NEW YORK TECH

PROJECT REPORT

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Circular film: Simulation

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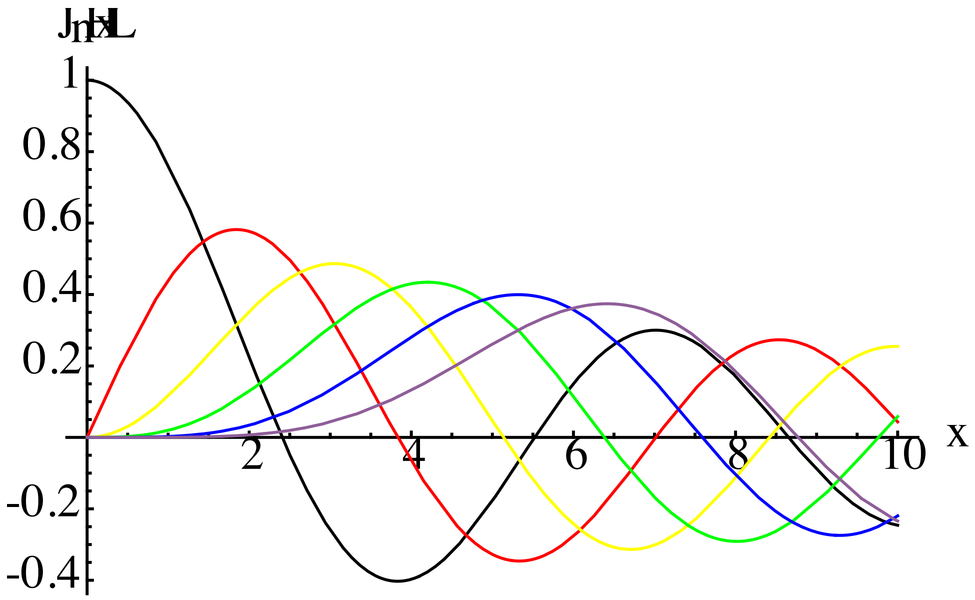
1 Introduction

This project intends to animate a wave inside a circular film using Mathematica. The Bessel functions of the first kind are defined as the solutions to the Bessel differential equation. Figure, below, gives plots of various Bessel functions. The Bessel function of the first kind is used to model the motion of a vibrating membrane. For example, a drum. Subscript is the solution of the Bessel differential equation below that is non-singular at the origin. $J_n(z)$)

The solution $R((\omega * r)/v)$ to the Bessel's function at the o'th order .

