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Gauss-Legendre - 4 pontos

1) RAIZES de P_4

$P_4 = \frac{1}{8}(35x^4 - 30x^2 + 3)$

$35t^2 - 30t + 3$

$t = \frac{15 \pm 2\sqrt{30}}{35}$

$\alpha_1 = -\sqrt{\frac{15+2\sqrt{30}}{35}}$

$\alpha_2 = -\sqrt{\frac{15-2\sqrt{30}}{35}}$

$\alpha_3 = \sqrt{\frac{15-2\sqrt{30}}{35}}$

$\alpha_4 = \sqrt{\frac{15+2\sqrt{30}}{35}}$

2) Cálculo de $x(\alpha)$

$x(\alpha) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2}\alpha_k$

3) Cálculo dos pesos ω_k

$$L_1(\alpha) = \frac{(\alpha-\alpha_2)(\alpha-\alpha_3)(\alpha-\alpha_4)}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)(\alpha_1-\alpha_4)} = \frac{\left(\alpha+\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(\alpha-\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(\alpha-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}{\left(-\sqrt{\frac{15+2\sqrt{30}}{35}}+\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(-\sqrt{\frac{15+2\sqrt{30}}{35}}-\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}$$

$$L_1(\alpha) = \frac{\left(\alpha^2 - \frac{15-2\sqrt{30}}{35}\right)\left(\alpha-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}{2\left(\frac{15+2\sqrt{30}}{35}-\frac{15-2\sqrt{30}}{35}\right)\cdot\left(-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}$$

$$L_1(\alpha) = \alpha^3 - \sqrt{\frac{15+2\sqrt{30}}{35}}\alpha^2 - \frac{15-2\sqrt{30}}{35}\alpha + \frac{15-2\sqrt{30}}{35}\sqrt{\frac{15+2\sqrt{30}}{35}}$$

$$\left(-\frac{8\sqrt{30}}{35}\right)\left(\sqrt{\frac{15+2\sqrt{30}}{35}}\right)$$

$$\omega_1 = \int_{-1}^{1} L_1(\alpha) = \frac{15\sqrt{30}-25}{30^{3/2}} = 0,347854 = \omega_4$$

$$L_2(\alpha) = \frac{(\alpha-\alpha_1)(\alpha-\alpha_3)(\alpha-\alpha_4)}{(\alpha_2-\alpha_1)(\alpha_2-\alpha_3)(\alpha_2-\alpha_4)} = \frac{\left(\alpha+\sqrt{\frac{15+2\sqrt{30}}{35}}\right)\left(\alpha-\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(\alpha-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}{\left(-\sqrt{\frac{15-2\sqrt{30}}{35}}+\sqrt{\frac{15+2\sqrt{30}}{35}}\right)\left(-\sqrt{\frac{15-2\sqrt{30}}{35}}-\sqrt{\frac{15-2\sqrt{30}}{35}}\right)\left(-\sqrt{\frac{15-2\sqrt{30}}{35}}-\sqrt{\frac{15+2\sqrt{30}}{35}}\right)}$$

$$L_2(\alpha) = \frac{\left(\alpha^2-\frac{15+2\sqrt{30}}{35}\right)\left(\alpha-\sqrt{\frac{15-2\sqrt{30}}{35}}\right)}{(-2)\left(\frac{-4\sqrt{30}}{35}\right)\left(\sqrt{\frac{15-2\sqrt{30}}{35}}\right)}$$

$$w_2 = \int_{-1}^{1} L_2(\alpha)\,d\alpha = \frac{(3\sqrt{30}+5)}{6\sqrt{30}} = 0,652145 \quad w_3$$

Com todos os igredientes diponiveis,

$$I = \frac{x_f - x_i}{2}\left[\sum_{k=1}^{4} f(x(\alpha_k))\,w_k\right]$$