the (Clifford) dynamics preserve stabilizer form, but noise and scheduling produce classical (indeed) correlations in detection events. Those correlations are the operational counterpart of the underlying entanglement structure, and are precisely what MWPM-based decoders typically exploit, [5, 6] I think. Our TEE and MI analysis offers a lightweight diagnostic to tune decoder graphs or window sizes.

Holography and holographic QEC

RT/HRT and entanglement geometry In AdS/CFT, the Ryu–Takayanagi (RT) proposal [1] and its covariant generalization relate boundary entanglement entropy to minimal (extremal) bulk surfaces. I personally have worked with this conjecture through my whole academic history, see our papers on the topic [7, 8, 9, 10] reviews [11, 12]. This geometrization of entropy inspired viewing bulk$\leftrightarrow$boundary duality through the lens of QEC: bulk operators are redundantly represented on boundary regions (complementary recovery), naturally phrased in operator-algebra QEC [13].

Holographic quantum codes Tensor-network toy models (like HaPPY) realize bulk-to-boundary isometries with QEC properties [14, 15]. Recent work continues to connect stabilizer code structure with holographic bulk–boundary features, like for example a holographic view of *topological stabilizer codes* [16], software tools for analyzing holographic codes [17], and studies of holographic (zero-rate) codes and thresholds [18]. These provide modern references tying TEE/topological order intuition to holographic QEC toy models, and there papers are our starting point and some of the main references going into the project.

How it ties to our pipeline Our state-level TEE module mirrors the constant-term diagnostics of topological order that also underlie many holographic code constructions. The advanced repo includes a tiny AdS_3 RT script showing $S \sim \frac{c}{3}\log(\ell/\epsilon)$ scaling for a boundary interval (the correct and expected result, that is). Note that it does not include the fully runnable code but instead, provides a visual contrast between geometric (holographic) and lattice (stabilizer) sources of constant terms and scaling, useful for talks bridging QEC and holography.

Related work, literature

- **Holographic codes and tools:** A software tool for analyzing holographic quantum codes (2024) [17]; thresholds and hashing-bound comparisons for holographic (zero-rate) codes (2024) [18].
- **Stabilizer/holography interface:** A holographic view of topological stabilizer codes (2023) [16].
- **Operator-algebra QEC (modern stabilizer framing):** Stabilizer formalism for OAQEC [13, 19].
- **Decoding surveys/updates:** Overviews of surface-code decoding and MWPM context (see [6], for example).

How to run the examples

```
# TEE (state-level)
qec ee tee --L 5 --kp_square "1,1,2 2,2,2 1,3,2"

# Stim -> shots/DEM -> MI -> MWPM decode
qec stim build  --rounds 50 --p 2e-3 --out circuit.stim
qec stim sample --in circuit.stim --shots 1e5 --shots_out shots.npy --dem_out model.dem
qec stim mi     --shots shots.npy --pairs 5000 --out mi.parquet
qec decode basic --dem model.dem --shots shots.npy
```

Takeaway

Entanglement-guided design + correlation-aware decoding connects information-theoretic structure (TEE, stabilizer entanglement) to engineering metrics (logical error rate, decoder performance). The paired holographic perspective offers complementary intuition for constant terms and redundancy.

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