Analysis of time interaction games and agent-based dynamic models of evolutionary games

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Abstract

As an emerging field based on the development of traditional game theory, evolutionary game theory improves on the mathematical theory in game theory by combining it with dynamical systems and applying it to explain animal interactions in the field of biology. A effective modeling technique used in the study of evolutionary game theory is agent-based modeling, by which the real world can be clearly described and explained, and even future behavior can be predicted.

Key words: game theory; evolution game theory; two-strategy game; agent-based model;

Contents

1 Introduction

Evolutionary game theory simulates and analyses the outcomes of interactions between individuals in a population by mathematically modeling the population in its natural environment. The mathematical models created by evolutionary game theory, which in general combines dynamical systems, are more closely matched to real-world behavioral characteristics than traditional game theory. As traditional game theory lacks consideration of dynamical systems, evolutionary game theory can be seen as a complement to traditional game theory in dynamics.

The structure of this thesis is divided into four parts:

It begins with a description of the development, definitions and types of game theory and talks about the historical development of game theory and evolutionary game theory. The primary reference is to the explorations of von Neumann and Josh Forbes Nash in their investigations of game theory, which made it possible to translate game theory into a mathematical theory of systems and made game theory a completely new field.[1] [2] An overview is included of the development of evolutionary game theory in addition to game theory.

The second part analyzes the hawk-dove game problem concretely and explains the analysis method, pure strategy and mixed strategy in the game problem. Simultaneously, new constraints of time interaction are added to the hawk-dove game to analyze the effect of different of interaction times on the outcome of the Hawk strategy.

The third part creates Agent-Based models using Netlogo to simulate the interaction of different individuals choosing strategies in real-world species, and analyses the changes in the models by changing the ecological situation or the parameters of the individuals.

Finally, a summary is given of the work in this thesis on evolutionary game theory, and we present some ideas and notable issues of evolutionary game theory.

The specific objective of this research is to explore the development of evolutionary game theory and the issues that remain to be addressed in the future by linking agent-based models to evolutionary game theory.

2 Background

2.1 Game Theory

2.1.1 The development of Game Theory

In general, game theory refers to the method of two players considering each other's strategies and updating their confrontation strategies in an equal competition to gain victory. Original game theory refers to strictly competitive games, that is, zero-sum games, where the gain obtained by the player means a loss for the other. However, this mathematical method is valid for games like chess, gambling and other competitive behaviors, but the results are not ideal when it comes to more complex mathematical models in biology.

Therefore, in the early 20th century, the field of game theory was relatively isolated in mathematics and economics, and researchers only had a basic understanding of game theory, without a detailed framework and necessary concepts. However, research continues to be active and the approach of game theory is still developing. Furthermore game theory continues to spread into many other fields and disciplines.

With the collaborative efforts of mathematicians and economists, in 1944, John von Neumann and Oskar Morgenstern published a book called "Theory of Games and Economic Behavior", which explained the principles and applications of game theory in mathematics. They use strategic form and matrix form to represent the game process, propose the concept of stable sets solutions, and give the expression method of game theory, which marks the initial formation of modern game theory. In this book, they discuss zero-sum games in detail and consider some new areas of research in game theory. They cleverly applied game theory to economics and, because of the broad nature of economics, their research provided a tremendous contribution to the subsequent development of game theory.[1]

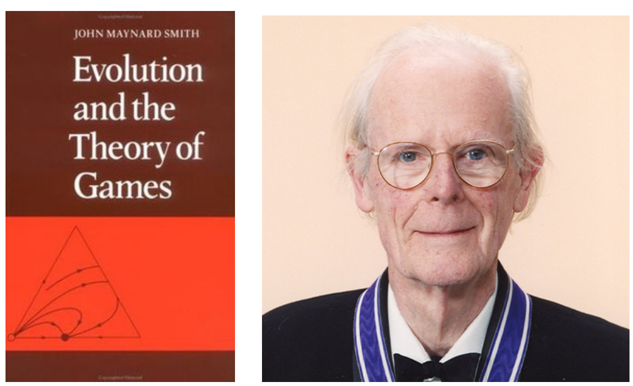
In the Theory of Game And Economic Behaviour, they start with a concrete analysis of some examples of games, then give a probabilistic description of the game and develop a simple two-player zero-sum game theory, on which they proceed to the theory of conditional zero-sum multi-player games, by which they theoretically solve the general game problem. At the same time, they argued that game theory can give a similar explanation of economic behavior in the market, by stating what the rational behavior of traders in the market is through the solution of a game, and that finding the solution to the game is the same as finding the optimal strategy for the traders.[1]

One of von Neumann's chief innovations was the assumption and proof of the Minimax theorem. He pointed out that nothing in the field of game theory was worth investigating until the Minimax theorem was proved. It is because basic concepts in game theory, such as pure strategy and strategy expressions, are based on the Minimax Theorem. If a participant in a game chooses a strategy based on the Minimax Theorem, the participant first considers the smallest payoff for each strategy, and then chooses the strategy with the largest payoff among the available payoffs. For a two-player game, each participant will have one or more mixed strategies such that they will have the same payoff when they adopt the strategy so that these mixed strategies are optimal for both participants. The magical thing about these strategies is that no player can deviate from them to obtain a higher payoff. In other words, a participant can choose a strategy ahead of his opponent and will not reduce his payoff due to his opponent's choice of strategy.[3]

Figure 2.1: On the left the book is Theory of Game And Economic Behaviour by von Neumann and Morgenstern. In the center a picture of von Neumann and on the right a picture of Oskar Morgenstern. See [4].

Rather than von Neumann and Oskar Morgenstern focusing on cooperative games, John Forbes Nash chose to focus his research on non-cooperative games. Building on their work, Nash improved the concept of game theory and proposed that cooperative games are a special case of non-cooperative games in his 1951 paper "Non-cooperative Games". In this article, he used a mathematical method called Nash equilibrium to analyze the balance of non-cooperative games in game theory. Nash equilibrium refers to a non-cooperative game with two or more players, assuming that the strategy adopted by each player is the optimal strategy among all strategies, when other players do not change their strategies, in order to maintain their payoffs, a conceptual solution that will not change their strategy. The proposition of a Nash equilibrium provides a reasonable explanation for a lose-lose outcome that occurs in game theory. At the same time, he also used the fixed-point theorem to prove the existence of equilibrium points, laying a vital foundation for the development of game theory.[2]

Later John Maynard Smith considered the connection between game theory and biology. He introduced the concept of evolutionary game theory in the 1970s by using game theory to analyze the strategic choices of species in nature, and the associated analysis of changes in population dynamics. Evolutionary game theory suggests that decision-makers achieve maximum payoff by constantly trying different strategies, which is similar to the principles of biological evolution seen in nature. It is worth noting that this theory can also provide an effective model of natural selection in Darwinism.[5]



(a) (b)

Figure 2.2: Maynard-Smith's groundbreaking book (a) and his picture (b). See [6].

After half a century of vigorous development, game theory is becoming popular and is being used in economics, management, sociology and may other disciplines. Through game theory, many consequences in the real world can be adequately explained. For example, many businesses have an agreement to monopolize the market by setting a reasonable price and not allowing them to reduce the price of their products, but there are always some businesses who secretly reduce the price of their products in order to sell them faster. This situation is exactly like the famous prisoner's dilemma model in game theory.

In the prisoner's dilemma, suppose two criminals are caught and if they both remain silent they are likely to receive the lightest possible punishment, if one confesses and the other does not, then the one who confesses gets a reduced punishment and the one who does not confess gets an increased punishment. In the absence of prior cooperation, often both criminals will choose to confess. Although the best option is to both keep silent, they will still choose to confess and to betray each other to reduce their punishment.

This result gives a warning to the strategy of price fixing. Even if there is no price reduction, each merchant will get the maximum benefit. Unless there is a great deal of experience in cooperation, merchants will still choose the optimal strategy for themselves, rather than the overall (global) optimal strategy.

As a branch of applied mathematics, game theory plays an important role in various disciplines such as economics, computer science, and political science.[7]

2.1.2 Definition of Game Theory

A simple model can be analyzed to give a better idea of what game theory is: assuming that there is a game of rock-paper-scissors and the winner can get a huge bonus, the players who participate in the game will consider the strategies that other players may choose and choose the most beneficial one in order to win the bonus. This method of considering whether there is an optimal strategy for individuals in different game processes and how to find that strategy mathematically is called game theory.

Figure 2.3: Rock-paper-scissors game refers to simulating three items of rock-paper-scissors by changing gestures. see [8]

In a game, there are generally three different elements:

- There are two or more participants in the game.
- There is a strategy set, individuals can choose different strategies in the strategy set to play the game.
- Payoff function, each individual's choice of strategy will cause the individual to get each a particular payoff. This payoff is affected by the choice of other individuals in the interaction, and the payoff of different participants in the same group of strategies may be different.

In a rock-paper-scissors game, the participants can be two people or multiple people. In addition, they can choose three different strategies, rock, scissors, and paper, and they do not know each other's chosen strategies. Their payoff matrices can be derived from the three decisions: rock wins against scissors, paper wins against the rock, and scissors wins against the paper.

Typically, the payoff matrix is used to express the payoff under different strategies:

$$\begin{array}{ccc} & & \text{Player } Y \\ & & \begin{array}{cc} A & B \end{array} \\ \text{Player } X & \begin{array}{c} A \\ B \end{array} & \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \end{array}$$

Through the concept of game theory, three elements can be identified and modeled and analyzed from the game of rock-paper-scissors. Suppose there are two players, player x and player y. In the payoff matrix, player X and player Y have three strategies, the payoff for a victory is 1, the payoff for a tie is 0, and the payoff for a defeat is -1. By observing the return matrix between them, it can be concluded that the probability distribution of their choice of different strategies is (1/3, 1/3, 1/3), which means that players will randomly choose strategies in the game. Likewise, the probability of winning is also (1/3, 1/3, 1/3), which means that the game is set up fairly, that is, everyone has the same probability of winning.

	rock	paper	scissors
rock	0	-1	1
paper	1	0	-1
scissors	-1	1	0

It is worth noting that the elements in the game theory depend on the complexity of the model and can be increased according to the requirements. The types of games are also different by changing the payoff in the game or adding different conditions.

2.2 Types of Game Theory

Games can be categorized in different ways, for example, the order in which different individuals in a game choose their strategies, how well they know the other individuals participating in the game, whether they are fully informed about the payoffs and whether they are rational in their analysis, all result in different game outcomes. In this case, the number of strategies, the payoffs and the components of the game will also change.

