Ordinal and Multinomial Classification of Graduate School Interest

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0.1 Introduction

This project analyzes the interest of undergraduate students in applying to graduate school. The response variable **apply** has three ordered categories:

- *unlikely*
- *somewhat likely*
- *very likely*

We study whether this interest is associated with:

- **public**: type of university (public or private)
- **pared**: whether at least one parent completed graduate studies
- **gpa**: current grade point average

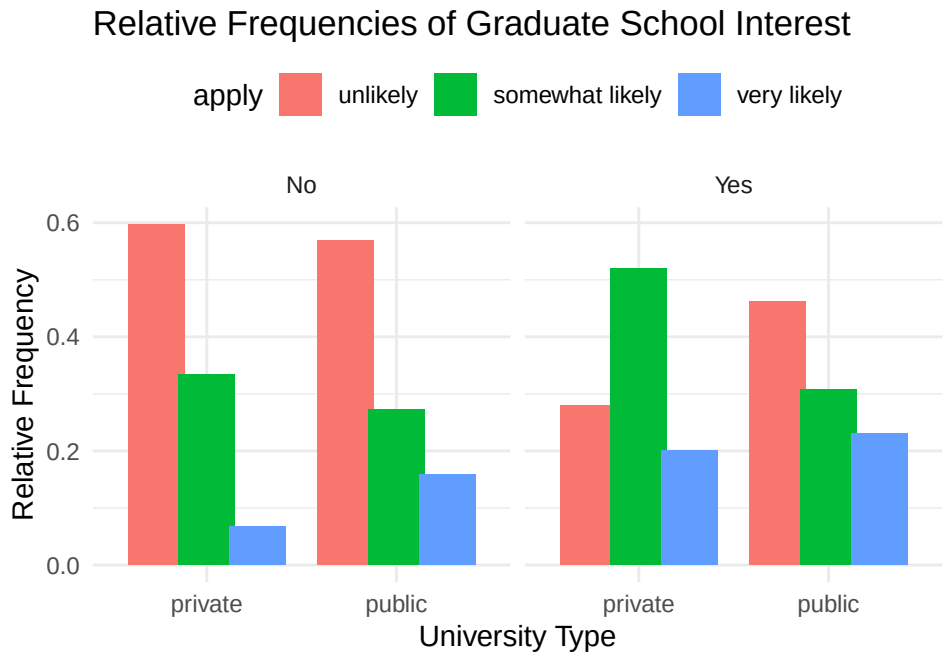
Both **multinomial** and **ordinal** generalized linear models are considered.

0.2 Data

The dataset contains observations on undergraduate students, including academic performance, family background, and institutional characteristics.

