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A Model for Predicting Average Fire Engine Travel Times

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We propose, motivate, and test a model for predicting *ET*, the expected fire engine travel time in a region, given the region's area, *A*; the number of fire engines stationed there, *n*; the alarm rate, λ; and the expected total service time per alarm, *ES*. The model is $ET=\alpha+\beta[A/(n-\lambda ES)]^{\gamma}$. Estimates of the values of parameters α, β, and γ are given for New York City, where the model has been validated, and used in analyzing fire engine deployment problems. Recent changes in the number and location of fire engines in New York City were based partially on this analysis.

WE PROPOSE and test a simple model for predicting expected fire engine travel times. The model states that, in a given region of a city, the expected travel time of the closest responding fire engine, ET, is given by

$$ET=\alpha+\beta[A/(n-\lambda ES)]^{\gamma}, \tag{1}$$

where A is the physical area of the region, n is the number of fire engines stationed there, λ is the expected number of alarms received per hour, and ES is the expected total service time (in hours) of all the fire engines that respond to and work at an alarm. α, β, and γ are parameters whose values depend on physical characteristics of the region. Field measurements and simulation experiments have validated the model for New York City and have shown that, in most regions of the city, the model is approximately

$$ET=2.2[A/(n-0.5\lambda)]^{0.3}. \tag{2}$$

The model has been used as a rule of thumb in estimating how average travel times change with increases or reductions in fire engine allocations, increases in alarm rate, etc. It has been employed in studies that have led to fire engine deployment changes in New York City resulting in improved fire-fighting effectiveness.[3]

Equation (1) is based upon two earlier models—one relating expected response distances to the square root of the area served per available company,[4] and the other relating expected travel time to response distance.[5,6] Both models are motivated by simple physical reasoning, are quite robust, and have been tested empirically under a wide variety of conditions. Although most of our testing has been in New York City, the robustness of the model and fragmentary data from other cities make it appear likely that the model is valid in many other places.

Several caveats are in order with regard to applications of the model to real problems. First, the model is only an approximation; and, while it is certainly useful in narrowing the range of alternatives considered and suggesting possibly desirable policies, its results should be double-checked and corroborated by more detailed analyses before any concrete actions are taken. Second, average response time is only one of many possible criteria of fire-fighting effectiveness. Maximum response times, the probability of running short of fire engines, and fire engine workloads are among other measures to be considered.

We have been using the term "fire engine" somewhat imprecisely. Actually, there are two main types of equipment used by fire departments. *Engines* are trucks equipped with pumps and hoses and are manned by a team whose main function is to extinguish fires by putting water on them. *Ladders* are trucks equipped with ladders and breaking and entering and rescue equipment. They are manned by a team whose main functions are to gain access to the fire premises to enable the enginemen to extinguish the fire, to ventilate the premises so that hot gases escape, and to make rescues. Engine and ladder functions are complementary. Both are dispatched to fires, but according to different rules and from different locations. Accordingly, in Section 3 we will at times consider engine and ladder operations separately.

1. BACKGROUND: THE RESPONSE DISTANCE AND TRAVEL TIME MODELS

It is hypothesized[4] that ED, the expected distance traveled by the closest responding fire engine to a typical alarm, is given by

$$ED = k[A/(n-\lambda ES)]^{1/2}. \tag{3}$$

Since λES is approximately the expected number of busy fire engines, the denominator $n-\lambda ES$ is approximately the expected number of available engines and (3) states that expected travel distance decreases with the square root of the number of available engines. This equation was motivated by several mathematical models and was tested extensively, but the main motivation is essentially a dimensional argument: area = (length)^2 and area served per engine decreases linearly with the number of engines added.

It is further hypothesized[6] that expected travel time, ET, given D, the distance traveled, obeys a simple function whose motivation follows: suppose that for short runs the fire engine never reaches a full cruising velocity, but rather increases its speed for the first half of the trip as it accelerates, gets onto main thoroughfares, etc., and then decelerates for the last half of the trip as the process reverses. For longer runs, we hypothesize that there is a similar initial 'acceleration' phase, but that then the engine runs at, or near, its cruising speed for some distance, finally 'decelerating' as it nears the fire scene.