2.2.1 Cooperative or Non-cooperative games

One common classification of game theory is that of cooperative and non-cooperative games.

A cooperative game is generally one in which both players in the game increase or at least one player's payoffs increase without the other player's payoffs decreasing, resulting in an overall increase in payoffs, and is primarily used to study how to distribute the payoffs reasonably when cooperating.

There are two conditions for a cooperative game to be achieved: on the one hand, for the collective, the payoffs of cooperation will be higher than the payoffs of each operating individually, and on the other hand, each individual will receive more payoffs when operating as part of the collective. To ensure that the conditions for a cooperative game are achieved, information is exchanged between each individual, that is, information is symmetrical, and the protocols reached need to be enforced.

A non-cooperative game is one in which the players do not reach an enforceable protocol, such as the prisoner's dilemma mentioned earlier. The players only need to consider whether the decision they choose maximizes the payoff, not the overall payoff.

In terms of the game process and strategy choice, the cooperative game is a special case of the non-cooperative game, which reduces the investigation of the game process and mode and focuses on the allocation of resources between individuals and the whole, so this game approach can be used to explore the problem of fair distribution in society or the choice between individuals and the whole. In the case of non-cooperative games, the process of the game, the choice of strategy or other issues related to one's payoffs are explored as such games only consider the individual's payoffs and their choice of strategy. In short, a cooperative game considers the behavioral characteristics of the collective more than a non-cooperative game considers the payoffs of the individuals themselves.

It is worth noting that in reality, not all protocols are enforceable, so non-cooperative games are more common than cooperative games, and therefore their application to practical problems will be different.

2.2.2 Static or Dynamic game

A game in which different players choose strategies at the same time and the payoffs for these players depend on different combinations of strategy is known as a static game. It means that individuals cannot know the strategy chosen by other individuals and thus change their strategy to maximize their payoffs. A case in point is a game of rock-paper-scissors where each participant does not know the choices of the others.

On the contrary, a dynamic game means that the strategies adopted by the individuals participating in the game are in sequence, and the individuals can know the strategy chosen by the strategist to take first, so the individuals who take the game later can compare their strategies to obtain the maximum payoffs. In games such as chess, participants can decide their next strategy by observing the strategies their opponents are taking. The order of the game is therefore also considered to be an important element in modeling dynamic games.

2.2.3 Complete Information or Incomplete Information game

In game theory, the information set refers to the set of possible actions of the participants observing the game. There are different game models for different states of information, which are generally divided into complete information games, incomplete information games, perfect information games and imperfect information games.

A complete information game means all benefits of the game are known to every player, the design of the game, the number of players, the set of strategies, and the payoff function. However, the players only know the probability that their opponents will choose a strategy, without knowing which strategy they will choose.

In an incomplete information game, some situations in the game are not fully understood by the players. For example, players do not know who the opponent is, do not know the payoff function of the opponent or do not fully know the profit function of the opponent.

2.3 Evolutionary Game Theory

Biologists realized that game theory can explain the complex relationship between biological adaptation and biological behavior in nature after traditional game theory was widely accepted and developed.

Fisher found in the 1930s that most males do not mate but many species have roughly equal sex ratios. Through this problem, he realized that the fitness of individuals in a population depends on the ratio of males to females, that when there are more males, females will have higher individual fitness, and when there are more females, males will have higher individual fitness. Fisher's research shows that game theory can explain Darwin's theory of natural selection without other assumptions. Therefore, in order to be more suitable for studying the game of biological populations, evolutionary game theory has carried out some development on the elements of the game.[9]

On the element of players, evolutionary game theory transforms it from rational decision-maker humans into different strategies chosen by the same population in the natural environment. In other words, players are regarded as only carriers of fixed strategies. Furthermore, in a natural environment where resources are assumed to be limited, interactions between organisms such as cooperation and competition can be viewed as different strategic choices for players. At the same time, when they interact, they will gain or consume energy to acquire resources or cooperate to bring enough resources to survive and reproduce, or only compete to survive and reproduce, depending on the payoff functions of their actions.

According to game theory, the payoff function of any one of these individuals depends not only on its own strategy, but also on the strategies of other individuals involved, biological interactions involve two or more decision-makers, so game theory can be used to explain interactions in evolutionary consequences. The payoff function can show that the equilibrium of the ratio can be achieved by natural selection. Evolutionary game theory plays an important role in biology by redefining players, strategies, and payoff function.

The reason why evolutionary game theory is important is that when the strategy changes due to evolution or mutation, the payoff function of adaptation changes, making the optimal strategy change, and a strategy that was not feasible may become optimal in the process of evolution or mutation.

Essentially, evolutionary game theory is a dynamic game with complete information, emphasizing equilibrium in dynamic systems. In repeated games, an equilibrium between biological populations will eventually emerge, perhaps only the player who chooses the optimal strategy survives.

2.3.1 Evolutionarily stable strategy

After 1973, ecologists Maynard Smith and Price proposed evolutionarily Stable Strategies to give the evolutionary game theory a concrete theoretical basis. If the majority of individuals choose this evolutionarily Stable Strategy in nature, then it is unlikely that a small population of mutants will invade the population. In other words, mutants either change their strategy to choose an evolutionarily stable strategy under natural selection or withdraw from the ecosystem and disappear entirely in the process of evolution.

Evolutionarily stable strategies are not the result of rational selection among alternative strategies by animals, but the result of natural selection in the evolutionary process. The choice of this strategy is the most appropriate in the environment in which the animal lives, so it will be in the offspring of reproduction.

In fact, evolutionarily stable strategy is not the most ideal strategy, but a strategy that is more suitable than other strategies in the current situation, it is a dynamic solution that helps us explain the behavioral fitness of species in a specific environment.[10]

One important potential of an evolutionarily stable strategy is that it does not require the assumption that the individuals acting in the game are rational, and it is adaptable to many situations. But the evolutionarily stable strategy does not explain how the population reaches stability, but only shows that once this stable state is reached, the original population is more resistant to the mutant population. That is to say, when the system is in the domain of attraction of a certain equilibrium point, with the evolution of time, the system will tend to this equilibrium point.

However, when the system has multiple domains of attraction, it is difficult to give a reasonable answer to the evolutionarily stable strategy. In addition, evolutionary stabilization strategies need to wait until the effect of one mutational factor on the entire population disappears before another mutational factor appears. When the related mutational factors overlap, it is difficult to explain which factor caused the stability.

Although evolutionarily stable strategies still need to be developed, it is indisputable that their emergence provides further explanations for the biological behavior of populations.

2.3.2 Replicator Dynamics

The replicator equation is an important game dynamics study related to evolutionary game theory. The main function of this equation is to calculate the convergence of Nash equilibrium and the strategy of evolutionary stability.

The main idea of this equation is that when the system is in a stable state, most members of the group adopt a relatively advantageous strategy, gradually forming a stable set that can resist the random influence of other strategies. In addition, When the system leaves the stable state, the original stable set will disappear, but there will be a new stable set from the original stable set to the new stable set. In this new stable set, each strategy has an advantage over the previous one. Therefore, the original policy distribution will gradually approach another policy distribution, thus realizing the system transfer from another equilibrium.[11]

In the replicator equation, individuals who choose different strategies are compared to replicators, different types of replicators account for different proportions in the population, and the interaction of different replicators is like the different strategies used in the game to generate a revenue function, that is, fitness. Replicators reproduce according to the fitness associated with the fitness of others. Replicators with a fitness greater than the average fitness of the population will increase their ratio of the population, and replicators with a fitness less than the average fitness of the population will decrease their ratio of the population.

Evolutionary stable strategies in evolutionary game theory can be calculated by the replicator equation. Consider a game with two strategies called 1 and 2. Let $x_i(t)$ be the number of players playing strategy i and $s_i(t)$ be the proportion of players playing strategy i. The system evolves according to:

$$\dot{x}_1 = x_1 f_1(x_1, x_2) \tag{1}$$

$$\dot{x}_2 = x_2 f_2(x_1, x_2) \tag{2}$$

f_1 and f_2 are given payoff function(fitness) in the game. It could be note that the proportions is:

$$s_i = \frac{x_i}{x_1 + x_2}$$

$$x_i = s_i(x_1 + x_2)$$

Given that $s_1 + s_2 = 1$:

$$\begin{aligned}
\dot{s}_i &= \frac{(x_1 + x_2)\dot{x}_i - x_i(\dot{x}_1 + \dot{x}_2)}{(x_1 + x_2)^2} \\
&= \frac{(s_1 + s_2)(x_1 + x_2)s_i(x_1 + x_2)f_i - s_i(x_1 + x_2)(x_1 + x_2)(s_1 f_1 + s_2 f_2)}{(x_1 + x_2)^2} \\
&= (s_1 + s_2)s_i f_i - s_i(s_1 f_i + s_2 f_2) \\
&= s_i(f_i - \phi)
\end{aligned} \tag{3}$$

Where $\phi = s_1 f_1 + s_2 f_2$ is the average fitness of the population. In the two-player game we assume that $s = s_1$:

$$\begin{aligned}
\dot{s}_i &= s(f_1 - (s f_1 + (1 - s) f_2)) \\
&= s(f_1 - s f_1 - f_2 + s f_2) \\
&= s(1 - s)(f_1 - f_2)
\end{aligned} \tag{4}$$

For example, giving an evolutionary game:

		Player Y	
		A	B
Player X	A	a	b
	B	c	d

In this situation, it could be obtain that the fitness is:

$$f_1 = as_1 + bs_2 = as + b(1 - s)$$

$$f_2 = cs_1 + ds_2 = cs + d(1 - s)$$

Applying replicator dynamic equation to find Nash equilibrium:

$$\dot{s} = s(1 - s)((as + b(1 - s)) - (cs + d(1 - s)))$$

$$= s(1 - s)(s(a - b - c + d) + b - d)$$

3 Two-strategy game with symmetric interaction times

With a basic understanding of game theory, it is known that game theory is related to many factors and this section would like to investigate the relationship between two-player strategy games and interaction time. Through this game, the influence of interaction time in the course of the game will be discussed.

In the first place, create a symmetric matrix game with two strategies e_1 and e_2, π represents payoff in different situations

$$\begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \end{array} & \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \end{array}$$

π_{ij} represents the expect payoff obtained by e_i after encountering e_j, by analyzing this matrix, we can get different payoff under different strategies.