$$R((\omega * r)/v) = J_0((\omega * r)/v) = 0 \tag{1.1}$$

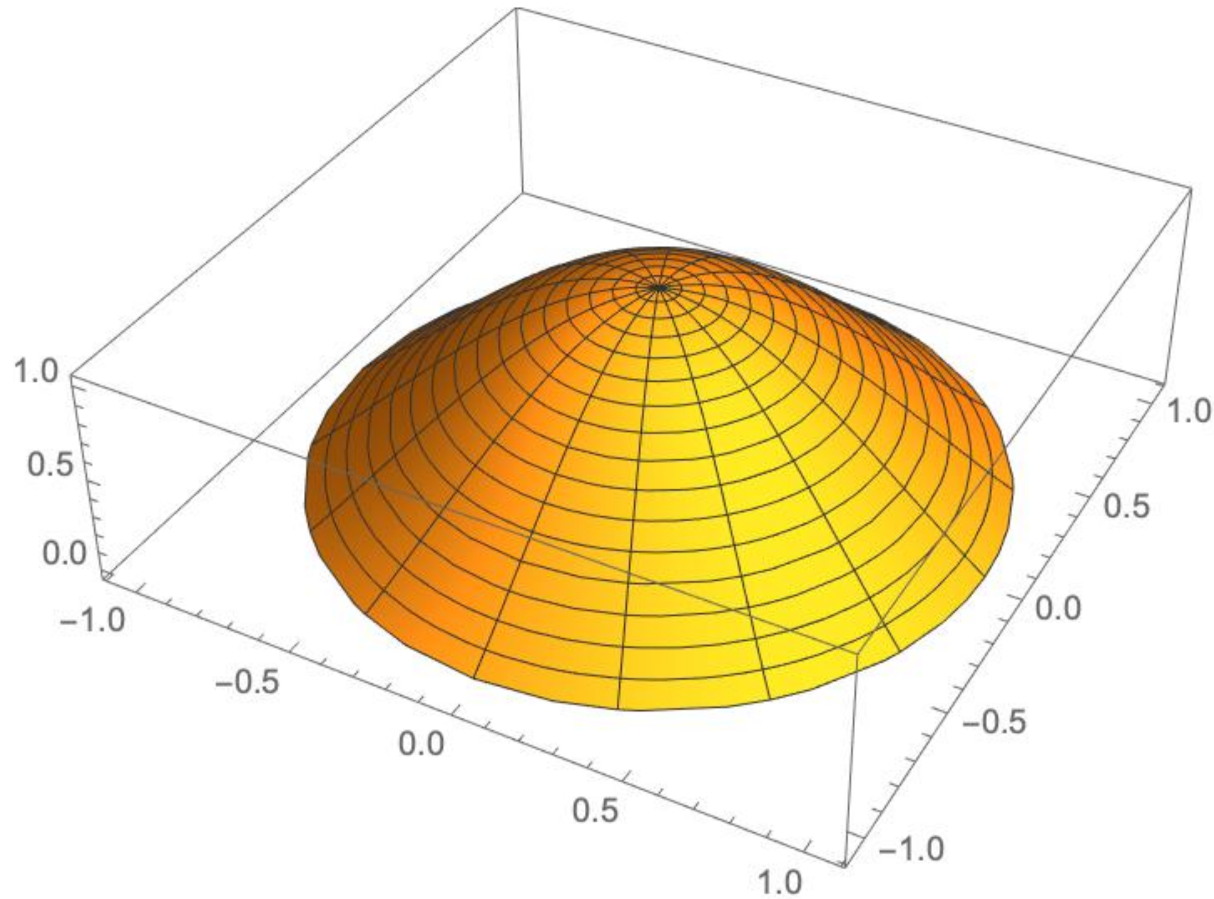


Using the powerful range of tools offered by Mathematica, I was able to plot the circular membrane generated by the Z displacement within the real-time domain using the command Manipulate.