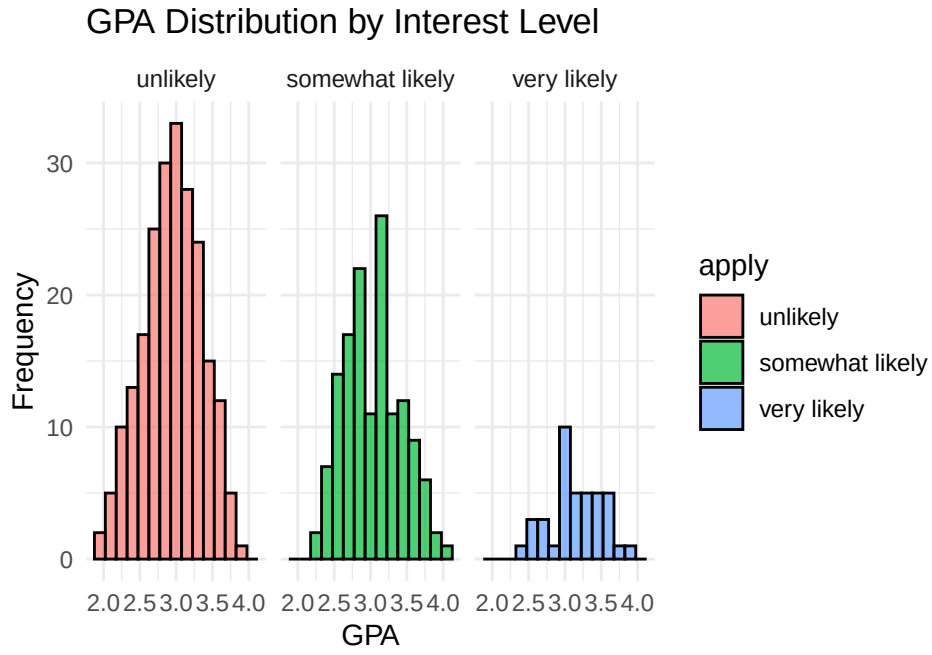
0.3 Exploratory Analysis

0.3.1 Relative Frequencies for Categorical Covariates



Interpretation. Interest levels vary across combinations of parental education and university type. Students whose parents completed graduate studies tend to show higher interest, particularly among private university students.

0.3.2 GPA Distribution by Interest Level



Interpretation. GPA distributions overlap across interest levels, although students with stronger interest exhibit a slight shift toward higher GPA values.

0.4 Multinomial Logistic Regression

0.4.1 Model Specification

0.4.2 Likelihood Ratio Tests

```
## Analysis of Deviance Table
##
## Model 1: apply ~ 1
## Model 2: apply ~ pared * public * gpa
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1       798     741.21
## 2       784     701.16 14    40.04 0.0002514 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Analysis of Deviance Table
##
## Model 1: apply ~ pared + public + gpa
## Model 2: apply ~ pared * public * gpa
```

```
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1       792     713.99
## 2       784     701.16  8   12.829   0.1179
```

Interpretation. The null model is rejected in favor of the full model. However, there is no evidence that interaction terms significantly improve model fit.

0.4.3 Model Selection via AIC

```
##     Model      AIC
## 1    Null 745.2053
## 2 Reduced 729.9940
## 3    Full 733.1648
```

The reduced multinomial model achieves the lowest AIC and is selected.

0.5 Ordinal Logistic Regression

Given the natural ordering of the response variable, cumulative logit models are considered.

0.5.1 Likelihood Ratio Tests

```
## Analysis of Deviance Table
##
## Model 1: apply ~ 1
## Model 2: apply ~ pared + public + gpa
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1       798     741.21
## 2       795     717.02  3    24.18 2.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Analysis of Deviance Table
##
## Model 1: apply ~ 1
## Model 2: apply ~ pared + public + gpa
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1       798     741.21
## 2       792     713.01  6   28.194 8.637e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Analysis of Deviance Table
##
## Model 1: apply ~ pared + public + gpa
## Model 2: apply ~ pared + public + gpa
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1       795     717.02
## 2       792     713.01  3   4.0138     0.26
```

Interpretation. There is no statistical evidence against the proportional odds assumption.

0.5.2 AIC Comparison

```
##                Model      AIC
## 1      Ordinal Null 745.2053
## 2 Proportional Odds 727.0249
## 3  Non-Proportional 729.0111
```

The **proportional odds model** is preferred.

0.6 Final Model Comparison

```
##                          Model      AIC
## 1       Multinomial (Reduced) 729.9940
## 2 Ordinal (Proportional Odds) 727.0249
```

The ordinal proportional odds model provides the best trade-off between fit and parsimony.

0.7 Model Interpretation

The final cumulative logit model is given by

$$\log\left(\frac{\mathbb{P}(Y \leq j)}{1 - \mathbb{P}(Y \leq j)}\right) = \beta_0^{(j)} + \beta_1 \text{pared} + \beta_2 \text{public} + \beta_3 \text{gpa}, \quad j = 1, 2$$

Higher GPA and having at least one parent with graduate education increase the probability of higher interest in graduate studies.