Similarly, create a time matrix to get the time spent interacting under different strategies, τ represents time in different situations

$$\begin{array}{cc} & \begin{array}{cc} e_1 & e_2 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \end{array} & \begin{pmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{pmatrix} \end{array}$$

Assuming that the interaction times are positive and the interaction time matrix is symmetric, the interaction is performed using the given payoff matrix and time matrix. In addition, all individuals can be paired randomly and immediately. The number of pairs is denoted as n_{11}, n_{12} and n_{22}. n_{11} is when both individuals use strategy e_1, n_{22} is when both individuals use strategy e_2, and n_{12} is when both individuals use strategy e_1 and strategy e_2, respectively. It is worth noting that the total number of individuals is constant as $N = 2(n_{11}+n_{12}+n_{22})$.

A pair n_{ij} splits after a Poisson process with parameter τ_{ij}, the number of pairs that disband is n_{ij}/τ_{ij}. The individuals who choose strategy e_1 per unit time are $2n_{11}/\tau_{11} + n_{12}/\tau_{12}$, and the individuals who choose strategy e_2 per unit time are $2n_{22}/\tau_{22}+n_{12}/\tau_{12}$, they will re-form a new pair after disbanding, forming a new pair of individuals The total is $2(n_{11}/\tau_{11} + n_{12}/\tau_{12} + n_{22}/\tau_{22})$. [12]

Besides, Poisson Process is a model for a series of discrete events where the average time between events is known, but the exact timing of events is random.

Therefore, the proportion of individuals who choose the e_1 strategy in unit time to the population is

$$\frac{2n_{11}/\tau_{11} + n_{12}/\tau_{12}}{2(n_{11}/\tau_{11} + n_{12}/\tau_{12} + n_{22}/\tau_{22})}$$

The proportion of individuals who choose the e_2 strategy in unit time to the population is

$$\frac{2n_{22}/\tau_{22} + n_{12}/\tau_{12}}{2(n_{11}/\tau_{11} + n_{12}/\tau_{12} + n_{22}/\tau_{22})}$$

In this case the proportion of newly formed n_{11} pairs among all newly formed pairs will be

$$\frac{n_{11}}{\tau_{11}} = (\frac{2n_{11}/\tau_{11} + n_{12}/\tau_{12}}{2(n_{11}/\tau_{11} + n_{12}/\tau_{12} + n_{22}/\tau_{22})})^2$$

Similarly, considerations for n_{12} and n_{22} pairs lead to the following pair dynamics.

In the process that individuals choose different strategies to interact and split in order to form new interactions, we can obtain that:

$$\frac{dn_{11}}{dt} = -\frac{n_{11}}{\tau_{11}} + \frac{(\frac{2n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}})^2}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})} \tag{5}$$

$$\frac{dn_{12}}{dt} = -\frac{n_{12}}{\tau_{12}} + \frac{2(\frac{2n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}})(\frac{n_{12}}{\tau_{12}} + \frac{2n_{22}}{\tau_{22}})}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})} \tag{6}$$

$$\frac{dn_{22}}{dt} = -\frac{n_{22}}{\tau_{22}} + \frac{(\frac{n_{12}}{\tau_{12}} + \frac{2n_{22}}{\tau_{22}})^2}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})} \tag{7}$$

When individuals split up after an interaction to form a new process, it can be observed that their probabilities are

$$\frac{n_{11}}{\tau_{11}} = \frac{(\frac{2n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}})^2}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})}, \tag{8}$$

$$\frac{n_{12}}{\tau_{12}} = \frac{2(\frac{2n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}})(\frac{n_{12}}{\tau_{12}} + \frac{2n_{22}}{\tau_{22}})}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})}, \tag{9}$$

$$\frac{n_{22}}{\tau_{22}} = \frac{(\frac{n_{12}}{\tau_{12}} + \frac{2n_{22}}{\tau_{22}})^2}{4(\frac{n_{11}}{\tau_{11}} + \frac{n_{12}}{\tau_{12}} + \frac{n_{22}}{\tau_{22}})}. \tag{10}$$

Given that

$$\frac{n_{11}}{\tau_{11}}\frac{n_{22}}{\tau_{22}} = \frac{1}{4}(\frac{n_{12}}{\tau_{12}})^2 \tag{11}$$

It could be seen that their ratios are in Hardy-Weinberg proportions. In the Hardy-Weinberg principle, they express through the formula that the frequency of genes and the type of genes will remain in a stable state after many reproductions of a population in an ideal state.

Suppose AA and aa are a pair of alleles, where A is the dominant allele and a is the recessive allele, and that A occurs with a frequency p and a with a frequency q. If the biological population is large enough and paired randomly, then the gene ratio of the next generation will be:

$$\begin{array}{ccc|c|c|} & & \multicolumn{2}{c}{Y} \\ & & \multicolumn{1}{c}{A} & \multicolumn{1}{c}{a} \\ \cline{3-4} \multirow{2}{*}{X} & A & pp & pq \\ \cline{3-4} & a & pq & qq \\ \cline{3-4} \end{array}$$

It is not difficult to observe that the final gene frequency is $AA : Aa : aa = p^2 : 2pq : q^2$. Therefore, without the influence of other factors, no matter what the initial frequencies p and q are, the ratio will be the same in the second generation, that is, it will reach an equilibrium state.

Therefore, this two-strategy game obeys the Hardy–Weinberg principle, which means population under ideal conditions (unaffected by specific confounding factors such as population migration, mutation, or limited population size), through For multiple generations, the gene frequency will remain constant and in a stable equilibrium state. In other words, in this game, the strategy selected by the individual in unit time tends to be stable.[13]

Due to the overall number of individuals is $N = 2(n_{11} + n_{12} + n_{22})$, in order to calculate the equilibrium we need to know how many individual chose e_1 (or e_2). Set $n_1 = n_{12} + 2n_{11}$ and $n_2 = n_{12} + 2n_{22}$ be numbers of e_1 and e_2, respectively, strategists in the population ($n_1 + n_2 = N$).

If

$$\tau_{12}^2 \neq \tau_{11}\tau_{22}$$

According to Hardy-Weinberg proportions

$$\frac{n_{11}}{\tau_{11}}, \frac{n_{22}}{\tau_{22}} = \frac{1}{4}(\frac{n_{12}}{\tau_{12}})^2$$

we could obtain the overall number of individuals

$$n_{11} = \frac{n_1(\tau_{12}^2 - \tau_{11}\tau_{22}) - \tau_{12}^2\frac{N}{2} + \tau_{12}\sqrt{n_1(n_1 - N)(\tau_{12}^2 - \tau_{11}\tau_{22}) + (\frac{N}{2})^2\tau_{12}^2}}{2(\tau_{12}^2 - \tau_{11}\tau_{22})} \tag{12}$$

$$n_{12} = \frac{\tau_{12}^2\frac{N}{2} - \tau_{12}\sqrt{n_1(n_1 - N)(\tau_{12}^2 - \tau_{11}\tau_{22} + (\frac{N}{2})^2\tau_{12}^2)}}{\tau_{12}^2 - \tau_{11}\tau_{22}} \tag{13}$$

$$n_{22} = \frac{N}{2} - n_{11} - n_{12} \tag{14}$$

If

$$\tau_{12}^2 = \tau_{11}\tau_{22}$$

the equilibrium has unique solution

$$n_{11} = \frac{n_1^2}{2N}, \tag{15}$$

$$n_{12} = \frac{n_1 n_2}{N}, \tag{16}$$

$$n_{22} = \frac{n_2^2}{2N} \tag{17}$$

which means it satisfy the classic Hardy-Weinberg equilibrium

$$p_{11} = p_1^2,$$

$$p_{12} = 2p_1p_2,$$

$$p_{22} = p_{p_2^2}$$

$p_i = n_i/N$ are proportions of the two alleles and $p_{ij} = 2n_{ij}/N$ are phenotype proportions.

If all interaction times are the same (i.e. $\tau_{ij} = \tau$ for all i, j = 1, 2 which corresponds to the implicit assumptions underlying the classic matrix model), the equilibrium is given by random pair formation of all individuals.

3.1 Payoffs, fitness and evolutionary outcome

3.1.1 Approach

Consider fitnesses are calculated as the expected payoff per unit of interaction time. It can be seen from the figure 3.1 that in an individual, assuming that the probability of playing strategy e_1 is p_1, then the probability (e_1e_1) of the individual interacting with another individual playing strategy e_1 is $\frac{2n_{11}}{2n_{11}+n_{12}}$ while the probability (e_1e_2) of another individual playing strategy e_2 to form an interaction is $\frac{n_{12}}{2n_{11}+n_{12}}$.

similarly, the probability of playing strategy e_2 is p_2 and $e_2e_1 = \frac{n_{12}}{2n_{11}+n_{12}}$, $e_2e_2 = \frac{2n_{22}}{2n_{22}+n_{12}}$, so

$$\Pi_1 = \frac{2n_{11}}{2n_{11}+n_{12}}\frac{\pi_{11}}{\tau_{11}} + \frac{n_{12}}{2n_{11}+n_{12}}\frac{\pi_{12}}{\tau_{12}}$$
$$\Pi_2 = \frac{2n_{22}}{2n_{22}+n_{12}}\frac{\pi_{22}}{\tau_{22}} + \frac{n_{12}}{2n_{22}+n_{12}}\frac{\pi_{21}}{\tau_{12}}$$

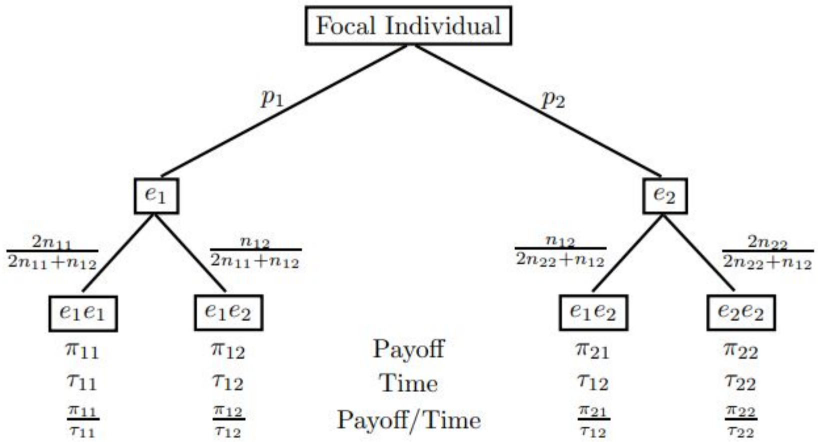


Figure 3.1: payoff, time and payoff/time for each individual in game [12]

3.1.2 Pure strategy

One possibility in this time-constrained game is that it is a pure strategy Nash equilibrium. A simple explanation is that pure strategy means that the player always chooses the only certain strategy, no matter what strategy the opponent chooses. In rock-paper-scissors game, if you ensure that your opponent's chosen strategy will always be scissors, then obviously you will win by choosing rock, in which case you will not change your strategy because you can always win.