Let a = acceleration, d_c = distance required to achieve cruising velocity, and v_c = cruising velocity. Then, using basic physical relations and assuming a constant acceleration and deceleration, a, during the initial and final phase, and a constant cruising velocity, v_c, during the middle phase, one obtains

$$E[T|D]=\begin{cases}2(D/a)^{1/2} & \text{if} \quad D\leqq 2d_c,\\ v_c/a+D/v_c & \text{if} \quad D>2d_c.\end{cases} \tag{4}$$

The model has been extensively tested and validated for New York City. Details may be found in reference 5.

2. COMBINING THE TRAVEL TIME AND RESPONSE DISTANCE MODELS

Now we combine the travel time model with the response distance model. By so doing, we relate expected travel time in a region to a few easily measured parameters. To motivate the approximation and the supporting data analysis that follows, we begin with some mathematical formalities.

Consider a particular region of the city. Let $T(x)$ denote the conditional expected travel time when the response distance is x, and $F_D(x)$ denote the cumulative probability distribution of the response distance. Then ET, the unconditional expected travel time (to an alarm an arbitrary distance away), is $ET=\int_0^\infty T(x)\,dF_D(x)$. Assuming the validity of the travel time model, we obtain

$$ET=\int_0^{2d_c} 2(x/a)^{1/2}dF_D(x)+\int_{2d_c}^{\infty} (v_c/a+x/v_c)\; dF_D(x). \tag{5}$$

Recognizing that both segments of the $T(x)$ function are concave, we have, by Jensen's inequality, that

$$ET\leqq\begin{cases}2(ED/a)^{1/2}, & \text{if} \quad ED\leqq 2d_c,\\ v_c/a+ED/v_c, & \text{if} \quad ED>2d_c.\end{cases} \tag{6}$$

Some limited empirical data suggested that this upper bound on ET might be tight. So that we might use (6) to approximate ET. In addition, we replace ED by (3) and obtain the approximation

$$ET\cong\begin{cases}c_1[A/(n-\lambda/ES)]^{1/4}, & \text{if} \quad ED \text{ is 'small,'} \quad (7a)\\ c_2+c_3[A/(n-\lambda/ES)]^{1/2}, & \text{if} \quad ED \text{ is 'large.'} \quad (7b)\end{cases}$$

From (7) we are led to hypothesize the general form for expected travel time given by equation (1). [We note that approximations (1) and (7) were derived from two inequalities of opposite direction.]

3. VALIDATING THE MODEL

It would be very difficult and possibly very dangerous to carry out a real-world experiment to test these models. Fire departments would be reluctant, to say the least, to change significantly the number of engines assigned to a region so that analysts could collect the appropriate data. Reducing the number of engines in a region could entail increased risk of loss of life and property, while increases would be uneconomical or just impossible. So, in order to test the validity of these models, we ran a series of simulations similar to those discussed in reference 4. We used the same simulation model of fire-fighting operations in the Bronx that has been documented in references 1 and 2.

Simulations were done at different alarm rates and with different numbers of companies assigned to one region of the city, the borough of the Bronx. Each simulation was run for about 3,000 alarms, and data were collected separately for engines and ladders and for two parts of the borough—north Bronx and south Bronx. In this series of simulations, the travel times to alarms were generated as follows:

1. The response distance was calculated using an empirically validated function whose inputs are the coordinates of the house of the responding engine and of the alarm, and the orientation of the street grid.

2. The travel time was calculated by applying the piecewise square root-linear travel time vs. response distance function (4) with empirically determined parameter values for the Bronx.

