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Gauss-Hermite

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

fazendo $n = 4$:

$$H_4(x) = e^{x^2} \cdot \frac{d^3}{dx^3}\left(-2x \cdot e^{-x^2}\right)$$

$$= e^{x^2} \cdot \frac{d^2}{dx^2}\left(-2e^{-x^2} + 4x^2 e^{-x^2}\right)$$

$$= e^{x^2} \cdot \frac{d}{dx}\left(4xe^{-x^2} + 8xe^{-x^2} + (4x^2)\cdot(-2x)e^{-x^2}\right)$$

$$= e^{x^2} \frac{d}{dx}\left(-2\left(-6xe^{-x^2} + 4x^3 e^{-x^2}\right)\right)$$

$$= e^{x^2}\left[-2\left(-6x(-2x)e^{-x^2} - 6e^{-x^2} \right.\right.$$
$$\left.\left. 12x^2 e^{-x^2} + 4x^3(-2x)e^{-x^2}\right)\right]$$

$$= \cancel{e^{x^2}}\left[-2\left(24x^2 \cancel{e^{-x^2}} - 6\cancel{e^{-x^2}} - 8x^4 \cancel{e^{-x^2}}\right)\right]$$

$$= -2\left(24x^2 - 8x^4 - 6\right)$$

Raizes: $-1{,}6507; -0{,}52465; 0{,}52465; 1{,}6507$

$$w_k = \frac{2^{4-1} \cdot 4! \sqrt{\pi}}{4^2 \cdot (8x_k^3 - 12x_k)^2}$$ Substituindo os valores,

$w_1 = 0{,}0813022$
$w_2 = 0{,}80491$
$w_3 = 0{,}80491$
$w_4 = 0{,}0813022$

Gauss - Laguerre

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(e^{-x}x^n\right)$$

fazendo $n = 4$

$$\frac{d^4}{dx^4}\left(e^{-x}x^4\right) = \frac{d^3}{dx^3}\left(4x^3e^{-x} - x^4e^{-x}\right) = \frac{d^2}{dx^2}\left(12x^2e^{-x}\right.$$

$$\left.- 4x^3e^{-x} - \left(4x^3e^{-x} - x^4e^{-x}\right)\right) = \frac{d^2}{dx^2}\left(12x^2e^{-x} - 8x^3e^{-x} + x^4e^{-x}\right)$$

$$= \frac{d}{dx}\left(24xe^{-x} - 12x^2e^{-x} - 24x^2e^{-x} + 8x^3e^{-x} + 4x^3e^{-x} - x^4e^{-x}\right)$$

$$= \frac{d}{dx}\left(-x^4e^{-x} + 12x^3e^{-x} - 36x^2e^{-x} + 24xe^{-x}\right)$$

$$= -4x^3e^{-x} + x^4e^{-x} + 36x^2e^{-x} - 12x^3e^{-x} - 72xe^{-x} + 36x^2e^{-x}$$

$$24e^{-x} - 24xe^{-x} = x^4e^{-x} - 16x^3e^{-x} + 72x^2e^{-x} - 96xe^{-x} + 24e^{-x}$$

$$L_4(x) = \left(x^4 - 16x^3 + 72x^2 - 96x + 24\right)/4!$$

RAIZES: 0,3225; 1,7458; 4,5366; 9,3951

$$w_k = \frac{x_k}{5^2 \cdot \left(L_3(x_k)\right)^2}$$

$w_1 = 0{,}603115$

$w_2 = 0{,}357347$

$w_3 = 0{,}038895$

$w_4 = 0{,}000539$

Gauss - Chebyshev

~~$T_n(x) =$~~

As raizes podem ser obtidas por $\cos\left(\frac{2k-1}{2n}\pi\right)$

fazendo $n=4$ temos:

$x_1 = -0{,}923879$
$x_2 = -0{,}382683$
$x_3 = 0{,}382683$
$x_4 = 0{,}923879$

e $w_k = \frac{\pi}{n}$. Portanto, $w_1 = \cdots = w_4 = 0{,}785398$