In this game of time interaction, one person who does not change his strategy does the same, and the other does not change his strategy.

Suppose all individuals play strategy e_1, then $n_{22} = n_{12} = 0$, $\Pi_1 = \frac{\pi_{11}}{\tau_{11}}$. If all individuals play strategy e_2, then $n_{11} = n_{12} = 0$, $\Pi_2 = \frac{\pi_{22}}{\tau_{22}}$. As the number of n_{11} pairs tends to 0 much faster than 0, the fitness Π_1 when be $\frac{\pi_{12}}{\tau_{12}}$. In Π_2, if $\frac{\pi_{22}}{\tau_{22}} > \frac{\pi_{12}}{\tau_{12}}$, e_2 will be stable (strict NE in game-theoretic) because all individuals are likely to play strategy e_2 eventually. Conversely, if $\frac{\pi_{22}}{\tau_{22}} < \frac{\pi_{12}}{\tau_{12}}$, then e_2 is unstable

3.1.3 Mixed strategy

Another possibility is that it is a mixed strategy if each pure strategy is assigned a probability and the individual is allowed to choose a strategy randomly.

Different from pure strategy, no matter which combination of strategies the two sides adopt, the losing party can always change its strategy to win again, so there is no pure strategy Nash equilibrium. Practically, pure strategy is a mixed strategy game with probability of one.

Going back to the game of rock-paper-scissors, if you are not sure which opponent will shoot, but you are sure that the opponent will play rock with 20% probability, scissors with 30% probability, and paper with 40% probability. How to choose the right strategy to win in this situation? A mixed strategy is to analyze this situation. In short, a mixed strategy is to add different probabilities to pure strategies. In a game, players choose a pure strategy to play with different probabilities.

The payoff expected by the individual is the payoff of the pure strategy multiplied by the probability of this payoff occurring and summed over each game payoff, that is, Π_1 and Π_2. In this case, the mixed strategy Nash equilibrium p_1 can obtain when $\Pi_1 = \Pi_2$ is satisfied.

Through that method, we could obtain 4 different cases.[12]

Case 1

Strategy e_1 is stable and e_2 is unstable.

The focal individual receives payoff per unit of time $\frac{\pi_{12}}{\tau_{12}} > \frac{\pi_{22}}{\tau_{22}}$ and $\frac{\pi_{11}}{\tau_{11}} > \frac{\pi_{21}}{\tau_{12}}$, that is, the number of n_{22} pairs tends to 0 much faster than n_{12}, the number of n_{21} pairs tends to 0 much faster than n_{11}.

Case 2

Strategies e_1 and e_2 are unstable.

The focal individual receives payoff per unit of time $\frac{\pi_{12}}{\tau_{12}} > \frac{\pi_{22}}{\tau_{22}}$ and $\frac{\pi_{11}}{\tau_{11}} < \frac{\pi_{21}}{\tau_{12}}$, that is, the number of n_{22} pairs tends to 0 much faster than n_{12}, the number of n_{11} pairs tends to 0 much faster than n_{21}.

Case 3

Strategies e_1 and e_2 are stable.

The focal individual receives payoff per unit of time $\frac{\pi_{12}}{\tau_{12}} < \frac{\pi_{22}}{\tau_{22}}$ and $\frac{\pi_{11}}{\tau_{11}} > \frac{\pi_{21}}{\tau_{12}}$, that is, the number of n_{12} pairs tends to 0 much faster than n_{22}, the number of n_{21} pairs tends to 0 much faster than n_{11}.

Case 4

Strategy e_1 is unstable and e_2 is stable.

The focal individual receives payoff per unit of time $\frac{\pi_{12}}{\tau_{12}} < \frac{\pi_{22}}{\tau_{22}}$ and $\frac{\pi_{11}}{\tau_{11}} < \frac{\pi_{21}}{\tau_{12}}$, that is, the number of n_{12} pairs tends to 0 much faster than n_{22}, the number of n_{11} pairs tends to 0 much faster than n_{21}.

3.2 Hawk-Dove Game

3.2.1 Instruction for the Hawk-Dove Game

The Hawk-Dove Game is a game theory model proposed by John Maynard Smith, the father of evolutionary game theory. In his paper The Logic of Animal Conflict, he used the phenomenon of fighting and avoidance in different situations in nature to create a game model and explains why there are animals of the same species that use different strategies and under what conditions the animals choose a strategy that achieves an evolutionarily stable strategy. For example, if the hawk strategy is an evolutionarily stable strategy, then the payoff from choosing the hawk strategy in that population will receive a higher payoff than if the other strategy is chosen, and therefore the frequency of that strategy will increase in the next generation under natural selection.[14]

Figure 3.2: The cowardly dove and the aggressive hawk in nature are used to express two opposed strategies in a game. see [15]

In a resource-limited survival environment, assuming that hawks and doves aim for the same food 2V, aggressive hawks will tend to hog the resource alone and struggle against other hawks. The struggle requires a large amount of food C for energy, so each hawk needs to consume V-C of food. When a dove and a dove find the same food, the milder dove will prefer to share, so each dove gets an equal share of food V. If the hawk and the dove find the same food 2V, then the cowardly dove will not choose to fight the hawk but choose other food, in which case the hawk gets all the food 2V. Thus the model create to a payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} H & D \end{array} \\ \begin{array}{c} H \\ D \end{array} & \begin{pmatrix} V-C & 2V \\ 0 & V \end{pmatrix} \end{array}$$

In this matrix, it is shown that the hawk always seems to get more resources during the hawk-dove encounter, so why are there still some individuals in the population who choose the dove strategy? It is because there is also a need for the moderate dove strategy in the model.

Assuming that all individuals in the population choose the hawk strategy, i.e. fighting each other to gain resources, the key is whether the energy gained from fighting individuals with the same hawk strategy is greater than the energy consumed in the fight, which is difficult to do in an environment with limited natural resources. It is also worth considering that it is difficult for all individuals in the population to choose the hawk strategy due to their age and the fact that they may be disabled and unable to fight.

Another possibility is that if all the individuals in the population choose the dove strategy, then since at this point the individual chooses the hawk strategy, its opponent should ideally be the dove strategy, then they will be able to gain the most resources without the battle consuming them, so with the choice of payoff, there must be some individuals who choose the hawk strategy to invade the population where all the individuals choose the dove group.

From both hypotheses, it is clear that it is difficult to have either an all hawk strategy or an all dove strategy. For individuals, some individuals may not change their strategy or may not be able to change their strategy due to their age and physical condition, but for the population as a whole, the ratio of individuals choosing the hawk strategy to those choosing the dove strategy will remain relatively stable, thus achieving an evolutionarily stable strategy.

Whether an individual adjusts their chosen strategy depends on the game situation. In the hawk-dove game, there are always three game situations, where the hawk meets the hawk, the hawk meets the dove and the dove meets the dove. The proportion of individuals who choose the hawk strategy and the proportion of individuals who choose the dove strategy is determined by these three situations. If a certain number of individuals choose the hawk strategy, it is more profitable to choose the dove strategy. Conversely, the hawk strategy is more profitable when a certain number of individuals choose the dove strategy.

3.2.2 Hawk-Dove Game with symmetric interaction times

At the same time, it is expected their interaction time t to be the same except for the time t11 at which the hawk and hawk engage in combat, and the different combat times t11 will have a significant impact on the efficiency of food acquisition by the hawk. It is conceivable that when the battle time is too long, the efficiency of food acquisition by the hawk and the hawk during the battle decreases, while on the contrary the efficiency increases and the time interaction matrix is:

$$
\begin{array}{cc}
 & \begin{array}{cc} H & D \end{array} \\
\begin{array}{c} H \\ D \end{array} & \begin{pmatrix} \tau_{11} & \tau \\ \tau & \tau \end{pmatrix}
\end{array}
$$

Now analyze the different cases depending on the parameters C and V.

Case A (V >C)

When the hawk needs to consume food V-C greater than 0, for example, the cost of obtaining food is greater than the cost of fighting, in this case:

$$
\frac{V-C}{\tau_{11}} > \frac{0}{\tau}
$$

$$
\frac{2V}{\tau} > \frac{V}{\tau}
$$

which means the dove cannot invade the hawk equilibrium, so in this pure strategy game, the choice to become a hawk would be the optimal choice in this model.

Considering the case of a mixed strategy game, according to the replicator dynamical equation we can obtain the solution equation for the mixed Nash equilibrium, that is:

$$
\begin{aligned}
\frac{ds}{dt} =& s(1-s)(\Pi_1 - \Pi_2) \\
=& \frac{1}{2\tau_{11}\tau^2(-\tau_{11}+\tau)}(-2s^2(\tau_{11}-\tau)\tau(\tau_{11}V+(C-V)\tau)+ \\
& (2\tau_{11}V+(C-V)\tau)(\tau^2-\sqrt{\tau^3(-4(-1+s)s(\tau_{11})+\tau)})+ \\
& s(\tau_{11}V+(C-V)\tau)(2\tau_{11}\tau-3\tau^2+\sqrt{\tau^3(-4(-1+s)s(\tau_{11}-\tau)+\tau)}))
\end{aligned}
$$

(s represents the proportion of the entire population selected to become hawk)

We could find different steady states when $\frac{ds}{dt} = 0$ and when it passes through the steady state, if the slope of the function is positive, the point is unstable. If the slope of the function is negative, the point is stable.

Setting time τ to 1, combat cost C to 1 and food V to 2, consider the relationship between hawk-hawk combat time τ_{11} and the Nash equilibrium point.