The simulation outputs used here are average travel times, denoted by $\bar{T}$, and average number of available units, denoted by $\bar{N}$. These data were used to fit the following models by linear or nonlinear least squares

$$\bar{T}=\alpha+\beta[A/\bar{N}]^{\gamma}, \tag{8}$$

$$\bar{T}=\alpha+\beta[A/\bar{N}]^{0.50}, \tag{9}$$

$$\bar{T}=\alpha+\beta[A/\bar{N}]^{0.25}, \tag{10}$$

$$\bar{T}=\beta[A/\bar{N}]^{\gamma}, \tag{11}$$

$$\bar{T}=\beta[A/\bar{N}]^{0.50}, \tag{12}$$

$$\bar{T}=\beta[A/\bar{N}]^{0.25}. \tag{13}$$

Analyses were performed where $\bar{T}$ was the average travel times of closest ladders and $\bar{N}$ was the average number of ladders available for the follow-

TABLE I

Results of Regressions of Simulated Travel Times vs. Availability for Ladders

Models	Set 1: South Bronx				Set 2: North Bronx				Set 3: All Bronx				Set 4: North & South Bronx (Set 1 ∪ Set 2)			
	α	β	γ	r^2	α	β	γ	r^2	α	β	γ	r^2	α	β	γ	r^2
(8) $\overline{T}=\alpha+\beta(A/\bar{N})^{\gamma}$	0.33	1.85	0.36	0.96	−1.91	4.00	0.19	0.87	1.30	0.69	0.69	0.96	−1.29	3.45	0.20	0.94
(9) $\overline{T}=\alpha+\beta(A/\bar{N})^{0.50}$	0.95	1.22	0.50	0.96	1.37	0.93	0.50	0.85	0.81	1.15	0.50	0.96	1.18	1.03	0.50	0.92
(10) $\overline{T}=\alpha+\beta(A/\bar{N})^{0.25}$	−0.67	2.85	0.25	0.96	−0.52	2.68	0.25	0.87	−1.06	2.96	0.25	0.96	−0.50	2.68	0.25	0.94
(11) $\overline{T}=\beta(A/\bar{N})^{\gamma}$	—	2.17	0.32	0.96	—	2.19	0.29	0.89	—	1.92	0.35	0.95	—	2.18	0.29	0.94
(12) $\overline{T}=\beta(A/\bar{N})^{0.50}$	—	1.92	0.50	0.62	—	1.60	0.50	0.37	—	1.63	0.50	0.78	—	1.69	0.50	0.49
(13) $\overline{T}=\beta(A/\bar{N})^{0.25}$	—	2.25	0.25	0.92	—	2.30	0.25	0.85	—	2.12	0.25	0.87	—	2.28	0.25	0.92

ing data sets:

Set 1: The south Bronx. This set consists of 12 (pairs of) observations ($\bar{T}$, $\bar{N}$) for closest ladders from the south Bronx region where each observation is the average over one entire simulation.

Set 2: The north Bronx. This set is as above except that the observations are from the north Bronx region.

Set 3: The entire Bronx. This set is as above except that the observations are for the entire Bronx.

Set 4: A concatenation of data sets 1 and 2: This set consists of the 24 observations comprising sets 1 and 2. Note that it is different from set 3.

No engine data are included in the above nor were any engine data used

TABLE II

CONFIDENCE INTERVALS FOR PARAMETERS OF MODEL 11

95% CONFIDENCE INTERVALS ON β AND γ FOR MODEL $\bar{T}=\beta(A/\bar{N})^{\gamma}$

Data set	Interval on β			Interval on γ		
	L_β[a]	$\hat{\beta}$	U_β	L_γ	$\hat{\gamma}$	U_γ
1	2.127	2.173	2.219	0.2908	0.3158	0.3393
2	2.112	2.187	2.262	0.2639	0.2874	0.3090
3	1.871	1.918	1.966	0.3290	0.3512	0.3728
4	2.142	2.184	2.225	0.2746	0.2923	0.3093

[a] The L_β and U_β columns contain observations on random variables L_β and U_β having the property that $P\{(L_\beta \leqq \beta) \cap (U_\beta > \beta)\} = 0.95$.

in model fitting. Instead, engine data were held in reserve and used subsequently to test the models fitted to the ladder data.

Table I summarizes the results of the regressions. It contains the parameter estimates and proportion of original variance explained (r^2) for each model using each of the four data sets. Inspection of the tabled results will disclose that

(a) Model (12), $\bar{T}=\beta(A/\bar{N})^{0.50}$, is distinctly inferior to the others as measured by the proportion of the original variance explained. This result holds for each data set.