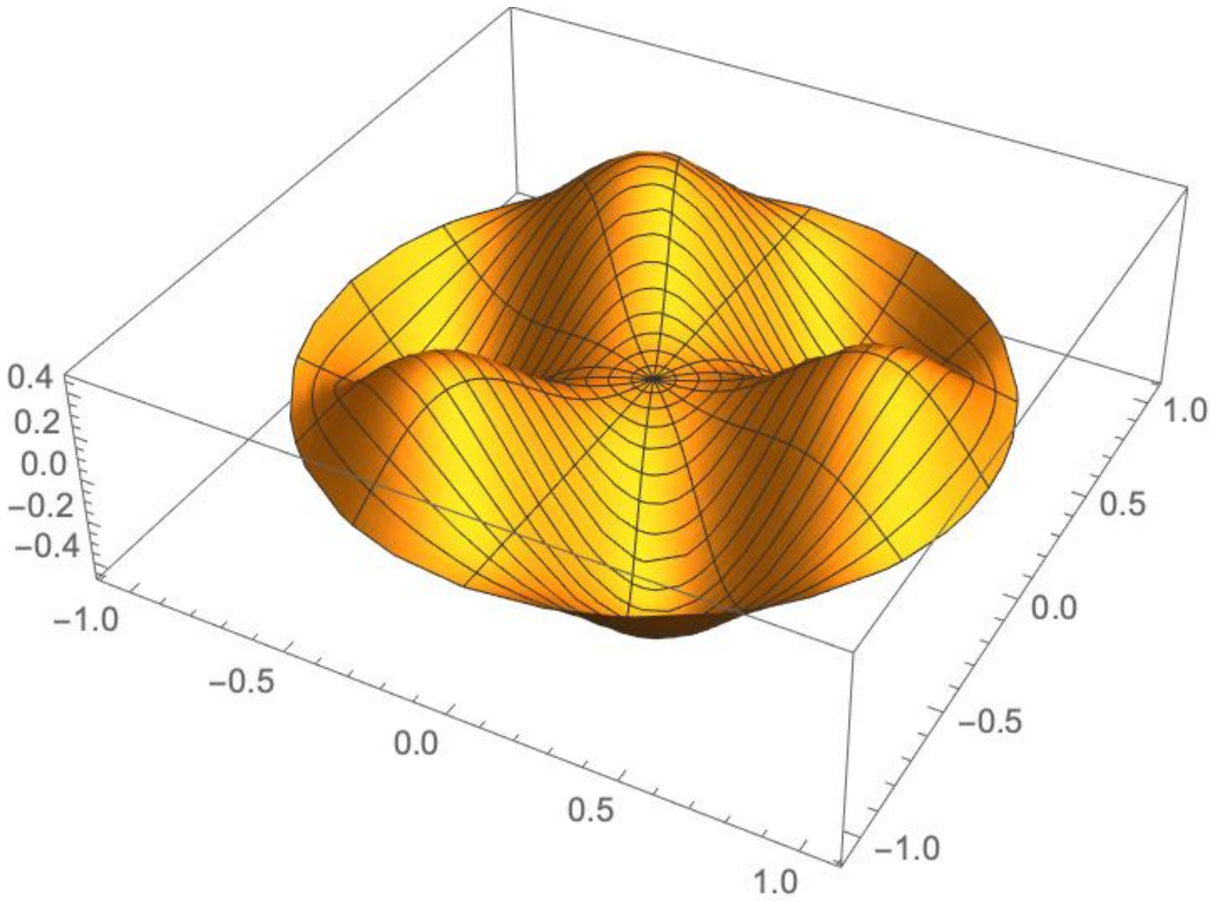
1.1 Generating the circular film

To generate the desired circular output, I started by retrieving the necessary fundamental data to recreate a sample of the simulation. For this purpose, I plotted the phases of the displacement using the command RevolutionPlot3D resulting in the following.

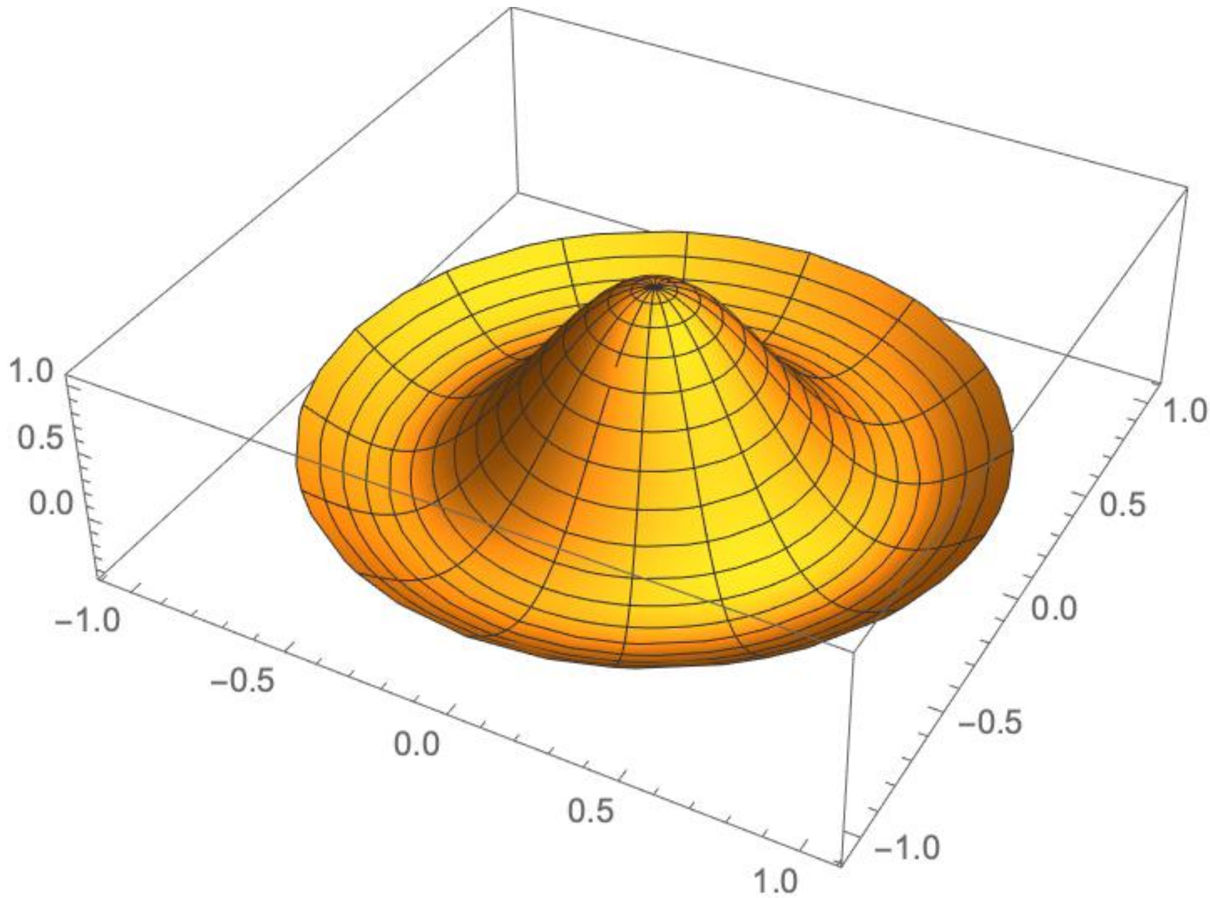
```

RevolutionPlot3D[BesselJ[0, 2.4 r]*Cos[0*a], {r, 0, 1}, {a, 0, 2 Pi}]
```



```
RevolutionPlot3D[BesselJ[2, 5.1 r]*Cos[2*a], {r, 0, 1}, {a, 0, 2 Pi}]
```



```
RevolutionPlot3D[BesselJ[0, 5.3 r]*Cos[0*a], {r, 0, 1}, {a, 0, 2 Pi}]
```