When $\tau_{11} = 3$, as we can see in figure A, there are two equilibria $p_1 = 0$ and $p_2 = 1$. p_1 is unstable point and p_2 is stable. Since there is only one point of stability, the proportion of the entire population selected to become hawk at the hawk-hawk interaction time $\tau_{11} = 3$ will only tend to move towards p_2 until stability

When τ_{11} up to 4, there a 4 equilibria we could find in figure B. In this case, $p_1 = 0$, $p_3 = \frac{6}{7}$ is unstable, $p_2 = \frac{5}{7}$, $p_4 = 1$ is stable. In this case, there are two stable points, so that in the interval [0, 6/7], the proportion of the entire population selected to become hawks tends to move towards stable point p_2, while in the interval [6/7, 1], the entire population selected to become hawks tends to move towards stable point p_4.

It is worth noting that mixed Nash equilibria will occur if τ_{11} is higher enough. Due to V, C, τ and τ_{11} are all positive and the Nash equilibrium belongs to the interval [0, 1] when

$$\tau_{11} \geq \frac{-C\tau + V\tau}{2V} + 2\sqrt{\frac{-C\tau^2 + V\tau^2}{V}}$$

4 equilibria will be exist, there are 0, 1,

$$\frac{\tau_{11}V^2+2C\tau_{11}V\tau+C^2\tau^2-V^2\tau^2-(\tau_{11}V+C\tau-V\tau\sqrt{\tau_{11}^2V^2+2C\tau_{11}V\tau-6\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+5V^2\tau^2})}{2(\tau_{11}^2V^2+2C\tau_{11}V\tau-2\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+V^2\tau^2)},$$

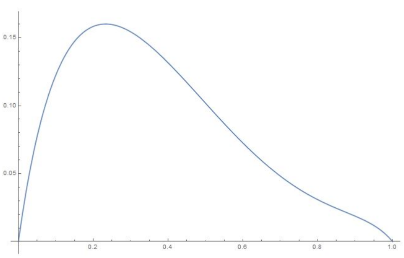
$$\frac{\tau_{11}V^2+2C\tau_{11}V\tau+C^2\tau^2-V^2\tau^2+(\tau_{11}V+C\tau-V\tau\sqrt{\tau_{11}^2V^2+2C\tau_{11}V\tau-6\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+5V^2\tau^2})}{2(\tau_{11}^2V^2+2C\tau_{11}V\tau-2\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+V^2\tau^2)},$$

and each equilibria belong to interval [0, 1].

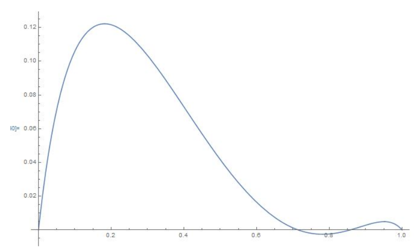
The vector plot figure C can be derived by comparing the hawk-hawk interaction time τ_{11} with the Nash equilibrium point.

Assuming that the interaction time in this model is gradually reduced from

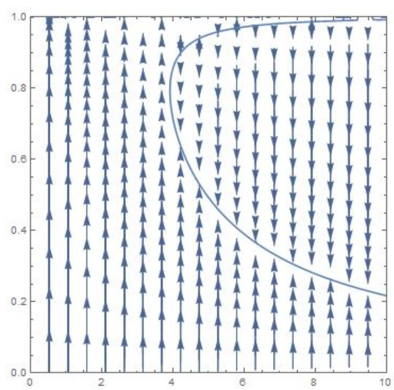
6 to 0 and then restored to 6, it can be seen from Figure D that the Nash equilibrium will not change even if it returns to the original time τ_{11}, which proves that when the model has reached a Nash equilibrium, The individuals in the entire model have reached the maximization of their playoff, so they will not change their chosen strategy, thus will not return to the original Nash equilibrium.



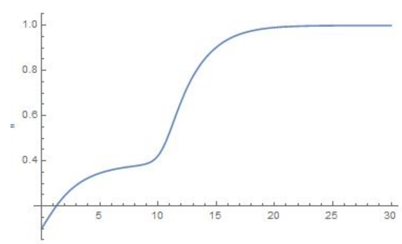
(a) Figure A: $\tau_{11} = 3$



(b) Figure B: $\tau_{11} = 4$



(c) Figure C: vector plot



(d) Figure D: NE when τ_{11}= 6→0→6

Case B (V <C)

When the hawk needs to consume food V-C less than 0, in this case

$$\frac{V-C}{\tau_{11}} < \frac{0}{\tau}$$
$$\frac{2V}{\tau} > \frac{V}{\tau}$$

which means the dove can invade the hawk equilibrium, so we are in no position to find Nash equilibrium in pure strategy, the equilibrium is not stable when it equals 1.

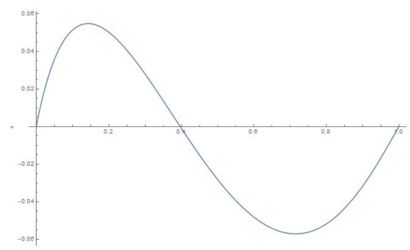
Setting time τ to 1, combat cost C to 2 and food V to 1, consider the relationship between hawk-hawk combat time τ_{11} and the Nash equilibrium point.

When $\tau_{11} = 3$, there are only one stable point in figure E. There will only be a Nash equilibrium, which suggests that the proportion of hawks in the population will always converge to p1 to reach stability.

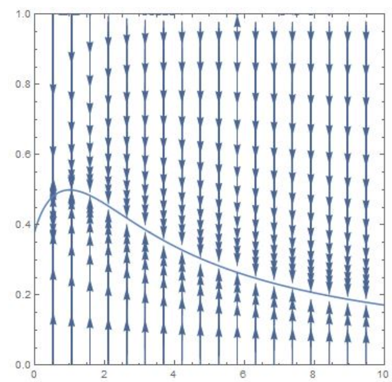
This unique Nash equilibrium can be derived from the replicator dynamic equation as:

$$\frac{\tau_{11}V^2+2C\tau_{11}V\tau+C^2\tau^2-V^2\tau^2-(\tau_{11}V+C\tau-V\tau\sqrt{\tau_{11}^2V^2+2C\tau_{11}V\tau-6\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+5V^2\tau^2})}{2(\tau_{11}^2V^2+2C\tau_{11}V\tau-2\tau_{11}V^2\tau+C^2\tau^2-2CV\tau^2+V^2\tau^2)}$$

Furthermore, the vector plot in figure F shows that when τ_{11} is equal to τ, the proportion of Nash equilibrium is maximum V/C. When τ_{11} is less than τ, the proportion of hawks in the entire population rises until it reaches the maximum V/C, and when τ_{11} is greater than τ, the proportion of hawks in the entire population decreases until it reaches 0.



(e) Figure E: $\tau_{11} = 3$



(f) Figure F: vector plot

4 Agent-Based Modeling in evolution game theory

Agent-based models are simulation methods used to study the interactions between species, things, environments, and time, by simulating the composition of the real world through computers and setting different information as a basis to build mathematical models and model-related problems. It can be verified, evaluated and analyzed on real-world experimental data by conducting simulation experiments. In addition, experiments that cannot be realized in the real world or experiments that are considered to be unethical, such as the speed of virus spread in the human body and the extinction of species, can also be achieved through this method.

Typically, agent-based models are implemented in programming languages and manipulated by computers. Due to the flexibility of the agent-based model, it is possible to create relevant models for analysis according to the needs of actual experiments. It can ignore other factors and focus on the factor that wants to be confirmed as the main reason. Besides, it could add other factors to complicate the model to make it more like the real world.

For biologists, the agent-based modeling approach, which allows different assumptions to be chosen at will and the relevant strategies to be changed, is suited to evolutionary game theory.[16]

4.1 About Netlogo

Netlogo is an open-source software whose purpose is to create a simple and efficient model for analysis. This software was designed by Professor Wilensky in 1999 based on the logo language. NetLogo modeling can well simulate individual behavior and overall performance and the relationship between the two. At present, Netlogo has been widely used in mathematics, physics, chemistry and biology and other related experiments. For evolutionary game theory, Netlogo is one of the useful tools. It is common that an agent-based evolutionary game theory model in Netlogo should have these four building blocks in short:

- Lots of agents
- A game played by agents
- Set the rules of the game and assign different strategies to agents
- Play and update the strategy chosen by the agent based on the payoff function

In the model, any number of agents could be chosen to play this game. These agents will play this game continuously, selecting opponents randomly, and modifying their probabilities with a certain probability according to the payoff function. If another agent earns more than the agent by payoff function, the agent will imitate the other agent's strategy in order to gain a better payoff function.[16]

Figure 4.1: Logo for netlogo

4.2 Hawk-Dove Game in Netlogo

4.2.1 Goal of the model

In Netlogo a natural ecological model is created in which there is a natural resource, grass, which is renewable and stops growing when it reaches a maximum level. Set the initial and maximum resources for the grass and use a frequency delta to control the growth rate of the grass, which can be judged by the color of the grass in the area.

In addition, create a group of sheep that eat grass to sustain their life activities. Sheep consume their energy by moving, fighting and giving birth. In this model, it is assumed that each sheep can choose different energy consumption for fighting. For example, consuming one unit of energy is strategy one, and consuming ten units of energy is strategy ten. Compared with the sheep that consume less energy, the strategy chosen in the game is similar to the cowardly dove in the hawk-dove game. On the contrary, the strategy of the sheep that use more energy to fight with each other is similar to the aggressive hawk in the hawk-dove game. Similarly, the color of the sheep can be used to determine the strategy of combat energy consumption chosen by the sheep.

If too little energy is spent on fighting, they will die because they cannot win the fight, and if too much energy is spent, they will die when the energy obtained from the resource is less than the energy consumed.

The optimal strategy under the different conditions of the model can be determined by looking at the strategy chosen by the surviving sheep.

4.2.2 Main Code and Implement

Define variables

For a start, use the breed statement to define the species of sheep, which will move continuously in the model, where the sheep have properties such as energy and cost with energy. Then use the definition of the patch to replace the grass in the ecosystem, the grass itself has properties such as quantity and maximum value. Finally, use global variables to define the average fight cost and calculate the sheep size below average cost and the sheep above average cost.

```
breed [ sheep a-sheep ]
sheep-own [
  energy
  cost_fight
]
patches-own [
  grass-amount
  max_resource
```

```
]
globals [
  meancost
  countlessmean
  countgreatermean
]
```

Create setup and go process

Set the setup process to initialize the model, clear all variables and administer commands to each patch in the model with the ask statement, set the variable max-resource to the maximum energy the grass has, which means the grass will grow to the maximum and will not increase energy anymore and set the grass initially contained energy. At the same time, use the scale-color statement to report the change in the energy and color of the grass.