(b) The other models are essentially equivalent in the proportion of variance explained, but notice that models (8), (9), and (10)—all of which have constant terms—have only negligibly higher r^2 values. Notice also that the estimated parameter values vary quite a bit across the different data sets, even though the fits are good. On the basis of this fact, we eliminate them from further consideration.

(c) Models (11) and (13) are the remaining contenders. Letting γ be a free parameter results in values near to, but different from 0.25, the hypothesized value. The differences are statistically, but perhaps not operationally, significant. [See Table II, which gives approximate 95 percent confidence intervals on the parameters of model (11).]

Figures 1, 2, and 3 show models (11) and (13) fitted to data sets 1 through 3. The fits of each are good, and it is interesting and significant to note that the resulting curves of $\bar{T}$ vs. $\bar{N}$, although different, are quite

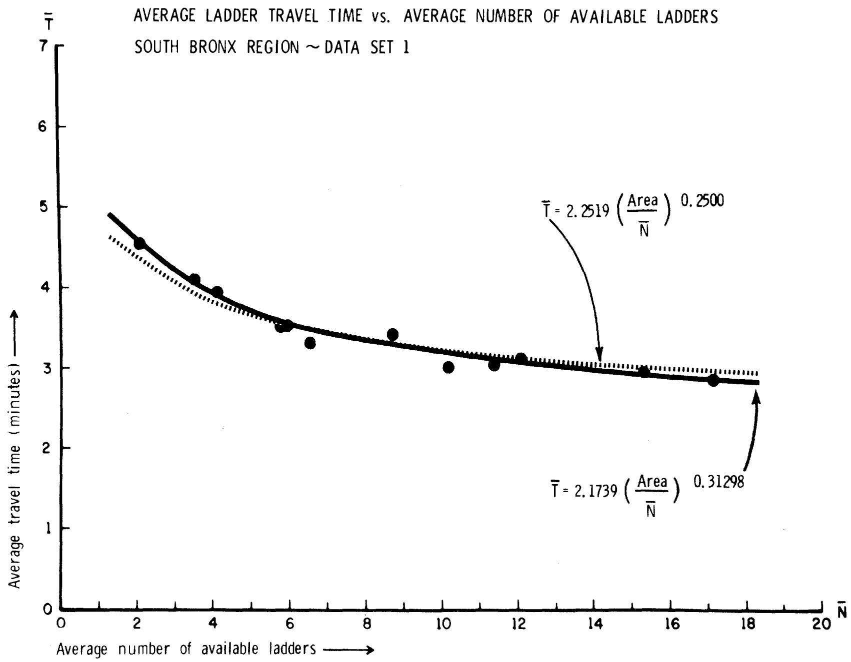

Figure 1

close. In addition, Fig. 4 shows models (11) and (13) fitted to data set 4. (The curves appear reversed from the convex form of Figs. 1 through 3 because, since two data sets with different areas were combined, we used A/N as the abscissa instead of N.) Again, the difference between the curves is very slight. Finally, Fig. 5 shows the curves (11) and (13) fitted from ladder data plotted with travel time data for engines. The engine data used come from none of the aforementioned data sets for closest ladders. Instead they are 12 observations for closest engines generated by concatenating data from the north and south Bronx at three different simulated alarm rates. These data points were not used to fit the functions, yet the correspondence is quite good.