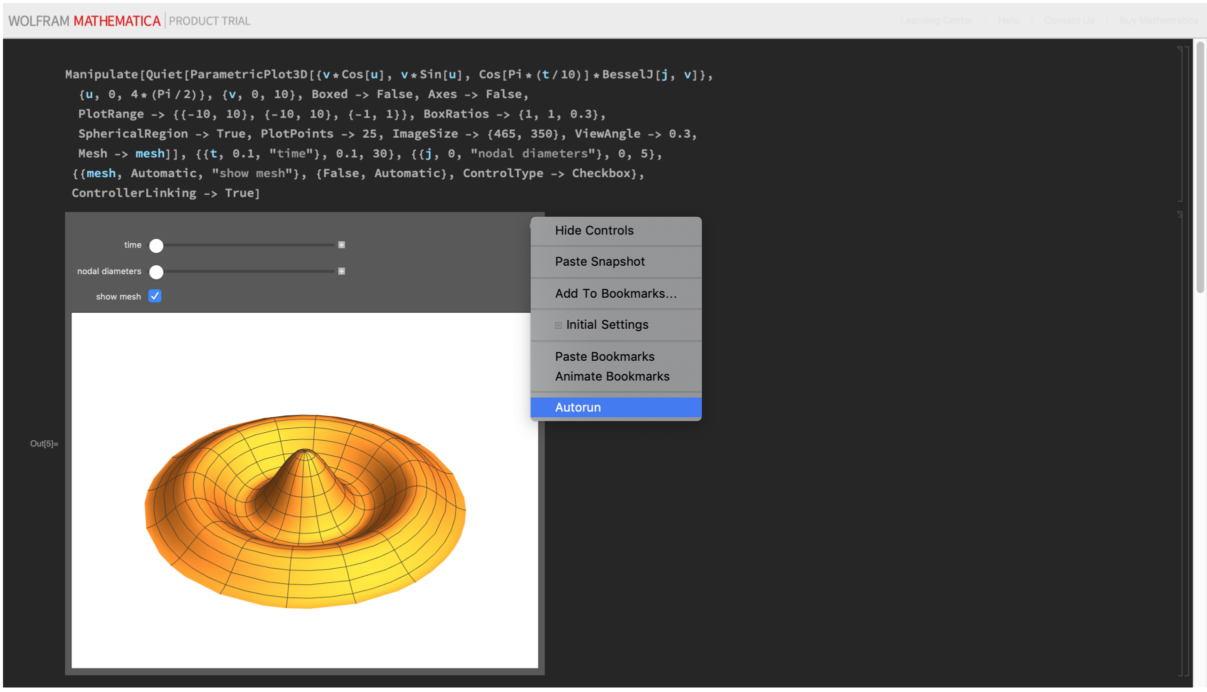
1.2 Procedure and Objectives

- Plot the 3d components of a the circular film created
- Evaluate at boundary conditions.
- Understand patterns regarding angular dependency.
- Simulate the circular membrane within a given timeframe using autorun

2 Results

As discussed earlier, the code generates a simulation for the membrane movement using the Mathematica command Manipulate. This command creates an input cell that would be evaluated in the notebook. The following code has been used to generate such simulation. A video of the manipulation has been attached to the project file for the reader's reference.

```
Manipulate[Quiet@ParametricPlot3D[
   {v Cos[u], v Sin[u], Cos[(\[Pi] t)/10] BesselJ[j, v]},
   {u, 0, (4
    \[Pi])/2}, {v, 0, 10}, Boxed -> False, Axes -> False,
   PlotRange -> {{-10, 10}, {-10, 10}, {-1, 1}},
   BoxRatios -> {1, 1, .3}, SphericalRegion -> True, PlotPoints -> 25,
    ImageSize -> {465, 350}, ViewAngle -> .3, Mesh -> mesh],
 {{t, 0.1, "time"}, 0.1, 30},
 {{j, 0, "nodal diameters"}, 0, 5},
 {{mesh, Automatic, "show mesh"}, {False, Automatic},
  ControlType -> Checkbox}]
```