When the energy of the grass is 0, it means that the grass of this patch has been eaten, and it needs to wait for time to grow again, then the color of this patch will change. When the energy of the grass is the maximum resource, it means that the grass will not increase its energy anymore, then the color in this patch will turn white. It could predict the growth of the grass by the shade of the color.

Furthermore, the initial number of sheep is created during the setup process, the location where the sheep move is set, the sheep can move to different patches to be active, and their initial energy initial-energy and energy cost-fight are set.

in terms of how to set the fighting energy, a random number is reported via random-float 1 random floating number greater than or equal to 0 but less than 1. On top of that, multiply this number by the value of the maximum fight energy minus the minimum fight ability, the reason for doing this is to ensure that each sheep's chosen fight energy expenditure is randomly assigned at the start of the model. What is more, the sheep's energy is subtracted from the fighting energy spent and the sheep's color is then updated by a scale-color statement.

```
to setup
  clear-all
  ask patches [
    set max_resource  maximum_resource
    set grass-amount initial_resource
    set pcolor scale-color green grass-amount
    0 max_resource
  ]
  create-sheep initial-population [
    setxy random-xcor random-ycor
    set shape "sheep"
    move-to one-of patches;
    set energy initial_energy
    set cost_fight ((random-float 1) *
```

```
    (max_cost_fight - min_cost_fight) + min_cost_fight)
    set energy (energy - cost_fight)
    sheep-recolor
  ]
  reset-ticks
end
```

Detect the number of sheep during the go process and stop the process if no sheep survive. Use the ask statement to command the sheep to move, eat and reproduce, check death and other activities and set the conditions for the grass to regrow.

Moreover, calculate the average value of the energy consumption of sheep fighting, the number of sheep whose fighting consumption is less than the average and the number of sheep whose fighting consumption is greater than the average in each process.

```
to go
  if not any? sheep [stop]
  ask sheep [
    move
    check-if-dead
    eat
    reproduce
  ]
```

```
  regrow-grass
  set  meancost mean [cost_fight] of sheep
  set countlessmean count sheep
  with [cost_fight < meancost]
  set countgreatermean count sheep
  with [cost_fight > meancost]
  tick
end
```

Create different modules in the go process

In NetLogo, a simple statement is scale-color to change the color of grass and sheep as mentioned before. In this process, set the initial color of the grass to green, black to reach the minimum value of 0, and white to reach the maximum value of max-resource. Similarly, set the initial color of the sheep to red, and dark red sheep choose less energy for To fight, the bright red sheep fight to choose more energy. Set up with the recolor-grass and sheep-recolor processes respectively

```
to recolor-grass
  set pcolor scale-color green grass-amount
  0 max_resource
end
to sheep-recolor
```

```
  set color scale-color red cost_fight
  min_cost_fight max_cost_fight
end
```

After that, set the sheep's range of motion and energy consumption during the move. rt random-float 360 means that the sheep can move in any direction, and stepsize is used to limit the movement distance of the sheep. Then subtract the energy consumed by the movement and the extra energy generated by the distance from the energy of the sheep itself.

A conceivable explanation is that the farther the sheep moves, the more energy it needs. If the stepsize is relatively small, it means that the sheep can only move in a small range. Since the total energy of the grass is limited, the sheep in this range may use more energy to fight to obtain energy. This means that sheep can move to a patch with enough grass to graze, in which case they will be assessed by the energy cost of moving and the energy cost of fighting, which can be determined by looking at the energy cost of their chosen fight.

```
to move
    rt random-float 360
    set xcor xcor + (stepsize * dx)
    set ycor ycor + (stepsize * dy)
    set energy energy - move_cost - stepsize
end
```

Similarly, set the sheep's eating process, and set the amount of energy the sheep can get from the grass, if there is enough energy for the grass to exist on the patch, the grass on that patch will be eaten by the sheep while the sheep's own energy will increase, then update the color of the grass on that patch.

```
to eat
  if ( grass-amount >= energy-from-grass ) [
    set energy energy + energy-from-grass
    set grass-amount grass-amount - energy-from-grass
    recolor-grass
  ]
end
```

In the reproduction process, the sheep's production conditions are set, and when they gain a certain amount of energy they reproduce, giving birth to offspring that inherit the energy consumption their parents chose during the battle, and the offspring can also move in any direction in search of food, just like their parents.

```
to reproduce
  if energy > birth_conditions [
    set energy (energy - birth_cost)
    hatch 1 [
    rt random-float 360
```

```
    set xcor xcor + (stepsize * dx)
    set ycor ycor + (stepsize * dy) ]
  ]
end
```

What's coming up is the regrow-grass process, each patch is checked and if the energy of the grass is less than the maximum that the grass can grow, it can continue to grow at a certain rate of delta-growth until it reaches the maximum, and likewise update the color of the grass.

```
to regrow-grass
  ask patches [
    if grass-amount < max_resource [
      set grass-amount (grass-amount + delta_growth)
    ]
    recolor-grass
  ]
end
```

Setup model interface

What's next is setup the model interface. On the left-hand side, it could be seen that the inputs related to different model parameters, including the minimum and maximum energy value of the grass, the growth rate of the grass, the initial population of sheep, the initial energy value, the parameters related to sheep eating, movement, fighting and birth, and by adjusting the different parameters it can observe that the strategy chosen by the sheep in the model. There are three different buttons for model manipulation, corresponding to the setup process and the go process in the model code, where the go once button refers to the model going through the go process once. If the user wants to stop the model, the user can click on the go process once.

In the middle of this interface is the model in which the sheep interact with the grass. The distribution and color shades of the sheep and the grass can be seen in this image, and the model is used to simulate realistic sheep movements to observe the sheep's choice of fighting and consumption strategies.

On the right-hand side of the user interface there are three plots showing the distribution of energy consumed by the sheep choosing to fight, the average energy of the sheep and the development of the number of sheep and grasses over time.

Additionally, Several monitors are set up to observe the specific quantities in the plots in order to facilitate the observation of the sheep as they make their strategic choices. Also, several monitors are designed to represent the above-average and below-average consumption of choice fights in the sheep and their share of the total sheep.

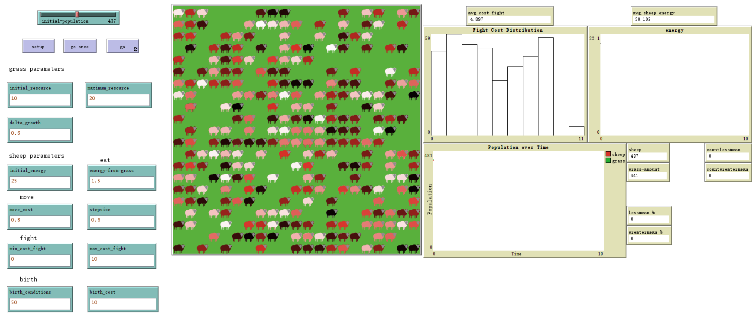


Figure 4.2: The initial interface of the model

4.2.3 Model Testing and analysis

Model Testing

Subsequently, it is necessary to test the model, randomly set parameters for grass and flock action, and observe whether the changes in the model image and different plots fit real-world conditions.

The parameters worth considering in this model are the total number of sheep, the rate of grass growth, the energy gained by the sheep when eating and the energy consumed by the sheep when making their movements and the energy consumed by production.

To test the growth rate of the grass, set the initial number of sheep to a maximum of 1000 and the growth rate of the grass to 0. In this case, if all the grass and sheep in the model die out, the model is as predicted. After setting the parameters, perform a setup operation to see if the model image works properly.

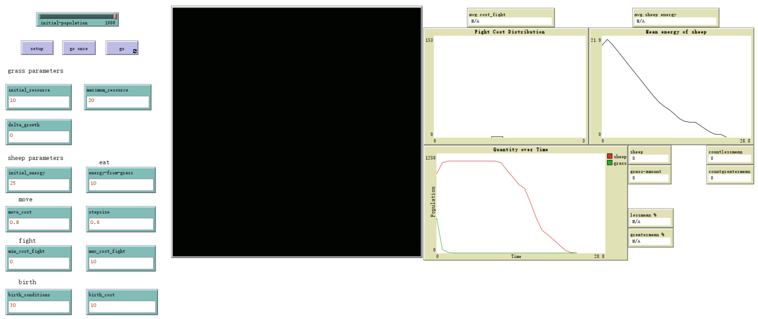


Figure 4.3: Model change over time when setting the growth rate of grass to 0

It can be seen that the model image in figure 4.3 is as expected, the grass and sheep are all dead. Since the sheep population is the maximum, the grass growth rate is increased to 0.5 and the model works perfectly and outputs the corresponding plots.

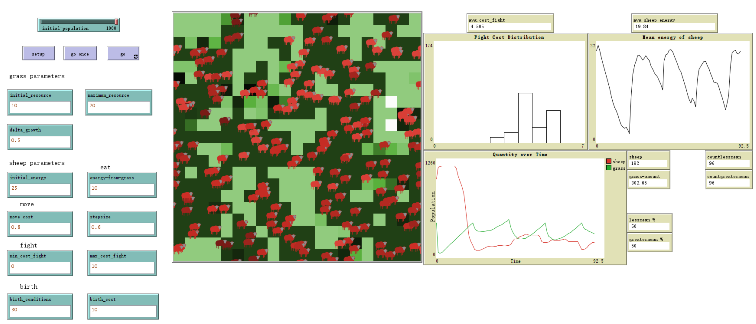


Figure 4.4: Model runs normally according to set parameters

To test the parameter energy obtained by sheep eating grass, set the energy obtained from grass to 0, which means the sheep cannot get any energy from the grass, if the sheep in the model are dead and the grass grows to maximum, the model is perfect. Similarly, after setting the parameters, perform the setup operation to observe whether the model image can run properly.