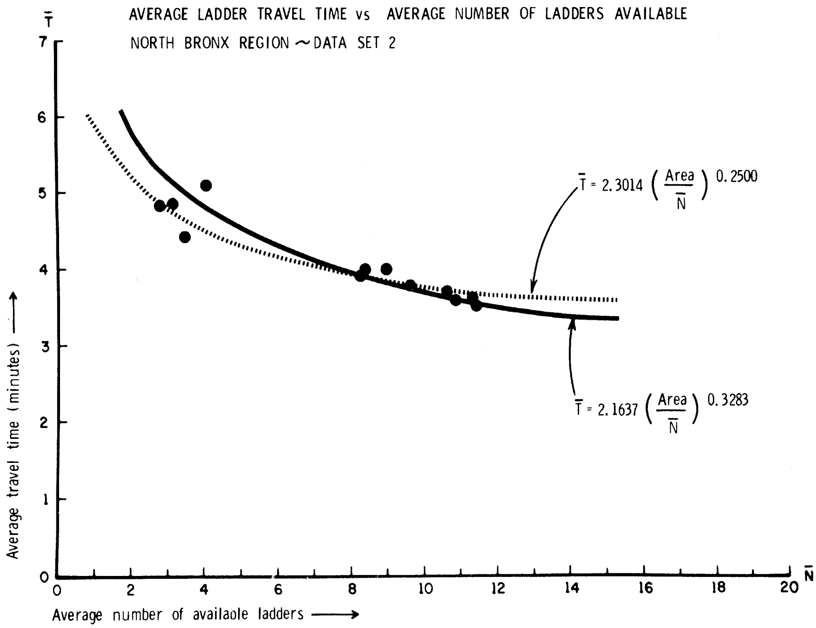

Figure 2

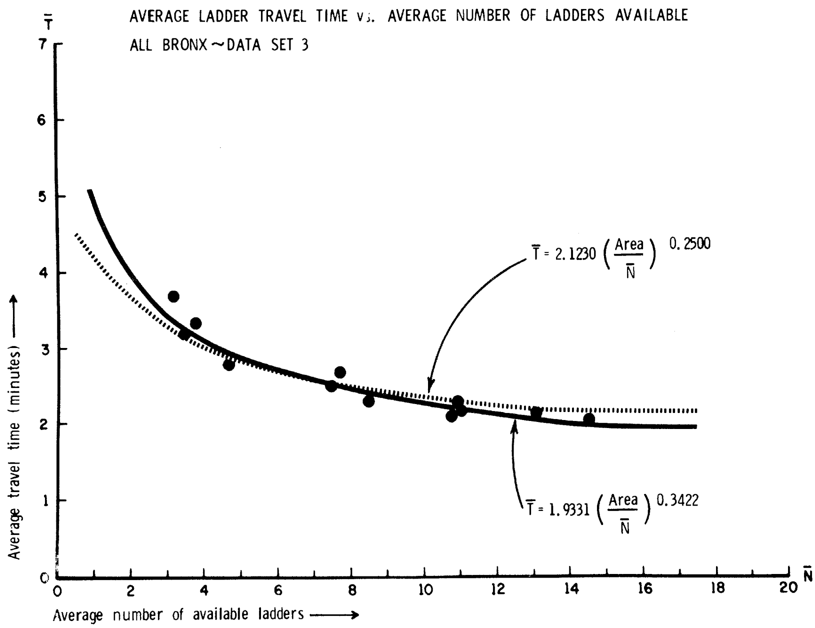

Figure 3

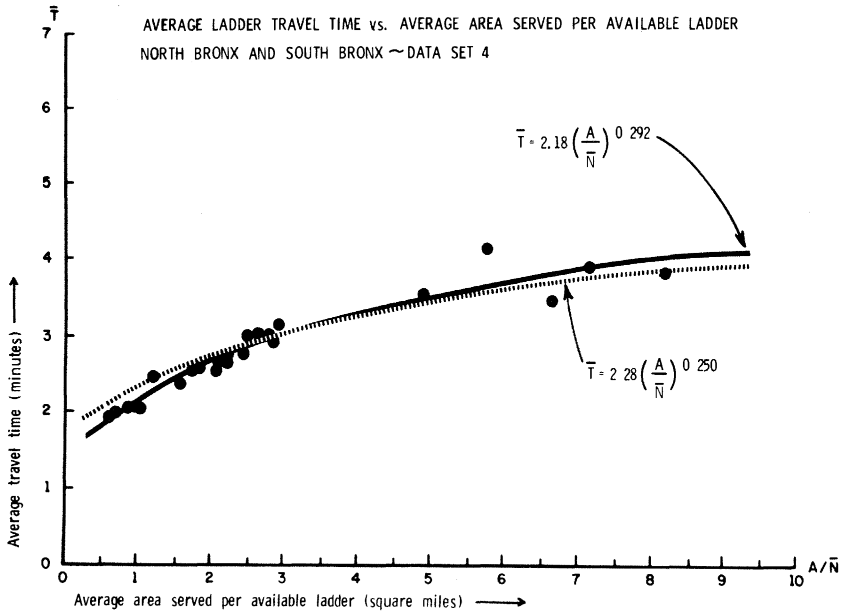

Figure 4

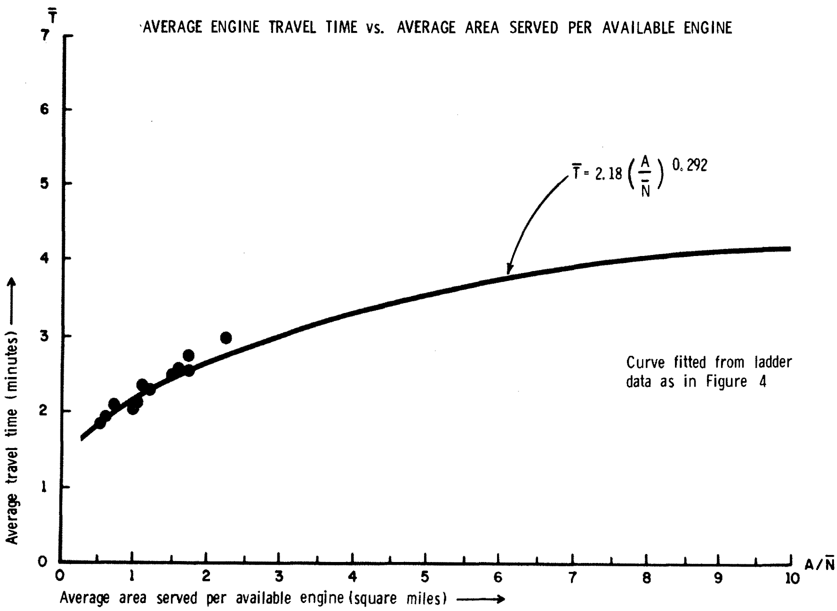

Figure 5

4. CONCLUSIONS

The results presented in the previous section show that the postulated power models give good estimates of simulated average fire engine travel times for the Bronx, New York. In the Bronx, travel distance is short—about one-half mile on the average—and so, as suggested by equation (7a), we found that model (13) gave a good fit. Still better estimates were given by model (11) with a parameter value of β of about 0.3. This result is not surprising since equation (7a) is a lower bound on expected travel time, and a better approximation will be obtained with coefficient greater than 0.25.

Because several of the components of the power model have been validated in other parts of New York City, we are confident that it gives good estimates of average travel times throughout the city, and that model (11) applies in regions where, like the Bronx, average travel distances are short. Because of the robustness of the model and partial validation of its components in other cities, we have little reason to doubt its wider applicability. Of course, in those regions where average travel distances are long, we expect that model (9) would give the best estimates.

Finally, we make some remarks about estimating the parameters of the model. The results of Section 3 show that there was little difference between models (11) and (13), so a precise estimate of the exponent γ is not crucial, particularly when using the model as a rule of thumb. However, 0.3 seems a good choice. The coefficient β can be derived from (3) and (4) and is equal to $(k/a)^{1/2}$. Both k and a can be estimated for any region of interest from simple field experiments as discussed in references 4 and 5. Actually, in our experience, the values of k and a have been quite invariant across several regions and cities; and one cannot go far wrong using the value of β estimated in New York, namely $\beta \approx 2.2$.

In cities or regions where average response distances are longer, (7b) can be expected to apply, that is $ET \approx c_2 + c_3[A/(n-\lambda/ES)]^{1/2}$. The values of c_2 and c_3 may be estimated by relating them to the parameters of the two component models. First, c_2 is v_c/a and c_3 is k/v_c. As mentioned above, our experience in several cities and in several regions within New York City indicates that the estimates derived in New York are quite robust. In the absence of better data, they can be used to give first-cut results for other cities. The numerical values obtained in New York are that $c_2 = 1.35$ minutes, and $c_3 = 0.76$ minutes/mile.

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