2.1 Generated output: Angular periodicity Anti-nodes

This Demonstration illustrates the normal vibrational modes of a drum head fixed along a circular edge. Each mode with elongation (in polar coordinates) is related to Bessel functions of the first kind using the radial coordinate, the polar angle, and the time. Selecting the order n determines the angular periodicity of the circular film. Selecting the zero of determines the number of anti-nodes along a radial line connecting the center and outer edge evaluating $J_n(z)$

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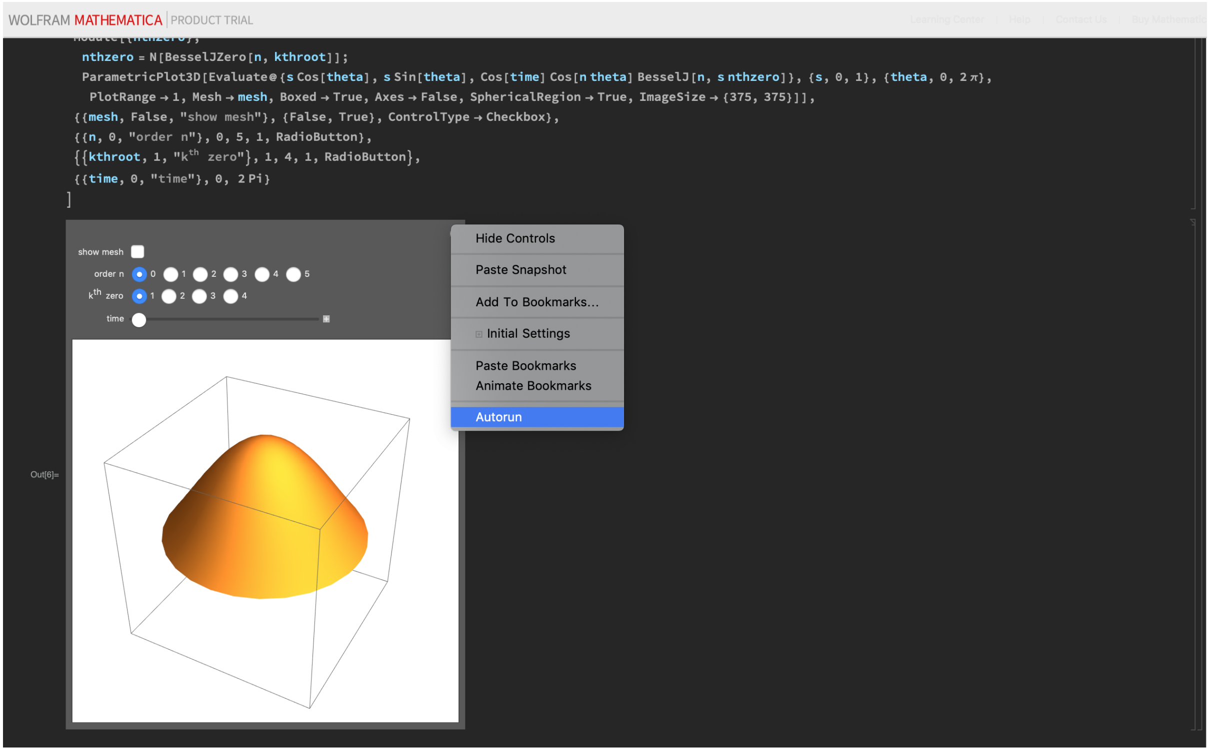

3 Conclusion and comments

This project has been instrumental in reaching some of my own goals as a student. With a healthy bit of programming, I was able to reproduce a visualization of circular film with an initial condition of the function δ. I don't have a direct background using Mathematica, however, the language is well documented and doesn't require an intensive knowledge of computer science to tweak the engineering a reproduce plots. Moreover, I used concepts from Signals and Systems class like Fourier transform and the Mathematical approach to analyze the signal, shifting when necessary, and finally scaling the signal for convenience. The main challenges of this Project are digitization, using knowledge of simple signal processing, practice with tuning the frequency for sine wave functions to obtain a clear output, and working with large arrays to generate the desired result.