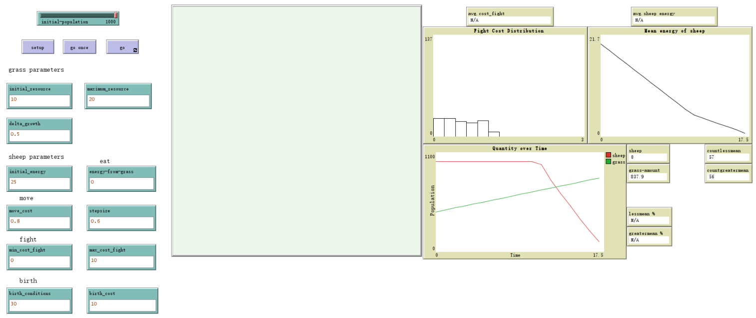


Figure 4.5: Set the energy gained from the grass to 0 and observe the model running over time

It can be seen that the model image in figure 4.5 is as expected, the sheep in the model is dead and the grass grows to the maximum. Reset this parameter to 10 and the model works properly.

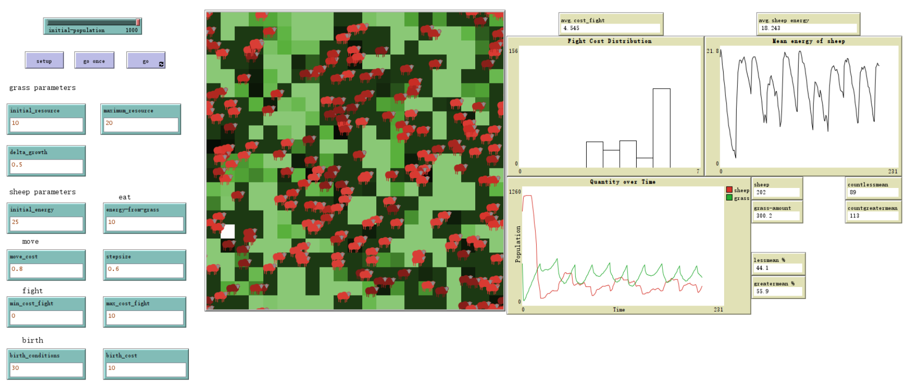


Figure 4.6: Reset the energy from grass parameter and the model runs normally

Test the energy consumed by the sheep's movement, and set the parameter step size to 0. In this case, the sheep can only search for grass on the current patch, and cannot go to other patches for eating. If the energy of a patch with sheep on the grass in the model decreases to 0 and all sheep die over time, and then the patch without sheep on the grass grows to its maximum energy, it could be said that the model runs as expected.

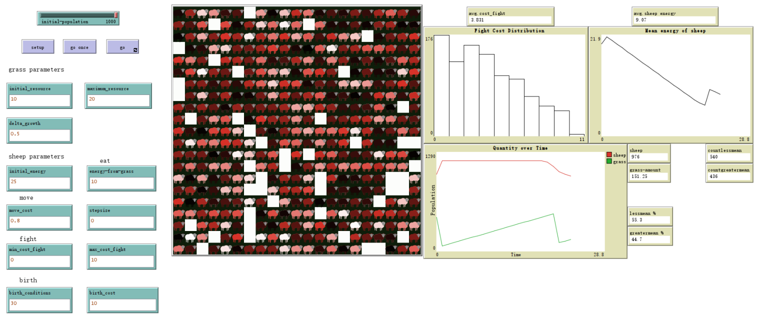


Figure 4.7: Set the parameter stepsize to 0 to observe the change of the model over time

It can be seen that the model image in figure 4.8 grows to the maximum energy in the grass without sheep, as expected, and in the grass. With sheep, the energy decreases to zero and the sheep gradually die out over time.

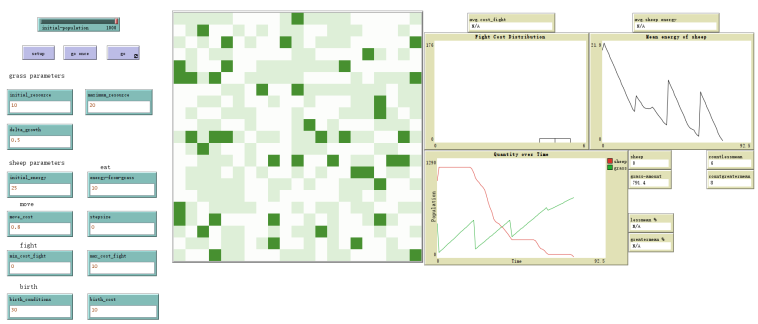


Figure 4.8: When stepsize is 0, the sheep survived for some time but eventually died out

Model analysis

First, analyze the stepsize in the model, and observe whether the movement range in the sheep and the battle consumption of the flock selection have any influence.

The initial number of sheep was set to 500 for the parameter selection, and the other parameters were chosen arbitrarily within a reasonable range. Only the parameter stepsize, the range within which the sheep could move, was changed, observing the movement of the sheep and the growth of the grass on the model interface.

In the early stages of the model, it can be expected that the number of sheep decreases significantly in figure 13 because there is not enough grass for the sheep to feed on and the amount of grass decreases rapidly until the number of sheep is reduced to the extent that the grass can provide. After a massive reduction in sheep the grass grows, at which point the number of sheep gradually rises, as the model is set up in such a way that the sheep will randomly and evenly choose a strategy for battle consumption, so there are still different battle strategies at this point. Let the sheep continue to interact and observe the change in battle strategy.

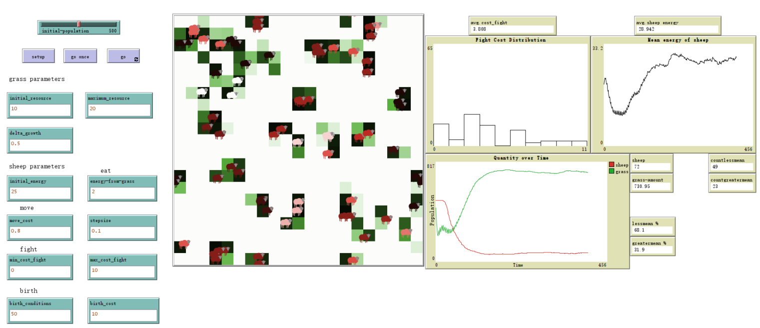


Figure 4.9: Due to the small stepsize parameter, the sheep are confined to a tight range of life activities

It is obvious that from figure 4.10 that after a certain period it was found that the fighting strategies of the sheep changed, from 10 strategies at the beginning to only 4 strategies existing, this is because the sheep that had chosen other fighting strategies in the model had died, in other words, the sheep that had chosen other fighting strategies were eliminated from the model. It can be observed through the model that the grasses where no sheep are become white, which means that the grasses in that patch have grown to their maximum energy. In addition, the sheep in the grasses where sheep are present are divided into different species according to their chosen fighting strategy, due to the small range of movement of the sheep and partly because the offspring of the sheep will inherit their parents' choice of fighting strategy.

Within this range, if the sheep choose a fighting strategy that consumes little energy, then they can fight in this area without expending too much energy. At the same time be able to obtain food peacefully, like the dove strategy encountered in the hawk-dove game. However, in some areas where the sheep have chosen a fighting strategy that is so energy intensive that the sheep will die if they do not fight, the sheep in this area have to fight even if the fight is expensive, a situation where they compete with each other like an hawk strategy encounter in the hawk-dove game.

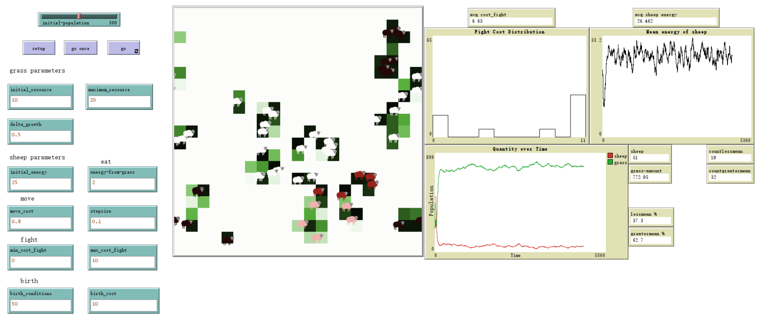


Figure 4.10: As time changes, a proportion of the sheep that do not have food become dead

Continuing to run the model in figure 4.11, only 3 battle strategies were selected by the sheep and 2 strategies were of high consumption as we can see in figure 15. It can be seen that the percentage of sheep choosing choosing combat strategies that are less expensive than the average in the model is 15.9% and 84.1% above the average by using the monitor in the interface.

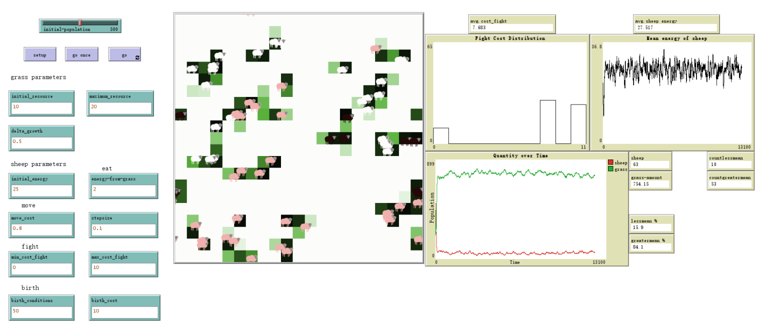


Figure 4.11: Some sheep that choose a fighting strategy that is not adapted to the current environment die and only those that choose three fighting strategies survive

A logical explanation is that the sheep's range of movement is limited so they need to choose a battle strategy that consumes more to obtain grass resources. In this case, although the sheep that chose the less costly fighting strategy were able to feed and reproduce by moving slowly to reach grass with plenty of energy, the sheep that chose the dove strategy had difficulty surviving once the sheep met another sheep that chose the hawk strategy.

Changing the parameter step size to 0.5 allows the sheep to have a larger range of movement and observe the change in the choice of strategy of the sheep in the model in figure 4.12. Through the model, we see that the proportion of sheep choosing to fight with less energy is instead higher at this point. As the range of movement becomes larger, the sheep with less energy can move to grass with more energy and avoid competing with the sheep with more energy to fight for resources. Therefore, with a larger range of activity, 83.2% of the sheep in the model chose a fighting strategy that consumed less than the average and 16.8% chose an over-average strategy, giving more sheep the dove strategy than the hawk strategy in this situation.

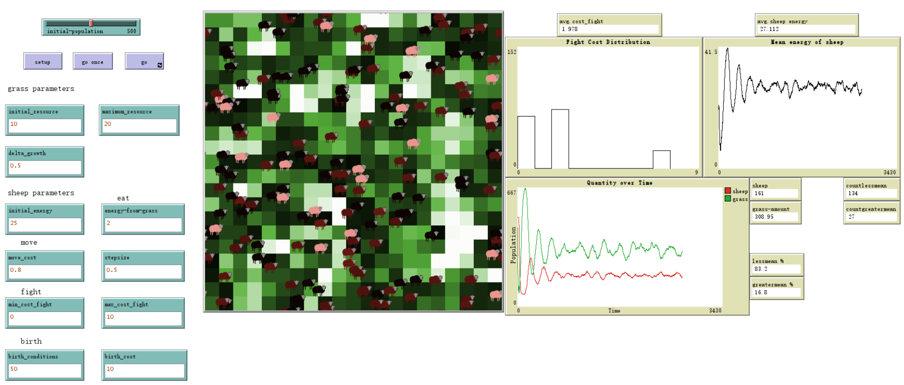


Figure 4.12: As the stepsize increases it seems that the sheep will choose the less costly fighting strategy

It seems to suggest that as the parameter step size increases, the sheep can have more opportunities to find other grasses and so will choose the less battle-consuming strategy to fight. To analyze this result more accurately, it can be analyzed using BehaviorSpace in Netlogo, a tool that allows different variables to be set simultaneously to run the model repeatedly and report the results.

In BehaviorSpace, the model is run at intervals of 0.2 by setting the stepsize from 0.1 to 1.9 and repeated 5 times to reduce the running error, reporting the mean value of the sheep selection strategy, and ending the run after 10,000 interactions to observe the results of the model run.

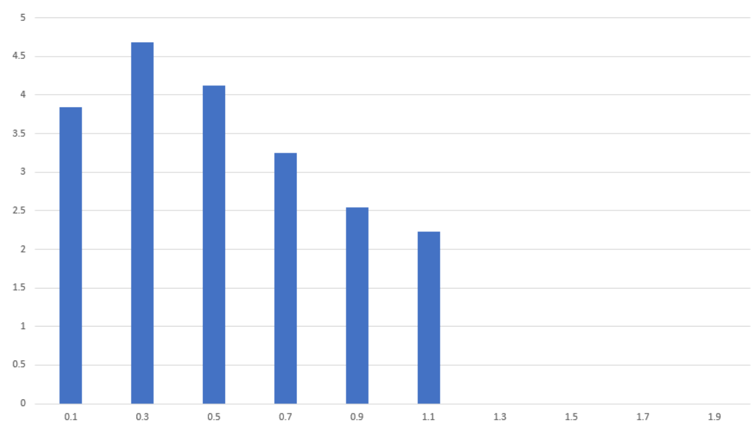


Figure 4.13: Compare the parameter stepsize with the combat strategy chosen by the sheep in the BehaviorSpace

Figure 4.13 shows that the sheep used less energy to fight in the range of 0.3 to 1.1, but at 1.3 there were massive deaths. It is because the sheep also used too much energy to move around in the setup, and when the sheep moved more energy to find food than the sheep had available, the sheep died.

In addition, the energy gained from sheep feeding was analyzed and given that sheep do not fully convert the energy of grass into their own energy during feeding and that the maximum energy that grass can grow is 20, the variable energy-from-grass was to set using BehaviorSpace, setting this parameter from 2 to 18, running the model at intervals of 2 and repeating it five times to reduce model error, report the mean of the sheep selection strategy and end the run after 10,000 interactions to observe the results of the model run.

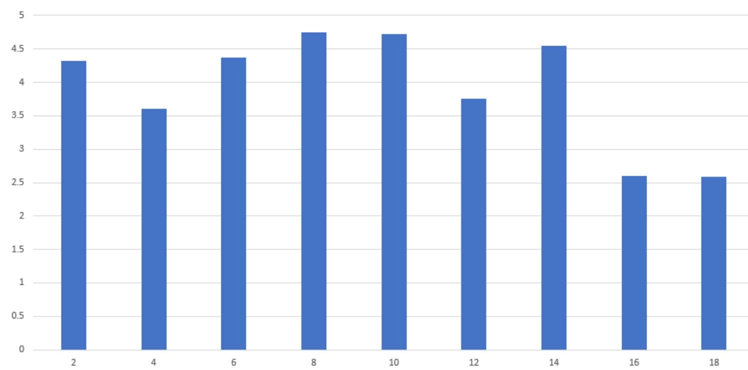


Figure 4.14: Compare the parameter energy gained from sheep with the combat strategy chosen by the sheep in the BehaviourSpace

Through figure 4.14, it can be seen that the relationship between the energy obtained by sheep from grass and the average value of the battle strategy selected by the sheep is oscillating, and the combat strategy selected by the sheep is different during each model operation. It seems that the energy gained from the grass is not directly related to the sheep's choice of combat strategy.

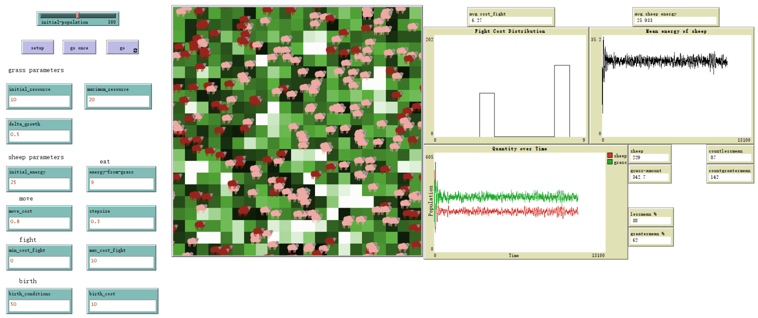


Figure 4.15: energy-from-grass is 9

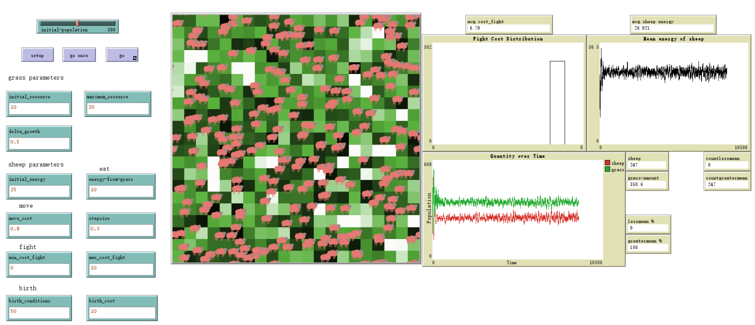


Figure 4.16: energy-from-grass is 10

It is worth noting that when the sheep's energy from grass is set to 10 it happens that all sheep choose only one battle strategy, no sheep choose the hawk's strategy and no sheep choose the dove's strategy, probably because in the model the sheep reach some kind of delicate equilibrium where there is one strategy among all the available battle strategies that all the sheep do not change.

5 Conclusion

After presenting some examples and models of game theory and evolutionary game theory, it is necessary to discuss the advantages and disadvantages of evolutionary game theory and its future development.

It is true that the theoretical study of evolutionary game theory and its applications in economics and biology has become an increasingly popular field. However, there are still many issues and theories that need to be addressed and validated in the context of game theory research, especially evolutionary game theory based on game theory. Although evolutionary game theory improves on the assumption of perfect rationality in traditional game theory, builds on Darwin's theory of biological natural selection to explore the effects of interactions between individuals in a population, and models the population as a dynamic system, the theory is not yet mature enough and specific applications are worth exploring.

One important issue lies in the fact that Nash equilibria often have multiple stable points in mixed strategies, and how the most suitable Nash equilibrium should be chosen when there are multiple Nash equilibria in a game or even an infinite number of Nash equilibria. Moreover, when the Nash equilibrium shifts from one stable point to another, it is difficult to give a scientific explanation for this situation, we know that it happens, but we do not know how it happens.[17]

Secondly, an important problem in Agent-based modeling and related mathematical analysis is that a large number of the results in the analysis are not actually derived from the model itself but from rational assumptions made by the modeler. The modeler will modify the model to obtain reasonable results, but in reality, the model is entirely the modeler's assumptions and difficult to match the reality of the situation. Some modelers only use their life experience or rational guesses to design models and produce results that meet the requirements, and it is difficult for them to give reasonable explanations when the results deviate.

With the development of evolutionary game theory, the key concern in evolutionary game theory is how to construct a self-consistent, scientific model and conduct proper scientific analysis.

Appendix

Detailed Code

```
breed [ sheep a-sheep ]

sheep-own [
  energy
  cost_fight
]

patches-own [
  grass-amount
  max_resource
]

globals[
  meancost
  countlessmean
  countgreatermean
]


to setup
```

```
  clear-all
  ask patches [
    set max_resource  maximum_resource
    set grass-amount initial_resource
    set pcolor scale-color green grass-amount
    0 max_resource
  ]

  create-sheep initial-population [
    setxy random-xcor random-ycor
    set shape "sheep"
    move-to one-of patches;
    set energy initial_energy
    set cost_fight ((random-float 1) *
      (max_cost_fight - min_cost_fight) + min_cost_fight)
    set energy (energy - cost_fight)
    sheep-recolor
  ]
  reset-ticks
end

to sheep-recolor
  set color scale-color red cost_fight
```

```
  min_cost_fight max_cost_fight
end

to go
  if not any? sheep [stop]
  ask sheep [
    move
    check-if-dead
    eat
    reproduce
  ]
  regrow-grass
  ;gather stats
  set  meancost mean [cost_fight] of sheep
  set countlessmean count sheep with
  [cost_fight < meancost]
  set countgreatermean count sheep with
  [cost_fight > meancost]
  tick
end


to recolor-grass
```

```
  set pcolor scale-color green grass-amount
  0 max_resource
end


to move
    rt random-float 360
    set xcor xcor + (stepsize * dx)
    set ycor ycor + (stepsize * dy)
  set energy energy - move_cost - stepsize
end

to check-if-dead

  if energy < 0  [
    die
  ]
end

to eat
  if ( grass-amount >= energy-from-grass ) [
    set energy energy + energy-from-grass
    set grass-amount grass-amount - energy-from-grass
```

```
    recolor-grass
  ]
end

to reproduce
  if energy > birth_conditions [
    set energy (energy - birth_cost)
    hatch 1 [
    rt random-float 360
    set xcor xcor + (stepsize * dx)
    set ycor ycor + (stepsize * dy) ]
  ]
end

to regrow-grass
  ask patches [
    if grass-amount  < max_resource [
      set grass-amount (grass-amount + delta_growth)
    ]
    recolor-grass
  ]
end
```

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